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Mesoscopic digital control for Practical String Stability of vehicular platoons

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Abstract— **In this paper, we propose a new piecewise constant feedback for string-stability of a platoon under sampled and quantized measurements. The design is based on a mesoscopic approach and is carried out over the sampled-data model associated to each vehicle. The proposed feedback ensures string-stability in the practical sense independently of the effect of sampling and quantization. Simulations show the effectiveness of the results.**

Index Terms— **Traffic control; Sampled-data control; Autonomous vehicles; String stability; Quantization; Mesoscopic controller; Micro-macro traffic control systems.**

I. Introduction

In the context of the strategies for intelligent transportation systems, it is crucial to find efficient ways to address the growing demand for effectiveness and safety. Within this context, current scholarly research is prioritizing the establishment of a crucial characteristic for the development of efficient and secure CACC systems: String Stability (SS) [1], [2]. This concept refers to the ability of a series of vehicles in a platoon, or *string*, to maintain a target distance between each other while moving along a roadway and not amplifying the effect of disturbances over the states. The type of transmitted data is of crucial importance in ensuring SS. Recent research explores the applications of such transmitted data for achieving SS by using macroscopic information. Specifically, it was demonstrated in [3], [4] that SS can be enforced within a platoon by utilizing local microscopic measurements (pertaining to the preceding vehicle) and aggregate (macroscopic) information solely about the platoon. The corresponding control strategy is thus mesoscopic. Transmitting data to the platoon faces a challenge due to communication channels with finite precision, requiring signal quantization. Quantizers have been widely studied: [5] proposes dynamic quantizer adjustment, while [6], [7] suggest a logarithmic quantizer.

The aim of this paper is to make further steps in this direction by considering an homogenous platoon of vehicles, communicating with each other, but allowing measurements to be quantized while being transmitted. More precisely, each vehicle receives, at certain sampling instants, quantized values of both the microscopic and macroscopic information.

Several applications of quantization in traffic platoon exist in the literature. For instance, in [8] a linear quantizer is applied, while [9] employs dynamic quantization. The main issue with quantization is related to global stability, which is in general not guaranteed [10]. One contribution of this paper consists in moving away from the dependence on a particular quantizer, making reference to a whole family of quantization functions. In detail, we first design a nominal piecewise constant controller making the platoon string-stable based on micro and macroscopic sampled measurements. Then, we analyze the effect of quantization and sampling on SS when implementing such a controller over sampled and quantized microscopic and macroscopic measurements. The design and analysis we propose consider a whole family of quantization devices. More precisely, the design is based on a suitable sampled-data model of the platoon when assuming samples of the measured information affected by quantization. In doing so, invoking cascaded arguments, a piecewise constant static control law is defined, which guarantees practical SS (pSS) under quantization and sampling. As one might expect, quantization makes SS practical in the sense that convergence to the desired distance and velocity profile are up to a steady-state offset. Such an offset depends on the quantization efficiency and naturally converges to 0 as the effect of quantization is mild. This is done by considering mild assumptions on the involved macroscopic functions and for the class of constant spacing policies. To the best of the authors' knowledge, no results are available for solving the problem in such a scenario despite its practicality and significance in applications.

The rest of the paper is organized as follows. Section II provides the considered models and hypotheses, as well as the problem statement. Section III introduces the main result with respect to the aforementioned spacing policy. Section IV shows the effectiveness of the proposed sampled-data

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quantized solution in simulations, while Section V outlines concluding remarks.

NOTATIONS.

$\mathbb{C}$, $\mathbb{M}$ and $\mathbb{R}$ denote the set of complex, real and natural numbers including 0 respectively. $\mathbb{R}^+$ denotes the set of positive real numbers. I and 0 denote respectively the identity and zero matrices of suitable dimensions. Given a matrix $A \in \mathbb{R}^{n\times n}$, $\sigma(A) \subset \mathbb{C}$ is its spectrum. For a complex number $\lambda \in \mathbb{C}$, $\mathrm{Re}(\lambda)$ represents its real part. A is said to be Schur if its spectrum is included in the open unit circle of the complex plane (i.e. all its eigenvalues are with norm strictly less than 1 and none is at the origin). $|\cdot| \in \mathbb{R}$ denotes, depending on the argument, ether the cardinality of a set $\mathcal{S}$, the absolute value of a complex number $\lambda \in \mathbb{C}$ or the norm of a matrix. Given a continuous-time signal $w : \mathbb{R}^+ \mapsto \mathbb{R}$ we define $\|w\|^{[0,\bar{t}]} = \sup_{t\in[0,\bar{t}]} |w(t)|$. Accordingly, for a discrete time signal $w_d : \mathbb{N} \mapsto \mathbb{R}$ we define $||w_d||^{[0,\bar{k}]} = \sup_{k\in[0,\bar{k}]} |w_d(k)|$. By quantizer we mean a piecewise constant function $q : \mathbb{R} \mapsto \mathcal{Q}^l$, where $\mathcal{Q}^l$ is a finite subset of $\mathbb{R}$, leading to a partition of $\mathbb{R}$ into a finite number of quantization regions of the form $\{x \in \mathbb{R} : q(x) = i\}, i \in \mathcal{Q}^l$. These quantization regions are not assumed to have any particular shape. Any quantizer is characterized by the parameters M, μ, respectively the range and the quantization error, such that the following implications hold: (i) if $|x| \le M \Rightarrow |q(x) - x| \le \mu$; (ii) if $|x| > M \Rightarrow |q(x)| > M - \mu$. By sampling instants we mean all the discrete time instants defined as $t_k = kT$, for $k \in \mathbb{N}$.

