2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Maintaining a relevant dataset for data-driven MPC using Willems' fundamental lemma extensions

Alexandre Faye-Bédrin, Stanislav Aranovskiy, Paul Chauchat, and Romain Bourdais

Abstract— **This work explores the recent formulation of non-linear Data-driven Model Predictive Control in the case of dynamic references. Indeed, the state-of-the-art methods rely on Willems fundamental lemma, and freeze the used dataset at some point. While this ensures consistent behavior, i.e., excitation and accuracy, for a given setpoint, this will likely fail when the reference, and thus the operating point, changes. To this end, we propose refined heuristics for dataset management. First, a singular value-based method induces regular dataset updates but still guarantees a minimum excitation level. Then, a double-dataset formulation aims at decoupling accuracy and excitation issues and leverages the singular value-based one. These heuristics are validated in real-time experiments on a heat-blower system.**

I. Introduction

Model Predictive Control (MPC) is a mature research area and one of the most industrially accepted advanced control solutions [1] including, e.g., building energy management systems [2], [3]. For MPC, the accuracy of the process model is crucial, which motivates and gives rise to research on Adaptive MPC capable of online adjustments for model uncertainties and variations. As in Adaptive Control, the adaptation in MPC can be model-based or non-parametric (data-driven). For the first one, an explicit parametric model of a plant is estimated, and then an MPC strategy is applied. The Data-Driven MPC (DD MPC) does not include such an explicit parametric model. In contrast, it uses a dataset of past measurements to formulate an optimization problem; such a dataset can be thus seen as an implicit non-parametric model. The absence of a-priori knowledge of the process is actually advantageous, as it reduces the risk of making wrong assumptions [4].

One possible approach in DD MPC is based on Willems *et al*'s fundamental lemma [5]. It says that any trajectory of a discrete-time linear time-invariant system can be derived from a sufficiently large dataset of past trajectories if these trajectories satisfy the persistence of excitation condition. Then an optimization problem can be formulated for the future trajectory using this dataset and avoiding explicit dynamics formulation.

This idea has been used to develop control and simulation methods [6], [4], [7], [8], [9], [10], going so far as in [11], where a DD MPC solution for a non-linear system with stability guarantees is developed. To this end, a record of recent input-output measurements is used as a linearization around the current position. Such a data-driven linearization is then used in the tracking MPC formulation; see also [12], [13] for more details. In [11], the crucial assumption to prove the stability and tracking capabilities is that the dataset remains persistently excited when operating in the closed loop. Such an assumption requires a dataset-maintaining strategy. The authors highlight that developments of efficient dataset update heuristics remain an open question. Specifically, they propose the following approach: measurements are collected until the system arrives at the desired position, and then the collected dataset is frozen; no future measurements are used. While this approach works for constant reference tracking, it may yield an error when the reference changes. The dataset collected around a linearization point may be irrelevant when used for regulation around *another* linearization point (another reference). Thus, the dataset-maintaining problem remains open.

The contribution of this paper is as follows. We apply the DD MPC [11] to a non-linear MIMO testbed representing a heating/ventilation system, where the temperature and the airflow set points (references) are piece-wise constants. Due to the changes in the set points, the heuristic proposed in [11] yields a steady-state tracking error. To this end, we develop a novel dataset-maintaining strategy that ensures the use of the most recent sufficiently excited dataset while keeping track of the offset at all times. The proposed strategy tracks the smallest singular value of the Hankel matrix introduced in Willems' lemma. Our experimental results show that the proposed solution tracks the set point variations.

The paper has the following structure. Section II presents some preliminary results: a Willems' lemma-based DD MPC for affine systems and its extension for non-linear systems; Section II-C highlights a limitation of the discussed approaches to be addressed in this paper. Section III presents our main result: the data management strategy. Section IV describes the experimental results on a heating/ventilation system regulation illustrating the performance of the proposed solution. Finally, conclusions and future directions are given in Section V.

Notation:

- The operator $\mathrm{col}(\cdot)$ stacks up its vector or matrix arguments.
- For a sequence $\{z_k\}_{k=m}^{M}$ and $a, b \in [m, M]$ we use the following notation for a stacked window:

$$z_{[a,b]} = \mathrm{col}(z_a, z_{a+1}, \ldots, z_b)$$

With a slight abuse of notations, we write z instead of $z_{[m,M]}$.
- For a sequence $\{z_k\}_{k=0}^{N-1}$ with $z_k \in \mathbb{R}^{n_z}$, we denote the

The authors are with IETR, CentraleSupélec, Rennes, France alexandre.faye-bedrin@centralesupelec.fr

Hankel matrix $H_L(z) \in \mathbb{R}^{Ln_z \times (N-L+1)}$ as

$$H_L(z) = \begin{bmatrix} z_{[0,L-1]} & z_{[1,L]} & \dots & z_{[N-L,N-1]} \end{bmatrix}.$$

- A vector or sequence computed at time t is denoted by $\cdot(t)$, e.g., $\{z_k(t)\}_{k=m}^M$.
- A vector of ones of length q is denoted by $\mathbb{1}_q$.
- The Kronecker product is denoted by $\otimes$.
- For a matrix A, the vector of singular values is denoted by $\sigma(A)$, and $\sigma_{min}(A) := \min_i(\sigma_i(A))$.

