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Energy-Optimal Trajectory-Based Traveling Salesman Problem for Multi-Rotor Unmanned Aerial Vehicles

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Abstract— **The trajectory-based traveling salesman problem represents an extension of the classical traveling salesman problem, aimed at determining the most optimal trajectory that passes through a designated set of points. This paper introduces a novel formulation, termed the Energy-Optimal Trajectory-Based Traveling Salesman Problem (EOTB-TSP), which is grounded in an innovative energy assessment model. This model takes into account the intricate dynamics of unmanned aerial vehicles (UAVs). In addition, the EOTB-TSP is cast as a bilevel optimization challenge. To tackle this complex problem, we introduce a modified genetic algorithm specifically tailored for its resolution. To validate the effectiveness of our proposed approach, we conduct a series of experiments and apply it to real-world scenarios. Our evaluation and comparative analyses unequivocally demonstrate the high efficiency of our method in minimizing energy consumption.**

I. INTRODUCTION

The Traveling Salesman Problem (TSP) stands as a classic combinatorial optimization challenge. Its primary objective revolves around identifying the most efficient tour within a provided set of cities, ensuring that each city is visited precisely once. The TSP has a wide range of applications, including search and rescue [1], [2], [3], infrastructure inspection [4], [5], [6], [7], and logistics [8], [9], [10]. However, in many real-world scenarios, the generated path may not be directly executable on robots due to dynamic constraints such as velocity and acceleration limits. This highlights the need for incorporating dynamic constraints into TSP to generate feasible trajectories for real robot systems.

For trajectory-based TSP, the path of robots is often formulated as Dubins path or Bézier curves. In [11], to address the air-ground coordination problem, the UAV is modeled as a Dubins vehicle. The path planning of the UAV is defined as a Dubins traveling salesman problem with the dynamic neighborhood to determine the shortest path that permits the UAV to visit all moving unmanned ground vehicles (UGVs). They also proposed an efficient memetic algorithm to solve this problem. In contrast with Dubins path where only the turning radius and forward velocity are considered in the generated trajectory, Jan and Petr [12] proposed to use Bézier curves to simulate the movement trajectory of the UAV. This approach seeks to find a fast and smooth trajectory in three-dimensional space, enabling more precise control over the speed and curvature of the trajectory than Dubins path. For any shape trajectory-based TSP, Fabian and Katharina [13] discrete the velocity and direction when traversing each point to find the trajectory with the shortest distance. This approach is effective in improving the performance of TSP problems involving complex trajectories or non-uniform motion patterns.

The above works aim to find the trajectory with the shortest distance or minimum time, which can be considered as the simplification of energy consumption since it reduces the amount of work required by the propulsion system to move the robot from one point to another. However, these approaches do not capture all of the factors that influence energy consumption, such as the robot's mass, the efficiency of its propulsion system, and environmental conditions like wind resistance or friction, to better estimate energy consumption, many researchers focus on building accurate energy models to generate the energy optimal trajectory. It is claimed in [14] that a large amount of energy is consumed by the propeller during the operation of the UAV, and the optimal energy is to minimize the product of torque generated by the motor and its speed. Fabio et al. [15] built the energy model of the UAV based on electrical consumption. This model integrates the UAV's dynamic model with the electrical model of its brushless DC motor, allowing for the calculation of energy consumption using current and voltage measurements through the motor. However, implementing it in online trajectory optimization can be challenging due to the complexity of the energy consumption models. As a result, Rashid et.al [16] proposed a simplified energy consumption model which has been verified through lots of experiments.

For the integration of energy optimal trajectory optimization and trajectory-based traveling salesman problem, Wang et.al [17] proposed a general energy minimization traveling salesman problem, where the energy consumption is related to travel distance and payload. Kevin et al. [18] used a generalized traveling salesman problem with neighborhoods to model the mission of UAV and use a genetic algorithm with the incorporation of an energy model to solve it. However, these approaches stay in the path stage, the dynamic constraints of robots are not considered in the formulation. The EOTB-TSP is a particularly challenging problem, as it involves both trajectory optimization and TSP, both of which are known to be NP-hard problems. Developing an effective formulation that accounts for dynamic constraints while remaining computationally tractable is an ongoing research challenge in the field of robotics.

