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Epidemic Population Games for Policy Design: Two Populations with Viral Reservoir Case Study

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***Abstract*— We extend to two populations a recently proposed system theoretic framework for studying an epidemic influenced by the strategic behavior of a single population's agents. Our framework couples the well-known susceptible-infected-susceptible (SIS) epidemic model with a population game that captures the strategic interactions among the agents of two large populations. This framework can also be employed to study a situation where a population of *nonstrategic* agents (such as animals) serves as a disease reservoir. Equipped with the framework, we investigate the problem of designing a suitable control policy that assigns dynamic payoffs to incentivize the agents to adopt costlier and more effective mitigating strategies subject to a long-term budget constraint. We formulate a non-convex constrained optimization program for minimizing the disease transmission rate at an endemic equilibrium, and explain how to obtain an approximate solution efficiently. A solution to the optimization problem is an aggregate strategy distribution for the population game which minimizes the basic reproduction number, hence the disease transmission rate, at the corresponding endemic equilibrium. We then propose a dynamic payoff mechanism and use a Lyapunov function to prove the convergence of i) the aggregate strategy distribution, ii) infection levels, and iii) the dynamic payoffs; the aggregate strategy distribution of the population converges to an (approximate) solution to the optimization problem, and the infection levels in the two populations converge to the endemic equilibrium associated with the solution of the optimization.**

I. INTRODUCTION

Policymakers seek to mitigate the effects of an epidemic by using data and model-based precepts to devise effective interventions. Successful policies should limit the consequences and spread of the epidemic subject to curbs on the policies' economic costs [1]. Furthermore, sound policies should take into account human behavior and the strategic interactions among individuals, which determine their decisions over time in response to their (perceived) payoffs and risks.

Here, we extend the work in [2], [3], which studied the problem of designing a policy that steers the epidemic toward a more desirable endemic equilibrium. Unlike [2], [3] that only considered a single population, we study an epidemic in two populations that interact with each other. Hence, the decisions by individuals in one population affect the epidemic process in both populations.

A. Overview of [2] and [3]

The study in [2] introduced a new framework that combines the strategic decision-making process of agents (evolutionary dynamics) and a compartmental epidemic model (SIRS model) for a *single* population. This framework allowed the authors to design a dynamic payoff mechanism that ensures the convergence to an endemic equilibrium where the disease transmission rate is minimized subject to a budget constraint under an assumption that the disease death rate is negligible. A follow-up study [3] relaxed this assumption and proposed an analogous dynamic payoff mechanism that achieves the same goal. A key contribution of these studies is that they provide *anytime* bounds on the peak infection, which are universal and hold for any protocol that meets certain assumptions. A main difference between them is, however, while [2] obtains the anytime bound by solving a quasi-convex problem, when the disease death rate is non-negligible, [3] requires solving a set of convex problems to obtain an *approximated* bound with arbitrary accuracy.

B. Contributions

We generalize the framework and main results of [2], [3] to two population scenarios, in which agents interact with other agents from both populations, but possibly at different rates. Considering two populations of strategic agents in an epidemic leads to significant changes in the analysis relative to the previous studies on a single population [2], [3], including the construction of a new Lyapunov function to prove convergence. Moreover, computing an optimal social state that minimizes the disease transmission rate for a given budget becomes harder; the optimization problem is non-convex, and only an approximate solution can be obtained via a line search over an equivalent problem constructed on the basis of a key observation (Lemma 1). Finally, with the help of a Lyapunov function, we design a stabilizing policy that leads any arbitrary initial state to an optimal social state where the disease transmission rate is minimized subject to a long-term budget constraint.

C. Related works

Several studies investigated the problem of managing an epidemic using control theory: di Lauro et al. [4] and Sontag [5] studied the problem of identifying the optimal timing for non-pharmaceutical interventions (NPIs), such as quarantine and lockdowns, to minimize the peak infections. Al-Radhawi et al. [6] examined the problem of tuning NPIs to regulate infection rates as an adaptive control problem and investigated the stability of disease-free and endemic steady states. Godara et al. [7] studied the problem of controlling the infection rate to minimize the total cost till herd immunity is attained as an optimal control problem subject to a constraint

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on the fraction of infectious population. However, these studies did not consider the strategic decision-making.

Game theory provides a natural framework to study interactions among strategic agents. For this reason, several recent studies employed game theory, including evolutionary or population games, to study epidemic processes with strategic agents [8]–[13]. For instance, [8] studied the effect of risk perception on whether individuals choose to self-quarantine or not, and how increased perceived risks could lead to multiple infection peaks. Khazaei et al. [13] adopted the SEIR epidemic model with the replicator dynamics to study the interplay between the underlying epidemic state and the behavioral response of a single population. They showed that as the disease prevalence changes over time, the level of public cooperation varies as well in response, which results in oscillations of infection level.

The dynamics of epidemic processes on networks have been studied extensively, e.g., [14]–[18]. Other studies also considered epidemics with multiple populations [19], [20]; each population represents a group of similar agents, a community or a geographic area (e.g., a city). The interactions among populations are often modeled using a graph, in which an edge weight indicates the contact or interactions rates across different populations. Considering multiple populations complicates the analysis of epidemic models, even without modeling agents' strategic interactions and leads to richer dynamics [19]. The impact of asymptomatic infections over complex networks has also been studied, along with seasonal transmission rate changes, e.g., a high tourist season or the start of a new school year [21].

Kuniya and Muroya [20] studied a multi-group SIS model with population migration and established global convergence to the endemic equilibrium when the basic reproduction number exceeds one, and to the disease free equilibrium otherwise. These studies, however, assume fixed transmission rates that do not depend on the strategies chosen by the agents in different populations.

Organization of the paper: Section II introduces the population game theory, followed by the description of the two-population epidemic population game in Section III. Section IV outlines the problem formulation and the goal of our study. Our proposed policy and its analysis is discussed in Section V. A special case in which one of the two populations serves as a disease reservoir is studied in Section VI. We conclude in Section VII.

II. EVOLUTIONARY DYNAMICS MODELS

We consider two populations, and each agent belongs to only one population. The agents are not permitted to move between the two populations (over the time horizon of interest). However, they interact with agents in both populations at fixed, but possibly different contact rates. These two populations can be viewed, for example, as two disjoint communities to which the agents belong.

