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# Globally Asymptotically Stable Control of Integrators with Long Dead Time in the Presence of Actuator Constraints

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***Abstract*— Different concepts to control integrators with long dead time have been presented over the last 40 years. Although this particular plant model is of practical interest, another practical issue of high interest, i.e. actuator constraints, has often enough been neglected in the treatises. This paper presents a control scheme (the so-called Conditioned Smith-Åström Predictor) that guarantees global asymptotic stability for integrators with long dead time. Furthermore, it offers a straightforward design procedure and—yet another practical aspect of increasing interest—an explicit disturbance estimate (independent of constraints). A highlight of the proposed stability proof is that it uses well-established graphical methods that allow for an easy verification of closed-loop stability even in the presence of unmodelled dynamics and dead time mismatch, the latter of which is illustrated in the paper. This makes it an appealing alternative to other methods found in literature.**

## I. INTRODUCTION

Integrators with (long) dead time are commonly found in practical applications, either directly (e.g. if mass and/or energy balances are involved) or—ever since the days of Ziegler and Nichols—as approximations of more complex models [1]. The control of such plants has been investigated for more than 40 years (see e.g. [2]), but often enough actuator constraints were either not considered or were neglected in stability analysis. In [3], the region of attraction was estimated for the so-called Filtered Smith Predictor by means of the circle criterion, yet global asymptotic stability has not been established for the integrator with long dead time. The main ideas of that article were generalised in [4] and global asymptotic stability was established for two (out of three) exemplary tunings of the Filtered Smith Predictor by means of linear matrix inequalities. In [5] and [6], the circle criterion was used to assess closed-loop stability for control of an integrator with long dead time, yet global asymptotic stability has not been rigorously established.

The present paper is based on the so-called Conditioned Smith-Åström Predictor—a widely recognised, yet underrated modification of the Smith Predictor—, which was recently re-evaluated in [1]. It offers a straightforward design procedure, considers actuator constraints, and can be tuned to yield the same input-output behaviour as the Filtered Smith Predictor in many cases [1]; additionally, it provides an explicit disturbance estimate (independent of constraints), which gains importance in modern industrial applications. This makes the Conditioned Smith-Åström Predictor an interesting alternative to the Filtered Smith Predictor.

The main contribution of this paper is that it presents the Smith-Åström Predictor in a rigorous manner (whereas [1] is written from a practical and didactic point of view), including—for the first time—stability analysis for the constrained case. Furthermore, global asymptotic stability for the integrator with long dead time is established for a whole class of controllers by means of the circle criterion; this approved graphical tool makes it easy to also guarantee stability for perturbed cases, which is illustrated for the case of a dead time mismatch of $\pm 20\,\%$.

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## II. THE CONDITIONED SMITH-ÅSTRÖM PREDICTOR

The Conditioned Smith-Åström Predictor [1] is an extension of the so-called Smith-Åström Predictor—originally published in [7], [8]—to deal with actuator saturation. Its structure for the case without saturation is shown in Fig. 1. The plant model $P(s)$ consists of a dead-time free part $P^*(s)$ and a delay with (dead) time $L$, i.e.

$$P(s) = P^*(s)\mathrm{e}^{-sL}\,. \tag{1}$$

The Smith-Åström Predictor consists of two controllers: $K(s)$ and $M(s)$, where the former determines the set-point response transfer function

$$T(s) = \frac{y(s)}{r(s)} = \frac{K(s)P^*(s)}{1+K(s)P^*(s)}\mathrm{e}^{-sL} \tag{2}$$

and the latter determines the load response transfer function

$$S_d(s) = \frac{y(s)}{d(s)} = \frac{P^*(s)}{1+M(s)P(s)}\mathrm{e}^{-sL} \tag{3}$$

(notice the decoupling of set-point and load response). The controller $M(s)$ is a modification of the famous Smith Predictor, i.e.

$$M(s) = \frac{R^*(s)}{1+R^*(s)P^*(s)\left[F(s)-\mathrm{e}^{-sL}\right]}\,, \tag{4}$$

where $R^*(s)$ is a "nominal" controller (see Fig. 2) and the filter $F(s)$ constitutes the modification (the original Smith Predictor is recovered for $F(s)=1$). The design procedure proposed in [1] is summarised in:

*Proposition 1:* Consider the control system shown in Fig. 1 with

$$P^*(s) = \frac{b}{s}\,, \tag{5}$$

$b \neq 0$ being a real constant, $L > 0$ being the dead time, and $M(s)$ be chosen according to (4). Let

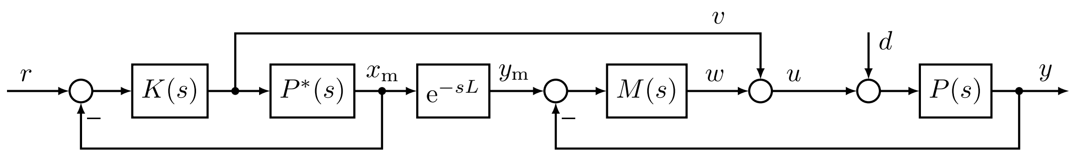


Fig. 1. Structure of the Smith-Åström Predictor; $P^*(s)$ is the dead-time free part of $P(s)$.

(i) $K(s)$ be a biproper rational function chosen such that

$$T^*(s) = \frac{K(s)P^*(s)}{1 + K(s)P^*(s)} \tag{6}$$

corresponds to a desired (i.e. at least BIBO stable) set-point response behaviour,

(ii) $\tilde{R}(s)$ be a proper rational function chosen such that

$$\tilde{S}_d(s) = \frac{P^*(s)}{1 + \tilde{R}(s)P^*(s)} \tag{7}$$

is BIBO stable and has a zero at $s = 0$, and

(iii) $F(s)$ be a biproper rational function chosen such that

$$H(s) = P^*(s)[F(s) - \mathrm{e}^{-sL}] \tag{8}$$

is BIBO stable (by cancellation of unstable plant poles) and has a zero at $s = 0$. Furthermore, let $F(s)$ and its inverse $1/F(s)$ be BIBO stable. If $R^*(s)$ is chosen as

$$R^*(s) = \frac{\tilde{R}(s)}{F(s)}\,, \tag{9}$$

then the closed-loop system is BIBO stable, the set-point response is a time-shifted version of the desired response, i.e.

$$T(s) = T^*(s)\mathrm{e}^{-sL}\,, \tag{10}$$

and constant input disturbances $d$ are rejected; the poles of $S_d(s)$ are given by the poles of $\tilde{S}_d(s)$ and the zeros of $F(s)$.

Additionally, if $F(s)$ and $\tilde{R}(s)$ share the same zeros, then these zeros are no longer poles of $S_d(s)$.

*Proof:* The set-point behaviour is trivial, compare (10) with (2). As for the load response, insert (4) into (3) to find

$$S_d(s) = \tilde{S}_d(s)[1 + R^*(s)H(s)]\mathrm{e}^{-sL}\,. \tag{11}$$

Denote the numerator and denominator polynomial of the following transfer functions with (arguments omitted for the sake of space)

$$P^*(s) = \frac{\mu_P}{\nu_P}\,, \quad \tilde{R}(s) = \frac{\mu_R}{\nu_R}\,, \quad F(s) = \frac{\mu_F}{\nu_F}\,. \tag{12}$$

Now observe that

$$\tilde{S}_d(s) = \frac{\mu_P \nu_R}{\nu_R \nu_P + \mu_R \mu_P} = \frac{\mu_P \nu_R}{\nu_{S_d}}\,, \quad R^*(s) = \frac{\mu_R \nu_F}{\nu_R \mu_F}\,, \tag{13}$$

and with $H(s) = h(s)/\nu_H(s)$ ($h(s)$ is not a polynomial)

$$1 + R^*(s)H(s) = \frac{\nu_R \mu_F \nu_H + \mu_R \nu_F h(s)}{\nu_R \mu_F \nu_H}\,. \tag{14}$$

Since $\nu_H(s) = \nu_F(s)$ (the pole of $P^*(s)$ being cancelled) and $\mu_P(s) = b$, the load response transfer function is

$$S_d(s) = b\frac{\nu_R \mu_F + \mu_R h(s)}{\nu_{S_d}\mu_F}\mathrm{e}^{-sL}\,, \tag{15}$$

which is BIBO stable (since $\nu_{S_d}$ and $\mu_F$ are Hurwitz) and its dynamics are governed by the poles of $\tilde{S}_d(s)$ and the zeros of $F(s)$. ■

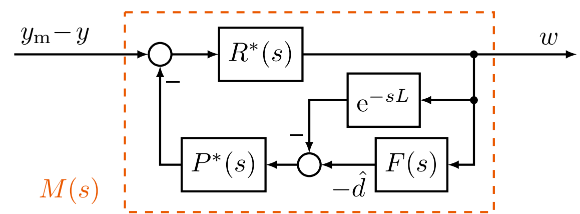


Fig. 2. A practicable implementation of the feedback controller $M(s)$ for integrating plants.