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Motivated by the challenges and limitations of existing approaches, this paper proposes a new formulation for solving the energy-optimal trajectory-based traveling salesman problem in UAV applications. Our approach represents an improvement over previous methods by explicitly accounting for dynamic constraints and energy evaluation in the optimization process. Specifically, the TSP and trajectory generation are integrated as a bilevel optimization problem that can be solved efficiently using a modified genetic algorithm. To evaluate the effectiveness of our proposed approach, we conduct a series of real-world experiments and also apply the approach in real building inspection applications. The results of these experiments demonstrate the better performance of our approach compared to other methods, including classical TSP algorithms and other trajectory-based TSP algorithms. The major contributions of this paper are summarized as follows: i) The formulation of the energy-optimal trajectory-based traveling salesman problem, which considers dynamic constraints and trajectory feasibility of the UAV; ii) the decomposition of the complex EOTB-TSP into two optimization problems and their solutions with an efficient modified genetic algorithm; and iii) the implementation of the proposed technique for tackling a real industrial application.

The remainder of this paper is organized as follows. In Section II, we give some preliminaries of the traveling salesman problem and the energy consumption model of UAV. Section III shows the problem formulation of EOTB-TSP. The proposed modified genetic algorithm is presented in Section IV. Section V presents the experiments. The real application of the proposed approach is shown in Section VI. Conclusions are presented in Section VII.

II. Preliminaries

In this section, we give a brief introduction to the traveling salesman problem and the energy consumption model of UAV. In TSP, there are n cities located at different locations, the goal is to find the shortest path that travels through all these cities without repetition. Define the cost of traveling from city v_i to the city v_j, $i,j = 1,\dots,n, i \neq j$ as c_{ij}, the TSP can be formulated into the following optimization problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}v_{ij} \\ \text{s.t.} \quad & \sum_{j} v_{ij} = 1, \quad \forall i \\ & \sum_{i} v_{ij} = 1, \quad \forall j \\ & u_i - u_j + n v_{ij} \leq n-1, \quad \forall i, j \neq 0 \\ & v_{ij} \in \{0,1\}, \quad \forall i,j \\ & u_i, u_j \in \mathbb{R}, \quad \forall i,j. \end{aligned} \tag{1}$$

The first two constraints are used to guarantee that each city is visited and only visited once, and the third constraint ensures that there is no sub-tour. In classical TSP, c_{ij} represents the Euclidean distance between two cities. As a result, by solving the above optimization problem, the shortest path can be found.

For the energy consumption model, we adopt the model presented in [16]. The rate at which a UAV consumes battery power is dependent on the sum of forces acting against it, namely the forces of acceleration and aerodynamic drag. This can be simplified by expressing it as a linear combination of variables including velocity, acceleration, and mass. Define the energy consumption as E, it can be written in the following form:

$$E = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \left\|\vec{v}_{xy}\right\| \\ \left\|\vec{a}_{xy}\right\| \\ \left\|\vec{v}_{xy}\right\|\left\|\vec{a}_{xy}\right\| \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_5 \\ \beta_6 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \|\vec{v}_z\| \\ \|\vec{a}_z\| \\ \|\vec{v}_z\|\|\vec{a}_z\| \end{bmatrix} + \begin{bmatrix} \beta_7 \\ \beta_8 \\ \beta_9 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} m \\ \vec{v}_{xy}\cdot\vec{w}_{xy} \\ 1 \end{bmatrix} \tag{2}$$

where $\beta_1,\dots,\beta_9$ are the coefficients to be calculated, $\vec{v}_{xy}$ and $\vec{a}_{xy}$ are the velocity and acceleration in the horizontal plane, $\vec{v}_z$ and $\vec{a}_z$ are the velocity and acceleration in the vertical plane, m is the payload weight, and $\vec{w}_{xy}$ is the wind speed in horizontal plane.