II. MODELING AND PROBLEM FORMULATION

A. Microscopic modeling

Let $\mathcal{I}_0^N$ be the set of N vehicles composing a platoon. Each vehicle is described by its longitudinal position, $p_i \in \mathbb{R}^+$, and its longitudinal speed, $0 \le v_i \le v_{\max}$, $v_{\max} \in \mathbb{R}^+$, $\forall\ i \in \mathcal{I}_0^N$. The state of the i^{th} vehicle is defined as $x_i = [\ p_i \quad v_i\ ]^\top$. Without loss of generality, the low level dynamics describing the power-train can be considered linear, possibly under feedback (see [11]–[13]). This allows one to consider the longitudinal dynamics only for describing heterogeneous platoons [14], i.e., platoons composed by non identical vehicles. The corresponding dynamical system is given by [11], [12], [15], [16]

$$\dot{x}_i = [\ \dot{p}_i \quad \dot{v}_i\ ]^\top = [\ v_i \quad u_i\ ]^\top, \qquad i \in \mathcal{I}_0^N, \quad (1)$$

where: $|u_i| \le u_{\max}$, $u_{\max} \in \mathbb{R}^+$, is the control input of the i^{th} vehicle, corresponding to the acceleration. In order to describe inter-vehicular interactions, we adopt the leader-follower model (see [17]), with respect to which we derive a global description of the platoon. For the derived model to include vehicle $i = 0$, we consider the presence of a virtual leader, $i = -1$, that precedes the entire platoon, with dynamical model

$$\dot{x}_{-1} = [\ \dot{p}_{-1} \quad \dot{v}_{-1}\ ]^\top = [\ v_{-1} \quad u_{-1}\ ]^\top \qquad (2)$$

where $u_{-1} \equiv 0$. Then, the state of each car-following pair between vehicle $i-1$ and i is defined as

$$\chi_i = x_i - x_{i-1} = \begin{bmatrix} \Delta p_i \\ \Delta v_i \end{bmatrix} = \begin{bmatrix} p_i - p_{i-1} \\ v_i - v_{i-1} \end{bmatrix},\ i \in \mathcal{I}_0^N. \quad (3)$$

The resulting microscopic dynamical model of the i^{th} car-following pair is

$$\dot{\chi}_i = A\chi_i + B\left(u_i - u_{i-1}\right),\ \ i \in \mathcal{I}_0^N \qquad (4)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \qquad (5)$$

To each vehicle $i \in \mathcal{I}_0^N$ we associate the sampling sequence $\Delta = \{t_0, t_1, \dots, t_k, \dots\}$ with $t_{k+1} - t_k = T$, $T > 0$. At this point, the following assumption is set.

Assumption 1 (Microscopic Sampling): The input u_i of each vehicle $i \in \mathcal{I}_0^N$ is a piecewise constant signal over the sampling period of length $T > 0$, i.e.,

$$u(t) = u(t_k) \quad t \in [t_k, t_{k+1}), \quad t_k, t_{k+1} \in \Delta. \qquad (6)$$

Under the assumption above, the dynamics of each vehicle (4) at all sampling instants $t_k = kT$ is equivalently described by the corresponding sampled- equivalent model [18], i.e.,

$$\chi_i(t_{k+1}) = A_{\mathsf{d}}\chi_i(t_k) + B_{\mathsf{d}}(u_i(t_k) - u_{i-1}(t_k)) \qquad (7)$$

with

$$A_{\mathsf{d}} = e^{AT} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad B_{\mathsf{d}} = \int_0^T e^{As}\mathrm{d}sB = T\begin{bmatrix} \frac{T}{2} \\ 1 \end{bmatrix}.$$