Each agent in the l-th population, $l \in \{1, 2\}$, must choose from a finite set of available strategies $\mathbb{A}^{(l)} := \{1, \dots, n_l\}$. Throughout the paper we will assume that $n_l \geq 2$ for both populations except for in Section VI where one of the two populations serves as a disease reservoir and has only one available strategy. Each strategy in $\mathbb{A}^{(l)}$ will have an impact on the transmission rates of the epidemic model, which will be defined shortly. At every time t in $\mathbb{R}_{\geq 0}$, each agent follows one strategy, but can revise it repeatedly on the basis of a payoff vector. The payoff vector for the l-th population is a vector in $\mathbb{R}^{n_l}$, which is defined as

$$p^{(l)}(t) := r^{(l)}(t) - c^{(l)}, \tag{1}$$

where $c^{(l)} = (c_i^{(l)} : i \in \mathbb{A}^{(l)})$ comprises the intrinsic costs of available strategies in $\mathbb{A}^{(l)}$, and $r^{(l)}(t) = (r_i^{(l)}(t) : i \in \mathbb{A}^{(l)})$ is the incentive (or reward) vector that stipulates the rewards provided by a policymaker for each strategy at time t. In other words, $c_i^{(l)}$ is the inherent cost associated with employing the i-th strategy, and $r_i^{(l)}(t)$ is the reward offered for the i-th strategy designed to incentivize agents to adopt safer, yet costlier strategies. Hereafter, we assume that the available strategies are ordered by decreasing intrinsic cost, i.e., $c_1^{(l)} > c_2^{(l)} > \cdots > c_{n_l}^{(l)}$, $l = 1, 2$. Moreover, without loss of generality, we assume $c_{n_l}^{(l)} = 0$, $l = 1, 2$.

The *population state* of the l-th population at time t is denoted by $x^{(l)}(t)$, where $x_i^{(l)}(t)$ indicates the fraction of the l-th population which adopts the i-th strategy in $\mathbb{A}^{(l)}$ at time t. It takes values in the standard simplex $\mathbb{X}^{(l)}$ defined as follows:

$$\mathbb{X}^{(l)} := \left\{ x \in [0,1]^{n_l} \,\middle|\, \textstyle\sum_{j \in \mathbb{A}^{(l)}} x_j = 1 \right\}, \quad l = 1, 2. \tag{2}$$

We define $x(t) := (x^{(1)}(t), x^{(2)}(t))$, which takes values from $\mathbb{X} := \mathbb{X}^{(1)} \times \mathbb{X}^{(2)}$, to be the *social state* consisting of the population states.

Following the approach in [22, Sec. 4.1.2], for each population l, the dynamics of its population state $x^{(l)}$ is governed by the following *evolutionary dynamics model* **(EDM)** in the large-population limit:

$$\dot{x}^{(l)}(t) = \mathcal{V}^{(l)}(x^{(l)}(t), p^{(l)}(t)), \quad t \geq 0, \tag{EDMa}$$

where the i-th component of $\mathcal{V}^{(l)}$ is specified as

$$\mathcal{V}_i^{(l)}(x^{(l)}(t), p^{(l)}(t)) := \underbrace{\textstyle\sum_{j=1, j \neq i}^{n_l} \; x_j^{(l)}(t) \mathcal{T}_{ji}^{(l)}(x^{(l)}(t), p^{(l)}(t))}_{\text{inflow switching to strategy } i}$$
$$- \underbrace{\textstyle\sum_{j=1, j \neq i}^{n_l} \; x_i^{(l)}(t) \mathcal{T}_{ij}^{(l)}(x^{(l)}(t), p^{(l)}(t))}_{\text{outflow switching away from strategy } i}. \tag{EDMb}$$

A Lipschitz continuous map $\mathcal{T}^{(l)} : \mathbb{X}^{(l)} \times \mathbb{R}^{n_l} \to [0, \overline{\mathcal{T}}]^{n_l \times n_l}$, with upper bound $\overline{\mathcal{T}}^{(l)} > 0$, is referred to as the *revision protocol* for the l-th population and models the agents' strategy revision preferences. In [22, Part II] and [23, Sec. 13.3-13.5] there is a comprehensive discussion on protocols types and the classes of bounded rationality rules they model. The use of (EDM) as a deterministic approximation when a dynamic payoff mechanism generates the payoff vector $p(t) := (p^{(1)}(t), p^{(2)}(t))$, as it is the case in our study, is established in [24, Sec. IV].

III. Two-Population Epidemic Population Game

We adopt a well-known epidemic model, namely susceptible-infected-susceptible (SIS) model, to approximate the transmission of a disease in the two populations: at any time t each agent is at either the 'susceptible' or 'infected' state. To facilitate our analysis, we employ the following normalized SIS model: $I(t) := (I^{(1)}(t), I^{(2)}(t))$ and $S(t) := (S^{(1)}(t), S^{(2)}(t))$, where $I^{(l)}(t)$ and $S^{(l)}(t)$ are the fraction of the l-th population that are infected and susceptible, respectively, at time t. Note that since $S^{(l)}(t) = 1 - I^{(l)}(t)$ for both $l = 1, 2$, it suffices to model either $I(t)$ or $S(t)$.

Our earlier work in [2] and [3] adopted the SIRS model for a single population. Employing the SIRS model for our problem with two populations would lead to a similar analysis with additional technical complications, especially when the disease-related death rate is non-negligible. For this reason, we chose to use the SIS model instead. This simplifies our analysis, while still providing insights into the studied problem.

Suppose that θ, ζ, γ and δ denote the birth rate, natural death rate, recovery rate, and disease-related death rate, respectively. We assume that these parameters are non-negative and common to both populations. The population size vector $N(t) := (N^{(1)}(t), N^{(2)}(t))$, where $N^{(l)}(t)$ approximates the l-th population's cardinality at time t, is obtained as the solution of $\dot{N} = (\theta - \zeta - \delta I) \circ N$, where $\circ$ denotes the Hadamard product, i.e., the element-wise multiplication of two vectors. We assume that $N^{(l)}(0)$, $l = 1, 2$, are large and the population sizes do not change much within the time interval of interest. Below we describe the *epidemic population game*:

$$\begin{aligned} \dot{I}(t) &= -(\theta + \gamma) I(t) + S(t) \circ (B(t) I(t)) \\ &\quad - \delta I(t) \circ S(t) && \text{(EPGa)} \\ \dot{q}^{(l)}(t) &= G^{(l)}(I(t), x(t), q(t)), && \text{(EPGb)} \\ r^{(l)}(t) &= H^{(l)}(I(t), x(t), q(t)), && \text{(EPGc)} \end{aligned}$$

Here, $B(t)$ is a 2×2 matrix whose element $B_{l,l'}(t)$ denotes the rate at which susceptible agents in population l contract the disease from (contacts with) infected agents in population l' at time t. This is described in Fig. 1. Equations (EPGb,c), where G and H are Lipschitz continuous functions, describe a dynamic payoff mechanism that we aim to design, where $r(t) := (r^{(1)}(t), r^{(2)}(t))$ is the reward vector in (1), and $q(t) := (q^{(1)}(t), q^{(2)}(t)) \in \mathbb{R}^m$, $m \geq 2$. In our design, $q^{(l)}(t)$, $l = 1, 2$, are scalars and $m = 2$.