*Remark 1 (Internal Stability):* Under the assumption that the controllers $K(s)$ and $R^*(s)$ are designed such that the feedback loop is internally stable for $L = 0$ (dead-time free case)—which is a sensible assumption in deed—, the internal stability of the Smith-Åström Predictor depends on the implementation of the transfer function $H(s)$ given in (8) [1], [9]. In a discrete-time implementation, $H(z)$ is a rational function and the unstable plant pole(s) can easily be cancelled out; however, this is not true for the continuous-time case. For integrating plants (one pole at zero, the others "left"), a possible implementation of $M(s)$—and thus $H(s)$—is given in Fig. 2. At steady state ($r$ and $d$ being constant), the constant signal $w$ is "blocked" by the zero of $F(s) - \mathrm{e}^{-sL}$ and thus, the input to the integrator of $P^*(s)$ is zero. Note that swapping the position of $P^*(s)$ and $F(s) - \mathrm{e}^{-sL}$ would lead to a ramp function at the output of $P^*(s)$ at steady state and hence to an internally unstable feedback loop.

*Remark 2 (Disturbance Estimate):* The signal $w$ is an explicit estimate of $-d$, since in the closed loop

$$w = -\frac{M(s)P(s)}{1 + M(s)P(s)}d = \frac{-1}{F(s)} \cdot \frac{\tilde{R}(s)P^*(s)}{1 + \tilde{R}(s)P^*(s)}d\,, \tag{16}$$

which can easily be verified by observing that

$$w = M(s)[P(s)v - P(s)(d + u)] \tag{17}$$

and $u = v + w$. Note that $w$ does not depend on $r$ but only on $d$. Further note that the step response of $1/F(s)$ may not be agreeable; thus, $w$ may be filtered by means of $F(s)$ to obtain a more acceptable disturbance estimate $\hat{d}$, see Fig. 2.

The *Conditioned Smith-Åström Predictor*, shown in Fig. 3, is obtained by applying the so-called *conditioning technique* due to Hanus [10], [11]. The idea is to calculate a reference that would—in the current time instance—lead to $u$ being equal to the constrained actuating signal $u^*$; this "realisable" reference $r^*$ is then applied to the *dynamic part* of the controller (thus the controller states are "kept consistent" with the constrained actuating signal, so to speak).

With $V(s)$ and $Q(s)$ defined such that

$$u(s) = V(s)r(s) - Q(s)y(s) \tag{18}$$

holds and assuming that $V(s)$ has a direct transmission term, i.e.

$$\lim_{s\to\infty} V(s) = v_0 \neq 0\,, \tag{19}$$

one gets

$$u^*(s) = v_0 r^*(s) + [V(s) - v_0]r(s) - Q(s)y(s) \tag{20}$$

and thus (subtracting (20) from (18) and rearranging)

$$r^*(s) = r(s) - \frac{1}{v_0}[u(s) - u^*(s)]\,. \tag{21}$$

Applying $r^*$ to the dynamic part of the controller gives

$$\begin{aligned} u(s) &= v_0 r(s) + [V(s) - v_0]r^*(s) - Q(s)y(s) \\ &= V(s)r(s) - \frac{V(s) - v_0}{v_0}[u(s) - u^*(s)] - Q(s)y(s)\,. \end{aligned} \tag{22}$$

In the unconstrained case ($u^* = u$), the original control law is recovered; in the constrained case ($u^* \neq u$), there is no feedback and $u$ is given by

$$u(s) = v_0 r(s) + \left[1 - \frac{v_0}{V(s)}\right] u^*(s) - v_0\frac{Q(s)}{V(s)}y(s)\,. \tag{23}$$

Note that $V(s)$, i.e. the path from $r$ to $u$, is inverted.