III. Problem formulation

In this section, we formulate the problem of finding the energy optimal trajectory that travels through a set of points as a bilevel optimization problem. Let x_i be the ith point, $i = 1,\dots,\mathrm{N}$, and $p(x_i) \in \mathbb{R}^3$ be the position of x_i. To calculate the trajectory, let S be the set of the order of the tours as follows:

$$S = \left\{ X \mid X = [x_1,\dots,x_{\mathrm{N}}], \bigcup_{k=1}^{\mathrm{N}} x_k = A \right\} \tag{3}$$

where $A = \{x_1,\dots,x_{\mathrm{N}}\}$ is the set of all points. $X = [x_1,\dots,x_{\mathrm{N}}]$ represents all the possible touring orders that satisfy each point is visited only once. As a result, there are a total of N! elements in the set S, and each element represents one order of touring.

Consequently, the EOTB-TSP can be formulated as an energy minimization problem over the set S, seeking to identify the trajectory of the least energy consumption. The mathematical formulation of this optimization problem is provided below:

$$\begin{aligned} \min \quad & E(\mathscr{T}(X)) \\ \text{s.t.} \quad & X \in S \\ & p(X_i) \in \mathscr{T}(X), \quad \forall i \in \mathrm{N}. \end{aligned} \tag{4}$$

where X_i denotes the ith element of X, while $\mathscr{T}(X)$ represents the optimal energy trajectory that traverses all points in the order stipulated by X. The second constraint ensures that the energy-optimal trajectory must pass through all positions corresponding to the set of points. However, this problem is too complex to be solved; consequently, we employ a decoupling strategy to address it through a bilevel optimization procedure.

For the energy optimal trajectory generation of one sequence of touring X. It can be divided into $\mathrm{N}-1$ segments. Let T denote the total travel time and T_i denote the elapsed time of X_i. To define the trajectory that travels these N points, we design a polynomial function of order n as follows:

$$f(t)=\begin{cases}\sum_{k=0}^{n}\lambda_{1,k}t^{k} & 0\le t<T_1\\ \sum_{k=0}^{n}\lambda_{2,k}t^{k} & T_1\le t<T_2\\ \vdots & \\ \sum_{k=0}^{n}\lambda_{\mathrm{N}-1,\mathrm{k}}t^{k} & T_{\mathrm{N}-1}\le t\le T_{\mathrm{N}}.\end{cases} \tag{5}$$

The velocity and acceleration of the trajectory can be defined by Eq. 6:

$$v(t)=\frac{\partial f}{\partial t},\quad a(t)=\frac{\partial^2 f}{\partial t^2}. \tag{6}$$

According to Eq. 2, the energy consumption of segment i can be defined as follows:

$$E_i=\int_{T_{i-1}}^{T_i}(\mathrm{C}_1 v(t)+\mathrm{C}_2 a(t)+\mathrm{C}_3 v(t)a(t))dt \tag{7}$$

where $\mathrm{C}_1,\mathrm{C}_2,\mathrm{C}_3\in\mathbb{R}^3$ are the coefficients determined by the UAV. Combining Eqs. 5–7, E_i can be rewritten as the following equation:

$$E_i=\frac{1}{2}\lambda_i^{\mathrm{T}}H_i\lambda_i+F_i^{\mathrm{T}}\lambda_i \tag{8}$$

where

$$\lambda_i=[\lambda_{i,0},\lambda_{i,1},\cdots,\lambda_{i,n}]^{\mathrm{T}}, \tag{9}$$

and

$$F_i=\int_{T_{i-1}}^{T_i}\begin{bmatrix}0\\ C_1\\ \vdots\\ C_1kt^{k-1}+C_2k(k-1)t^{k-2}\end{bmatrix}dt. \tag{10}$$