In the following, we will refer to (7) as the microscopic sampled-data equivalent model. To define the equilibrium point of the platoon, we consider that the virtual leader moves at a constant speed with no disturbances acting over. Thus, the virtual vehicle's speed can be considered as the reference speed of $i = 0$. Assuming that $\Delta\bar{p} > 0$ is the desired inter-vehicular distance at steady state and that, for all $t_k = kT$ and $k \in \mathbb{N}$, $\Delta p_0(t_k) = -\Delta\bar{p}$ then the equilibrium point for the i^{th} system corresponds to the case where all the vehicles have the same speed and are at the same distance, i.e., $\chi_{e,i} = \bar{\chi} = [-\Delta\bar{p}\ 0]^\top$. Since the state vector is defined with respect to the follower vehicle, then the distance Δp_i and the relative speed Δv_i have opposite sign. For this reason, the equilibrium distance is $-\Delta\bar{p} < 0$. From the platoon point of view, we define the lumped state and the lumped equilibrium for $u_{i-1} = 0$ respectively as

$$\chi = \left[\chi_0^\top\ \chi_1^\top\ \cdots\ \chi_N^\top\right]^\top, \chi_e = \left[\bar{\chi}^\top \quad \bar{\chi}^\top \quad \dots \quad \bar{\chi}^\top\right]^\top. \quad (8)$$

B. Macroscopic modeling

In general, each vehicle $i \in \mathcal{I}_0^N$ receives partial aggregate information on the platoon states beyond the ones on the corresponding state and of its own predecessor. More in detail, for all $i \in \mathcal{I}_0^N$, such an information is endowed within the so called *macroscopic* function

$$\psi_{i-1}(\chi_0(t_k), \cdots, \chi_{i-1}(t_k)) : \mathbb{R}^2 \times \cdots \times \mathbb{R}^2 \mapsto \mathbb{R}^2 \quad (9)$$

such that $\psi_0 = 0$ and for all $i \in \mathcal{I}_0^N$

$$|\psi_{i-1}(\chi_0(t_k), \cdots, \chi_{i-1}(t_k))| \leq c \max_{i \in \mathcal{I}_0^N} |\chi_{i-1}(t_k)|. \quad (10)$$

These functions represent the state of the leading vehicles, indicating the presence of either a transient or a steady state phase. In the following, we let this quantity be spread over the network only at the sampling instants.

Assumption 2: Each vehicle $i \in \mathcal{I}_0^N$ receives or measures the corresponding microscopic and macroscopic quantities (i.e., the state x_i, the state x_{i-1}, the input u_{i-1} of its predecessor $i-1 \in \mathcal{I}_0^N$ and ψ_{i-1}) at the sampling instants only, i.e., $t_k = kT \in \Delta$. In addition, all measured quantities are subject to quantization, i.e., at all $t_k \in \Delta$ with $k \in \mathbb{N}$, each vehicle measures $q(u_{i-1}(t_k))$, $q(\psi_{i-1}(\chi_0(t_k), \ldots, \chi_{i-1}(t_k))$ and $q(\chi_{i-1}(t_k))$ denoting, respectively, the quantized input, macroscopic information and state.

C. Problem Statement

Let $\Delta\bar{p}$ denote the constant inter-vehicular distance. Then, the following constant spacing policy is adopted

$$\Delta p_i^r(t_k) = -\Delta\bar{p}, \quad t_k \in \Delta. \quad (11)$$

Under the assumptions stated above, the goal is to derive a piecewise constant controller that asymptotically tracks the desired distance using micro and macroscopic information, with a quantized signal of the state, macroscopic information, and control actions of other vehicles. More in detail, for all $t_k, t_{k+1} \in \Delta$ we propose a sampled-data and piecewise constant controller of the form

$$\begin{aligned} u_i(t_k) =& q(u_{i-1}(t_k)) - K_{\mathtt{d}} q(\chi_i(t_k)) \\ &+ F_{\mathtt{d}} q(\psi_{i-1}(q(\chi_0(t_k)), \ldots, q(\chi_{i-1}(t_k)))), \end{aligned} \quad (12)$$

where $t_k = kT$ and $K_{\mathtt{d}}, F_{\mathtt{d}}$ are matrices of suitable dimensions to be specified later on. Such a controller must ensure, despite the effect of sampling and quantization, practical String Stability (pSS) in the sense of the definition below:

Definition 2.1 (Practical String Stability): For all $i \in \mathcal{I}_0^N$, system (7) is said to be *practically string stable (pSS)* if there exists $\vartheta_\mu \geq 0$ such that the following conditions hold:

(i) for all $\varepsilon > 0$ there exists $\alpha_\varepsilon > 0$ such that for all $N \in \mathbb{N}$, $t_k \in \Delta$

$$\max_{i \in \mathcal{I}_0^N} |\chi_i(0) - \chi_{e,i}| < \alpha_\varepsilon \Rightarrow \max_{i \in \mathcal{I}_0^N} |\chi_i(t_k) - \chi_{e,i}| < \varepsilon + \vartheta_\mu; \quad (13)$$

(ii) the trajectories asymptotically approach the set $\mathcal{B}_{\vartheta_\mu}(\chi_e) = \{\chi \in \mathbb{R}^2 : |\chi - \chi_e| \leq \vartheta_\mu\}$, i.e.,

$$\lim_{t_k \to \infty} |\chi_i(t_k)|_{\mathcal{B}_{\vartheta_\mu}(\chi_e)} = 0. \quad (14)$$

Remark 2.1: When $\vartheta_\mu \to 0$, Definition 2.1 recovers the standard one in [4] (that is standard String Stability). Whereas String Stability ensures asymptotic convergence to the equilibrium, practical string stability ensures convergence to a neighborhood of the origin with the radius depending on the quantizer parameter μ.

The problem addressed is formalized as follows.

pSS Digital Control Problem: Consider a platoon of vehicles described by (4) under Assumptions 1 and 2 (piecewise constant control and quantized measurements) with the macroscopic information (9) verifying (10). Design a feedback law as in (12) making the equilibrium χ_e of the closed-loop platoon pSS with respect to μ in the sense of Definition 2.1. ▲

III. Control Design

System (7) under control (12) takes the form

$$\begin{aligned} \chi_i(t_{k+1}) =& A_{\mathtt{d}}\chi_i(t_k) - B_{\mathtt{d}}K_{\mathtt{d}}q(\chi_i(t_k)) \\ &+ B_{\mathtt{d}}F_{\mathtt{d}_e}q(\psi_{i-1}(t_k)) \\ &+ B_{\mathtt{d}}(q(u_{i-1}(t_k)) - u_{i-1}(t_k)). \end{aligned} \quad (15)$$

At this point, the main result can be stated.

Theorem 3.1: The *pSS Digital Control Problem* is solved by the control (12) with $\psi_{i-1}(t_k)$ as in (9) and the matrices $F_{\mathtt{d}}, K_{\mathtt{d}}$ verifying the conditions below:

1) the matrix $A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}}$ is Schur;
2) there exists $c \in \mathbb{R}^+$ such that (9) verifies, for all $i \in \mathcal{I}_0^N$
$$|\psi_{i-1}(\chi_0(t_k), \cdots, \chi_{i-1}(t_k))| \leq c \max_{i \in \mathcal{I}_0^N} |\chi_i(t_k)|;$$
3) the parameter
$$\gamma = (1-\alpha)^{-1} c\beta r g \quad (16)$$
verifies $\gamma \in (0,1)$ for $\alpha = \max_{\lambda \in \sigma(A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})} |\lambda|, \beta = \alpha^{-1}|A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}}|$, $g = |B_{\mathtt{d}}|, r = |F_{\mathtt{d}}|$.

Then, setting $\kappa = |K_{\mathtt{d}}|$, the closed-loop platoon is pSS with

$$\vartheta_\mu = (1 - (\alpha + grc\beta))^{-1}\beta g\mu(\kappa + r(c+1) + 1). \quad (17)$$

Proof: For all $i \in \mathcal{I}_0^N$, the closed-loop platoon reads

$$\begin{aligned} \chi_0(t_{k+1}) &= (A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})\chi_0(t_k) + B_{\mathtt{d}}w_0(t_k) \\ \chi_1(t_{k+1}) &= (A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})\chi_1(t_k) + B_{\mathtt{d}}F_{\mathtt{d}}\psi_0(t_k) + B_{\mathtt{d}}w_1(t_k) \\ &\vdots \\ \chi_i(t_{k+1}) &= (A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})\chi_i(t_k) + B_{\mathtt{d}}F_{\mathtt{d}}\psi_{i-1}(t_k) + B_{\mathtt{d}}w_i(t_k) \end{aligned}$$

with

$$\begin{aligned} w_i(t_k) =& K_{\mathtt{d}}(q(\chi_i(t_k)) - \chi_i(t_k))) + q(u_{i-1}(t_k)) - u_{i-1}(t_k) \\ &+ F_{\mathtt{d}}(q(\psi_{i-1}(t_k)) - \psi_{i-1}(t_k)) \end{aligned}$$

being the *virtual perturbation* induced by the quantization errors associated to each signal.