Applying the conditioning technique to the Smith-Åström Predictor, one gets the structure shown in Fig. 3, compare [1], i.e. the Conditioned Smith-Åström Predictor; its main properties are summarised in the following proposition.

*Proposition 2:* Consider the Conditioned Smith-Åström Predictor, i.e. the control system shown in Fig. 3, and let the assumptions of *proposition 1* be satisfied. Then, the Conditioned Smith-Åström Predictor is asymptotically stable in the unconstrained case ($u^* = u$). In the constrained case ($u^* \neq u$), bounded signals $r$, $u^*$, and $y$ lead to a bounded signal $u$, if the numerator polynomial of $K(s)$ is Hurwitz.

*Proof:* The unconstrained case is trivial. For the constrained case, it has to be shown—according to (23)—that $1/V(s)$ and $Q(s)/V(s)$ are BIBO stable, with

$$V(s) = \frac{K(s)[1 + M(s)P(s)]}{1 + K(s)P^*(s)}\,, \quad Q(s) = M(s)\,, \tag{24}$$

and

$$v_0 = k_0 = \lim_{s\to\infty} K(s)\,, \tag{25}$$

which is non-zero by assumption.

As for $1/V(s)$, observe that

$$\frac{1}{V(s)}\mathrm{e}^{-sL} = \frac{S_d(s)}{T^*(s)}\,. \tag{26}$$

Thus, it is sufficient to show that the numerator polynomial of $T^*(s)$ is Hurwitz and that $1/V(s)$ is biproper.

$$T^*(s) = \frac{b\,\mu_K(s)}{s\,\nu_K(s) + b\,\mu_K(s)}\,, \tag{27}$$

where $\mu_K$ and $\nu_K$ denote the numerator and denominator polynomial of $K(s)$, respectively; $\mu_K(s)$ is Hurwitz by assumption. With (13) one sees that $1/V(s)$ is biproper iff

$$\frac{\tilde{S}_d(s)}{T^*(s)} = \frac{s\,\nu_K(s) + b\,\mu_K(s)}{b\,\mu_K(s)} \cdot \frac{b\,\nu_R(s)}{s\,\nu_R(s) + b\,\mu_R(s)} \tag{28}$$

is biproper, which is the case since $K(s)$ is biproper by assumption.

As for $Q(s)/V(s)$, observe that with (24), (4), and (12)

$$\begin{aligned} \frac{Q(s)}{V(s)} &= \frac{1}{T^*(s)} \cdot \frac{1}{F(s)} \cdot \frac{\tilde{R}(s)P^*(s)}{1 + \tilde{R}(s)P^*(s)} \\ &= \frac{s\,\nu_K(s) + b\,\mu_K(s)}{b\,\mu_K(s)} \cdot \frac{\nu_F(s)}{\mu_F(s)} \cdot \frac{b\,\mu_R(s)}{s\,\nu_R(s) + b\,\mu_R(s)}\,, \end{aligned} \tag{29}$$

which is a proper transfer function; since $\mu_K(s)$, $\mu_F(s)$, and $s\nu_R(s) + b\mu_R(s) = \nu_{S_d}(s)$ are Hurwitz by assumption, $Q(s)/V(s)$ is BIBO stable. ■

*Remark 3 (Disturbance Estimate):* The signal $w$ still depends on $d$ alone, regardless of the actuator constraints, i.e. (16) holds true even in the constrained case.

## III. STABILITY IN THE PRESENCE OF ACTUATOR CONSTRAINTS

Stability of the feedback loop given in Fig. 3, i.e. the Conditioned Smith-Åström Predictor, was proven for the unconstrained case only. For the constrained case, it was shown that bounded signals $r$, $u^*$, and $y$ lead to a bounded signal $u$, which does not necessarily imply closed-loop stability. To establish closed-loop stability in the constrained case, the notion of absolute stability is used. To that end, the feedback loop without inputs, i.e. $r = d = 0$, is considered. The Conditioned Smith-Åström Predictor thus reduces to the feedback loop shown in Fig. 4, with

$$L(s) = k_0 P^*(s) + \frac{k_0}{K(s)} - 1\,, \tag{30}$$

which follows from (23) with $y = P(s)u^*$. The static nonlinearity $\varphi(u)$ is the saturation function, i.e.

$$u^* = \varphi(u) = \begin{cases} u & |u| \leq u_{\max}\,, \\ u_{\max} \cdot \operatorname{sign}(u) & |u| > u_{\max}\,, \end{cases} \tag{31}$$

$u_{\max} > 0$. With these preliminary remarks it can be shown that for $P(s)$ being an integrator with long dead time, one can design a Conditioned Smith-Åström Predictor that results in a globally asymptotically stable feedback loop.

