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Model Predictive Control for the Scheduling of Seedings in an Adaptive Vertical Farm

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***Abstract*— A model predictive control approach is presented for the scheduling of sowings in an adaptive vertical farm, i.e., an innovative vertical greenhouse in which the spacing between shelves is automatically adapted to crop growth. First, a dynamic model describing the evolution of occupancy and shelf height is developed. The model is affected by disturbances to account for possible deviations of crop growth from the nominal pattern. Then, an optimal control problem over a given timeframe is defined to determine the best time instants to perform seedings in the various shelves with the goal of maximizing production yield. The repeated solution of the optimal control problem over a shorter, moving window over time, according to the receding horizon paradigm, allows to devise robust control strategies with respect to disturbances, even in the absence of predictions about their future realizations. Preliminary simulation results are reported for different control horizons and type of disturbances to showcase the effectiveness of the proposed approach in maximizing production yield while exploiting almost all the available vertical space.**

I. INTRODUCTION

Growing plants under fully controlled conditions in vertical farming systems, or "vertical agriculture," is attracting interest and investments in many countries, but sustainability and widespread diffusion of this paradigm is still limited by large energy requirements and high production costs, which make the final product more expensive than traditional agriculture. Reducing costs and increasing energy efficiency are therefore key points in making vertical farming sustainable (see, e.g., [1], [2]).

In industrial vertical farms, crops are grown indoors in a controlled environment, typically in multi-story buildings. The farm adopts advanced technologies such as artificial lighting, climate control, as well as hydroponic or aeroponic growing methods to optimize plant growth and increase productivity [3]. The vertical design of the greenhouse allows an efficient use of space and resources, making it possible to obtain high production yields in a small area. By growing crops in a controlled environment, the use of pesticides and fertilizers can be significantly reduced or even eliminated. Vertical farms also require less water and land use as compared to traditional agriculture, and the adoption of renewable energy sources can further reduce their carbon footprint. In a vertical farm, a variety of crops can be grown, depending on the used technology and the specific market demand. However, leafy greens and herbs are the most commonly grown crops due to their fast growth cycle and high value per unit area. Examples include lettuce, spinach, kale, arugula, and microgreens. These crops are ideal for indoor growing since they require minimal space and can be harvested in few weeks [4].

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In [5]–[7], we introduced a new concept of industrial vertical farm based on the possibility of adapting the vertical space available for crop growth, hereinafter simply called adaptive vertical farm (AVF). The idea behind the AVF concept is that crops require a volume in height that depend on their actual growth level [8]. Crops at the beginning of their growth stage demand less volume than they need at a later stage. The shelves where cultivation takes place can therefore be moved vertically to provide only the volume actually required for the plant to grow, thus optimizing the occupancy of the available vertical space. Therefore, in an AVF, the available volume of each shelf is automatically adapted according to the growth stage of the plants grown therein. The growth level is estimated by a proper set of sensors, while the movement of shelves is implemented by some actuators. This allows cultivating more shelves per unit volume than existing vertical farms with fixed shelves. Moreover, the AVF idea relies on the presence of an aeration formwork in each shelf allowing uniform aeration that can decrease the leaf distance, i.e., the distance between the top of the crops and the top shelf containing the lighting system (see [8] for details). Although preliminary results from [9] indicate that adjusting the distance between shelves based on the growth level of plants may result in energy savings per unit of production, here we only focus on the production yield. In this respect, a careful scheduling of seedings, i.e., the selection of the best time instant when to perform sowings, is needed in the AVF to fully leverage the adaptive principle and maximize the overall production. In fact, if sowings were done simultaneously in all shelves, as typically done in fixed-shelves vertical farms, the plants would outgrow the greenhouse before harvest due to the higher number of shelves. Thus, planning of sowings is crucial to enforce the optimal use of the available volume and resources. To address this issue, an optimal, static scheduling algorithm based on the solution a mixed-integer linear programming (MILP) problem was devised in [5]–[7].

In the absence of disturbances, such a static scheduling algorithm allows for proper planning of seedings, maximizing production and ensuring to have the necessary volume for the growth of crops until harvest. In the case one or more crops experience a deviation from the expected nominal growth rate due to, for instance, non-ideal humidity or temperature conditions, identified through sensors that measure the volume occupied in each shelf in real time, a proper tuning of the static scheduling is needed to ensure that the maximum available volume is never exceeded and the crops always reach the target height before harvest. Toward this end, in this work we define a dynamic optimization approach based on model predictive control (MPC), which enables robust scheduling of seedings with respect to disturbances, even in the absence of predictions of their future realizations. In more detail, according to the MPC paradigm, a sequence of optimal control problems over a moving window over time are solved at each time step based on the actual growth level of crops. Each control problem is based on a dynamic model that describes the evolution of shelf occupancy and distance between shelves over time. In the presence of disturbances that cause plant growth to deviate from the nominal growth, MPC implements a feedback control that generates robust decisions to maximize the production yield, without exceeding the total height of the greenhouse.