We proceed iteratively and by induction considering the leader and the first vehicle (i.e., $i = 0$ and $i = 1$) at first. Let us start with $i = 0$ evolving as

$$\chi_0(t_{k+1}) = (A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})\chi_0(t_k) + B_{\mathtt{d}}w_0(t_k)$$

By condition 1), there exist $\beta > 0$ and $\alpha \in (0,1)$ such that $|A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}}| \leq \beta\alpha^k$ thus implying

$$|\chi_0(t_k)| \leq \beta\alpha^k|\chi_0(0)| + \frac{\beta}{1-\alpha} g\|w_0(t_k)\|^{[0,k]}.$$

For $i = 1$, by 1), the corresponding dynamics is given by

$$\chi_1(t_{k+1}) = (A_{\mathtt{d}} - B_{\mathtt{d}}K_{\mathtt{d}})\chi_1(t_k) + B_{\mathtt{d}}F_{\mathtt{d}_e}\psi_0(t_k) + B_{\mathtt{d}}w_1(t_k)$$

and verifies, for $\kappa = |K_{\mathsf{d}}|$

$$|\chi_1(t_k)| \leq \beta\alpha^k |\chi_1(0)| + \frac{\beta}{1-\alpha} gr|\psi_0(t_k)| + \frac{\beta}{1-\alpha} g\kappa \|w_1(t_k)\|^{[0,k]}.$$

At this point, the inequality below holds

$$|\psi_{i-1}(\chi_0(t_k), \cdots, \chi_{i-1}(t_k))| \leq c \max_{i\in\mathcal{I}_0^N} |\chi_i(t_k)|$$

implying that

$$|\psi_0(\chi_0(t_k))| \leq c|\chi_0(t_k)| \leq c\beta\alpha^k|\chi_0(0)| + c\frac{\beta g}{1-\alpha}\|w_0(t_k)\|^{[0,k]}$$

and thus

$$|\chi_1(t_k)| \leq \beta\alpha^k|\chi_1(0)| + \frac{c\beta gr}{1-\alpha}\big(\beta\alpha^k|\chi_0(0)| + \frac{\beta g}{1-\alpha}\|w_0(t_k)\|^{[0,k]}\big) + \frac{\beta g}{1-\alpha}\|w_1(t_k)\|^{[0,k]}.$$

Using now (16) yields

$$\begin{aligned}|\chi_1(t_k)| \leq& \beta\alpha^k[|\chi_1(0)| + \gamma|\chi_0(0)|] + \frac{\beta g}{1-\alpha}[\|w_1(t_k)\| + \gamma\|w_0(t_k)\|] \\ \leq& \beta(1+\gamma)\max_{l=0,1}\{|\chi_l(0)|\} + \frac{\beta g(1+\gamma)}{1-\alpha}\max_{l=0,1}\left\{\|w_i(t_k)\|^{[0,k]}\right\}.\end{aligned} \tag{18}$$

In this case, the perturbation verifies

$$\|w_1(t_k)\|^{[0,k]} \leq \kappa\mu + r\left(\mu + c\max_{j\in\mathcal{I}_0^N}|q(\chi_0(0)) - \chi_0(0)|\right) \leq \mu\left(\kappa + r(c+1) + 1\right).$$

Combining the inequality above with (18) yields

$$|\chi_1(t_k)| \leq \beta(1+\gamma)\max_{l=0,1}\{|\chi_l(0)|\} + \frac{\beta g(1+\gamma)}{1-\alpha}\mu(\kappa + r(c+1)+1), \tag{19}$$

where, by 3), $\gamma \in (0,1)$.

Proceeding by induction for the i^{th} vehicle, 1) yields

$$\begin{aligned}|\chi_i(t_k)| \leq& \beta\alpha^k|\chi_i(0)| + \frac{\beta gr}{1-\alpha}|\psi_{i-1}(t_k)| + \frac{\beta g}{1-\alpha}\max_{i\in\mathcal{I}_0^N}\left\{\|w_i(t_k)\|^{[0,k]}\right\} \\ \leq& \beta\alpha^k|\chi_i(0)| + \frac{c\beta gr}{1-\alpha}\max_{j\in\mathcal{I}_0^N}\{|\chi_j(0)|\} + \frac{\beta g}{1-\alpha}\max_{i\in\mathcal{I}_0^N}\left\{\|w_i(t_k)\|^{[0,k]}\right\} \\ \leq& \beta\alpha^k(1+\gamma+\cdots+\gamma^i)\max_{j\in\mathcal{I}_0^N}\{|\chi_j(0)|\} + \frac{\beta g(1+\gamma+\cdots+\gamma^i)}{1-\alpha}\max_{i\in\mathcal{I}_0^N}\left\{\|w_i(t_k)\|^{[0,k]}\right\}.\end{aligned}$$