In our model, the matrix $B(t)$ depends on the social state $x(t)$ and thus is time-varying in general:

$$B(t) \equiv \bar{B}(x(t)) = \mathrm{diag}\left(f(x(t))\right) \Theta, \tag{5}$$

where $f(x) = \left(f^{(1)}(x^{(1)}), f^{(2)}(x^{(2)})\right)$ with $f^{(l)}(x^{(l)}) = \beta^{(l)\prime} x^{(l)}$, $l = 1, 2$. The vector $\beta^{(l)} = (\beta_i^{(l)} : i \in \mathbb{A}^{(l)})$, $l = 1, 2$, specifies the disease transmission rates for susceptible agents adopting various strategies available for the l-th population, which are assumed strictly positive. Since the available strategies are ordered by decreasing intrinsic cost, we assume $\beta_1^{(l)} < \beta_2^{(l)} < \cdots < \beta_{n_l}^{(l)}$, $l = 1, 2$. This is consistent with the expectation that costlier strategies are more effective at preventing the transmission of disease. Finally, Θ is a 2×2 positive matrix and models the contact rates among the agents within each population and across the two populations, and also captures the relative sizes of the two populations. Since we assume that the population sizes do not change much during the time interval of interest, we consider Θ to be constant. We note that, since $\bar{B}(x(t))$ is strictly positive, it is also irreducible.

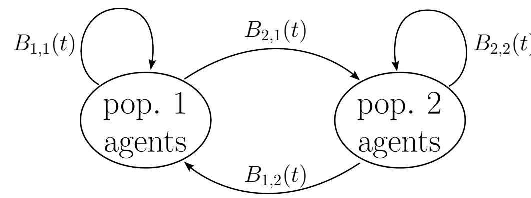

Fig. 1: Disease transmission rates described by $B(t)$.

We are interested in scenarios where it is too costly to eradicate the disease from the populations and the disease outbreak becomes endemic. As we will show shortly, the following assumption ensures the existence of an endemic equilibrium of (EPGa) for any x in $\mathbb{X}$.

Assumption 1: $0 < \delta + \theta + \gamma < \min\{\beta_1^{(1)} \Theta_{11}, \beta_1^{(2)} \Theta_{22}\}$.

IV. Problem Formulation

We seek to minimize the disease transmission rate at an endemic state by providing suitable incentives for available strategies via a dynamic payoff mechanism. In order to formulate our problem, we make use of the observation that the spectral radius of the transmission rate matrix corresponds to the basic reproduction number of the epidemic model in (EPGa) [20].

We formulate our problem as a constrained optimization problem in which the objective function is the spectral radius of the transmission rate matrix (or the basic reproduction number) subject to a long-term budget constraint on rewards offered to offset cost differentials:

$$\begin{aligned} \text{minimize}_{x \in \mathbb{X}} \quad & \lambda_{\max}\left(\bar{B}(x)\right) \\ \text{subject to} \quad & \textstyle\sum_{l=1}^{2} c^{(l)\prime} x^{(l)} \leq c^* \end{aligned} \tag{6}$$

where $c^* > 0$ is a budget. We denote an optimal point of this optimization problem by x^* with $B^* := \mathrm{diag}\left(f(x^*)\right) \Theta$.

The dynamic payoff mechanism in (EPGb,c) should ensure the convergence of the social state to an optimal point x^* where the disease transmission is minimized subject to the budget constraint. To this end, we introduce the following assumption on the intrinsic costs, which can be viewed as the law of diminishing returns.

Assumption 2: For each population l, if $n_l \geq 3$, the following holds:

$$\frac{c_i^{(l)} - c_{i+1}^{(l)}}{\beta_{i+1}^{(l)} - \beta_i^{(l)}} > \frac{c_{i+1}^{(l)} - c_{i+2}^{(l)}}{\beta_{i+2}^{(l)} - \beta_{i+1}^{(l)}}, \qquad 1 \leq i \leq n_l - 2 \tag{7}$$

Assumption 2 means that as the agents attempt to curtail the risk of contracting the disease by adopting costlier strategies, it gets more expensive to reduce the risk further.

The optimization problem (6) is non-convex and cannot be efficiently solved even when the number of available strategies for each population is modest. In order to devise a computationally efficient method for approximately solving (6), we take advantage of the following observation: for each x in $\mathbb{X}$, define $h(x) = \sum_{l=1}^{2} f^{(l)}(x^{(l)})$.

Lemma 1: If x^* in $\mathbb{X}$ solves (6), it also solves the following optimization problem:

$$\begin{aligned} \text{minimize}_{x\in\mathbb{X}} \quad & h(x) \\ \text{subject to} \quad & c^{(l)'} x^{(l)} \leq \eta^{(l)},\ l = 1, 2 \end{aligned} \tag{8}$$

where $\eta^{(1)} = c^{(1)'} x^{*(1)}$ and $\eta^{(2)} = c^* - \eta^{(1)}$.

Note that the optimization problem (8) has a separable objective function with independent constraints for the two populations and can be solved separately for each population. This observation suggests the following efficient approach to solving (6) : for each $\eta \in [0, c^*]$, let $\bar{h}(\eta)$ be the optimal value of (8) with budget constraints $\eta^{(1)} = \eta$ and $\eta^{(2)} = c^* - \eta$ for populations 1 and 2, respectively. For fixed $\eta \in [0, c^*]$, the solution to (8) for each population can be found efficiently from [2, Remark 3]: fix $\eta \in [0, c^*]$ and denote the optimal point of (8) by $\bar{x}(\eta)$. **(Case 1):** If $\eta < c_1^{(1)}$, there is $i^* \in \{1, \ldots, n_1 - 1\}$ such that $c_{i^*+1}^{(1)} < \eta \leq c_{i^*}^{(1)}$ and we have $\bar{x}_{i^*}^{(1)}(\eta) = (\eta - c_{i^*+1}^{(1)})/(c_{i^*}^{(1)} - c_{i^*+1}^{(1)})$, $\bar{x}_{i^*+1}^{(1)}(\eta) = 1 - \bar{x}_{i^*}^{(1)}(\eta)$ and $\bar{x}_i^{(1)}(\eta) = 0$ for $i \notin \{i^*, i^*+1\}$. **(Case 2):** If $\eta \geq c_1^{(1)}$, then $\bar{x}_1^{(1)}(\eta) = 1$ and $\bar{x}_i^{(1)}(\eta) = 0$ for all $i > 1$. The optimal point for population 2 can be found analogously.