H_i can be calculated by the following equation:

$$H_i=\int_{T_{i-1}}^{T_i}C(Q_i^{\mathrm{T}}+Q_i)dt \tag{11}$$

where

$$Q_i=\begin{bmatrix}0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 2 & \cdots & 5nt^{n-2}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 2nt^{n-1} & \cdots & 20nt^{n+2}\end{bmatrix}_{(n+1)\times(n+1)}. \tag{12}$$

As a result, the problem of energy optimal trajectory generation of one sequence can be written in the following optimization problem:

$$\begin{aligned}\min\quad & \frac{1}{2}\lambda^{\mathrm{T}}H\lambda+F^{\mathrm{T}}\lambda\\ \text{s.t.}\quad & f_1(0)=P(X_1)\\ & f_{\mathrm{N}-1}(T_{\mathrm{N}})=P(X_{\mathrm{N}})\\ & f_{i+1}(T_i)=f_i(T_i)\\ & v_{i+1}(T_i)=v_i(T_i)\\ & a_{i+1}(T_i)=a_i(T_i)\\ & \|v(t)\|<v_{max}, \qquad \forall t\in[0,T_{\mathrm{N}}]\\ & \|a(t)\|<a_{max}, \qquad \forall t\in[0,T_{\mathrm{N}}]\\ & i=1,\ldots,\mathrm{N}-2\end{aligned} \tag{13}$$

where $\lambda^{\mathrm{T}}=[\lambda_1^{\mathrm{T}},\ldots,\lambda_{\mathrm{N}-1}^{\mathrm{T}}]$, $F^{\mathrm{T}}=[F_1^{\mathrm{T}},\ldots,F_{\mathrm{N}-1}^{\mathrm{T}}]$, and $H=$ block diag$(H_1,\cdots,H_{\mathrm{N}-1})$. The first and second constraints ensure that the generated trajectory must pass through the first and last points. f_i, v_i, and a_i represent the ith segment of function f, v, and a. The third to fifth constraints ensure the continuity of position, velocity, and acceleration between any two trajectories. The variables v_{max} and a_{max} represent the maximum velocity and acceleration, respectively. The sixth and seventh constraints impose bounds on the velocity and acceleration of the UAV, limiting their magnitudes within the values specified by v_{max} and a_{max}. This problem is a quadratic programming problem, in which the objective function is to minimize the energy cost of the trajectory. Solving the aforementioned optimization problem, we can get the minimum energy cost value of each X in S. As a result, the EOTB-TSP becomes the problem of finding the element of S with the smallest value.

IV. Proposed Modified Genetic Algorithm

In this section, we present the modified genetic algorithm that can solve EOTB-TSP efficiently. The overall framework of the proposed problem and solver is shown in Fig. 1. Given a set of points that need to be traveled, every order of visit is regarded as one gene. Every gene is checked to see if it is present in the gene graph for each iteration. If it is a brand-new one, an energy-optimal trajectory generation process will be performed to determine the adaption of this gene and record it in the gene graph. The genes with high adaption are more likely to be chosen as the next generation. After the result convergence, the optimal trajectory that travels through all these points can be generated.

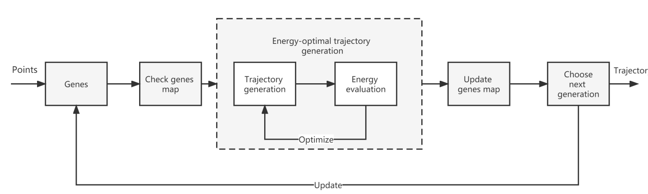

Fig. 1. Overall framework of the proposed EOTB-TSP.

