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Decentralized lateral and longitudinal control of vehicle platoons with constant headway spacing

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Abstract— **The formation of platoons, where groups of vehicles follow each other at close distances, has the potential to increase road capacity. In this paper, a decentralized control approach is presented that extends the well-known constant headway vehicle following approach to the two-dimensional case, *i.e.*, lateral control is included in addition to the longitudinal control. The presented control scheme employs a direct vehicle following approach where each vehicle in the platoon is responsible for following the directly preceding vehicle according to a nonlinear spacing policy. The proposed constant headway spacing policy is motivated by an approximation of a delay-based spacing policy and results in a generalization of the constant headway spacing policy to the two-dimensional case. By input-output linearization, necessary and sufficient conditions for the tracking of the nonlinear spacing policy are obtained, which motivate the synthesis of the lateral and longitudinal controllers of each vehicle in the platoon. By deriving an internal state representation of the follower vehicle and showing input-to-state stability, the internal dynamics for each leader-follower subsystem are shown to be well-behaved in case the leader drives in steady state conditions (*i.e.*, the leader vehicle's trajectory is unexcited). The results are illustrated by a simulation.**

I. INTRODUCTION

Limited highway capacity and the associated congestion problems have been a key motivation for the research into vehicle automation and intelligent transportation systems. The formation of vehicle platoons, which are closely packed formations of vehicles driving at short inter-vehicle distances, has shown to be effective to increase the road capacity [1]. Potentially more than a doubling of highway capacity can be achieved when all vehicles are platooning [2]. In addition to the increased highway capacity, the small inter-vehicle distances result in reduced aerodynamic drag that leads to fuel savings for all vehicles in the platoon [3] [4].

Motivated by these advantages the problem of vehicle platooning has been studied extensively. In particular longitudinal control of vehicle platoons has received a lot of attention. See [5]–[8] for early works and [9]–[12] for more recent contributions.

Using a cooperative control strategy, where vehicle-to-vehicle (V2V) communication is used in addition to the on-board forward looking sensors, has the benefit that short following distances can be achieved, while still attenuating disturbances in the vehicle string [13]. The attenuation of disturbances through the vehicle string is referred to as string stability, for which in the one dimensional platooning application (i.e., only considering the longitudinal behavior on a straight line) definitions have been proposed in [6] and [13].

To fully automate the vehicles in a platoon, lateral control is required in addition to the longitudinal control. To this end, the lateral control can be treated independently from the longitudinal control, by using a lane-keeping or path following algorithm in combination with Cooperative Adaptive Cruise Control (CACC) as done in [7] and [4]. However, the close distances in a platoon prove to be a difficulty to obtain accurate readings of the lane markings [14]. In [8] magnetic markers are used to obtain a reference for the lateral controller that includes future information about the road geometry. Although such an approach would be capable to work in close vehicle following situations, it requires significant changes to the road infrastructure. Other (path following) approaches require trajectory information of the leader vehicle, or a history of the leader vehicle's path [15], increasing demands on the memory and computational power that is required for the control algorithm. Moreover, the obstructed field of view of the forward looking sensors can still pose a difficulty for the localization of the ego vehicle with respect to the path. In such situations, a direct vehicle following approach is beneficial, as [4] showed by switching to a direct vehicle following approach as a backup for the vision based lane keeping algorithm.

The main contributions in this paper are the following. Firstly, we formulate a constant headway spacing policy in a two-dimensional setup, which allows for lateral and longitudinal vehicle platoon control, and show that the spacing policy can be regarded as a first order Taylor approximation of the delay-based spacing policy in [9]. Secondly, we suggest a decentralized controller design approach using a predecessor-follower control structure. Following the idea from [10]–[12], the follower vehicle is responsible for maintaining the desired spacing with respect to its predecessor, regardless of the lateral and longitudinal control of the predecessor. A definition of string stability that suits within this framework is suggested, and it is shown that string stability follows from the choice of spacing policy. Thirdly, we show that the controller that ensures tracking of the spacing policy can be regarded as an input-output linearizing feedback controller. It is shown that the resulting internal dynamics are input-to-

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state stable with respect to the spacing errors and the states of the predecessor, for an unexcited leader vehicle trajectory. Furthermore, it is demonstrated that tracking of the spacing policy does not necessarily imply proper behavior of the follower vehicle. Accordingly, conditions on the standstill distances are stated such that proper follower behavior is guaranteed.

The remainder of the paper is structured as follows. In Section II the spacing policy is introduced and motivated, followed by the problem statement. The results on controller design are formulated in Section III, with an analysis of the internal dynamics in Section IV. Some illustrative simulation results are provided in Sections III and IV, and the main results are summarized in Section V with some recommendations for future research.

II. MOTIVATION AND PROBLEM FORMULATION

Consider a platoon of $N+1$ vehicles modeled by the unicycle model on a Cartesian coordinate system. That is, the dynamics of each vehicle is assumed to be given by

$$\begin{aligned} \dot{x}_i &= v_i\cos\theta_i, & \dot{\theta}_i &= \omega_i,\\ \dot{y}_i &= v_i\sin\theta_i, & \dot{\omega}_i &= \alpha_i,\\ \dot{v}_i &= a_i, & \dot{\alpha}_i &= -\alpha_i + u_{i,2},\\ \tau_i\dot{a}_i &= -a_i + u_{i,1}, \end{aligned} \tag{1}$$

for $i \in \{0,1,...,N\}$. Here $x_i, y_i \in \mathbb{R}$ are the lateral and longitudinal position of the centre of mass of the i^{th} vehicle and v_i and a_i (both in $\mathbb{R}$) denote its velocity and acceleration. The variable $\theta_i \in [0,2\pi)$ denotes the local orientation of the vehicle with respect to the x-axis. Furthermore, $\omega_i \in \mathbb{R}$ and $\alpha_i \in \mathbb{R}$ are the angular velocity and the angular acceleration, respectively. The control inputs $u_{i,1}, u_{i,2} \in \mathbb{R}$ can be regarded as the desired acceleration and desired angular acceleration. The time constant $\tau_i > 0$ represents the engine dynamics which is not necessarily identical for each vehicle. Hence, a heterogeneous platoon is considered. This is a slight extension of the model used in [16], [17].