An optimal point of (6) can now be obtained by solving the following equivalent problem:

$$\text{minimize}_{\eta\in[0,c^*]} \quad \lambda_{\max}\big(\bar{B}(\bar{x}(\eta))\big) \tag{9}$$

Although an exact solution may be difficult to obtain, an approximate solution can be found via a line search. Since the objective function of (6) is continuous, a small perturbation of the optimization variables causes a minor change in the spectral radius.

A. Lyapunov Stability with a Fixed Social State

Before we consider a general setting with time-varying social state, we first establish the existence of a unique endemic equilibrium of (EPGa) for each fixed social state x in $\mathbb{X}$. Note from (5) that when the social state x is fixed, so is the transmission rate matrix $\bar{B}(x)$.

Lemma 2: Suppose that Assumption 1 holds. For each fixed social state x in $\mathbb{X}$, there is a non-empty set $\mathbb{I}_x \subset (0, 1]^2$ such that every I_x in $\mathbb{I}_x$ satisfies

$$-(\theta + \gamma) I_x + S_x \circ \big(\bar{B}(x) I_x\big) - \delta I_x \circ S_x = \mathbf{0}, \tag{10}$$

where $S_x = \mathbf{1} - I_x$, and $\mathbf{1}$ and $\mathbf{0}$ are vectors of 1's and 0's, respectively, of appropriate dimension.

The proof of the lemma follows steps similar to those used in [25, Sec. 2.2] to establish the existence of a strongly endemic equilibrium and is omitted here.

Lemma 2 states that, for fixed social state x in $\mathbb{X}$, there is at least one endemic equilibrium of (EPGa). We will now show that, for fixed social state x, $I(t)$ converges to a *unique* equilibrium in $(0, 1]^2$, which we call the *endemic state* and denote by $\bar{I}_x$. Let $I_*(x)$ be an element of $\mathbb{I}_x$, thus an equilibrium of (EPGa) for fixed x in $\mathbb{X}$. Define a function $g : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$ with $g(z) := z - 1 - \ln(z), z > 0$. Note that $g(z) \geq 0$ for $z > 0$, and $g(z) = 0$ only for $z = 1$. Consider the following function, which is a modification of the Lyapunov function used in [20].

$$\mathcal{U}_x(I) := \textstyle\sum_{l=1}^{2} w_l(x) I_*^{(l)}(x) g\big(I^{(l)}/I_*^{(l)}(x)\big) \tag{11}$$

where $w_1(x) := \tilde{B}_{2,1}(x)$, $w_2(x) := \tilde{B}_{1,2}(x)$, and $\tilde{B}_{l,l'}(x) := S_*^{(l)}(x) \bar{B}_{l,l'}(x) I_*^{(l')}(x)$.

Lemma 3: For (EPGa) with a fixed social state x in $\mathbb{X}$, $\mathcal{U}_x(I)$ is a Lyapunov function.

Proof: Note from (5) that $w_1(x)$ and $w_2(x)$ are positive. Since $g\big(I^{(l)}/I_*^{(l)}(x)\big) = 0$ if and only if $I^{(l)} = I_*^{(l)}(x)$, we have $\mathcal{U}_x(I) > 0$ for all $I \in (0, 1]^2 \setminus \{I_*(x)\}$. To simplify our notation, we do not indicate the dependence of $\bar{B}, S_*, I_*$ and w_l, $l = 1, 2$, on the social state x below.

The derivative of $\mathcal{U}(I(t))$ along trajectories is

$$\begin{aligned} \dot{\mathcal{U}}_x(I(t)) = & -\sum_{l=1}^{2} w_l S_*^{(l)} \sum_{l'=1}^{2} \bar{B}_{l,l'} I_*^{(l')} g\left(\frac{I^{(l')}(t) I_*^{(l)}}{I^{(l)}(t) I_*^{(l')}}\right) \\ & + \sum_{l=1}^{2} w_l \left(\delta - \sum_{l'=1}^{2} \bar{B}_{l,l'} \frac{I^{(l')}(t)}{I^{(l)}(t)}\right) (I_*^{(l)} - I^{(l)}(t))^2. \end{aligned} \tag{12}$$

By Assumption 1, $\delta < \min_l \bar{B}_{l,l}$, and $\dot{\mathcal{U}}_x(I(t)) < 0$ for all $I(t) \in (0, 1]^2 \setminus \{I_*(x)\}$. Hence, $\mathcal{U}_x$ is a Lyapunov function, and $\bar{I}_x = I_*(x)$ is the *unique* element of $\mathbb{I}_x$. ∎

The following lemma tells us that $\bar{I}_x$ is continuously differentiable in social state x. Its proof can be obtained using Lemma 3 and the implicit function theorem.

Lemma 4: The unique endemic state $\bar{I}_x$ is continuously differentiable in x. Also, from (10) for all x in $\mathbb{X}$ we have $\bar{I}_x \geq \underline{I} := \bar{I}_{((1,0,\ldots,0),(1,0,\ldots,0))} > \mathbf{0}$. This follows from the fact that $\bar{I}_x$ is non-decreasing in each $f^{(l)}(x^{(l)})$.

V. Lyapunov Function and Determining a Stabilizing Policy

A. Nash Stationarity and δ-Passivity Assumption

The following two assumptions on (EDM) will be crucial to our analysis. The first assumption guarantees that the population state of a population remains constant if it is a best response to its current payoff vector, while the second assumption will be useful when analyzing the long-term behavior of $(I, x, p)(t)$ and originates from the δ-passivity concept originally proposed in [26] and generalized in [24], [27], [28].

Assumption 3: **(Nash Stationarity)** We assume that $\mathcal{T}^{(l)}$, $l = 1, 2$, is "Nash stationary", i.e., it satisfies the following:

$$\mathcal{V}^{(l)}(x^{(l)}, p^{(l)}) = \mathbf{0} \ \Leftrightarrow\ x^{(l)} \in \mathscr{M}^{(l)}(p^{(l)}),\ p^{(l)} \in \mathbb{R}^{n_l} \tag{NS}$$

where $\mathscr{M}^{(l)}:\mathbb{R}^{n_l}\to 2^{\mathbb{X}^{(l)}}$ is the following best response map:

$$\mathscr{M}^{(l)}(p^{(l)}):=\underset{x^{(l)}\in\mathbb{X}^{(l)}}{\arg\max}\ p^{(l)\prime}x^{(l)},\quad p^{(l)}\in\mathbb{R}^{n_l}.$$

For a revision protocol that satisfies (NS), a social state x is an equilibrium of (EDM) if and only $x^{(l)}$ is a best response to $p^{(l)}$ for both populations. Any impartial pairwise comparison (IPC) protocol (see [29]) satisfies (NS), as do other large classes of protocols (see [23, Sec. 13.5.3] [24, Sec. 13.5.3]). For instance, the so-called Smith's protocol is an example of IPC revision protocol [30].