The process of the proposed modified genetic algorithm is detailed in Alg. 1. The input of the algorithm is the points $\mathscr{X}$ that needs to be visited, the maximum number of iterations $iter$, the number of total genes N_g, the rate of cross γ_c, and the rate of mutation γ_m. According to [19], the performance of the genetic algorithm is related to the initial genes. As a result, at initialization configuration, we use the distance between two points to seek potential good genes. The detailed process is shown in Alg. 2. Initially, one point is randomly selected from $\mathscr{X}$ as the departing point (lines 3-4). The other point in $\mathscr{X}$ that has the shortest distance to x_s is selected as the next point and recorded in set *Gene* until all the points are visited (lines 5-12). card() denotes the number of elements of a set. After genes initialization, iterations are performed to update and select genes gradually, meanwhile a gene map is maintained to record the order of visits, generated trajectory, and energy cost. Each step checks each gene in G to see if it is listed on the gene map. The energy cost and the trajectory can be directly acquired if they have already been recorded (lines 4-6). The method greatly reduces calculation complexity. For those genes that have not been recorded, an energy-optimal trajectory generation process is used to get trajectory and energy cost (line 8). The trajectory generation is formulated into an optimization problem using Eq. 13. After solving the optimization problem, the adaption of each gene and the gene map can be updated (lines 9-10). The genes with higher adaption are chosen as the next generation (line 11). According to the rate of the cross, genes are randomly selected to exchange their sequences (line 12). Furthermore, a mutation process is performed to avoid the local optimal (line 13). Finally, the trajectory with the lowest energy cost can be extracted from the gene map (line 14).

Algorithm 1: Modified Genetic Algorithm

Input: $\mathscr{X}, iter, N_g, \gamma_c, \gamma_m$
Output: $\mathscr{T}$
1 $G \leftarrow GreedInit(\mathscr{X}, N_g)$;
2 $M \leftarrow \{\}$;
3 **while** $i \leq iter$ **do**
4   **foreach** gene $g \in G$ **do**
5   **if** $InGeneMap(g, M)$ **then**
6       $E_g, \mathscr{T}_g \leftarrow CheckGeneMap(g, M)$;
7   **else**
8       $E_g, \mathscr{T}_g \leftarrow TrajectoryGeneration(g)$;
9     $adap_g \leftarrow \frac{1}{E_g}$
10   $M \leftarrow UpdateGeneMap(E_g, \mathscr{T}_g)$;
11   $G \leftarrow Choose(G, adap)$;
12   $G \leftarrow Cross(G, \gamma_c)$;
13   $G \leftarrow Mutation(G, \gamma_c)$;
14 $E, \mathscr{T} \leftarrow FindLowestEnergy(M)$;

V. Evaluation and Comparison

In this section, we implement the proposed algorithm in outdoor environments using a self-developed UAV platform depicted in Fig. 2. The UAV has a gross weight of 1.5 kg and employs a Pixhawk flight controller. Xavier NX serves as the onboard computer for trajectory execution, the RTK module provides precise localization information, and the power measurement unit records the energy consumption. The velocity and acceleration limits are set as $v_{max} = [3.0, 3.0, 2.5]$ and $a_{max} = [2.5, 2.5, 2.0]$. Based on the data collected from experiments, the least square is used to get the value of coefficients. The coefficients of the self-develop UAV are as follows: $C_1 = [0.3652, 0.6612, 0.6819]$, $C_2 = [2.9709, 1.4287, 0.2507]$, and $C_3 = [-1.9457, -1.1289, -0.2041]$.

Algorithm 2: GreedInit

Input: $\mathscr{X}, N_g$
Output: G
1 $G \leftarrow \{\}$;
2 **while** card$(G) \leq N_g$ **do**
3   $x_s \leftarrow RandomSelect(\mathscr{X})$;
4   $Gene \leftarrow \{x_s\}$;
5   **while** card$(Gene) \leq$ card$(\mathscr{X})$ **do**
6     $mindistance \leftarrow 0$;
7     $x_l \leftarrow Last(Gene)$;
8   **foreach** $x \in \mathscr{X}, x \notin Gene$ **do**
9   **if** $Distance(x, x_l)$ **then**
10         $mindistance \leftarrow Distance(x, x_l)$;
11         $x_B \leftarrow x$;
12     $Gene \leftarrow \{Gene, x_B\}$
13   $G \leftarrow \{G, Gene\}$;