II. Preliminaries

MPC is an acclaimed control method as it can account for non-linear dynamics, input and output constraints, and versatile objective functions. However, this heavily relies on the accuracy of the process model. Adaptive MPC methods try to overcome this problem, and a promising line of research is Data-Driven MPC [11], which we recall in this section.

A. Data-driven MPC for affine systems

Consider the following affine system:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + e, \\ y_k &= Cx_k + Du_k + r, \end{aligned} \tag{1}$$

where $u_k \in \mathbb{R}^m$, $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^p$, A, B, C, and D are constant matrices of appropriate dimensions, and e, r are constants, e.g., resulting due to a linearization around a non-equilibrium point.

Most of the data-driven control methods designed so far, with guarantees on the accuracy of the prediction, rely on Willems' fundamental lemma [5] and its few extensions [6], [10], [11], which characterize all the system's possible trajectories with respect to a collection of past ones. Data-based methods have been developed prior to the lemma, e.g., [14], based on the idea of *subspace identification* [15]. In a way, Willems' lemma specifies when the subspace identification of a system is exact.

1) Willems' fundamental lemma for affine systems: Suppose that (1) defines a controllable and observable system.

Definition 1: A sequence $\{z_k\}_{k=0}^{N-1}$ with $z_k \in \mathbb{R}^q$ is said to be persistently exciting of order M if

$$rank(H_M(z)) = qM.$$

Let $\left(u^d = \{u_k^d\}_{k=0}^{N-1},\ y^d = \{y_k^d\}_{k=0}^{N-1}\right)$ be a series of inputs-outputs of the system.

Theorem 1 ([11], [10]): If u^d is persistently exciting of order $L+n+1$, then $(u,\, y)$ with $u = \text{col}(u_0, \dots, u_{L-1})$ and $y = \text{col}(y_0, \dots, y_{L-1})$ is a trajectory of length L of the system if and only if:

$$\exists \alpha \in \mathbb{R}^{N-L+1},\ \begin{bmatrix} u \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} H_L(u^d) \\ H_L(y^d) \\ \mathbb{1}_{N-L+1}^\top \end{bmatrix} \alpha \tag{2}$$

2) Control scheme: Let y^r be the reference. The goal of this scheme is to find a feasible setpoint (u^s, y^s) such that y^s comes close to y^r. Then, a trajectory from the current operating point to this setpoint is computed, and the first control step of this trajectory is applied. For every time k, input is subject to constraint $u_k \in \mathbb{U}$, while steady state input must satisfy $u^s \in \mathbb{U}^s \subseteq \text{int}(\mathbb{U})$.

Suppose that the dataset (u^d, y^d) is available at the time instance t, where u^d is persistently exciting of order $L+2n+2$. Then the open-loop optimal control problem is based on the following two-stage procedure.

a) First stage: The first stage computes a feasible steady-state $(u^{sr}(t), y^{sr}(t))$ close to the reference as a solution to the following problem:

$$\min_{\substack{\varepsilon^{sr}, \alpha^{sr}, \\ u^{sr}, y^{sr}}} \|y^{sr} - y^r\|_S^2 + \lambda_\alpha^s \|\alpha^{sr}\|_2^2 + \lambda_\varepsilon^s \|\varepsilon^{sr}\|_2^2 \tag{3a}$$

$$\text{st.} \begin{bmatrix} \mathbb{1}_{L+n+1} \otimes u^{sr} \\ \mathbb{1}_{L+n+1} \otimes y^{sr} + \varepsilon^{sr} \\ 1 \end{bmatrix} = \begin{bmatrix} H_{L+n+1}(u^d) \\ H_{L+n+1}(y^d) \\ \mathbb{1}_{N-L-n}^\top \end{bmatrix} \alpha^{sr} \tag{3b}$$

$$u^{sr}(t) \in \mathbb{U}^s \tag{3c}$$

Here, ε^{sr} is a feasibility term, useful for both the noisy and non-linear cases, i.e., it allows a deviation of the measurement from the model (1), and $\lambda_\alpha^s, \lambda_\varepsilon^s$ help regularising the problem. The computed values y^{sr} and u^{sr} are not actually used for the control input computations, but the goal of this stage is obtaining the α^{sr} value based on the data available at t; it is further denoted as $\alpha^{sr}(t)$. This value is further used as a regularization term in the second stage.