Assumption 4: For each population l, $l=1,2$, there exist (i) a differentiable function $\mathcal{S}^{(l)}:\mathbb{X}^{(l)}\times\mathbb{R}^{n_l}\to\mathbb{R}_{\geq 0}$ and (ii) a Lipschitz continuous function $\mathcal{P}^{(l)}:\mathbb{X}^{(l)}\times\mathbb{R}^{n_l}\to\mathbb{R}_{\geq 0}$, which satisfy the following inequality for all $x^{(l)}$, $p^{(l)}$ and $u^{(l)}$ in $\mathbb{X}^{(l)}$, $\mathbb{R}^{n_l}$ and $\mathbb{R}^{n_l}$, respectively:

$$\frac{\partial\mathcal{S}^{(l)}(x^{(l)},p^{(l)})}{\partial x^{(l)}}\mathcal{V}^{(l)}(x^{(l)},p^{(l)})+\frac{\partial\mathcal{S}^{(l)}(x^{(l)},p^{(l)})}{\partial p^{(l)}}u^{(l)} \leq -\mathcal{P}^{(l)}(x^{(l)},p^{(l)})+u^{(l)\prime}\mathcal{V}^{(l)}(x^{(l)},p^{(l)}) \quad \text{(13a)}$$

where $\mathcal{S}^{(l)}$ and $\mathcal{P}^{(l)}$ must also satisfy the equivalences below:

$$\mathcal{S}^{(l)}(x^{(l)},p^{(l)})=0 \quad\Leftrightarrow\quad \mathcal{V}^{(l)}(x^{(l)},p^{(l)})=\mathbf{0} \qquad \text{(13b)}$$

$$\mathcal{P}^{(l)}(x^{(l)},p^{(l)})=0 \quad\Leftrightarrow\quad \mathcal{V}^{(l)}(x^{(l)},p^{(l)})=\mathbf{0}. \qquad \text{(13c)}$$

In addition, the following inequality (not required in standard δ-passivity) must hold for all $x^{(l)}$ in $\mathbb{X}^{(l)}$ and $p^{(l)}$ in $\mathbb{R}^{n_l}$:

$$\mathcal{P}^{(l)}(x^{(l)},\alpha p^{(l)})\geq\mathcal{P}^{(l)}(x^{(l)},p^{(l)}),\quad \alpha\geq 1. \qquad \text{(13d)}$$

B. Specifying G, and H, and Lyapunov Function

Throughout this subsection, we fix an optimal point x^* to (6) and, to simplify the notation, omit the dependence of (i) $I,p,x,w,\bar{I}_x$ and $\bar{S}_x$ on t and (ii) w_1, w_2 and $\bar{B}$ on $x(t)$, and indicate the dependence only when necessary.

Define the function

$$\mathscr{S}(I(t),x(t))=\mathcal{U}_{x(t)}(I(t))+\upsilon\|\bar{B}(x(t))-B^*\|_F^2, \qquad (14)$$

where $B^*:=\bar{B}(x^*)$, $\mathcal{U}_{x(t)}(I(t))$ is defined in (11), υ is a design parameter, and $\|\cdot\|_F$ denotes the Frobenius norm. The derivative of $\mathscr{S}(I(t),x(t))$ along the trajectories is given by

$$\dot{\mathscr{S}}(I,x)=\frac{\partial\mathcal{U}_x(I)}{\partial I}\dot{I}+\sum_{l=1}^{2}\frac{\partial\mathscr{S}(I,x)}{\partial f^{(l)}(x^{(l)})}\beta^{(l)\prime}\dot{x}^{(l)}. \qquad (15)$$

Note that the first term cannot be positive from (12). Moreover, recall from (5) that the dynamics of $I(t)$ depends on the social state $x(t)$ only through $f(x(t))$. Thus, the second term in (15) captures $(\partial\mathscr{S}(I,x)/\partial x)\dot{x}$.

We can now define the dynamic payoff mechanism used in (EPGb,c). As in [2], we can choose for each population $l=1,2$ the following dynamic payoff mechanism

$$G^{(l)}(I,x,q)=-\frac{\partial\mathscr{S}(I,x)}{\partial f^{(l)}(x^{(l)})}, \qquad (16)$$

$$H^{(l)}(I,x,q)=q^{(l)}(t)\beta^{(l)}+\check{r}^{(l)}+c^{(l)}, \qquad (17)$$

where $\check{r}^{(l)}=(\check{r}_i^{(l)}:i\in\mathbb{A}^{(l)})$ with

$$\check{r}_i^{(l)}=\begin{cases}0 & \text{if } x_i^{*(l)}>0,\\ -\rho & \text{if } x_i^{*(l)}=0,\end{cases} \qquad (18)$$

and ρ is a design parameter. This leads to $p^{(l)}(t)=q^{(l)}(t)\beta^{(l)}+\check{r}^{(l)}$, $l=1,2$.

Let us define

$$\mathcal{L}(I,x,p)=\mathscr{S}(I,x)+\sum_{l=1}^{2}\mathcal{S}^{(l)}(x^{(l)},p^{(l)}), \qquad (19)$$

which is positive-definite by Assumption 4 and (16). From (12), (13a), (15), (16) and (17), we get

$$\begin{aligned}\dot{\mathcal{L}}(I,x,p)\leq&-\mathcal{P}^{(1)}(x^{(1)},p^{(1)})-\mathcal{P}^{(2)}(x^{(2)},p^{(2)})\\&-\sum_{l=1}^{2}w_l\bar{S}_x^{(l)}\sum_{l'=1}^{2}\bar{B}_{l,l'}\bar{I}_x^{(l')}g\left(\frac{I^{(l')}\bar{I}_x^{(l)}}{I^{(l)}\bar{I}_x^{(l')}}\right)\\&+\sum_{l=1}^{2}w_l\left(\delta-\sum_{l'=1}^{2}\bar{B}_{l,l'}\frac{I^{(l')}}{I^{(l)}}\right)(\bar{I}_x^{(l)}-I^{(l)})^2. \qquad (20)\end{aligned}$$

Recall that $\bar{I}_x$ denotes the unique endemic state associated with social state x and $\bar{S}_x^{(l)}=1-\bar{I}_x^{(l)}$, $l=1,2$.