MPC has been successfully applied in agriculture as regards flow and water level control in irrigation systems, autonomous tractor navigation, control of environmental parameters (temperature, humidity, CO_2 level, etc.), and energy regulation (see, e.g., [10] and the references therein). However, to the best of our knowledge, its use in vertical farming is not yet widely diffused, and it has never been used for optimal scheduling of sowings in the presence of disturbances, thus making our work innovative and challenging. Simulation results for different control horizons and intensity of disturbances are carried out to verify the effectiveness of the proposed MPC approach for the cultivation of two different types of crops one at a time, i.e., lettuce and basil.

The rest of this paper is organized as follows. The problem formulation and the related optimal solution under suitable conditions is reported in Section II. The MPC approach is described in Section III. Simulation results are showcased in Section IV. Conclusions are discussed in Section V.

II. Problem Statement

In this section, we present the discrete-time dynamic system used to model the evolution of the crops within the AVF. At least in principle, the AVF allows for different crops to be grown simultaneously on various shelves to ensure a variety of harvests. However, for the purpose of this study, it is assumed that all shelves are cultivated with the same type of plant, as it is typical in industrial vertical greenhouses. Once the crops have reached their maximum growth, they are harvested, and the shelves are ready for new seedings. We consider an AVF with N shelves and a total height H_{tot}, characterized by a modular structure, i.e., each shelf occupies a maximum height equal to H_m when the plant cultivated therein is at its maximum growth level (see Fig. 1). The maximum height H_m of each module is composed of a variable adaptive component H_c corresponding to the height of the crop, which reaches the value H at harvest, and a fixed part that is not adapted $H_{\text{fixed}} := H_t + H_s + H_l$, where H_t is the height of the technical space, containing also the aeration formwork and the lighting system, H_s is the height of the cultivation substrate (for instance, soil for traditional cultivation, water for hydroponic cultivation, root system for aeroponic cultivation), and H_l is the leaf distance of the top of the cultivation from the formwork to ensure proper air circulation and light supply. The presence of the adaptive component H_c makes the height occupied by a shelf equal to H_m only at harvest, while for the other time instants, when the plant has not yet reached its maximum height, the occupied vertical space is lower.

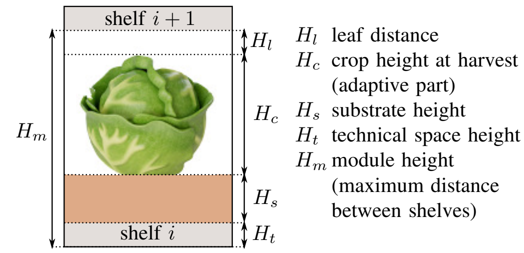

Fig. 1. Vertical space occupied by a module of the AVF.

We focus on the continuous time interval $[0, \mathcal{T}]$, where $\mathcal{T}$ is a given timeframe, which is discretized in T intervals of length Δt, i.e., we refer to the discrete time instants $t = 0, 1, \ldots, T$. We assume that the AVF is used for the cultivation of crops characterized by a nominal cultivation cycle, i.e., the typical growth time from seeding to harvest, equal to C time steps and height at harvest equal to H. A linear plant growth is considered, that is, an increase in plant height equal to H/C from a given time step to the following one is assumed. Such an assumption is considered as a first, yet effective, approximation of the crop growth due to the lack in the literature of precise relationships between height of crops and time. In real situations, whatever is the assumed pattern, the plant growth may deviate from the nominal curve due to several factors, such as, for instance, non-ideal humidity or temperature. We account for such deviations through a disturbance acting on the system.

The following state variables are defined for $t = 0, 1, \ldots, T$ to keep track of the evolution of crop growth over time within the AVF:

- $x_{i,t} \in \{0, 1\}$, $i = 1, \ldots, N$, denotes the occupancy of the shelves, and it is equal to 1 if the shelf i is cultivated at the time t, while it is equal to zero otherwise;
- $h_{i,t} \geqslant 0$, $i = 1, \ldots, N$, represents the height of the shelf i at the time t, i.e., the distance with respect to the shelf above.