b) Second stage: The second problem stage computes an open-loop trajectory in a more classic way. As well as in the model-based case [13], it relies on a constrained optimization problem using a quadratic cost function.

$$\begin{aligned} \min_{\substack{\varepsilon(t), \alpha(t), \\ u^s(t), y^s(t)}} \sum_{k=-n}^{L} & \|\bar{u}_k(t) - u^s(t)\|_R^2 + \|\bar{y}_k(t) - y^s(t)\|_Q^2 \\ & + \|y^s(t) - y^r\|_S^2 + \lambda_\alpha \|\alpha(t) - \alpha^{sr}(t)\|_2^2 \\ & + \lambda_\varepsilon \|\varepsilon(t)\|_2^2 \end{aligned} \tag{4a}$$

$$\text{st.} \begin{bmatrix} \bar{u}(t) \\ \bar{y}(t) + \varepsilon(t) \\ 1 \end{bmatrix} = \begin{bmatrix} H_{L+n+1}(u^d) \\ H_{L+n+1}(y^d) \\ \mathbb{1}_{N-L-n}^\top \end{bmatrix} \alpha(t) \tag{4b}$$

$$\begin{bmatrix} \bar{u}_{[-n,-1]}(t) \\ \bar{y}_{[-n,-1]}(t) \end{bmatrix} = \begin{bmatrix} u_{[t-n,t-1]} \\ y_{[t-n,t-1]} \end{bmatrix} \tag{4c}$$

$$\begin{bmatrix} \bar{u}_{[L-n,L]}(t) \\ \bar{y}_{[L-n,L]}(t) \end{bmatrix} = \begin{bmatrix} \mathbb{1}_{n+1} \otimes u^s(t) \\ \mathbb{1}_{n+1} \otimes y^s(t) \end{bmatrix} \tag{4d}$$

$$\bar{u}_k(t) \in \mathbb{U};\ u^s(t) \in \mathbb{U}^s \tag{4e}$$

Using notation $(.)_{[a,b]}$ with sometimes negative values of a, b allows keeping consistency with [11]. Here, at time t, we denote $\bar{u}(t) = \{\bar{u}_k(t)\}_{k=-n}^L$, likewise for $\bar{y}$. This centers the stacked sequences around time zero. Thus (4c) ensures, for $k < 0$, $\bar{u}_k(t) = u_{t+k}$, so that it represents the measured inputs, likewise for y. This induces a consistent trajectory initialization. Then $\bar{u}_k, \bar{y}_k,\ k \geq 0$, represent the predicted inputs and outputs from time t. Terminal constraint (4d) ensures closed-loop stability, and (4b) ensures, thanks to Willems' lemma, that $\bar{u}, \bar{y}$ is close to a trajectory of the system, with again feasibility and regularisation terms.

Once Problem 4 is solved, $\bar{u}_0(t)$ is the control to be applied.

3) Obtaining an acceptable dataset: The performance of DD MPC is directly linked with the quality of the dataset (u^d, y^d). It has to satisfy two criteria (CR):

- CR-i: its prediction must be accurate,
- CR-ii: it must ensure the controllability of the system by spanning all feasible state-control sequences.

For an affine system, Willems' lemma ensures that both properties are satisfied as soon as u^d is persistently exciting of sufficient order. This is why it was proposed to set up an initialization phase during which a sufficiently exciting input sequence u^d is injected, similar to a probe signal injection, and then use the corresponding outputs y^d to form the dataset. This single dataset will then ensure accuracy and controllability for an arbitrary long time [5], [12].

B. Data-driven MPC for non-linear systems

Consider a non-linear control-affine dynamic system $\mathcal{B}$ of the form

$$\begin{cases} x_{k+1} &= f(x_k) + Bu_k \\ y_k &= g(x_k) + Du_k \end{cases} \tag{5}$$

with $u_k \in \mathbb{R}^m$, $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^p$.

If the system satisfies some suitable assumptions (mainly, smoothness of f and g), it can be linearized around an operating point $\widetilde{x}$. The result is an affine system $\mathcal{B}_a(\widetilde{x})$ in the form (1), which can approximately predict the behavior of the non-linear system (5). The matrices B and D of the system $\mathcal{B}_a(\widetilde{x})$ are the same as in (5), and the matrices A, C and the vectors e, r in (1) are computed around $\widetilde{x}$,

$$\begin{aligned} A_{\widetilde{x}} &:= \frac{df}{dx}(\widetilde{x}), \qquad e_{\widetilde{x}} := f(\widetilde{x}) - A_{\widetilde{x}}\widetilde{x} \\ C_{\widetilde{x}} &:= \frac{dg}{dx}(\widetilde{x}), \qquad r_{\widetilde{x}} := g(\widetilde{x}) - C_{\widetilde{x}}\widetilde{x} \end{aligned} \tag{6}$$

Just as in the model-based case, this model can be used to get a control scheme around $\widetilde{x}$ [13]. In turn, non-linear DD MPC simply relies on the method designed for affine systems and presented in Section II-A.