Substitute (11) and (14) in (16) to obtain

$$\begin{aligned}G^{(l)}(I,x,q)=&-\sum_{l'=1}^{2}\Bigg(\left(\partial_{f^{(l)}}w_{l'}\right)\bar{I}_x^{(l')}g\left(\frac{I^{(l')}}{\bar{I}_x^{(l')}}\right)\\&+w_{l'}\left(\partial_{f^{(l)}}\bar{I}_x^{(l')}\right)g\left(\frac{I^{(l')}}{\bar{I}_x^{(l')}}\right)+w_{l'}\bar{I}_x^{(l')}\left(\partial_{f^{(l)}}g\left(\frac{I^{(l')}}{\bar{I}_x^{(l')}}\right)\right)\Bigg)\\&-2\upsilon f^{(l)}(x^{(l)}-x^{*(l)})(\Theta\Theta')_{l,l}, \qquad (21)\end{aligned}$$

where $\partial_{f^{(l)}}$ denotes the partial derivative with respect to $f^{(l)}(x^{(l)})$. The derivatives $\partial_{f^{(l)}}\bar{I}_x$ and $\partial_{f^{(l)}}w_{l'}$, $l,l'\in\{1,2\}$, can be obtained by solving the following system of linear equations:

$$\begin{aligned}\partial_{f^{(l)}}\bar{I}_x&=-J^{-1}\operatorname{diag}\left(\bar{S}_x\right)\operatorname{diag}\left(\partial_{f^{(l)}}f(x)\right)\Theta\bar{I}_x,\\ J&=(\theta+\gamma)\left(\operatorname{diag}\left(\bar{S}_x\right)D-\operatorname{diag}\left(D\bar{I}_x\right)-\mathbf{I}_2\right),\\ D&=\frac{\bar{B}(x)-\delta\mathbf{I}_2}{\theta+\gamma},\\ \partial_{f^{(l)}}w_1&=\bar{B}_{2,1}(x)\left(\bar{S}_x^{(2)}(\partial_{f^{(l)}}\bar{I}_x^{(1)})-(\partial_{f^{(l)}}\bar{I}_x^{(2)})\bar{I}_x^{(1)}\right)\\&\quad+\bar{S}_x^{(2)}\bar{I}_x^{(1)}(\partial_{f^{(l)}}\bar{B}_{2,1}(x)),\quad\text{and}\\ \partial_{f^{(l)}}w_2&=\bar{B}_{1,2}(x)\left(\bar{S}_x^{(1)}(\partial_{f^{(l)}}\bar{I}_x^{(2)})-(\partial_{f^{(l)}}\bar{I}_x^{(1)})\bar{I}_x^{(2)}\right)\\&\quad+\bar{S}_x^{(1)}\bar{I}_x^{(2)}(\partial_{f^{(l)}}\bar{B}_{1,2}(x)).\end{aligned}$$

C. Stability Notion and Main Result

Suppose that we adopt a dynamic payoff mechanism given by (17) and (21) for the fixed optimal point x^*. Let $I^*=\bar{I}_{x^*}$.

Theorem 1: Let the protocol defining (EDM) and the design parameters $\upsilon>0$, $\rho>0$ and $c^*>0$ be given such that, for both populations $l=1,2$, we have $c_1^{(l)}>c^{(l)\prime}x^{*(l)}>0$. If (NS) and Assumption 1-4 hold, then for G given by (21) the set $\mathbb{E}^*:=(I^*,x^*)\times\mathbb{Q}$ is globally asymptotically stable,

$$(I,x,q)(t)\xrightarrow[t\to\infty]{}(I^*,x^*)\times\mathbb{Q},$$

where $\mathbb{Q} := \mathbb{Q}^{(1)} \times \mathbb{Q}^{(2)}$ and, for each $l = 1, 2$, (a) if $c^{(l)\prime} x^{*(l)} \notin \{c_1^{(l)}, \ldots, c_{n_l}^{(l)}\}$, $\mathbb{Q}^{(l)} = \{0\}$ and (b) otherwise,

$$\mathbb{Q}^{(l)} = [-\rho(\beta_{n_l}^{(l)} - f^{(l)}(x^{*(l)}))^{-1}, \rho(f^{(l)}(x^{*(l)}) - \beta_1^{(l)})^{-1}].$$

Lastly, if $\mathbb{Q} = \{\mathbf{0}\}$, then $\lim_{t\to\infty} \sum_{l=1}^{2} r^{(l)}(t)' x^{(l)}(t) = c^*$.

Sketch of a proof for Theorem 1: To prove the theorem we first remark that $\mathcal{L}(I, x, q) \to \infty$ as any component of I approaches zero. Let $\mathcal{Y}(0) := (I, x, q)(0)$ be an element of

$$\mathbb{Y} := \{(I, x, q) \mid I \in (0, 1]^2,\ x \in \mathbb{X}, q \in \mathbb{R}^2\}.$$

For any $\mathcal{Y}(0) \in \mathbb{Y}$, from (20) we have $\mathcal{L}(\mathcal{Y}(t)) < \infty$ for all $t \geq 0$. Then, we can apply the steps used in [2, Appen. A.2-A.3] to each population to obtain that $q(t)$ remains bounded for all $t \geq 0$, and that there exist some positive ϵ and finite $\bar{t} \geq 0$ such that, for all $t \geq \bar{t}$, $q(t)$ lies in $\mathbb{Q}_\epsilon := \{q \mid \min_{q' \in \mathbb{Q}} \|q - q'\| \leq \epsilon\}$. Without loss of generality we can assume $\bar{t} = 0$ and study the stability of $\mathcal{Y}(t)$ that belongs to the bounded set

$$\mathbb{Y}_\epsilon := \{(I, x, q) \mid I \in (0, 1]^2,\ x \in \mathbb{X}, q \in \mathbb{Q}_\epsilon\},$$

for all $t \geq 0$.

By [31, Lemma 4.1] we have that for any trajectory, as $t \to \infty$, $\mathcal{Y}(t)$ will approach the positive limit set L^+, which is a nonempty, compact, invariant set contained in the closure of $\mathbb{Y}_\epsilon$. Since $\mathcal{L}(\mathcal{Y}(t))$ is continuous and non-increasing in t, it is bounded from below and has a limit as $t \to \infty$. Because the closure of $\mathbb{Y}_\epsilon$ is compact, we can find an increasing sequence $\{t_k : k \in \mathbb{N}\}$ such that, as $k \to \infty$, (a) $t_k \to \infty$ and (b) $\mathcal{Y}(t_k) \to y^+$ for some $y^+ = (I^+, x^+, q^+)$ in L^+. Note that $I^+ \geq \underline{I} > \mathbf{0}$ and, thus, y^+ lies in $\mathbb{Y}_\epsilon$ and $L^+ \subset \mathbb{Y}_\epsilon$.