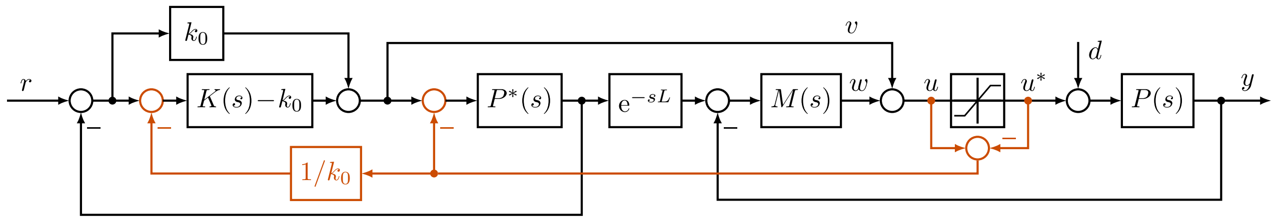


Fig. 3. Structure of the *Conditioned* Smith-Åström Predictor for $P^*(s)$ being strictly proper and $K(s)$ being biproper, with $k_0 = \lim_{s\to\infty} K(s)$.

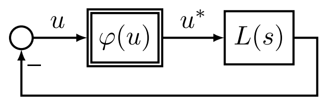


Fig. 4. Nonlinear feedback loop for $r = 0$, $d = 0$.

*Theorem 1:* Consider the Conditioned Smith-Åström Predictor, Fig. 3, and let the assumptions of *proposition 1* be satisfied. Let $\alpha_1, \alpha_2 \in \mathbb{C}$ be chosen such that

$$k_1 = \alpha_1 + \alpha_2 > 0 \qquad \text{and} \qquad k_2 = \alpha_1\alpha_2 > 0\,, \tag{32}$$

i.e. $\alpha_1, \alpha_2$ are either positive or complex-conjugate with positive real part, and let

$$K(s) = \frac{1}{b}\cdot\frac{k_2}{k_1}\,, \qquad \tilde{R}(s) = \frac{k_1 s + k_2}{b\,s}\,, \tag{33}$$

and

$$F(s) = \frac{k_1 s + k_2}{(k_1 + k_2 L)s + k_2}\,. \tag{34}$$

If $|d(t)| < u_{\max}$ for all $t$, then the resulting feedback loop is globally asymptotically stable both in the unconstrained ($u^* = u$) and in the constrained ($u^* \neq u$) case.

*Proof:* First observe that the assumptions of proposition 1 are in deed satisfied: (i) $K(s)$ is biproper and the set-point response transfer function (2) is

$$T(s) = \frac{k_2}{k_1 s + k_2}\,\mathrm{e}^{-sL}\,, \tag{35}$$

which is BIBO stable according to (32); (ii) $\tilde{R}(s)$ is proper and gives

$$\tilde{S}_d(s) = b\,\frac{s}{(s+\alpha_1)(s+\alpha_2)}\,, \tag{36}$$

which is BIBO stable with a zero at $s = 0$; (iii) both $F(s)$ and $1/F(s)$ are BIBO stable and $F(s) - \mathrm{e}^{-sL}$ has two zeros at $s = 0$. Furthermore, $F(s)$ and $\tilde{R}(s)$ share the same zeros, thus

$$R^*(s) = \frac{(k_1 + k_2 L)s + k_2}{b\,s} \tag{37}$$

and by proposition 1, $S_d(s)$ is BIBO stable and has two poles at $s = -\alpha_1$ and $s = -\alpha_2$. In fact, $S_d(s)$ is [1]

$$S_d(s) = \frac{b(1 - \mathrm{e}^{-sL})\mathrm{e}^{-sL}}{s} + \frac{b(s - \alpha_1\alpha_2 L)\mathrm{e}^{-s2L}}{(s+\alpha_1)(s+\alpha_2)}\,. \tag{38}$$

The unconstrained case is thus proven.