Note that the dynamics (1) act on the centre of mass of the vehicle. To incorporate the length of vehicle i and some standstill distance in the modeling, let $p_i := \begin{bmatrix} x_i & y_i \end{bmatrix}^\top$ and denote $q(\varphi) := \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \end{bmatrix}^\top$. The vectors

$$\overline{p}_i := p_i + q(\theta_i)d_{f,i} \qquad \underline{p}_i := p_i - q(\theta_i)d_{r,i} \tag{2}$$

represent a point of distance $d_{f,i}$ in front and distance $d_{r,i}$ behind vehicle i, respectively, relative to the orientation of the vehicle. Taking into account $d_{f,i}$ and $d_{r,i}$, the lateral and longitudinal distance between vehicle i and $i-1$ is given by

$$\Delta_i = \begin{bmatrix} \Delta_{x_i} \\ \Delta_{y_i} \end{bmatrix} = \begin{bmatrix} \underline{p}_{i-1,1} - \overline{p}_{i,1} \\ \underline{p}_{i-1,2} - \overline{p}_{i,2} \end{bmatrix} = \underline{p}_{i-1} - \overline{p}_i, \tag{3}$$

respectively.

The main design goal in vehicle platooning is to ensure the tracking of a *desired* or *reference* inter-vehicle distance. This desired inter-vehicle distance can be defined as a function $\Delta_i^{\text{ref}} : \mathbb{R}^{10} \to \mathbb{R}^2$ of $(v_j, a_j, \theta_j, \omega_j, \alpha_j)$ for $j \in \{i, i-1\}$ and is commonly referred to as the *spacing policy*. Common examples of spacing policies are the constant spacing policy $\Delta_i^{\text{ref}} = d_0$ for some standstill distance $d_0 \in \mathbb{R}^2$ and the delay-based spacing policy [9], which can be written as $\Delta_i^{\text{ref}} = \overline{p}_i(t+\lambda) - \overline{p}_i$ for some time-gap $\lambda > 0$.

In this paper we consider the constant headway spacing policy defined by

$$\Delta_i^{\text{ref}} := \lambda\dot{\overline{p}}_i \tag{4}$$

for some $\lambda > 0$. The choice for (4) is motivated by choosing Δ_i^{ref} as the first order Taylor approximation of the delay-based spacing policy from [9] as follows. If the delay-based spacing policy is tracked perfectly, then

$$\underline{p}_{i-1}(t) - \overline{p}_i(t+\lambda) = 0. \tag{5}$$

By taking a first-order Taylor approximation of (5) we obtain

$$\underline{p}_{i-1}(t) - \overline{p_i}(t) - \lambda\dot{\overline{p}}_i(t) = 0.$$

By arranging the terms we obtain

$$\Delta_i - \Delta_i^{\text{ref}} = 0, \tag{6}$$

where Δ_i and Δ_i^{ref} are defined as (3) and (4), respectively. Hence, if (4) is tracked perfectly, the point in front of vehicle i approximates the point behind vehicle $i-1$ after a time delay $\lambda > 0$.

This work pursues a decentralized approach towards controller design that extends the results in [10]. That is, we aim to design a controller for vehicle i that ensures the desired spacing with respect to the directly preceding vehicle $i-1$, using only local measurements, *i.e.,* information regarding the states of itself and its predecessor. Moreover, the controller for vehicle i is required to be robust with respect to the inputs $u_{i-1,1}$ and $u_{i-1,2}$ of its predecessor in the sense that the desired spacing should be achieved for all possible inputs of the preceding vehicle. As a consequence of this approach, a platoon consisting of only two consecutive vehicles needs to be considered. Therefore, we consider a platoon of two vehicles for the purpose of the analysis of this problem.

The inputs and states of the points in front of and behind vehicle i can be captured as $u_i = \begin{bmatrix} u_{i,1} & u_{i,2} \end{bmatrix}^\top$, and

$$\begin{aligned} \overline{\xi}_i &= \begin{bmatrix} \overline{p}_{i,1} & \overline{p}_{i,2} & v_i & a_i & \theta_i & \omega_i & \alpha_i \end{bmatrix}^\top,\\ \underline{\xi}_i &= \begin{bmatrix} \underline{p}_{i,1} & \underline{p}_{i,2} & v_i & a_i & \theta_i & \omega_i & \alpha_i \end{bmatrix}^\top, \end{aligned}$$

respectively. The state of the point behind vehicle $i-1$ and the state the point in front of vehicle i can be collected as

$$\xi_i = \begin{bmatrix} \overline{\xi}_i^\top & \underline{\xi}_{i-1}^\top \end{bmatrix}^\top \in \mathbb{R}^{14},$$

such that the dynamics of the platoon yields a nonlinear differential equation that is affine in the input

$$\dot{\xi}_i = \begin{bmatrix} \overline{f}_i(\xi_i) \\ \underline{f}_{i-1}(\xi_i) \end{bmatrix} + \begin{bmatrix} g_i(\xi_i) \\ 0 \end{bmatrix} u_i + \begin{bmatrix} 0 \\ g_{i-1}(\xi_i) \end{bmatrix} u_{i-1}, \tag{7}$$

where $\overline{f}_i$, $\underline{f}_i$ and g_i are defined as in (8). The spacing error between vehicles i and $i-1$ is naturally defined as

$$e_i(t) := \Delta_i - \Delta_i^{\mathrm{ref}} \in \mathbb{R}^2. \tag{9}$$

Observe from (3) and (4) that e_i can be expressed as a function of ξ_i.

The control objective of tracking an asymptotic stabilization of a given spacing policy can now be defined as follows.