By the continuity of $\mathcal{L}$ and $\dot{\mathcal{L}}(\mathcal{Y}(t)) \leq 0$ for any $\mathcal{Y}(t) \in \mathbb{Y}_\epsilon$ from (12) and (20), we have $\mathcal{L}(\mathcal{Y}(t)) \to \mathcal{L}(y^+)$ and $\dot{\mathcal{L}}(\mathcal{Y}(t)) \to 0$ as $t \to \infty$. Note that $\dot{\mathcal{L}}(y) = 0$ for any $y \in L^+$. Define M to be the largest invariant subset such that $\dot{\mathcal{L}}(y) = 0$ for all $y \in M$. Then, for any $\mathcal{Y}(0)$ in M,

$$I(t) = \bar{I}_{x(t)} \tag{22a}$$

$$x^{(l)}(t) \in \mathscr{M}(q^{(l)}(t)\beta^{(l)} + \check{r}^{(l)}),\ l = 1, 2. \tag{22b}$$

For trajectories starting in the invariant set M, by (NS) and (22b), we obtain that $\dot{x}(t) = 0$ and $x(t) = x(0)$ for all $t \geq 0$. This in turn means from (21) that $\dot{q}(t)$ is constant, as G depends only on $x(t)$ and $I(t)$, which are constant.

Suppose that $\dot{q}^{(l)}(t) \neq 0$ for some $l \in \{1, 2\}$. This implies that $q(t)$ will leave $\mathbb{Q}_\epsilon$ after a finite amount of time, which is a contradiction. Thus, $\dot{q}(t) = 0$ and $q(t) = q(0)$ for all $t \geq 0$. Since $q(t) = q(0)$ for all $t \geq 0$, using an argument similar to that used in [2, Appen. A.1 (Cases I & II)], we can conclude that $q(t)$ lies in $\mathbb{Q}$. This observation, in conjunction with (18) and (22b), tells us $x(0) = x(t) = x^*$. Because (a) $\mathcal{Y}(t)$ converges to the limit set $L^+ \subset M$ and (b) any y in M satisfies $I(0) = I^*$, $x(0) = x^*$ and $q(0) \in \mathbb{Q}$ and hence belongs to $\mathbb{E}^*$, we have $L^+ \subset \mathbb{E}^*$.

If both populations satisfy $c^{(l)\prime} x^{*(l)} \neq c_i^{(l)}$ for all i in $\mathbb{A}^{(l)}$, then we have $\mathbb{Q} = \{\mathbf{0}\}$ and the long-term budget constraint is satisfied from (17) and (18), i.e., $\lim_{t\to\infty} \sum_{l=1}^{2} r^{(l)}(t)' x^{(l)}(t) = c^*$. ■

Remark 1: It can be shown that $\mathcal{L}(I, x, p) = 0$ if and only if $I = \bar{I}_x$, each $x^{(l)} \in \mathscr{M}(q^{(l)}(t)\beta^{(l)} + \check{r}^{(l)})$ and $\bar{B}(x) = B^*$. This implies that $\mathcal{L}(I, x, p(q)) = 0$ if and only if $I = \bar{I}^*$, $x = x^*$ and $q \in \mathbb{Q}$. Thus, when $\mathbb{Q} = \{\mathbf{0}\}$, the system is Lyapunov stable.

Remark 2: Suppose that the optimal solution x^* of (6) is such that $x_{i^*}^{*(l)} = 1$ for some population l and a strategy i^* in $\mathbb{A}^{(l)}$, i.e, all agents of population l adopt strategy i^* at the optimal point. As pointed out earlier, since the objective function of (6) is continuous, minor perturbations to x^* will only have a small effect on the spectral radius. Thus, we can find a perturbation $\tilde{x}$ close to the optimal point x^*, which is a feasible point satisfying all constraints, so that, for both $l = 1, 2$, we have $\tilde{x}_{i_l}^{(l)}, \tilde{x}_{i_l+1}^{(l)} > 0$ for some $1 \leq i_l < n_l$ and $\tilde{x}_j^{(l)} = 0$ for $j \notin \{i_l, i_l + 1\}$. This will ensure $\lambda_{\max}(B(\tilde{x})) \approx \lambda_{\max}(B(x^*))$ and, since $\mathbb{Q} = \{\mathbf{0}\}$, the theorem guarantees the global asymptotic stability to the unique equilibrium.

D. Numerical Example

Example 1: We consider two populations with parameters $\theta = 0.0002$, $\delta = 0.0005$ and $\gamma = 0.14$ (mean recovery period ~ 7 days). Each population has access to two strategies: $\beta^{(1)} = \beta^{(2)} = (0.15, 0.19)$ and costs $c^{(1)} = (0.35, 0)$, and $c^{(2)} = (0.4, 0)$. The contact rate matrix is $\Theta = \begin{bmatrix} 1 & 0.3 \\ 0.1 & 1 \end{bmatrix}$, and the available budget is $c^* = 0.3$. This yields an optimal point of (9) equal to $x^{*(1)} \approx (46.4\%, 53.5\%)$ and $x^{*(2)} \approx (34.4\%, 65.6\%)$, and the corresponding endemic equilibrium is $I^* \approx (34.4\%, 28.8\%)$. We assume that each population was using its costlier strategy at $t = 0$ and $I(0) = (0.1\%, 0.11\%)$. By using the dynamic payoff, (EPGb) and (EPGc), we obtain that for any $\rho > 0$, the state converges to (I^*, x^*). For the simulation[1], we selected the parameter value $\upsilon = 4$.

From Figure 2 we observe that for the first 300 days both populations use the costlier strategy with a lower transmission rate. After the 300-day mark the social state approaches x^* and we observe a second hike in the infection level, as the social state converges $q(t)$ also approaches zero, as Theorem 1 indicates. Throughout the simulation both populations maintain a similar level of infection.

VI. Viral Reservoir Case

One interesting application of our model and results is the case in which only one population can be incentivized while the other population comprises nonstrategic agents that follow a single fixed strategy, which serves as a disease reservoir, e.g., animal disease reservoir. Without loss of generality we assume that the agents in the first population have more than one available strategies and are targeted with incentives, and the agents from the second population stay with a fixed strategy.