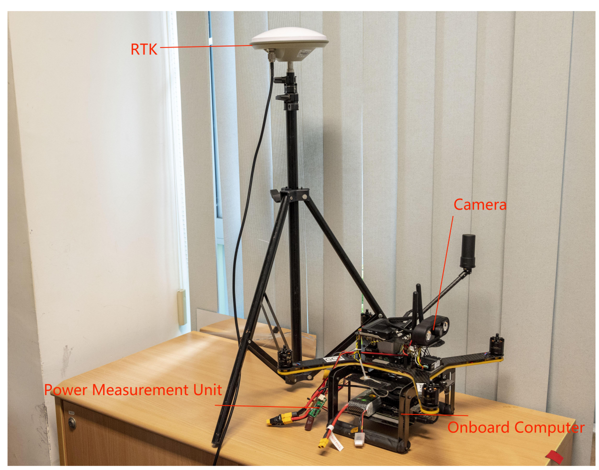

Fig. 2. Self-developed UAV equipped with RTK, FPV camera, power measurement unit, and onboard computer.

To evaluate the performance of the proposed algorithm, we conduct experiments in both small and large-scale environments. The small-scale scenario is defined as having a point-to-point distance of up to 10m, which aligns more closely with the requirements of power line and building inspection. Conversely, the large-scale environment is characterized by point-to-point distances exceeding 50m, which are more relevant to the needs of logistics, search, and rescue operations. We compare the proposed approach with several trajectory-based TSP algorithms. Euclidean-TSP is the traditional traveling salesman problem to minimize the total travel distance. Bézier-TSP-D [12] is another trajectory-based TSP where the points are connected with the Bézier curve, and the objective is to minimize the trajectory length. Bézier-TSP-E is the modified version of Bézier-TSP-D where we insert our energy evaluation model to find the Bézier curve with the lowest energy cost. TSP-TG [7] is a two-stage method, the shortest path is generated in the first stage by solving TSP, and then trajectory optimization is used to convert the path into a trajectory.

The detailed comparison result is shown in Table I. The proposed EOTB-TSP method performs better than the other trajectory-based TSP on both small and large scales, especially in small scales. This is due to the concentration of points within small-scale scenes, which necessitates continuous acceleration and deceleration by the UAV during movement, resulting in significant increases in energy consumption. In several experimental scenarios, our proposed approach demonstrated smaller finish times and energy costs than alternative methods, despite having a longer path length. This suggests that the EOTB-TSP method enables more mission points to be completed in a single flight, while also achieving faster execution speeds.

VI. Real Application

To verify the performance of the proposed EOTB-TSP in real applications, we deploy the proposed system for a real-world inspection task. Three UAVs equipped with RGB and infrared cameras are used to conduct an inspection and reconstruction task for a logistics center building with a dimension of 36m $\times$ 27m $\times$ 100m as shown in Fig. 3. There are numerous flaws of various degrees, some of which call for more thorough inspections to guarantee the security of the surrounding built environment. To comprehensively survey this building, approximately 3600 viewpoints must be reached by UAVs to capture images. However, the whole work is time-consuming in a such large-scale environment, batteries need to be changed frequently during the operation. As a result, we divide the task for each UAV and implement the proposed EOTB-TSP on each UAV to make full use of the battery to collect more images during one flight.

In this task, the viewpoints are generated in a vertical direction at a fixed distance of 10 meters above the building surface. Fig. 4 is the real flight trajectory generated by EOTB-TSP and Euclidean distance TSP. This trajectory travels through 66 viewpoints, the UAV takes pictures at each viewpoint. The velocity limit for the flight is set as 1 m/s. Notably, while the path distances generated by these two methods are similar, executing the EOTB-TSP trajectory requires only 6 minutes and 35 seconds, with an energy consumption of 1050 mAh, indicating a 19% improvement in efficiency compared to the Euclidean distance TSP method.

Fig. 3. Three UAVs cooperative inspection for a logistics center building.