Definition 1: Consider the platoon (1) and the spacing error (9). A (nonlinear) state feedback $u_i = k_i(\xi_i) + l_i(\xi_i)\nu_i$ is said to

i) track the spacing policies if, for any $u_{i-1}(\cdot)$ and $\xi_i(0)$ such that $e_i(0) = \dot{e}_i(0) = 0$, it holds that $e_i(t) = 0$ for all $t \geqslant 0$,

ii) asymptotically stabilize the spacing policies if for all $\xi_i(0) \in \mathbb{R}^{14}$ it holds that $\lim_{t\to\infty} e_i(t) = 0$,

iii) achieve string stability if, for any $u_{i-1}(\cdot)$ and $\xi_i(0)$ such that $e_i(0) = \dot{e}_i(0) = 0$, for all $T > 0$ the following inequalities hold:

$$\begin{aligned}\int_0^T |\overline{p}_{i,1} - \overline{p}_{i,1}(0)|^2 \,\mathrm{d}t &\leqslant \int_0^T |\underline{p}_{i-1,1} - \overline{p}_{i,1}(0)|^2 \,\mathrm{d}t,\\ \int_0^T |\overline{p}_{i,2} - \overline{p}_{i,2}(0)|^2 \,\mathrm{d}t &\leqslant \int_0^T |\underline{p}_{i-1,2} - \overline{p}_{i,2}(0)|^2 \,\mathrm{d}t.\end{aligned} \tag{10}$$

The definition of string stability in (10) ensures the attenuation of disturbances that propagate through the platoon in both the lateral and longitudinal direction. In particular, unstable swinging of the platoon when a lane shift occurs is mitigated if (10) is satisfied. Furthermore, if the platoon is driving along a straight line, (10) can equivalently be expressed in terms of the velocity. Hence, this definition is in line with the classical literature on string stability [5], [6], [18] and similar to the definitions adopted in [10], [12]. An alternative definition that is independent of the initial value of ξ_i, such as used in [13] and [9] could also have been adopted. However, the definition as in (10) is more suitable for the design approach taken in this paper. To make this explicit, we show at the end of this section, that if a controller ensures *i)* and *ii)* in Definition 1, that $iii)$ follows from the choice of spacing policy by design. As such, Definition 1 leads to the following problem.

Problem 1: Given (7) and the spacing error (9) with spacing policy Δ_i^{ref} as in (4), find a (nonlinear) feedback controller $u_i = k_i(\xi_i) + l_i(\xi_i)\nu_i$ such that the closed loop system satisfies the following properties for any $u_{i-1}(\cdot)$:

i) $e_i(0) = \dot{e}_i(0) = 0$ implies $e_i(t) = 0$ for all $t \geqslant 0$,

ii) for all $\xi_i(0) \in \mathbb{R}^{14}$, it holds that $\lim_{t\to\infty} e_i(t) = 0$.

Remark 1: Note that the system (7) can be interpreted as an affine nonlinear system with output (9). Consequently, Problem 1 can be interpreted as a disturbance decoupling problem (DDP) with output stabilization. For affine nonlinear single-input-single-output systems, a controller that solves the DDP can straightforwardly be designed. This is in contrast to multi-input multi-output systems, for which there does not exists a straightforward method to verify the existence of a controller that solves the DDP, let alone a design method. However, for the multi-input multi-output system (7) it will be shown that there exists a controller that solves Problem 1 and a design method is presented.

Properties *i)* and *ii)* in Problem 1 clearly correspond to the objective of tracking and asymptotic stabilization of the spacing policy as in items *i)* and *ii)* in Definition 1. We conclude this section by showing that if properties *i)* and *ii)* in Definition 1 are satisfied, property *iii)* of Definition 1 follows from the choice of spacing policy by design.

Lemma 2: Consider the spacing policy (4). Any controller that solves part *i)* of Problem 1 achieves string stability.

Proof: If $\xi_i(0)$ such that $e_i(0) = \dot{e}_i(0) = 0$, we have that $e_i(t) = 0$ for all $t \geqslant 0$ by virtue of *i)*. Without loss of generality we translate $\underline{p}_{i-1}$ and $\overline{p}_i$ such that $\overline{p}_i(0) = 0$. Consequently,

$$e_{i,j}(t) = \underline{p}_{i-1,j} - \overline{p}_{i,j} - \lambda\dot{\overline{p}}_{i,j} = 0,$$

for $j = 1, 2$. Considering for $j = 1, 2$ the storage functions $V_j(\overline{p}_{i,j}) = \lambda(\overline{p}_{i,j})^2$, then

$$\begin{aligned}\dot{V}_j(\overline{p}_{i,j}) &= 2\lambda\overline{p}_{i,j}\dot{\overline{p}}_{i,j} = 2\overline{p}_{i,j}\underline{p}_{i-1,j} - 2\overline{p}_{i,j}^2\\ &= \underline{p}_{i-1,j}^2 - (\overline{p}_{i-1,j} - \overline{p}_{i,j})^2 - \overline{p}_{i,j}^2 \leqslant \underline{p}_{i-1,j}^2 - \overline{p}_{i,j}^2.\end{aligned}$$

Therefore, we obtain

$$V_j(\overline{p}_{i,j}(T)) - V_j(\overline{p}_{i,j}(0)) \leqslant \int_0^T \underline{p}_{i-1,j}^2(t)\mathrm{d}t - \int_0^T \overline{p}_{i,j}^2(t)\mathrm{d}t.$$

Since $\overline{p}_i(0) = 0$, we have $V_j(\overline{p}_{i,j}(0)) = 0$. The result follows after rearranging terms and noting that $V_j(\overline{p}_{i,j}(T)) \geqslant 0$ for $j = 1, 2$. ■

III. CONTROLLER DESIGN

In this section we show that part *i)* of Problem 1 is solvable. To that extent we present the following result.