[1]The code used to generate Figs. 2 and 3 and an interactive simulation tool based on Example 1 can be found at github.com/jcert/E2PG/tree/v1.0.

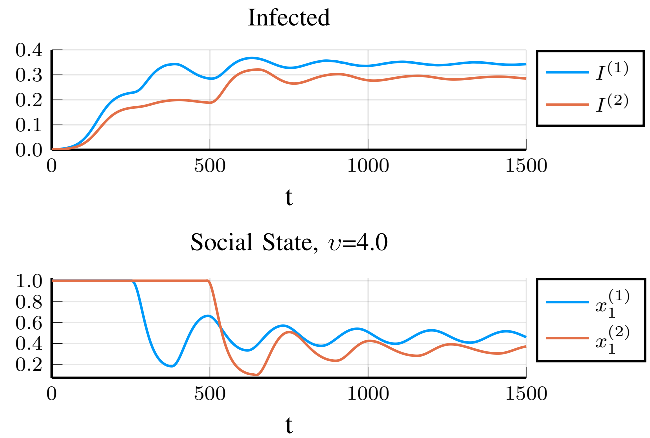

Fig. 2: Simulation for Example 1 using υ as shown, and a Smith's protocol specified by $\lambda = 0.1$ and $\bar{\mathcal{T}} = 0.1$.

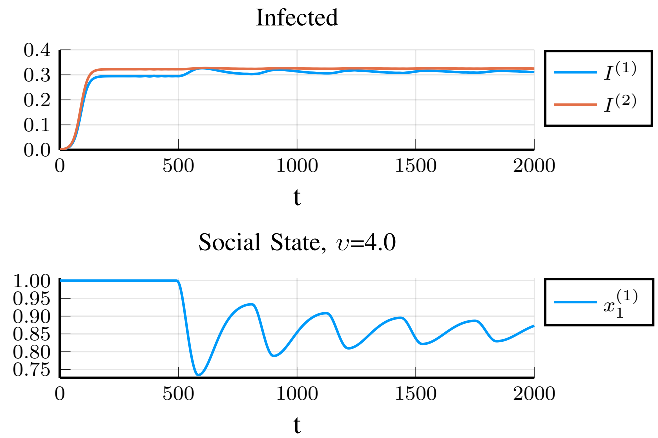

Fig. 3: Simulation for Example 2 using υ as shown, and a Smith's protocol specified by $\lambda = 0.1$ and $\bar{\mathcal{T}} = 0.1$.

Even though the agents from the second population cannot revise their strategies, modeling them is still important because of the interaction between the two populations: the infection rates seen by the agents of the first population depend on the fraction of infected agents in the second population, and vice versa. This special case of the two-population model is degenerate in that $q^{(2)}$ is not needed as no incentive is needed for the second population and can be dropped from model. Moreover, in this case the optimization problem in (6) for finding an optimal state x^* can be solved exactly, since it equates to solving (8) with $\eta^{(1)} = c^*$ and $\eta^{(2)} = 0$. The convergence of $(I, x, q^{(1)})(t)$ to $(I^*, x^*, q^{(1)*})$ in this case can be proved similarly to Theorem 1.

Corollary 1: Let the protocol defining (EDM) and the design parameters $\upsilon > 0$, $\rho > 0$ and $c^* > 0$ be given, and assume $c^* < c_1^{(1)}$. If (NS) and Assumption 1-4 hold, then for $G^{(1)}$given by (21) the set $\mathbb{E}^* := (I^*, x^*) \times \mathbb{Q}$ is globally asymptotically stable[2] and $(I, x, q^{(1)})(t) \xrightarrow[t\to\infty]{} (I^*, x^*) \times \mathbb{Q}$, where (a) if $c^* \notin \{c_1^{(1)}, \ldots, c_{n_1}^{(1)}\}$, then $\mathbb{Q} := \{0\}$ and (b) otherwise, we have

$$\mathbb{Q} = [-\rho(\beta_{n_1}^{(1)} - f^{(1)}(x^{*(1)}))^{-1}, \rho(f^{(1)}(x^{*(1)}) - \beta_1^{(1)})^{-1}].$$

Moreover, $\lim_{t\to\infty} r^{(1)}(t)' x^{(1)}(t) = c^*$ when $\mathbb{Q} = \{0\}$.

Example 2: For the numerical example, we use the same setup, parameter values and initial conditions in Example 1 except that the second population has only one available strategy, which is the second available strategy in Example 1 with $\beta^{(2)} = 0.19$. Solving (9), we obtain the optimal population state $x^{*(1)} \approx (85.7\%, 14.3\%)$, and the endemic state associated with it is given by $I^* \approx (31.2\%, 32.5\%)$. We assume that the first population was using the costlier strategy at $t = 0$. By using the dynamic payoff, (EPGb) and (EPGc), we obtain that for any $\rho > 0$, the state converges to (I^*, x^*). For simulation, we select $\upsilon = 4$ as in Example 1.

[2]Since the second population follows a fixed strategy, $G^{(2)}$ and $q^{(2)}$ have no impact on the trajectory $(I, x, q^{(1)})(t)$ and can be removed from the model.

We use the same budget $c^* = 0.3$ used for Example 1. Figure 3 shows that $(I, x, q^{(1)})(t)$ converges to $(I^*, x^*, 0)$, as stated in Corollary 3. In addition, Figure 3 indicates that the infection levels of the two populations differ more than they do in Example 1; the first population enjoys a lower infected level at the equilibrium by 3.2 percentage points, which comes at the expense of a higher infection level for the second population (by 3.7 percentage points). This is expected because only the first population is incentivized, and the second population receives no incentive.

Note that the second example is identical to the first example, except for the additional constraint on the second population; its population state is fixed at (0, 1) since only the second strategy is available to the agents of the second population. Hence, when we solve the optimization problem in (6), we need to impose this additional constraint. For this reason, the optimal value, which is the reproduction number corresponding to the optimal state, of Example 2 cannot be smaller than that of Example 1. In fact, the reproduction numbers associated with the optimal states for Examples 1 and 2 are 0.2039 and 0.2072, respectively.

VII. Conclusion

We studied the problem of designing a policy that can steer an epidemic in two populations toward a desired endemic equilibrium at which the basic reproduction number is minimized, subject to a long-term budget constraint. First, we extended to two populations the framework that was proposed for modeling the coupled dynamics between an epidemic state and the decision-making process of strategic agents in a single population. Using this new framework, we devised a policy with provable convergence to an optimal social state that we can select by solving a non-convex constrained optimization problem. Moreover, we showed that our framework can also be employed to study the interplay between a strategic population and a disease reservoir, e.g., animal disease reservoir.

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