For the contrained case, the idea is to show absolute stability by means of the circle cirterion for the (reduced) feedback loop given in Fig. 4; with (30) and (33) one gets

$$L(s) = \frac{k_2}{k_1 s}\,. \tag{39}$$

Due to the pole at $s = 0$, one cannot immediately apply the circle criterion, because $\varphi(u)$ globally belongs to the sector $[0, 1]$ as does the “nonlinearity” $\psi(u) = 0$, the latter of which does not lead to an asymptotically stable origin. To circumvent this technical difficulty, assume that

$$|u(t)| \leq \frac{u_{\max}}{\epsilon}\,, \qquad 0 < \epsilon \ll 1\,, \tag{40}$$

holds true for all $t$; thus, it is sufficient to show absolute stability for a nonlinearity in the sector $[\epsilon, 1]$. Since $L(s)$ has no poles with positive real part and

$$\mathrm{Re}\{1 + L(\mathrm{j}\omega)\} = 1 > 0\,, \tag{41}$$

the nonlinear feedback loop in Fig. 4 is absolutely stable [12, theorem 7.2, p. 270]. Although this should suffice for any practical application, the so-called off-axis circle criterion is now used to rigorously establish global asymptotic stability; assumption (40) can thus be abandoned. The off-axis circle criterion states that in order the establish global asymptotic stability in the considered case (the nonlinearity being monotonically increasing), it is sufficient to show that the Nyquist plot of $L(\mathrm{j}\omega)$ lies entirely to the right of a straight line passing through the point $(-1+\varepsilon, 0)$, with $\varepsilon > 0$ [13, theorem 2, p. 414]. This is the case according to (41) and thus, the Conditioned Smith-Åström Predictor is globally asymptotically stable for $r = d = 0$ and proposition 2 guarantees that it is globally asymptotically stable for bounded signals $r$ and $d$, where the latter satisfies $|d(t)| < u_{\max}$ for all $t$ by assumption. Note that the bound on $d$ is necessary, for if $u_{\max} < d = \text{const.}$, then $d + u^* > 0$ and $y$ approaches infinity. ■

*Remark 4 (Using No Anti-Windup Measure):* For the unconditioned Smith-Åström Predictor, $L(s)$ in the nonlinear feedback loop of Fig. 4 is

$$L(s) = M(s)P(s) = \frac{R^*(s)P(s)}{1 + R^*(s)P^*(s)\left[F(s) - \mathrm{e}^{-sL}\right]}\,. \tag{42}$$

Here, global asymptotic stability for $P(s)$, $\tilde{R}(s)$, and $F(s)$ according to theorem 1 cannot be guaranteed by means of the circle criterion. Two exemplary Nyquist plots are shown[1] in Fig. 5 and 6. In the first case, asymptotic stability can be guaranteed if $|u(t)| \leq u_{\max}/0.26$ holds true for all $t$ and in the second case if $|u(t)| \leq u_{\max}/0.3$ holds true. Simulations show that in the second case the feedback loop is unstable for $u(t{=}0) = -2 = -u_{\max}/0.05$ with $u_{\max} = 0.1$.

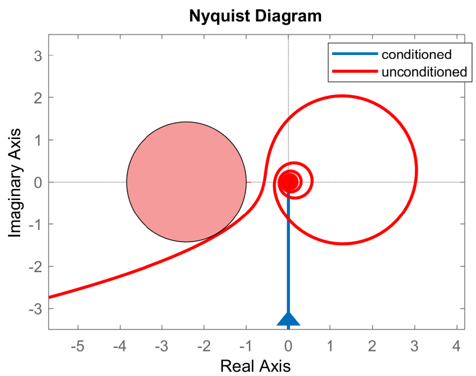


Fig. 5. Nyquist plot ($\omega \geq 0$) of $L(\mathrm{j}\omega)$ for the conditioned and unconditioned Smith-Åström Predictor; $b = 1$, $L = 5$, $\alpha_1 = \alpha_2 = 0.25$; the circle passes through $-1/0.26$ and $-1$.