Theorem 1: Consider the system (1) with the constant headway spacing policies for λ. A feedback controller solves part *i)* in Problem 1 *if and only if*

$$u_i = k(\xi_i) + \bar{u}_i, \tag{11}$$

$$\begin{aligned}\overline{f}_i(\xi_i) &= \begin{bmatrix} v_i\cos\theta_i - d_{f,i}\omega_i\sin\theta_i & v_i\sin\theta_i + d_{f,i}\omega_i\cos\theta_i & a_i & -\frac{1}{\tau_i}a_i & \omega_i & \alpha_i & -\alpha_i\end{bmatrix}^\top,\\ \underline{f}_i(\xi_i) &= \begin{bmatrix} v_i\cos\theta_i + d_{r,i}\omega_i\sin\theta_i & v_i\sin\theta_i - d_{r,i}\omega_i\cos\theta_i & a_i & -\frac{1}{\tau_i}a_i & \omega_i & \alpha_i & -\alpha_i\end{bmatrix}^\top,\\ g_i(\xi_i) &= \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau_i} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}^\top,\end{aligned} \tag{8}$$

where

$$k_1(\xi_i) = a_i + 3\tau_i d_{f,i}\omega_i\alpha_i + \tau_i v_i\omega_i^2 + \frac{\tau_i}{\lambda}\Big(- a_i + d_{f,i}\omega_i^2 + \cos(\theta_{i-1}-\theta_i)(a_{i-1}+d_{r,i-1}\omega_{i-1}^2) - \sin(\theta_{i-1}-\theta_i)(-d_{r,i-1}\alpha_{i-1}+v_{i-1}\omega_{i-1})\Big) \quad (12)$$

and

$$k_2(\xi_i) = \alpha_i + \omega_i^3 - \frac{1}{d_{f,i}}(2a_i\omega_i + v_i\alpha_i) + \frac{1}{\lambda d_{f,i}}\Big(- v_i\omega_i - d_{f,i}\alpha_i + \sin(\theta_{i-1}-\theta_i)(a_{i-1}+d_{r,i-1}\omega_{i-1}^2) + \cos(\theta_{i-1}-\theta_i)(-d_{r,i-1}\alpha_{i-1}+v_{i-1}\omega_{i-1})\Big) \quad (13)$$

and $\bar{u}_i$ is any function satisfying $\bar{u}_i = 0$ if $e_i = \dot{e}_i = 0$.

Proof: For notational convenience we denote

$$R(\theta_i) := \begin{bmatrix}\cos\theta_i & -\sin\theta_i\\ \sin\theta_i & \cos\theta_i\end{bmatrix}.$$

Given this notation, it can be verified that

$$\dot{\overline{p}}_i = R(\theta_i)\begin{bmatrix}v_i\\ d_{f,i}\omega_i\end{bmatrix}; \qquad \ddot{\overline{p}}_i = R(\theta_i)\begin{bmatrix}a_i - d_{f,i}\omega_i^2\\ d_{f,i}\alpha_i + v_i\omega_i\end{bmatrix};$$
$$\dddot{\overline{p}}_i = R(\theta_i)\begin{bmatrix}\dot{a}_i - 3d_{f,i}\omega_i\alpha_i - v_i\omega_i^2\\ d_{f,i}\dot{\alpha}_i + 2a_i\omega_i + v_i\alpha_i - d_{f,i}\omega_i^3\end{bmatrix}, \quad (14)$$

and analogously we can express $\ddot{\underline{p}}_{i-1}$ similarly. Due to (9), the spacing error satisfies

$$\ddot{e}_i = \ddot{\underline{p}}_{i-1} - \ddot{\overline{p}}_i - \lambda\dddot{\overline{p}}_i. \quad (15)$$

If $e_i = 0$ for all $t \geqslant 0$ then also $\dot{e}_i = \ddot{e}_i = 0$ for all $t \geqslant 0$. If $\ddot{e}_i = 0$, then multiplying (15) by $R(\theta_i)^{-1}$ and collecting the inputs on the left-hand side leads to

$$\lambda\begin{bmatrix}\tau_i^{-1}u_{i,1}\\ d_{f,i}u_{i,2}\end{bmatrix} = R(\theta_{i-1}-\theta_i)\begin{bmatrix}a_{i-1}+d_{r,i-1}\omega_{i-1}^2\\ -d_{r,i-1}\alpha_{i-1}+v_{i-1}\omega_{i-1}\end{bmatrix} - \begin{bmatrix}a_i - d_{f,i}\omega_i^2\\ d_{f,i}\alpha_i + v_i\omega_i\end{bmatrix} - \lambda\begin{bmatrix}-\tau_i^{-1}a_i - 3d_{f,i}\omega_i\alpha_i - v_i\omega_i^2\\ -d_{f,i}\alpha_i + 2a_i\omega_i + v_i\alpha_i - d_{f,i}\omega_i^3\end{bmatrix},$$

which proves the necessity. For the sufficiency, we have that

$$\ddot{e}_i = -\lambda R(\theta_i)\begin{bmatrix}\tau_i^{-1} & 0\\ 0 & d_{f,i}\end{bmatrix}\bar{u}_i, \quad (16)$$

due to our choice of u_i. Since $\bar{u}_i = 0$ whenever $e_i = \dot{e}_i = 0$, if $\xi(0)$ is such that $e_i(0) = \dot{e}_i(0) = 0$, we have that $e_i(t) = 0$ for all $t \geqslant 0$. ■

Remark 2: In the case that $d_{r,i-1} \neq 0$, a controller u_i for vehicle i that solves Problem 1 depends on the angular acceleration α_{i-1} of the preceding vehicle. In the case that α_{i-1} is not available or cannot be measured, the spacing policy cannot be tracked. However, an alternative approach is to require a different spacing policy where $d_{r,i-1} = 0$. In that case the controller that solves Problem 1 does not depend on α_{i-1}.

The result of Theorem 1 shows not only that part *i)* of Problem 1 is solvable, but also that the controller that solves part *i)* is not unique. Consequently, we can directly state the following result.

Corollary 1: Consider the platoon with dynamics (1) and the constant headway spacing policy (4). Then part *ii)* of Problem 1 is solvable.