For the sake of compactness, let us collect all the state variables in the vector $\underline{x}_t := \text{col}\,[x_{i,t}, h_{i,t}, i = 1, \ldots, N] \in \mathbb{R}^{2N}$ for $t = 0, 1, \ldots, T$.

The control inputs are given by the seedings performed at each time steps, i.e., the following ones are defined for $t = 0, 1, \ldots, T-1$:

- $s_{i,t} \in \{0, 1\}$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, is equal to 1 if a seeding occurs in the shelf i at the time t, while it is equal to zero otherwise.

We collect again all the control inputs in the vector $\underline{u}_t := \text{col}\,[s_{i,t}, i = 1, \ldots, N] \in \mathbb{R}^N$ for $t = 0, 1, \ldots, T-1$.

As previously pointed out, we also consider the presence of disturbances acting on the system that may delay or speedup the growth of the plants. In particular, the following disturbance is defined for $t = 0, 1, \ldots, T$:

- $\xi_{i,t} \in \mathbb{R}$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, is a disturbance affecting the growth of the plant cultivated in the shelf i at the time t. It does not affect the dynamics if the shelf is empty at time t. A positive value of $\xi_{i,t}$ denotes a speedup in the growth rate of the plant, whereas a negative one indicates a slowdown.

We collect all the disturbances in the vector $\underline{\xi}_t := \text{col}\,[\xi_{i,t}, i = 1, \ldots, N] \in \mathbb{R}^N$ for $t = 0, 1, \ldots, T$.

The evolution of the state variables over time is governed by the discrete-time dynamic system

$$\underline{x}_{t+1} = f\left(\underline{x}_t, \underline{u}_t, \underline{\xi}_t\right), \quad t = 0, 1, \ldots, T-1,$$

where the mapping $\mathbb{R}^{2N} \times \mathbb{R}^N \times \mathbb{R}^N \mapsto \mathbb{R}^{2N}$ defined by the function f is the following:

$$x_{i,t+1} = \begin{cases} 0 & \text{if } s_{i,t} = 0 \text{ and } h_{i,t} = H_{\text{fixed}}, \\ 0 & \text{if } s_{i,t} = 0 \text{ and } h_{i,t} \geqslant H + H_{\text{fixed}}, \\ x_{i,t} & \text{if } x_{i,t} = 1 \text{ and } h_{i,t} < H + H_{\text{fixed}}, \\ 1 & \text{if } s_{i,t} = 1, \end{cases}$$

$$h_{i,t+1} = \begin{cases} H_{\text{fixed}} & \text{if } s_{i,t} = 0 \text{ and } h_{i,t} = H_{\text{fixed}}, \\ H_{\text{fixed}} & \text{if } s_{i,t} = 0 \text{ and } h_{i,t} \geqslant H + H_{\text{fixed}}, \\ h_{i,t} + \frac{H}{C} + \xi_{i,t} & \text{if } x_{i,t} = 1 \text{ and } h_{i,t} < H + H_{\text{fixed}}, \\ H_{\text{fixed}} + \frac{H}{C} + \xi_{i,t} & \text{if } s_{i,t} = 1, \end{cases}$$

$$i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1. \tag{1}$$

For a given shelf i, if no sowing occurs at time t and the shelf is empty, then the shelf remains empty also at the time instant $t+1$; consequently, its height at $t+1$ is still equal to H_{fixed}. If a shelf is cultivated at time t, the crops have reached the height for the harvest, and no new seeding is performed, then harvest occurs and the shelf becomes free at time $t+1$, with height equal to the minimum one H_{fixed}. On the contrary, if a shelf is cultivated at time t but crops have not yet reached the height for harvest, then the shelf remains cultivated also at time $t+1$, with a nominal increase of the crops equal to H/C subject to the effect of the disturbance. Lastly, if a new sowing is performed at time t, then the shelf will be cultivated at time $t+1$, and the height will increase of the nominal growth rate H/C plus a disturbance.

The dynamic model (1) is completed by proper constraints, as detailed in the following. If a shelf is cultivated at time t and crops have not yet reached the height for harvest, then a new seeding cannot be performed, i.e.,

$$s_{i,t} = 0 \text{ if } x_{i,t} = 1 \text{ and } h_{i,t} < H + H_{\text{fixed}}, \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1. \tag{2}$$

Moreover, the sum of the heights of the various shelves cannot exceed the total height of the greenhouse, i.e.,

$$\sum_{i=1}^{N} h_{i,t} \leqslant H_{\text{tot}}, \quad t = 0, 1, \ldots, T. \tag{3}$$

With a little abuse of notation, we re-write the constraints (2) and (3) at each time t by defining a function $g : \mathbb{R}^{2N} \times \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}^{N+1}$ such that $g(\underline{x}_t, \underline{u}_t, \underline{\xi}_t) \leqslant \underline{0}$, where $\underline{0}$ is the zero vector with $N+1$ components.