C. Main limitation of DD MPC for non-linear systems: How to maintain the dataset?

Similarly to the affine case, (u^d, y^d) must satisfy criteria CR-i and CR-ii. However, in this case, Willems' lemma alone does not guarantee CR-i: the associated linearization point needs to be close to the current operating point.

An easy strategy to ensure this is to form u^d with the last N samples at a given time t: $u^d(t) = u_{[t-N,t-1]}$. However, the risk is to reduce the controllability of the system if these samples do not carry enough excitation, especially in a steady state. Thus, CR-i and CR-ii appear to be conflicting. A compromise was proposed in [11], [16] in the form of the following heuristic. Starting with an exciting initialization phase as in the affine case, the dataset is frozen once a steady state is reached, to ensure accuracy around this operating point.

However, this only works if the system has a unique setpoint during its lifetime. No guarantees can be given for a reference which would change over time. In the following, we propose refined heuristics to overcome this problem.

III. Proposed Dataset Management Solutions

In this section, we explore heuristics that can be used for non-linear DD MPC with changing references. We first propose a method to update a single dataset based on a compromise between accuracy and controllability requirements. Then, we leverage this solution to develop a new double dataset management heuristic, which aims at decoupling both of these requirements.

A. A simple singular value-based heuristics

Let (u^d, y^d) be the previously collected dataset used for DD MPC, and $(\widetilde{u}^d(t) := u_{[t-N,t-1]},\ \widetilde{y}^d(t) := y_{[t-N,t-1]})$ be the one formed from the last N steps. The question is: when do we replace (u^d, y^d) with $(\widetilde{u}^d, \widetilde{y}^d)$? Our rationale is as follows: a dataset update will be needed if the reference changes and if this change triggers enough excitation. Mathematically speaking, Willems' lemma is based on a Hankel matrix built from u^d and which has to be a full-rank matrix. We propose to track this condition over time through the smallest singular value of this matrix $\varsigma(u^d) := \sigma_{min}(H_{L+2n+2}(u^d))$. Since in practice, the presence of noise will almost always induce some excitation of measurements, we introduce the threshold $\bar{\varsigma} > 0$ and assume that the dataset (u^d, y^d) is sufficiently excited if $\varsigma(u^d) \geq \bar{\varsigma}$. Then, a simple heuristics can be formulated as always using the most recent sufficiently excited data, i.e., if $\varsigma(\widetilde{u}^d) \geq \bar{\varsigma}$. The current dataset $(\widetilde{u}^d, \widetilde{y}^d)$ is further used for DD MPC until a more recent sufficiently excited dataset is observed.

The threshold $\bar{\varsigma}$ has to be small enough to allow for frequent updates, but also large enough so that the data-based representation has a good "signal to noise" ratio, and is then able to control the system. In general, the choice of $\bar{\varsigma}$ is not easy, and requires prior knowledge of the system [17] or numerous trial-and-error runs.

B. Using two complementary datasets

Willems' fundamental lemma (in its original form, for linear systems [5]) has been generalized to the case of multiple datasets in [18]: instead of relying on a unique trajectory that represents the behavior of the system, it is possible to use a collection of trajectories, under some similar conditions.

This leads us to the following idea: it is possible to use multiple datasets with different properties, in order to fulfill the two criteria of a "good" dataset.

In our case, we propose the following method with two datasets:

- one that guarantees the sufficient excitation level,
- one that tracks the current operating point $\widetilde{x}$.

1) Defining two relevant datasets: We define the two following datasets:

- $(u^{d,a}(t) := u_{[t-N_a,t-1]},\ y^{d,a}(t) := y_{[t-N_a,t-1]})$ is a dataset with the most recent data; it is used to track the current operating point $\widetilde{x}$, specifically the offset $\gamma_{e,r,L}$ (defined hereafter in Remark 1) of the affine system $\mathcal{B}_a(\widetilde{x})$.
- $(u^{d,\ell}(t) := u_{[t_\ell-N_\ell,t_\ell-1]},\ y^{d,\ell}(t) := y_{[t_\ell-N_\ell,t_\ell-1]})$ is a dataset collected at the time $t_\ell \le t$ chosen such that the dataset is sufficiently excited, $\varsigma(u^{d,\ell}) \ge \bar{\varsigma}$, e.g., the initial excitation. However, since t_ℓ can be a –potentially far– past time, this dataset is used to "complete" the most recent one.