Remark 3: A canonical choice for $\bar{u}_i$ is given by

$$\bar{u}_i = \frac{1}{\lambda}\begin{bmatrix}\tau_i & 0\\ 0 & d_{f,i}^{-1}\end{bmatrix}\begin{bmatrix}\cos\theta_i & \sin\theta_i\\ -\sin\theta_i & \cos\theta_i\end{bmatrix}\begin{bmatrix}c_1e_{i,1}+c_2\dot{e}_{i,1}\\ c_3e_{i,2}+c_4\dot{e}_{i,2}\end{bmatrix} \quad (17)$$

for some constants $c_1, c_2, c_3, c_4 > 0$. Due to (16), such a choice for $\bar{u}_i$ leads to the spacing error dynamics

$$\begin{bmatrix}\dot{e}_{i,1}\\ \ddot{e}_{i,1}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -c_1 & -c_2\end{bmatrix}\begin{bmatrix}e_{i,1}\\ \dot{e}_{i,1}\end{bmatrix}, \quad \begin{bmatrix}\dot{e}_{i,2}\\ \ddot{e}_{i,2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -c_3 & -c_4\end{bmatrix}\begin{bmatrix}e_{i,2}\\ \dot{e}_{i,2}\end{bmatrix},$$

which are asymptotically stable. Fig. 1 illustrates the behavior of a platoon that adopts this control design. A heterogeneous platoon is considered, where the leader is modelled with a driveline constant $\tau_0 = 1$ and the following vehicles with $\tau_i = i\tau_0$. Each follower aims to follow its predecessor using the controller of Theorem 1 and Remark 3 at a headway $\lambda_i = 0.1$ seconds, with standstill distances $d_{f,i} = d_{r,i-1} = 0.5$ meter. The controller gains are chosen as $c_1 = 1$, $c_2 = 2$, $c_3 = 1$, $c_4 = 2$.

Remark 4: If the leading vehicle is driving along the x-axis, *i.e.*, $\theta_{i-1} = 0$ and $\underline{p}_{i-1,2} = \omega_{i-1} = \alpha_{i-1} = 0$ and the initial condition of the follower vehicle satisfies $\overline{p}_{i,2}(0) = 0$ and $\theta_i(0) = 0$, $\omega_i(0) = 0$ and $\alpha_i(0) = 0$, the controller (11) with $\bar{u}_i$ given by (17) satisfies

$$u_i = \begin{bmatrix}\frac{\tau_i}{\lambda}(a_{i-1}-a_i) - a_i - c_1e_{i,1} - c_2\dot{e}_{i,1}\\ 0\end{bmatrix}.$$

This recovers the controller synthesized in [10], [11].

IV. THE NOMINAL BEHAVIOR OF THE FOLLOWER VEHICLE AND INTERNAL DYNAMICS

As the controller of Theorem 1 can be regarded as the result of input-output linearization, it remains to be shown that the internal dynamics of (7) are stable. To address this problem, a representation of the internal state dynamics should be phrased, and shown to be input-to-state stable (*cf.* [19]) with respect to the state variables of the leader vehicle

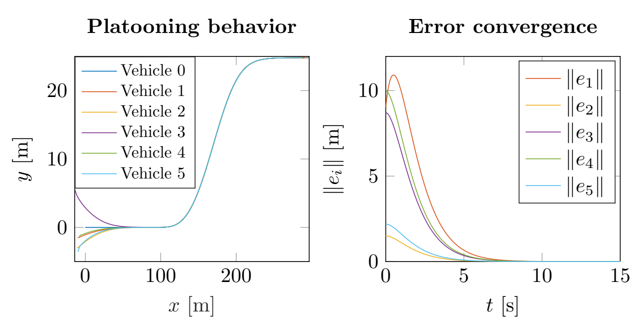

Fig. 1. A simulation of a platoon of vehicles that implement the controller of Theorem 1 and Remark 3. The leader of the platoon, vehicle 0, drives from the origin to the point $(300, 25)$. The left figure shows the spatial behavior of the platoon that follows the trajectory of the leader. The right figure shows the spacing errors of the following vehicles converge to zero asymptotically.

$i-1$ and the error dynamics e_i and $\dot{e}_i$. In this paper we give a partial answer to this problem by studying the case where the leader vehicle $i-1$ follows an unexcited trajectory, by which we mean that $a_{i-1} = \alpha_{i-1} = 0$. Although simulations indicate input-to-state stability as well in the case of an excited trajectory of the leader, it remains future work to prove this property for all trajectories.

When vehicle $i-1$ follows an unexcited trajectory, its velocity v_{i-1} and angular velocity ω_{i-1} are constant. Consequently, its trajectory is either a line if $\omega_{i-1} = 0$, or a circle if $\omega_{i-1} \neq 0$. To arrive at an internal state representation of the follower vehicle, we study the limit behavior of the follower vehicle subject to an unexcited leader, which we refer to as the nominal following behavior. In this case, we expect the follower vehicle i to drive at a fixed position relative to vehicle $i-1$ when $e_i = 0$. For this purpose we define the transformation $T(x)$ and its inverse by

$$\begin{aligned} T(x) &:= R(\theta_{i-1})^{-1}(x - \underline{p}_{i-1}), \\ T^{-1}(x) &:= \underline{p}_{i-1} + R(\theta_{i-1})x. \end{aligned}$$

The transformation $T(x)$ takes any point in the global frame and transports it to the corresponding point in the frame of $\underline{p}_{i-1}$, the point behind vehicle $i-1$. We let $q := T(\overline{p}_i)$ denote the point $\overline{p}_i$ in this local frame. The derivative of q satisfies

$$\begin{aligned} \lambda\dot{q} &= -\lambda\omega_{i-1}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} q + \lambda R(\theta_{i-1})^{-1}(\dot{\overline{p}}_i - \dot{\underline{p}}_{i-1}), \\ &= -\lambda\omega_{i-1}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} q + R(\theta_{i-1})^{-1}(\underline{p}_{i-1} - \overline{p}_i - \lambda\dot{\underline{p}}_{i-1}), \\ &= -\lambda\omega_{i-1}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} q - q - R(\theta_{i-1})^{-1}\lambda\dot{\underline{p}}_{i-1}, \end{aligned}$$

where we used (9) with $e_i = 0$. The intuition that the vehicle i drives at a fixed position behind vehicle $i-1$ is reflected by setting $\dot{q} = 0$. The obtained expression for q gives the relative position of vehicle i in the local frame. We find that setting $\dot{q} = 0$ implies

$$\begin{aligned} q &= -\lambda\left(\lambda\omega_{i-1}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} + I\right)^{-1} R(\theta_{i-1})^{-1}\dot{\underline{p}}_{i-1}, \\ &= -\frac{\lambda}{1+\lambda^2\omega_{i-1}^2}\begin{bmatrix}1 & \lambda\omega_{i-1}\\ -\lambda\omega_{i-1} & 1\end{bmatrix} R(\theta_{i-1})^{-1}\dot{\underline{p}}_{i-1}. \end{aligned}$$