Since $\gamma\in(0,1)$, one has $1+\cdots+\gamma^i \leq (1-\gamma)^{-1}$ and thus, noting that $\beta g(1-\gamma)(1-\alpha)^{-1} = \beta g(1-(\alpha+grc\beta))^{-1}$

$$|\chi_i(t_k)| \leq \frac{\beta\alpha^k}{1-\gamma}\max_{i\in\mathcal{I}_0^N}\{|\chi_i(0)|\} + \frac{\beta g}{1-(\alpha+grc\beta)}\max_{i\in\mathcal{I}_0^N}\left\{\|w_i(t_k)\|^{[0,k]}\right\}. \tag{20}$$

As far as the disturbance term is concerned, one has

$$\begin{aligned}\max_{i\in\mathcal{I}_0^N}\left\{\|w_i(t_k)\|^{[0,k]}\right\} =& \max_{i\in\mathcal{I}_0^N}\{\|K_{\mathsf{d}}(q(\chi_i(t_k)) - \chi_i(t_k)) + F_{\mathsf{d}}(q(\psi_{i-1}(t_k)) - \psi_{i-1}(t_k)) + (q(u_{i-1}) - u_{i-1})\|^{[0,k]}\} \\ \leq& \kappa\mu + r(\mu + c\max_{i\in\mathcal{I}_0^N}|q(\chi_i) - \chi_i|) + \mu \\ \leq& \mu(\kappa + r(c+1)+1).\end{aligned}$$

The latter inequality yields

$$|\chi_i(t_k)| \leq \frac{\beta\alpha^k}{1-\gamma}\max_{i\in\mathcal{I}_0^N}\{|\chi_i(0)|\} + \frac{\beta g\mu(\kappa + r(c+1)+1)}{1-(\alpha+grc\beta)}, \tag{21}$$

Accordingly, the inequality below holds

$$\max_{i\in\mathcal{I}_0^N}\{|\chi_i(t_k)|\} \leq \frac{\beta\alpha^k}{1-\gamma}\max_{i\in\mathcal{I}_0^N}\{|\chi_i(0)|\} + \frac{\beta g\mu(\kappa+r(c+1)+1)}{1-(\alpha+grc\beta)}. \tag{22}$$

Since $\alpha\in(0,1)$, pSS follows with ϑ_μ as in (17) and

$$\varepsilon = \beta(1-\gamma)^{-1}\alpha_\varepsilon, \quad \alpha_\varepsilon = (1-\gamma)\beta\varepsilon.$$

∎

Remark 3.1: From the expression in (17), it is clear that the proposed controller ensures that $\vartheta_\mu \to 0$ as $\mu \to 0$. Accordingly, when no quantization is implemented, the proposed control law guarantees standard SS.

Remark 3.2: As typical in practical stability problems at large (e.g., [10]), (17) provides an estimate, that might be conservative depending on the case, of the ball the trajectories converge to.

Remark 3.3: Condition (22) implies that the closed-loop platoon is Input-to-State Stable (ISS) with respect to the quantizer parameter μ. However, the reverse does not hold in general as both SS and pSS are stronger properties.

IV. SIMULATIONS

The proposed sampled-data quantized control strategy is here verified in simulations in MATLAB under realistic settings. We consider a platoon of $N = 10$ vehicles. The spacing policy is chosen as a constant reference distance, namely $\Delta\bar{p} = 20$ m and the initial desired speed of the leader is 20 m/s. The vehicles' acceleration is bounded by $-7 \leq u_i \leq 7$ m/s^2. The fifth and eight vehicles have initial distances equal to 22 m and 18 m respectively. All vehicles are initialized with the primer vehicle. We employ

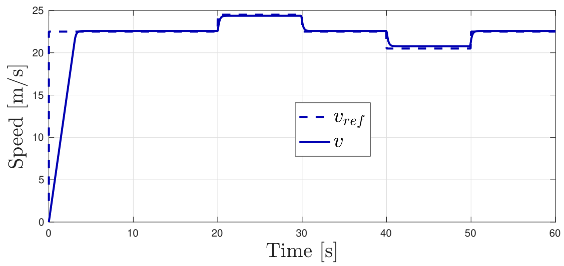

Fig. 1: Speed of the leader.

the dynamic quantizer in [9], with error bound $\mu = 0.1$ and $M = 11$. The simulation time is 60 s. To invoke the relationship among the macroscopic variables as seen in [19] and in analogy with [3], we consider the macroscopic function $\psi_{i-1}(t_k)$ as follows. For each $i \in \mathcal{I}, \mu_{l,i-1}$ and $\sigma^2_{l,i-1}$ denote the intervehicular distance $l = \Delta p$ and speed error $l = \Delta v$ mean and variance, respectively, computed from vehicle 0 to vehicle $i-1$. For $l \in [\Delta p, \Delta v]$ they are

$$\mu_{l,i-1}=\frac{1}{i+1}\sum_{j=0}^{i} l_j,\ \sigma^2_{l,i-1} = \frac{1}{i+1}\sum_{j=0}^{i}(l_j-\mu_{l,i-1})^2. \quad (23)$$