These two datasets are then combined to represent the approximation $\mathcal{B}_a(\widetilde{x})$ as discussed in the following extension to Willems' lemma.

Theorem 2: Let $\mathcal{B}$ be an affine system in the form (1), and let $(u^{d,\ell}, y^{d,\ell}), (u^{d,a}, y^{d,a})$ input-output trajectories of $\mathcal{B}$ as previously defined. Then (u, y) is a trajectory of length L of $\mathcal{B}$ if and only if there exists $\alpha \in \mathbb{R}^{N_a+N_\ell-2L+2}$ such that

$$\begin{bmatrix} u \\ y \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} H_L(u^{d,\ell}) & H_L(u^{d,a}) \\ H_L(y^{d,\ell}) & H_L(y^{d,a}) \\ \mathbf{0} & \mathbb{1}_{N_a-L+1}^\top \\ \mathbb{1}_{N_\ell-L+1}^\top & \mathbf{0} \end{bmatrix} \alpha. \tag{7}$$

Remark 1: Any trajectory of length L of the affine system $\mathcal{B}$ can be described as

$$y_{[t,t+L-1]} = \Phi_L x_t + \Gamma_{u,L} u_{[t,t+L-1]} + \gamma_{e,r,L} \tag{8}$$

with x_t the initial state, and

$$\Phi_L = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{L-1} \end{bmatrix};\ \gamma_{e,r,L} = \begin{bmatrix} r \\ Ce + r \\ \vdots \\ CA^{L-2}e + r \end{bmatrix};$$

$$\Gamma_{u,L} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{L-2}B & CA^{L-3}B & \cdots & D \end{bmatrix}.$$

In particular, this equality applies to the columns of the Hankel matrices used in (7).

Remark 2: Decompose $\alpha = \begin{bmatrix} \alpha^\ell \\ \alpha^a \end{bmatrix}$ with $\alpha^\ell \in \mathbb{R}^{N_\ell - L+1}$ and $\alpha^a \in \mathbb{R}^{N_a-L+1}$. Then the last two lines of (7) imply

$$\sum_{k=0}^{N_\ell-L} \alpha_k^\ell = 0,\ \sum_{k=0}^{N_a-L} \alpha_k^a = 1$$

which means the offset $\gamma_{e,r,L}$ is extracted only from $(u^{d,a}, y^{d,a})$.

Proof: Omitted for concision. Available in online extended version at `https://hal.science/hal-04073063`. ■

2) Changes in the control scheme: To use the two datasets in the control scheme, we simply adapt (4b) and (3b) to take the same form as (7). The resulting optimal control problem is then still defined in two stages.

a) First stage: The first stage computes a feasible steady-state $(u^{sr}(t), y^{sr}(t))$ close to the reference as a solution to the following problem, similar to the original Problem 3, where we define $\mathcal{L} := L + n + 1$ for concision:

$$\min_{\substack{\varepsilon^{sr}, \alpha^{sr}, \\ u^{sr}, y^{sr}}} \|y^{sr} - y^r\|_S^2 + \lambda_\alpha^s \|\alpha^{sr}\|_2^2 + \lambda_\varepsilon^s \|\varepsilon^{sr}\|_2^2 \tag{9a}$$

$$\text{st.} \begin{bmatrix} \mathbb{1}_\mathcal{L} \otimes u^{sr} \\ \mathbb{1}_\mathcal{L} \otimes y^{sr} + \varepsilon^{sr} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} H_\mathcal{L}(u^{d,\ell}) & H_\mathcal{L}(u^{d,a}) \\ H_\mathcal{L}(y^{d,\ell}) & H_\mathcal{L}(y^{d,a}) \\ \mathbf{0} & \mathbb{1}_{N_a-\mathcal{L}+1}^\top \\ \mathbb{1}_{N_\ell-\mathcal{L}+1}^\top & \mathbf{0} \end{bmatrix} \alpha^{sr} \tag{9b}$$

$$u^{sr}(t) \in \mathbb{U}^s \tag{9c}$$

Similarly to the original scheme, only the obtained α^{sr} is used in the second stage.

b) Second stage: Likewise, the second stage is very similar to the original one (Problem 4):

$$\begin{aligned} \min_{\substack{\varepsilon(t),\alpha(t), \\ u^s(t),y^s(t)}} \sum_{k=-n}^{L} & \|\bar{u}_k(t) - u^s(t)\|_R^2 + \|\bar{y}_k(t) - y^s(t)\|_Q^2 \\ & + \|y^s(t) - y^r\|_S^2 + \lambda_\alpha \|\alpha(t) - \alpha^{sr}(t)\|_2^2 \\ & + \lambda_\varepsilon \|\varepsilon(t)\|_2^2 \end{aligned} \tag{10a}$$