We recall that ω_{i-1} is constant since $\alpha_{i-1} = 0$. The matrix

$$\tfrac{1}{\sqrt{1+\lambda^2\omega_{i-1}^2}}\begin{bmatrix}1 & \lambda\omega_{i-1}\\ -\lambda\omega_{i-1} & 1\end{bmatrix} \tag{18}$$

constitutes a rotation matrix and is also constant. We define the constant angle ψ such that $R(\psi)$ coincides with (18). Transporting the point q back to the global frame yields

$$T^{-1}(q) = \underline{p}_{i-1} - \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\dot{\underline{p}}_{i-1},$$

which we refer to as the nominal trajectory of the follower vehicle. Our expectation is that $\overline{p}_i \to T^{-1}(q)$ asymptotically, and we therefore define

$$\zeta := \underline{p}_{i-1} - \overline{p}_i - \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\dot{\underline{p}}_{i-1}, \tag{19}$$

so that $\zeta = T^{-1}(q) - \overline{p}_i$. We take ζ as part of the internal state representation, and as such show that ζ is input-to-state stable. Note from (9) and (19) that

$$\lambda\dot{\overline{p}}_i = \zeta - e_i + \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\dot{\underline{p}}_{i-1} \tag{20}$$

and that due to (19) the derivative of ζ satisfies

$$\lambda\dot{\zeta} = \lambda\dot{\underline{p}}_{i-1} - \zeta + e_i - \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)(\dot{\underline{p}}_{i-1} + \lambda\ddot{\underline{p}}_{i-1}).$$

By defining the Lyapunov candidate $V := \frac{\lambda}{2}\zeta^\top\zeta$ we find

$$\begin{aligned} \dot{V} = \lambda\zeta^\top\dot{\zeta} &= -\zeta^\top\zeta + \zeta^\top e_i + \lambda\zeta^\top\dot{\underline{p}}_{i-1} \\ &\quad - \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}\zeta^\top R(\psi)(\dot{\underline{p}}_{i-1} + \lambda\ddot{\underline{p}}_{i-1}), \\ &\leqslant -\|\zeta\|^2 + \|\zeta\|\big(\|e_i\| + \lambda\|\dot{\underline{p}}_{i-1}\| \\ &\quad + \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}(\|\dot{\underline{p}}_{i-1}\| + \lambda\|\ddot{\underline{p}}_{i-1}\|)\big), \end{aligned}$$

using Cauchy-Schwartz' inequality. It follows that $\dot{V} \leqslant 0$ if

$$\|e_i\| + \lambda\|\dot{\underline{p}}_{i-1}\| + \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}(\|\dot{\underline{p}}_{i-1}\| + \lambda\|\ddot{\underline{p}}_{i-1}\|) \leqslant \|\zeta\|.$$

Since v_{i-1} and ω_{i-1} are constant, and since

$$\begin{aligned} \dot{\underline{p}}_{i-1} &= R(\theta_{i-1})\begin{bmatrix}v_{i-1}\\ -d_{r,i-1}\omega_{i-1}\end{bmatrix}, \\ \ddot{\underline{p}}_{i-1} &= R(\theta_{i-1})\begin{bmatrix}a_{i-1} + d_{r,i-1}\omega_{i-1}^2\\ -d_{r,i-1}\alpha_{i-1} + v_{i-1}\omega_{i-1}\end{bmatrix}, \end{aligned} \tag{21}$$

we have that $\|\dot{\underline{p}}_{i-1}\|$ and $\|\ddot{\underline{p}}_{i-1}\|$ are constant. This shows that ζ remains bounded and that ζ is input-to-state stable.

Whenever $e_i = 0$ and $\zeta = 0$, we expect that also the angular velocity of the follower vehicle i relative to the leader vehicle $i-1$, given by $\omega_i - \omega_{i-1}$, converges. If $\omega_{i-1} = 0$ and the leader trajectory is a straight line, we expect that $\omega_i \to 0$. Similarly, if $\omega_{i-1} = c \neq 0$ and the leader trajectory is a circle, we similarly expect that $\omega_i \to \omega_{i-1}$ and that $\theta_i - \theta_{i-1}$ converges to a constant angle ϕ. We refer to ϕ as the nominal orientation difference between the two vehicles. We determine the value of ϕ as follows. By taking $\zeta = 0$ in (19) and assuming that $\omega_i = \omega_{i-1}$ we observe that

$$\dot{\overline{p}}_i = \dot{\underline{p}}_{i-1} - \tfrac{\lambda}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\ddot{\underline{p}}_{i-1}.$$

By substituting (14) with $\omega_i = \omega_{i-1}$ and (21) with $a_{i-1} = \alpha_{i-1} = 0$, we find that

$$\begin{aligned} R(\theta_i)\begin{bmatrix}v_i\\ d_{f,i}\omega_{i-1}\end{bmatrix} &= R(\theta_{i-1})\begin{bmatrix}v_{i-1}\\ -d_{r,i-1}\omega_{i-1}\end{bmatrix} \\ &\quad - \tfrac{\lambda\omega_{i-1}}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)R(\theta_{i-1})\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}v_{i-1}\\ -d_{r,i-1}\omega_{i-1}\end{bmatrix}. \end{aligned}$$