We now formulate an optimal control problem with the goal of determining the best time instant when to perform seedings in the various shelves, in order to obtain the maximum number of sowings (and therefore of harvests) from time $t = 0$ up to time T. In more detail, the problem is the following:

$$\begin{aligned} &\max_{\underline{u}_0, \ldots, \underline{u}_{T-1}} E_{\underline{\xi}_0, \ldots, \underline{\xi}_T} \left\{ \sum_{t=0}^{T} \sum_{i=1}^{N} s_{i,t} \right\}, \\ &\text{subject to} \\ &\underline{x}_{t+1} = f(\underline{x}_t, \underline{u}_t, \underline{\xi}_t), \quad t = 0, 1, \ldots, T-1, \\ &g(\underline{x}_t, \underline{u}_t, \underline{\xi}_t) \leqslant \underline{0}, \quad t = 0, 1, \ldots, T, \\ &\underline{x}_0 = \underline{\hat{x}}, \end{aligned} \tag{4}$$

where $\underline{\hat{x}}$ is a given initial condition for the dynamic system (1) representing the occupancy and height of the shelves of the AVF at $t = 0$, and $E\{\cdot\}$ is the expectation operator performed with respect to the sequence of disturbances.

The optimal control problem (4) can be re-written as an MILP problem with decision variables $x_{i,t}$, $h_{i,t}$, $s_{i,t}$, and $\delta_{i,t}$, where $\delta_{i,t}$ is an auxiliary binary variable that is equal to 1 if $h_{i,t} < H + H_{\text{fixed}}$, otherwise it is equal to 0 (such a condition determines whether a crop can be harvested, and therefore its modeling plays a crucial role in the optimization problem). The various cases in the state equation (1) can be written in terms of linear constraints by introducing a very large, positive constant M, which makes each constraint active or trivially satisfied depending on the various cases in (1). The overall MILP problem is the following:

$$\max E_{\underline{\xi}_0, \ldots, \underline{\xi}_T} \left\{ \sum_{t=0}^{T} \sum_{i=1}^{N} s_{i,t} \right\}, \tag{5}$$

subject to

$$x_{i,t+1} \leqslant M(s_{i,t} + h_{i,t} - H_{\text{fixed}}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{6}$$

$$x_{i,t+1} \geqslant -M(s_{i,t} + h_{i,t} - H_{\text{fixed}}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{7}$$

$$h_{i,t+1} \leqslant H_{\text{fixed}} + M(s_{i,t} + h_{i,t} - H_{\text{fixed}}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{8}$$

$$h_{i,t+1} \geqslant H_{\text{fixed}} - M(s_{i,t} + h_{i,t} - H_{\text{fixed}}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{9}$$

$$x_{i,t+1} \leqslant M(s_{i,t} + \delta_{i,t}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{10}$$

$$x_{i,t+1} \geqslant -M(s_{i,t} + \delta_{i,t}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{11}$$

$$h_{i,t+1} \leqslant H_{\text{fixed}} + M(s_{i,t} + \delta_{i,t}), \quad i = 1, \ldots, N, \quad t = 0, 1, \ldots, T-1, \tag{12}$$

$$h_{i,t+1} \geqslant H_{\text{fixed}} - M(s_{i,t} + \delta_{i,t}),$$

$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{13}$$

$$x_{i,t+1} \leqslant x_{i,t} + M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{14}$$

$$x_{i,t+1} \geqslant x_{i,t} - M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{15}$$

$$h_{i,t+1} \leqslant h_{i,t} + H/C + \xi_{i,t} + M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{16}$$

$$h_{i,t+1} \geqslant h_{i,t} + H/C + \xi_{i,t} + M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{17}$$

$$x_{i,t+1} \leqslant 1 + M(1 - s_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{18}$$

$$x_{i,t+1} \geqslant 1 - M(1 - s_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{19}$$

$$h_{i,t+1} \leqslant H_{\text{fixed}} + H/C + \xi_{i,t} + M(1 - s_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{20}$$

$$h_{i,t+1} \geqslant H_{\text{fixed}} + H/C + \xi_{i,t} - M(1 - s_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{21}$$