$$\text{st.} \begin{bmatrix} \bar{u}(t) \\ \bar{y}(t) + \varepsilon(t) \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} H_\mathcal{L}(u^{d,\ell}) & H_\mathcal{L}(u^{d,a}) \\ H_\mathcal{L}(y^{d,\ell}) & H_\mathcal{L}(y^{d,a}) \\ \mathbf{0} & \mathbb{1}_{N_a-\mathcal{L}+1}^\top \\ \mathbb{1}_{N_\ell-\mathcal{L}+1}^\top & \mathbf{0} \end{bmatrix} \alpha(t) \tag{10b}$$

$$\begin{bmatrix} \bar{u}_{[-n,-1]}(t) \\ \bar{y}_{[-n,-1]}(t) \end{bmatrix} = \begin{bmatrix} u_{[t-n,t-1]} \\ y_{[t-n,t-1]} \end{bmatrix} \tag{10c}$$

$$\begin{bmatrix} \bar{u}_{[L-n,L]}(t) \\ \bar{y}_{[L-n,L]}(t) \end{bmatrix} = \begin{bmatrix} \mathbb{1}_{n+1} \otimes u^s(t) \\ \mathbb{1}_{n+1} \otimes y^s(t) \end{bmatrix} \tag{10d}$$

$$\bar{u}_k(t) \in \mathbb{U};\ u^s(t) \in \mathbb{U}^s \tag{10e}$$

Here, we did not split α (nor α^{sr}) into (α^ℓ, α^a) for concision. However, it may be relevant to do so and assign various weights to these two parts in the objective function.

The exciting dataset $(u^{d,\ell}, y^{d,\ell})$ can be updated, for example, according to the singular value criterion proposed in Section III-A. Update should help keeping the data relevant.

3) Control scheme: With our proposed double-dataset management solution, the complete control scheme is as described in Algorithm 1. Note that the scheme is purely *online*.

IV. Experimental results

We assess the performance of the various DD MPC schemes on a heat blower shown in Figure 1. The system is equipped with a blower fan and heating resistance, and is instrumented with a thermocouple and a flow meter. With the sample time of $0.3s$ and the prediction horizon $L = 50$ samples, we assumed a system of order $n = 12$. The objective function parameters are $Q = R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $S = (L+n+1)R$, $\lambda_\sigma = \lambda_\varepsilon = \lambda_\sigma^s = \lambda_\varepsilon^s = 1$. Datasets lengths are $N = N^\ell = 456$ and $N^a = 76$ samples. The admissible input set is $\mathbb{U} = \mathbb{U}^s = [0, 10] \times [0, 10]$, and the update threshold is chosen as $\bar{\varsigma} = 0.1$ via trial and error.

Algorithm 1 Data-driven MPC scheme

Initialization

Generate $u^{d,I}$, a control sequence of length N_ℓ which is persistently exciting of order $L + 2n + 2$.

Apply this sequence to the system: $u_{[0,N_\ell-1]} \leftarrow u^{d,I}$.

Initialize the two datasets:

$(u^{d,\ell}, y^{d,\ell}) \leftarrow (u^{d,I}, y_{[0,N_\ell-1]})$

$(u^{d,a}, y^{d,a}) \leftarrow (u_{[N_\ell-N_a,N_\ell-1]}, y_{[N_\ell-N_a,N_\ell-1]})$

End of initialization

for each time step t **do**

Compute and apply the optimal control:

Compute $\alpha^{sr}(t)$ by solving Problem 9

Compute $\bar{u}(t)$ by solving Problem 10

Apply the computed input: $u_t \leftarrow \bar{u}_0(t)$

Update the datasets:

$(u^{d,a}, y^{d,a}) \leftarrow (u_{[t-N_a+1,t]}, y_{[t-N_a+1,t]})$

$(\widetilde{u}^{d,\ell}, \widetilde{y}^{d,\ell}) \leftarrow (u_{[t-N_\ell+1,t]}, y_{[t-N_\ell+1,t]})$

Compute $\varsigma(\widetilde{u}^{d,\ell}) = \sigma_{min}(H_{L+2n+2}(\widetilde{u}^{d,\ell}))$

if $\varsigma(\widetilde{u}^{d,\ell}) \geq \bar{\varsigma}$ **then** update the exciting dataset:

$(u^{d,\ell}, y^{d,\ell}) \leftarrow (\widetilde{u}^{d,\ell}, \widetilde{y}^{d,\ell})$

end if

end for

Fig. 1. Instrumented heat blower. The actuators (blower fan and heating resistance) are situated on the left of the picture, while the flow meter and thermocouple are on the right.