Multiplication by $R(\theta_{i-1})^{-1}$ yields

$$\begin{aligned} &R(\theta_i - \theta_{i-1})\begin{bmatrix}v_i\\ d_{f,i}\omega_{i-1}\end{bmatrix} \\ &= \left(I - \tfrac{\lambda\omega_{i-1}}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\right)\begin{bmatrix}v_{i-1}\\ -d_{r,i-1}\omega_{i-1}\end{bmatrix} \\ &= \tfrac{1}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\begin{bmatrix}v_{i-1}\\ -d_{r,i-1}\omega_{i-1}\end{bmatrix} \end{aligned} \tag{22}$$

by direct computation. For this equality to hold, we require that the (squared) norms of both sides of the equation coincide. This is to say that we require

$$v_i^2 + d_{f,i}^2\omega_{i-1}^2 = \frac{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2}{1+\lambda^2\omega_{i-1}^2}. \tag{23}$$

Note that (23) is an equation in v_i and does not always have a solution. Specifically, this is the case only if $\omega_{i-1} \neq 0$ and the leader trajectory is a circle, and $d_{f,i}$ is too large or $d_{r,i-1}$ is too small. Fig. 2 shows the erratic behavior that results whenever a solution to (23) does not exist. The key observation here is that the radius of the circular trajectory of $\underline{p}_{i-1}$ should not exceed $d_{f,i}$. We remark that the existence of a solution to (23) is a necessary condition for the follower to converge to the nominal trajectory, but does not affect the asymptotic convergence of e_i to zero. The erratic behavior can be avoided by increasing $d_{r,i-1}$ and/or decreasing $d_{f,i}$, such that (23) is always solvable along the trajectory of the leader vehicle. However, this analysis is beyond the scope of this paper.

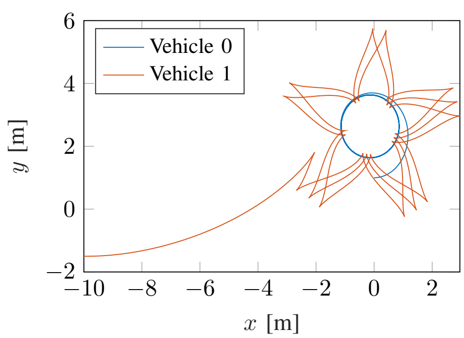

Fig. 2. Example of the erratic follower behavior that may occur if the leader vehicle drives on a circle and (23) does not have a solution. The plot shows the trajectory of the centers of mass of the two vehicles. The trajectory of the point $\underline{p}_{i-1}$ behind the leader vehicle is also circular. Since (23) has no solution, $d_{r,i-1}$ is too small compared to $d_{f,i}$, and consequently the radius of the trajectory of $\underline{p}_{i-1}$ that vehicle i wishes to track is too small compared to the following distance $d_{f,i}$. As a result, the follower cannot drive at a fixed position behind the leader, and erratic following behavior occurs.

Assuming that (23) is solvable for v_i, we define

$$\hat{v}_i := \sqrt{\frac{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2}{1+\lambda^2\omega_{i-1}^2} - d_{f,i}^2\omega_{i-1}^2}$$

as the nominal velocity of the follower vehicle. If $v_i = \hat{v}_i$ in (22), both sides of the equation have the same norm and there exists a choice for the relative orientation $\theta_i - \theta_{i-1}$ such that (22) holds. We let ϕ be this angle, which is to say that ϕ is defined by

$$R(\phi)\begin{bmatrix}\hat{v}_i \\ d_{f,i}\omega_{i-1}\end{bmatrix} = \frac{1}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\begin{bmatrix}v_{i-1} \\ -d_{r,i-1}\omega_{i-1}\end{bmatrix}. \tag{24}$$

It may be observed that this is equivalent to the equation

$$\begin{aligned} R(\phi)&\begin{bmatrix}\hat{v}_i & -d_{f,i}\omega_{i-1} \\ d_{f,i}\omega_{i-1} & \hat{v}_i\end{bmatrix} \\ &= \frac{1}{\sqrt{1+\lambda^2\omega_{i-1}^2}}R(\psi)\begin{bmatrix}v_{i-1} & d_{r,i-1}\omega_{i-1} \\ -d_{r,i-1}\omega_{i-1} & v_{i-1}\end{bmatrix}. \end{aligned}$$

We may invert the matrix after $R(\phi)$ to obtain an explicit expression for $R(\phi)$. Using (18), by direct computation we find that ϕ satisfies

$$\begin{aligned} \cos\phi =& \frac{v_{i-1}(\hat{v}_i - \lambda d_{f,i}\omega_{i-1}^2)}{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2} \\ &+ \frac{d_{r,i-1}\omega_{i-1}(\lambda\hat{v}_i\omega_{i-1} + d_{f,i}\omega_{i-1})}{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2}; \\ \sin\phi =& \frac{-d_{r,i-1}\omega_{i-1}(\hat{v}_i - \lambda d_{f,i}\omega_{i-1}^2)}{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2} \\ &+ \frac{v_{i-1}(\lambda\hat{v}_i\omega_{i-1} + d_{f,i}\omega_{i-1})}{v_{i-1}^2 + d_{r,i-1}^2\omega_{i-1}^2}. \end{aligned}$$

Note in particular that $\omega_{i-1} = 0$ implies $\phi = 0$.

To capture the remaining internal state variable that is complementary to ζ, we make the following definitions. Let $\alpha \in [-2\pi, 0)$ be the angle defined by

$$\alpha := \begin{cases} 2\arctan\left(\frac{-\hat{v}_i}{d_{f,i}\omega_{i-1}}\right) & \text{if } \omega_{i-1} > 0, \\ 2\arctan\left(\frac{-\hat{v}_i}{d_{f,i}\omega_{i-1}}\right) - 2\pi & \text{if } \omega_{i-1} < 0, \\ -\pi & \text{if } \omega_{i-1} = 0. \end{cases}$$

Furthermore, we define the variable

$$\begin{aligned} \delta(t) :=& \int_0^t \omega_i(s) - \omega_{i-1}(s)\,\mathrm{d}s + \theta_i(0) - \theta_{i-1}(0) - \phi, \\ =& \theta_i - \theta_{i-1} - \phi \end{aligned}$$

and for $\delta \in [\alpha, \alpha + 2\pi)$ the function

$$F(\delta) := \lambda\hat{v}_i(1-\cos(\delta)) + \lambda d_{f,i}\omega_{i-1}(\delta - \sin(\delta)).$$

We note the partial derivative of F with respect to δ satisfies

$$\tfrac{\partial F}{\partial \delta} = \lambda\hat{v}_i\sin(\delta) + \lambda d_{f,i}\omega_{i-1}(1-\cos(\delta)). \tag{25}$$

Consequently, the extrema of F are the local minimum at $\delta = 0$ and the local maxima at $\delta = \alpha, \alpha + 2\pi$.