$$h_{i,t} < H + H_{\text{fixed}} + M(1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T, \tag{22}$$

$$h_{i,t} \geqslant H + H_{\text{fixed}} - M\delta_{i,t},$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{23}$$

$$s_{i,t} \leqslant M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{24}$$

$$s_{i,t} \geqslant -M(1 - x_{i,t} + 1 - \delta_{i,t}),$$
$$i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{25}$$

$$x_{i,0} = \hat{x}_i, \quad i = 1, \dots, N, \tag{26}$$

$$h_{i,0} = \hat{h}_i, \quad i = 1, \dots, N, \tag{27}$$

$$\sum_{i=1}^{N} h_{i,t} \leqslant H_{\text{tot}}, \quad t = 0, 1, \dots, T, \tag{28}$$

$$x_{i,t} \in \{0, 1\}, \quad i = 1, \dots, N, \quad t = 0, 1, \dots, T, \tag{29}$$

$$h_{i,t} \geqslant H_{\text{fixed}}, \quad i = 1, \dots, N, \quad t = 0, 1, \dots, T, \tag{30}$$

$$s_{i,t} \in \{0, 1\}, \quad i = 1, \dots, N, \quad t = 0, 1, \dots, T-1, \tag{31}$$

$$\delta_{i,t} \in \{0, 1\}, \quad i = 1, \dots, N, \quad t = 0, 1, \dots, T. \tag{32}$$

Specifically, the cost function in (5) aims at maximizing seedings. Constraints (6)–(7) and (8)–(9), (10)–(11) and (12)–(13), (14)–(15) and (16)–(17), (18)–(19) and (20)–(21) implement the first, second, third, and fourth conditions for $x_{i,t+1}$ and $h_{i,t+1}$ in (1), respectively. Constraints (22)–(23) define the relationship between $\delta_{i,t}$ and $h_{i,t}$. Constraints (24)–(25) and (28) are equivalent to (2) and (3), respectively. Constraints (26)–(27) impose initial conditions $\hat{x}_i$ and $\hat{h}_i$ for the occupancy and height in all the shelves as in (1), while (29)–(32) define the range of variation of the variables.

III. MPC FOR OPTIMAL SEEDING

As pointed out also in Section II, in practice the growth curve of the plants may deviate from the nominal one due to several factors, such as, for instance, non-ideal humidity or temperature. Such a deviation is taken into account through the disturbance terms $\xi_{i,t}$, $i = 1, \dots, N$, $t = 0, 1, \dots, T$ in the dynamic system (1) and the corresponding MILP formulation (5)–(32). The presence of the disturbances entails the expectation operator in the cost function, which complicates finding an optimal solution since it requires the knowledge of the probability density function of the noise and/or the availability of suitable predictions of the values attained by the disturbances in the interval $[0, T]$.

In this section, we propose an approach based on MPC to determine the best value of the control inputs, i.e., the best time instants when to perform sowings in the various shelves, that are robust with respect to the disturbances. In more detail, we fix a horizon $\overline{T} < T$ and, for all $t = 0, 1, \dots, T$, we devise an optimal control problem over the interval $[t, t+\overline{T}-1]$ aimed at maximizing the number of sowings (and therefore also the number of harvests), in the same interval. Within each of these optimal control problems, crops are assumed to grow at their nominal rate, i.e., at each time step there is an increase of the height of crops equal to H/C. The unknown control inputs are the values of the vector $\underline{u}_\tau$ for $\tau = t, t+1, \dots, t+\overline{T}-1$. In more detail, we solve the following problem at each time instant $t = 0, 1, \dots, T$:

$$\begin{aligned}
&\max_{\underline{u}_t, \dots, \underline{u}_{t+\overline{T}-1}} \sum_{\tau=t}^{t+\overline{T}} \sum_{i=1}^{N} s_{i,\tau}, \\
&\text{subject to} \\
&\underline{x}_{\tau+1} = f(\underline{x}_\tau, \underline{u}_\tau, \underline{0}), \quad \tau = t, t+1, \dots, t+\overline{T}-1, \\
&g(\underline{x}_\tau, \underline{u}_\tau, \underline{0}) \leqslant \underline{0}, \quad \tau = t, t+1, \dots, t+\overline{T}, \\
&\underline{x}_t = \underline{x}_t^\star,
\end{aligned} \tag{33}$$

where $\underline{x}_t^\star$ is the state of the system at time t.