We generated an initial exciting control sequence $u^{d,I}$ as a vector of uniform random values in $[1, 5]$, and we designed a piece-wise constant reference $y^r(t)$. We then fed those two elements to all experiments, which shared the same control scheme and parameters up to the dataset management strategies. The following methods were compared:

- STR-1: Following [11] (recalled in Section II-A.2): the unique dataset is updated until the steady state is reached, after which it is indefinitely frozen.
- STR-2: With singular value criteria: as described in Section III-A, the unique dataset is updated every time the condition on the minimal singular value of a Hankel matrix is fulfilled.
- STR-3: With 2 datasets and singular value criteria: this is the complete strategy, that we propose in Section III-B.3 with Algorithm 1.

We show in Figure 2 the outcome of the experiments. Notice the initial excitation, between $t = 0$ and $t = 137s$, common to all strategies. We also compute the Root Mean Square (RMS) tracking error, in relation to the quadratic objective function used in the control scheme, for both output of the system and each strategy. Results are in Table I.

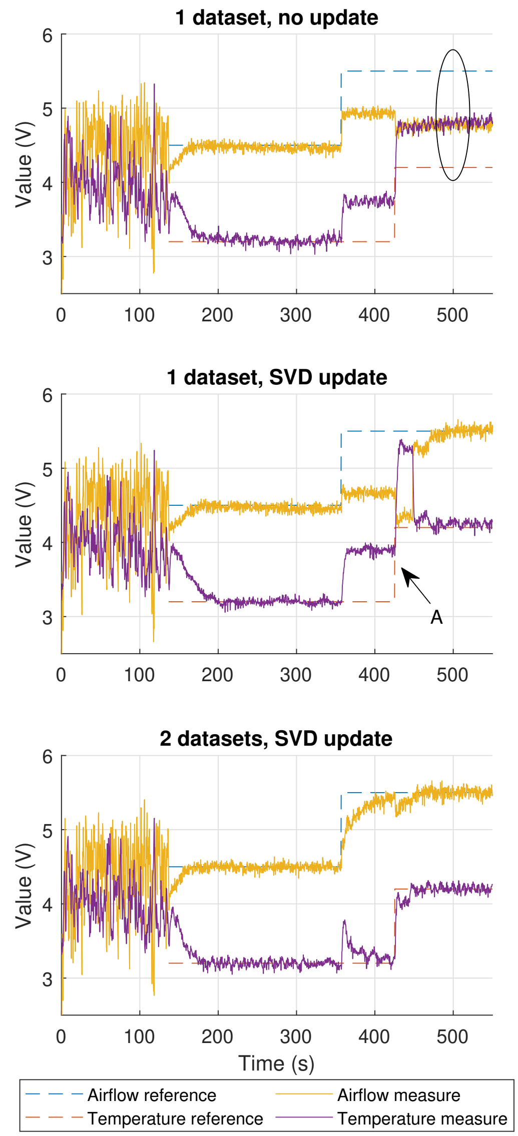

Fig. 2. Reference and output trajectories with the 3 considered strategies. STR-1 (top) performs well until the first change of reference ($t = 350s$): after that, a large static error (highlighted in the oval) stays between measures and reference. STR-2 (middle) starts similarly. However, as a change in reference (A, at $t = 420s$) brings enough excitation in the control law, the dataset can be updated to ensure a null static error. STR-3 (bottom) successfully converges to the reference after every change.

Data management strategy	Airflow	Temperature
STR-1 (No update)	0.59	0.51
STR-2 (SVD-based)	0.35	0.31
STR-3 (2 datasets)	0.12	0.13

TABLE I

RMS OF TRACKING ERROR DURING THE EXPERIMENT

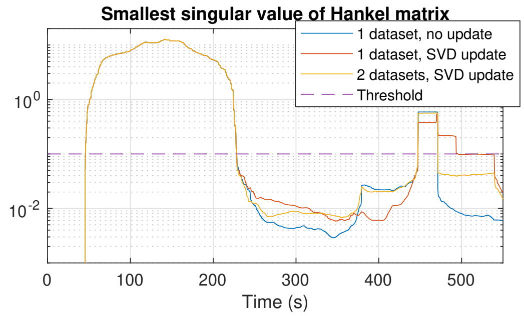

Fig. 3. Smallest singular value of the Hankel matrix used to verify persistency of excitation, for the 3 considered strategies. For STR-2 and STR-3, the dataset update is triggered by a singular value above the threshold. The initialization is intentionally exciting, hence the important values at the beginning.

We can notice in Figure 2 that the original strategy STR-1 is incapable of following a variation of reference. The proposed singular value based strategy STR-2 can overcome this limitation, but only once the second change of reference triggered enough excitation. Finally, the two-datasets strategy STR-3 is the best performing one, as it successfully tracks the reference. This clearly shows the interest of having this second dataset, which does not depend on excitation.