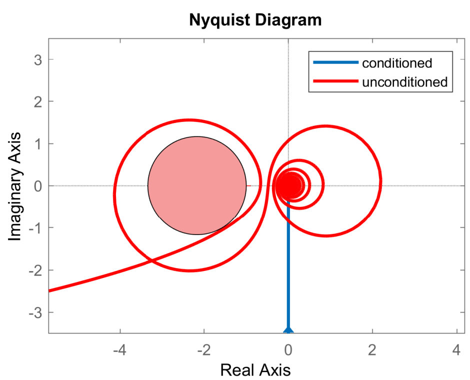


Fig. 6. Nyquist plot ($\omega \geq 0$) of $L(\mathrm{j}\omega)$ for the conditioned and unconditioned Smith-Åström Predictor; $b = 1$, $L = 5$, $\alpha_1 = \alpha_2 = 0.4$; the circle passes through $-1/0.3$ and $-1$.

## IV. THE PERTURBED CASE

The proposed method can easily be used to investigate the perturbed case. For the feedback loop considered in theorem 1, let the actual plant be given by

$$\hat{P}(s) = \frac{b}{s}\mathrm{e}^{-s\hat{L}}\,. \tag{43}$$

Then, the load response transfer function is

$$\hat{S}_d(s) = \frac{\hat{P}(s)}{1 + M(s)\hat{P}(s)}\,. \tag{44}$$

[1]Note that in both plots, the Nyquist criterion is satisfied for the conditioned and unconditioned predictor; for the latter, this follows from the fact that the feedback loop without saturation (linear case) is globally asymptotically stable.

Hence, with (4) and with the particular choice for $K(s)$, $\tilde{R}(s)$, and $F(s)$

$$\hat{S}_d(s) = \frac{b[s^2 + k_1 s + k_2 - (k_3 s + k_2)\mathrm{e}^{-sL}]\mathrm{e}^{-s\hat{L}}}{s[s^2 + k_1 s + k_2 - (k_3 s + k_2)(\mathrm{e}^{-sL} - \mathrm{e}^{-s\hat{L}})]}\,, \tag{45}$$

where

$$k_3 = k_1 + k_2 L\,. \tag{46}$$

Since the pole at $s = 0$ is cancelled by a zero, it has to be shown that

$$s^2 + k_1 s + k_2 - (k_3 s + k_2)(\mathrm{e}^{-sL} - \mathrm{e}^{-s\hat{L}}) = 0 \tag{47}$$

has only roots with negative real parts in order for $\hat{S}_d(s)$ to be BIBO stable; since $s^2 + k_1 s + k_2$ is Hurwitz, it is sufficient to show—similar to [14]—that

$$1 + W(s) = 1 + \frac{-(k_3 s + k_2)(\mathrm{e}^{-sL} - \mathrm{e}^{-s\hat{L}})}{s^2 + k_1 s + k_2} = 0 \tag{48}$$

has only roots with negative real parts, which is the case if the Nyquist criterion for $W(s)$—which has only poles with negative real part—is satisfied. Two cases with an exemplary dead time mismatch of $\pm 20\,\%$, i.e. $\hat{L} = 0.8L$ and $\hat{L} = 1.2L$, are considered; the corresponding Nyquist plots are shown in Fig. 7 and 8, respectively; the Nyquist criterion is satisfied in both cases, which concludes the unconstrained case.

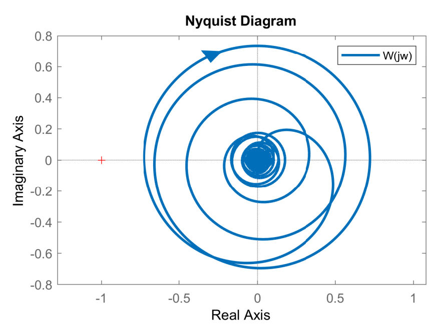


Fig. 7. Nyquist plot ($\omega \geq 0$) of $W(\mathrm{j}\omega)$; $b = 1$, $L = 5$, $\hat{L} = 4$, $\alpha_1 = \alpha_2 = 0.25$.