Problem (33) differs from problem (4) since the former is defined on the discrete interval $[t, t+\overline{T}]$ rather than in the entire horizon $[0, T]$, and with the disturbance vector $\underline{\xi}_\tau = \underline{0}$ for all $\tau = t, t+1, \dots, t+\overline{T}$. This allows to avoid the expectation operator in the cost function. Moreover, the initial condition $\underline{x}_t^\star$ is the state of the system at time t. Let $\underline{u}_t^\star, \dots, \underline{u}_{t+\overline{T}-1}^\star$ be the optimal control inputs obtained by solving (33). According to the receding horizon principle of MPC, we discard all the control inputs $\underline{u}_{t+1}^\star, \dots, \underline{u}_{t+\overline{T}-1}^\star$ and retain and apply only the first one $\underline{u}_t^\star$.

In more detail, starting from time $t = 0$ and an initial condition $\underline{x}_0^\star$ for the occupancy and height of the various shelves of the greenhouse, we solve the noise-free MPC problem (33) defined over the discrete interval $[0, \overline{T}]$ and obtain the optimal control input $\underline{u}_0^\star$, after having discarded all the subsequent control inputs within the interval. This control input is applied to the system, which evolves according to the state equation (1), with the possible effect of the disturbance $\underline{\xi}_0$, thus obtaining the new state vector $\underline{x}_1^\star$. At this point, the new control input $\underline{u}_1^\star$ is obtained by solving a new optimal control problem (33) with a one-step-forward shift of the control horizon, i.e., optimization is performed in the interval $[1, 1+\overline{T}]$. The procedure is iterated up to time T. The use of the updated state $\underline{x}_t^\star$ at each time step accounts for the presence of disturbances acting on the system and implements the typical feedback mechanism of MPC.

IV. Simulation Results

In this section, we report the results of the simulations performed to check the effectiveness of the proposed MPC approach. Simulations were performed in Matlab on a 2.6 GHz Intel Xeon CPU with 32 GB of RAM. We considered an overall horizon equal to 6 months with a sampling time equal to 1 day, i.e., we focused on the discrete time instants $t = 0, 1, \ldots, 180$.

We investigated the cultivation of two types of crops, i.e., lettuce and basil, characterized by different length of the cultivation cycle and height at harvest (such values are indicative since we do not provide details on the particular varieties that we considered). The nominal length C of the cultivation cycle was fixed to 25 and 40 days for lettuce and basil, respectively, while the height H at harvest was taken equal to 30 cm and 20 cm, respectively. The two types of crops were considered one at a time, i.e., first the AVF was used for the cultivation of lettuce alone, and then for the cultivation of basil, as it typically happens in industrial vertical farms. Concerning lettuce, we focused on an AVF with total height $H_{\text{tot}} = 600$ cm and $N = 14$ shelves. As regards basil, we considered again an AVF with total height $H_{\text{tot}} = 600$ cm and $N = 16$ shelves. Such numbers of shelves are the maximum ones that can be installed within the considered heights of the greenhouse. Such a height is composed of a substrate thickness $H_s = 10$ cm, a technical space height $H_t = 5$ cm, and a leaf distance $H_l = 10$ cm.

We evaluated the effectiveness of the proposed MPC scheme for different values of the control horizon $\overline{T}$ and of the disturbances. As the regards $\overline{T}$, we considered 30, 50, and 100 days. Concerning disturbances, besides the noise free case, we focused on random disturbances $\underline{\xi}_t$, $t = 0, 1, \ldots, T$, taken from Gaussian probability density functions with given mean $\mu_{\underline{\xi}}$ and standard deviation $\sigma_{\underline{\xi}}$. In more detail, we assessed performances in a large spectrum of cases, by considering $\mu_{\underline{\xi}} = -0.5$, $\sigma_{\underline{\xi}} = 0.1$ as well as $\mu_{\underline{\xi}} = 0.1$, $\sigma_{\underline{\xi}} = 1$ for the lettuce and $\mu_{\underline{\xi}} = -0.1$, $\sigma_{\underline{\xi}} = 0.5$ as well as $\mu_{\underline{\xi}} = 0.5$, $\sigma_{\underline{\xi}} = 0.1$ for the basil. In all cases, the greenhouse was assumed empty at $t = 0$. Performances were evaluated through the number of sowings n_s in the discrete interval $[0, T]$, the overall height $\overline{h}_t = \sum_{i=1}^{N} h_{i,t}$ occupied by crops at each time instant, and the average CPU time required to solve each MPC problem (33) at a certain time step.

Table I reports the results of the simulations in terms of the previously-introduced performance indexes. Both in the cases of the cultivation of lettuce and basil, the number of seedings grows if the control horizon $\overline{T}$ increases, as more effective schedules can be found over a larger time interval.