To provide the i^{th} vehicle with the macroscopic information embedded in $\mu_{l,i-1}$ and $\sigma^2_{l,i-1}$ the macroscopic functions are

$$\psi^l_{i-1}(t_k) = \text{sign}(\mu_{l,i-1} - \bar{\chi}_l)\sqrt{\sigma^2_{l,i-1}},\ l \in \{\Delta p, \Delta v\} \quad (24)$$

$\bar{\chi}_{\Delta p} = -\Delta\bar{p}$ and $\bar{\chi}_{\Delta v} = 0$. Then, $\psi_{i-1}(t_k)$ reads

$$\psi_{i-1}(t_k) = \begin{bmatrix}\psi^{\Delta p}_{i-1}(t_k) & \psi^{\Delta v}_{i-1}(t_k)\end{bmatrix}.$$

The proposed control strategy is here validated via experiments, in the unideal case in which the vehicles are subject to external disturbances which are not communicated and can possibly propagate along the platoon. The leader of the platoon tracks a piecewise constant reference speed profile $v_{ref}(t)$ that is split as follows: 1) for $t \in [0, 20)$, $v_{ref} = 20$ m/s; 2) for $t \in [20, 30)$, $v_{ref} = 22$ m/s; 3) for $t \in [30, 40)$, $v_{ref} = 20$ m/s; 4) for $t \in [40, 50)$, $v_{ref} = 18$ m/s; 5) for $t \in [50, 60]$, $v_{ref} = 20$ m/s. Performances are evaluated in a scenario in which the macroscopic information is communicated synchronously from the network at the sampling instants, with the sampling period fixed as $T = 0.1$ s. This choice, together with the one of $K_{\mathtt{d}} = \begin{bmatrix}0.9171 & 1.6356\end{bmatrix}$ and $F_{\mathtt{d}} = \begin{bmatrix}0.4039 & 0.4589\end{bmatrix}$ results $\gamma = 0.8049$ and $\vartheta_\mu = 0.5201$. In addition, to verify robustness of the proposed strategy, we assume an additive input disturbance affecting the vehicles. We assume those disturbances are not communicated between the vehicles and that they can possibly propagate among the platoon. The simulation time is divided into distinct phases: 1) for $t \in [10, 15) \cup [15, 20) \cup [25, 40) \cup [50, 60]$, no disturbance is acting over, i.e. $d_i(t) \equiv 0$ for all $i \in \mathcal{I}^{10}_0$; 2) for $t \in [10, 15)$, a constant disturbance $d_1(t) = 3$ acts on the first vehicle only; 3) for $t \in [20, 25)$ a constant disturbance $d_1(t) = -3$ acts on the first vehicle; 4) for $t \in [40, 50)$ a sinusoidal disturbance is acting on each vehicle of the platoon.

Fig. 1 shows the piecewise constant speed tracked by the leader, whereas Figures 2 and 3 show the performance of the proposed controller. Specifically, with reference to Fig. 2a, we notice the last vehicles of the platoon present a smoother transient than the one of the leading vehicles, as expected, thanks to the role played by the shared macroscopic information. On the other hand, looking at Fig. 3, we note that Practical String Stability is ensured: in fact, in $t = 20$ s, $t = 40$ s and $t = 60$ s, i.e. when the leader reference trajectory changes, the tail vehicles provide better performances with respect to the leading ones, which are still string stable. In fact, we can see that, getting closer to the tail vehicle, the effect of the displacement from the reference trajectory decreases. The efficiency of the proposed control law is outstanding, despite the effect of the quantizer, whose parameters are not set at the best on purpose. This is noticeable in Fig. 2a, where the offset is almost unnoticeable. Finally, we highlight that, in the simulated case, pSS holds with trajectories converging to a ball centered at the equilibrium of radius 0.05. As commented in Remark 3.2, it is not surprising that the actual radius is smaller than its estimate provided by (17) and being, in this case, 0.5201.

V. Conclusions

A new sampled-data quantized mesoscopic controller was proposed to ensure practical string stability of a vehicle platoon, independently of the effect of both sampling and quantization. The design is constructive and provides a control law the parameters of which can be freely tuned to enforce the required performances. Moreover, the introduction of quantization enlarges the applicability of the approach, adapting the control to a more likely scenario in which communication constraints arise due either to technical issues or due to finite precision of the used devices. Current work is addressing networks under asynchronous communication, input perturbations and, possibly, time-delayed and noisy measurements.