By substituting (14), (21) and (24) in (20), we find that

$$\lambda R(\theta_i)\begin{bmatrix}v_i \\ d_{f,i}\omega_i\end{bmatrix} = \zeta - e_i + \lambda R(\phi)R(\theta_{i-1})\begin{bmatrix}\hat{v}_i \\ d_{f,i}\omega_{i-1}\end{bmatrix},$$

which, through multiplying by $R(\theta_i)^{-1}$, yields

$$\begin{aligned} \lambda\begin{bmatrix}v_i \\ d_{f,i}\omega_i\end{bmatrix} &= R(\theta_i)^{-1}(\zeta - e_i) + \lambda R(\delta)^{-1}\begin{bmatrix}\hat{v}_i \\ d_{f,i}\omega_{i-1}\end{bmatrix}, \\ &= R(\delta + \theta_{i-1} + \phi)^{-1}(\zeta - e_i) + \lambda R(\delta)^{-1}\begin{bmatrix}\hat{v}_i \\ d_{f,i}\omega_{i-1}\end{bmatrix}. \end{aligned}$$

By writing out the second line of this equation, we obtain

$$\begin{aligned} \lambda d_{f,i}\omega_i =& -\sin(\delta + \theta_{i-1} + \phi)(\zeta_1 - e_{i,1}) \\ &+ \cos(\delta + \theta_{i-1} + \phi)(\zeta_2 - e_{i,2}) \\ &- \lambda\hat{v}_i\sin(\delta) + \lambda d_{f,i}\omega_{i-1}\cos(\delta), \end{aligned}$$

and substitution in (25) yields the partial derivative of F as

$$\begin{aligned} \tfrac{\partial F}{\partial \delta} =& \lambda d_{f,i}(\omega_{i-1} - \omega_i) - \sin(\delta + \theta_{i-1} + \phi)(\zeta_1 - e_{i,1}) \\ &+ \cos(\delta + \theta_{i-1} + \phi)(\zeta_2 - e_{i,2}). \end{aligned}$$

We define as our internal state variable $z := \omega_i - \omega_{i-1}$, together with the candidate Lyapunov function

$$W(z,t) := F\left(\int_0^t z(\tau)\mathrm{d}\tau + \theta_i(0) - \theta_{i-1}(0) - \phi\right).$$

Note that $W(z,t) = F(\delta)$, and therefore

$$\begin{aligned}\dot{W} = \tfrac{\partial F}{\partial \delta}\dot{\delta} &= -\lambda d_{f,i}(\omega_i - \omega_{i-1})^2\\ &\quad - \sin(\delta + \theta_{i-1} + \phi)(\zeta_1 - e_{i,1})(\omega_i - \omega_{i-1})\\ &\quad + \cos(\delta + \theta_{i-1} + \phi)(\zeta_2 - e_{i,2})(\omega_i - \omega_{i-1}),\\ &\leq -\lambda d_{f,i}|\omega_i - \omega_{i-1}|^2\\ &\quad + (|\zeta_1 - e_{i,1}| + |\zeta_2 - e_{i,2}|)|\omega_i - \omega_{i-1}|,\\ &= -\lambda d_{f,i} z^2 + (|\zeta_1 - e_{i,1}| + |\zeta_2 - e_{i,2}|)|z|.\end{aligned}$$

Consequently, if

$$|\zeta_1 - e_{i,1}| + |\zeta_2 - e_{i,2}| \leq \lambda d_{f,i}|z|,$$

then $\dot{W} \leq 0$. Thus, we can conclude that the variable $z = \omega_i - \omega_{i-1}$ is input-to-state stable with respect to ζ and e_i if z considered as part of the internal state representation.

Finally, when adjoining the internal state variables ζ and z to the error dynamics e_i, $\dot{e}_i$ and the state of the leader $\underline{\xi}_{i-1}$, this representation constitutes a coordinate transformation.

Lemma 3: The function defined by

$$S(\xi) := \begin{bmatrix}\underline{\xi}_{i-1}^\top & e_i^\top & \dot{e}_i^\top & \zeta^\top & z\end{bmatrix}^\top,$$

yields a coordinate transformation.

Proof: A direct computation of the determinant of the Jacobian of $S(\xi)$ yields $\det(\nabla S(\xi)) = 1$. Hence, $S(\xi)$ is invertible and thus a coordinate transformation. ■

Altogether we have proven the following result.

Theorem 2: Consider the spacing policy (4) and the dynamics (7), where the leader vehicle follows an unexcited trajectory. The remaining internal dynamics are input-to-state stable with respect to e_i, $\dot{e}_i$ and ξ_{i-1}.

V. CONCLUSIONS

In this paper we consider decentralized controller design for lateral and longitudinal control of heterogeneous vehicle platoons with constant headway spacing. A nonlinear spacing policy is obtained by approximation of a time delayed spacing policy. The results generalize the already existing constant headway spacing policy for longitudinal control to the two-dimensional case. Necessary and sufficient conditions for tracking of the spacing policy are presented, which motivate the synthesis of the longitudinal and lateral controllers. The resulting internal dynamics are shown to be input-to-state stable with respect to the spacing errors for an unexcited leader trajectory. The same analysis for a general leader trajectory is left for future research. Another future direction of research is to re-frame the results in terms of a body fixed frame, instead of an inertial frame, to enable practical implementation and experimental validation of the controller. The authors have obtained some results in this direction which seem promising. However, they are not sufficiently developed to incorporate in this paper. Additionally, detailed analysis of the choice of standstill distances and the effects on the (erratic) following behavior should be addressed in future research. Finally, it remains to investigate the performance of the controller in the case of imperfect knowledge of the state.

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