In more detail, as regards lettuce, the scheduling in the disturbance-free case where $\mu_{\underline{\xi}} = 0$ and $\sigma_{\underline{\xi}} = 0$ guarantees 84, 85, and 85 sowings for horizons 30, 50, and 100, respectively. Such numbers of sowings reduce in the presence of disturbances on the crop growth. The decay is larger for $\mu_{\underline{\xi}} = -0.5$ and $\sigma_{\underline{\xi}} = 0.1$ since in this case there is a

TABLE I
Simulation results.

	$\mu_{\underline{\xi}}$	$\sigma_{\underline{\xi}}$	$\overline{T}$	n_s	CPU time (s)
Lettuce	-0.5	0.1	30	48	11.99
	-0.5	0.1	50	48	13.76
	-0.5	0.1	100	49	35.19
	0	0	30	84	12.74
	0	0	50	85	17.71
	0	0	100	85	32.02
	0.1	1	30	67	9.81
	0.1	1	50	67	13.69
	0.1	1	100	70	29.99
Basil	-0.1	0.5	30	41	12.06
	-0.1	0.5	50	42	18.18
	-0.1	0.5	100	44	31.95
	0	0	30	55	12.78
	0	0	50	55	21.95
	0	0	100	57	35.23
	0.5	0.1	30	74	10.20
	0.5	0.1	50	94	18.29
	0.5	0.1	100	96	28.82

consistent average reduction of the growth rate (half a day), i.e., the time required from seeding to harvest increases. A moderate reduction in the number of sowings can be observed also for $\mu_{\underline{\xi}} = 0.1$ and $\sigma_{\underline{\xi}} = 1$. In this case, there is a speedup in the growth rate of 0.1 days on the average, but the large standard deviation vanishes such an advantage, causing an overall decay of performances as compared to the noise-free case. Concerning basil, the scheduling in the disturbance-free case where $\mu_{\underline{\xi}} = 0$ and $\sigma_{\underline{\xi}} = 0$ guarantees 55, 55, and 57 sowings for horizons 30, 50, and 100, respectively. For $\mu_{\underline{\xi}} = -0.1$ and $\sigma_{\underline{\xi}} = 0.5$, we observe a reduction on the overall number of sowings due to a small slowdown of the growth rate of the crops, while for $\mu_{\underline{\xi}} = 0.5$ and $\sigma_{\underline{\xi}} = 0.1$ we have a large increase of performances owing to a speedup in the growth rate of 0.5 days on the average. In this case, the difference in the performances when using a larger horizon (for instance, $\overline{T} = 100$ *versus* $\overline{T} = 30$) is more evident due to the large, almost constant nature of the disturbance (the standard deviation is equal to 0.1).

The average computational time to find a solution of each problem (33) at a certain time t increases with the control horizon $\overline{T}$, as expected, as more complex optimization problems have to be solved due to a larger number of unknowns. The mean and standard deviation of the disturbances have a negligible effect on the computational time.

Fig. 2 reports a pictorial description of selected results. The plots correspond to the case $\overline{T} = 50$, $\mu_{\underline{\xi}} = 0.1$, $\sigma_{\underline{\xi}} = 1$ for the lettuce (first row), and to $\overline{T} = 50$, $\mu_{\underline{\xi}} = -0.1$, $\sigma_{\underline{\xi}} = 0.5$ for the basil (second row). On the left, center, and right we find the disturbances affecting shelves 1 and 2, the scheduling of sowings and harvests as determined by MPC, and the total vertical space occupied by crops, respectively. Looking at the schedule of the lettuce (top-center plot), we observe a regularity in the sowings, especially for the "lower" shelves, and a higher variability for the "upper" ones. Such a