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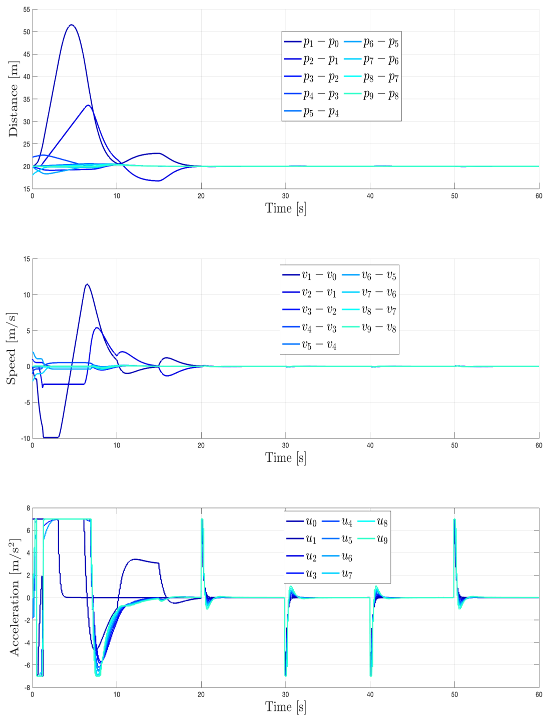
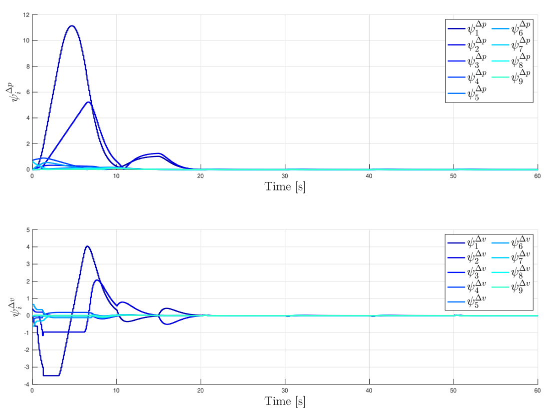


(a) Distance, velocity difference and control inputs of each vehicle



(b) Macroscopic information functions

Fig. 2: The color scale from dark to light blue represents the vehicles of the platoon from the head pair (0,1) (dark blue) to the tail one $(N-1, N)$ (light blue).

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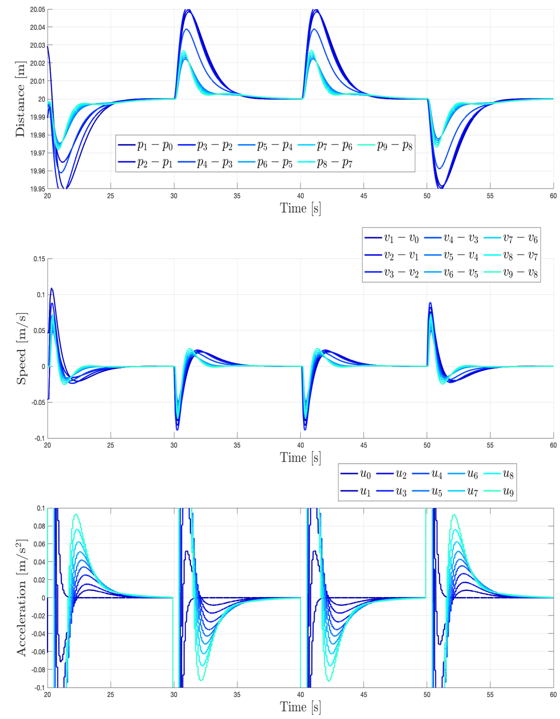
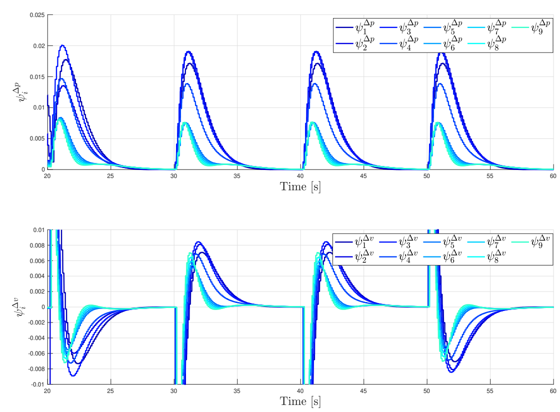


(a) Zoom of Fig. 2a.



(b) Zoom of Fig. 2b.

Fig. 3: Zoom of Fig. 2.

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