Figure 3 shows the value of $\varsigma(\widetilde{u}^d)$ (as defined in Section III-A) over time, and therefore illustrates the excitation of the candidate datasets and the times dataset updates happen given a threshold. In particular, we can clearly see that the second change of reference triggers enough excitation for STR-2 to update, which in turn improves the tracking accuracy.

V. Conclusion

In this paper, we proposed two strategies to update the dataset required for the Data-Driven MPC implementation in a relevant way. The results obtained in experimentation highlight the potential of this approach to track dynamic references. As a future direction, we propose investigating the persistent excitation order, which could be reduced to lower the requirements defining a good dataset, thus allowing more frequent updates.

References

[1] T. Samad, M. Bauer, S. Bortoff, S. Di Cairano, L. Fagiano, P. F. Odgaard, R. R. Rhinehart, R. Sánchez-Peña, A. Serbezov, F. Ankersen, P. Goupil, B. Grosman, M. Heertjes, I. Mareels, and R. Sosseh, "Industry engagement with control research: Perspective and messages," *Annual Reviews in Control*, vol. 49, pp. 1–14, 2020.

[2] Y. Wang, J. Kuckelkorn, and Y. Liu, "A state of art review on methodologies for control strategies in low energy buildings in the period from 2006 to 2016," *Energy and Buildings*, vol. 147, pp. 27–40, Jul. 2017.

[3] P. Stoffel, L. Maier, A. Kümpel, T. Schreiber, and D. Müller, "Evaluation of advanced control strategies for building energy systems," *Energy and Buildings*, vol. 280, p. 112709, Feb. 2023.

[4] J. Coulson, J. Lygeros, and F. Dörfler, "Data-Enabled Predictive Control: In the Shallows of the DeePC," in *2019 18th European Control Conference (ECC)*, Jun. 2019, pp. 307–312.

[5] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. M. De Moor, "A note on persistency of excitation," *Systems & Control Letters*, vol. 54, no. 4, pp. 325–329, Apr. 2005.

[6] I. Markovsky and P. Rapisarda, "Data-driven simulation and control," *International Journal of Control*, vol. 81, no. 12, pp. 1946–1959, Dec. 2008.

[7] J. Berberich, J. Köhler, M. A. Müller, and F. Allgöwer, "Data-Driven Tracking MPC for Changing Setpoints," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 6923–6930, Jan. 2020.

[8] I. Markovsky and F. Dörfler, "Behavioral systems theory in data-driven analysis, signal processing, and control," *Annual Reviews in Control*, vol. 52, pp. 42–64, Jan. 2021.

[9] I. Markovsky, L. Huang, and F. Dörfler, "Data-driven control based on behavioral approach: From theory to applications in power systems," *IEEE Control Syst.*, 2022.

[10] A. Martinelli, M. Gargiani, M. Draskovic, and J. Lygeros, "Data-Driven Optimal Control of Affine Systems: A Linear Programming Perspective," *IEEE Control Systems Letters*, vol. 6, pp. 3092–3097, 2022.

[11] J. Berberich, J. Köhler, M. A. Müller, and F. Allgöwer, "Linear Tracking MPC for Nonlinear Systems—Part II: The Data-Driven Case," *IEEE Transactions on Automatic Control*, vol. 67, no. 9, pp. 4406–4421, Sep. 2022.

[12] ——, "Data-Driven Model Predictive Control with Stability and Robustness Guarantees," *IEEE Transactions on Automatic Control*, vol. 66, no. 4, pp. 1702–1717, Apr. 2021.

[13] ——, "Linear Tracking MPC for Nonlinear Systems—Part I: The Model-Based Case," *IEEE Transactions on Automatic Control*, vol. 67, no. 9, pp. 4390–4405, Sep. 2022.

[14] B. L. M. De Moor, W. Favoreel, and M. Gevers, "SPC: Subspace Predictive Control," *IFAC Proceedings Volumes*, vol. 32, no. 2, pp. 4004–4009, Jul. 1999.

[15] P. Van Overschee and B. De Moor, *Subspace Identification for Linear Systems*. Boston, MA: Springer US, 1996.

[16] S. Baros, C.-Y. Chang, G. E. Colón-Reyes, and A. Bernstein, "Online data-enabled predictive control," *Automatica*, vol. 138, p. 109926, Apr. 2022.

[17] J. Coulson, H. J. van Waarde, J. Lygeros, and F. Dörfler, "A Quantitative Notion of Persistency of Excitation and the Robust Fundamental Lemma," *IEEE Control Systems Letters*, vol. 7, pp. 1243–1248, 2023.

[18] H. J. van Waarde, C. De Persis, M. K. Camlibel, and P. Tesi, "Willems' Fundamental Lemma for State-Space Systems and Its Extension to Multiple Datasets," *IEEE Control Systems Letters*, vol. 4, no. 3, pp. 602–607, Jul. 2020.