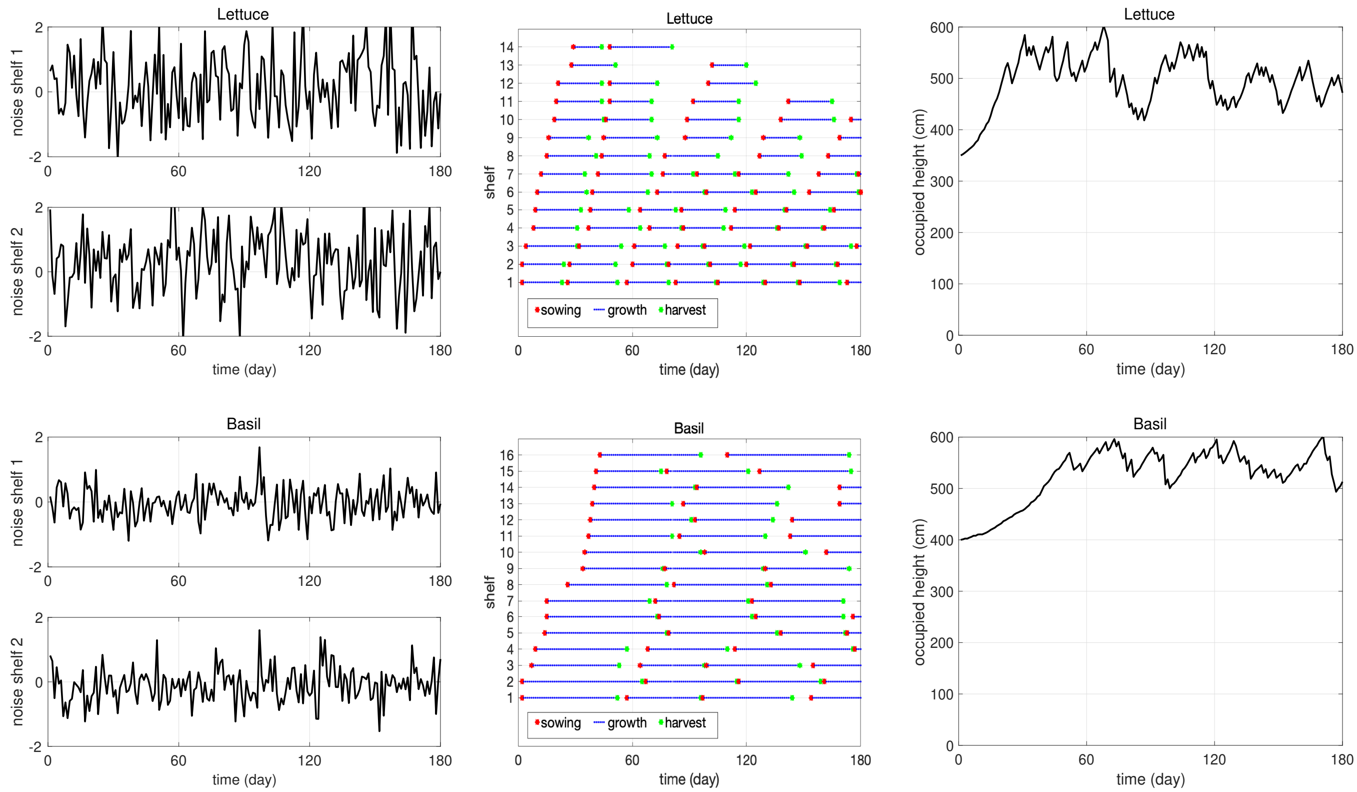

Fig. 2. Disturbances affecting shelves 1 and 2 (right), schedules of sowings and harvests (center), and total occupied height (right) for the cultivation of lettuce (top) and basil (bottom). The plots correspond to $\overline{T} = 50$, $\mu_{\underline{\xi}} = 0.1$, $\sigma_{\underline{\xi}} = 1$ for the lettuce and to $\overline{T} = 50$, $\mu_{\underline{\xi}} = -0.1$, $\sigma_{\underline{\xi}} = 0.5$ for the basil.

variability is due to the large standard deviation of the noise (top-left plot), which forces the optimal control actions to reduce sowings in order to avoid exceeding the total height H_{tot} of the greenhouse. As a consequence, the occupied height (top-right plot) presents oscillations. A more regular pattern can be observed in the schedule of the basil (bottom-center plot), as a noise with a reduced standard deviation is applied (bottom-left plot). Thus, also the occupied vertical space (bottom-right plot) presents a reduced number of oscillations as compared to the case of the lettuce.

V. Conclusions

We have presented an MPC scheme to schedule sowings in an adaptive vertical farm (AVF). Specifically, we have developed a dynamic model describing the evolution of the occupancy and height of the shelves. The presence of disturbances affecting the state equation accounts for possible deviations of the growth of the crops from the nominal pattern. An optimal control problem has been defined to determine the best time instants to perform seedings in the various shelves with the goal of maximizing production yield. The application of MPC has allowed to compute robust control actions with respect to disturbances, without requiring the availability of future predictions on the growth of the plants, which are very unlikely to be available. The reported simulation results have proved the effectiveness of the MPC approach to maximize the production yield without exceeding the total height of the greenhouse at the same time.

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