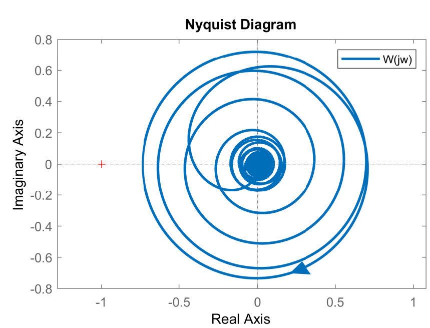


Fig. 8. Nyquist plot ($\omega \geq 0$) of $W(\mathrm{j}\omega)$; $b = 1$, $L = 5$, $\hat{L} = 6$, $\alpha_1 = \alpha_2 = 0.25$.

For the constrained case, the open-loop transfer function $L(s)$—corresponding to Fig. 4—is

$$L(s) = \left(k_0 P^*(s) + \frac{k_0}{K(s)}\right) \frac{1 + M(s)\hat{P}(s)}{1 + M(s)P(s)} - 1\,, \quad (49)$$

compare (30), which is equivalent to

$$L(s) = \frac{k_2}{k_1 s} - \frac{k_1 s + k_2}{k_1 s} \frac{(k_3 s + k_2)(\mathrm{e}^{-sL} - \mathrm{e}^{-s\hat{L}})}{s^2 + k_1 s + k_2}\,, \quad (50)$$

compare (39). For the two perturbed cases shown before, i.e. Fig. 7 and 8, the inequality

$$\mathrm{Re}\{1 + L(\mathrm{j}\omega)\} > 0 \quad (51)$$

still holds true as shown by Fig. 9 and 10, respectively, compare (41).

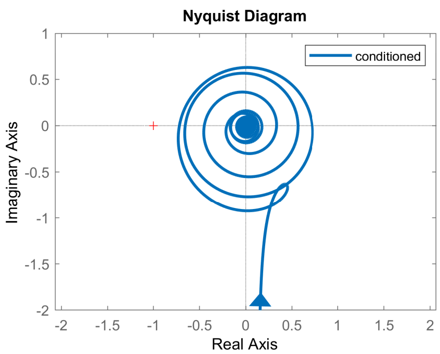


Fig. 9. Nyquist plot ($\omega \geq 0$) of $L(\mathrm{j}\omega)$, perturbed case; $b = 1$, $L = 5$, $\hat{L} = 4$, $\alpha_1 = \alpha_2 = 0.25$.

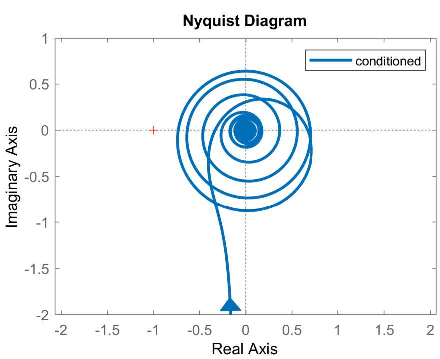


Fig. 10. Nyquist plot ($\omega \geq 0$) of $L(\mathrm{j}\omega)$, perturbed case; $b = 1$, $L = 5$, $\hat{L} = 6$, $\alpha_1 = \alpha_2 = 0.25$.

Thus, for $\hat{L} = 0.8L = 4$ and $\hat{L} = 1.2L = 6$, global asymptotic stability could easily be established by means of the tools shown in the previous section.

## V. CONCLUSIONS AND FUTURE WORKS

When it comes to controlling an integrator (or integrating plants) with long dead time, the Conditioned Smith-Åström Predictor is an appealing alternative to other control schemes found in literature (such as the so-called Filtered Smith Predictor, for instance). Its prime feature is that closed-loop stability can be guaranteed globally both in the unconstrained and constrained case. By means of the circle criterion, global asymptotic stability has be established quite easily (in contrast to other methods given in literature) even in the perturbed case. Another appealing feature is the availability of an explicit disturbance estimate that is not affected by actuator constraints.

The Conditioned Smith-Åström Predictor can easily be applied in a discrete-time setting by using corresponding transfer functions, replacing $\mathrm{e}^{-sL}$ by $z^{-\ell}$, and applying the conditions and methods analogously; however, a rigorous presentation is still considered future work. The same is true for the extension to unstable plants—in the continuous-time and/or discrete-time case—, which presents no conceptual difficulties, however. In fact, the method in this paper is presented with the general case in mind. Only the region of attraction will not contain the whole state space, in general; thus, the circle criterion needs to be complemented with a method that can establish rigorous bounds on the region of attraction.

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