In addition, we provide a summary of the correlation between

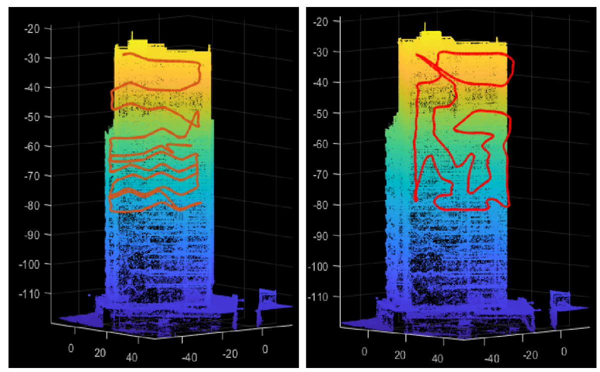

Fig. 4. The real flight trajectory of Euclidean-TSP and EOTB-TSP (66 viewpoints are given).

battery power and the number of feasible viewpoints that can be executed. Table II illustrates that with increasing battery power, the trajectory generated by the EOTB-TSP method can access a larger number of viewpoints. Specifically, when using a UAV with a 3850 mAh battery, our proposed approach can cover 63 additional points in a single flight. The experimental video is available at Youtube.

VII. Conclusion

We have introduced in this work an innovative EOTB-TSP, specifically tailored for real-world applications. Our approach seamlessly combines energy-efficient trajectory planning with the classic traveling salesman problem, thus framing them within the context of a bilevel optimization problem. To effectively tackle this intricate challenge, we have introduced a highly efficient modified genetic algorithm, custom-designed to address the unique demands of the EOTB-TSP. Our comparative analysis reveals that our approach not only demonstrates practical feasibility but also outperforms alternative methods. Notably, it achieves remarkable energy savings when contrasted with other approaches.

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TABLE I
THE COMPARISON OF THE ENERGY CONSUMPTION OF DIFFERENT VARIANTS OF TSP.

Method			Euclidean-TSP	Bezier-TSP-D	Bezier-TSP-E	TSP-TG	Ours
Small scale	30 points	Energy cost (mAh)	552.2	478.8	474.8	478.6	**431.9**
		Path length (m)	**196.36**	224.84	234.39	272.35	223.01
		Finish time (S)	122.32	97.28	98.56	98.1	**90.32**
	60 points	Energy cost (mAh)	936.4	774.0	737.2	742.4	**693.4**
		Path length (m)	**322.37**	366.71	375.07	380.17	377.3
		Finish time (S)	211.9	154.9	157.28	160.9	**150.4**
	90 points	Energy cost (mAh)	1292.0	871.2	868.4	973.3	**827.0**
		Path length (m)	**414.49**	436.43	454.50	531.19	440.78
		Finish time (S)	294.58	187.6	189.7	206.9	**180.84**
Large scale	30 points	Energy cost (mAh)	2185.9	2119.4	2144.0	2269.2	**2105.7**
		Path length (m)	**1175.6**	1176.9	1219.3	1270.4	1227.1
		Finish time (S)	504.5	484.78	488.58	507.96	**481.2**
	60 points	Energy cost (mAh)	4225.7	4180.7	4194.5	4691.9	**4060.7**
		Path length (m)	**2235.2**	2295.8	2434.5	2643.1	2385.4
		Finish time (S)	982.16	977.8	984.7	1087.6	**965.4**
	90 points	Energy cost (mAh)	5380.7	5542.6	5478.8	5861.4	**5322.9**
		Path length (m)	**2854.7**	3137.9	3149.8	3209.5	3029.9
		Finish time (S)	1252.3	1286.7	1272.68	1335.96	**1236.7**

TABLE II
THE CORRELATION BETWEEN BATTERY POWER AND THE NUMBER OF VIEWPOINTS THAT CAN BE EXECUTED.

Battery Usage	500mAh	1000mAh	2000mAh	3850mAh
Euclidean-TSP	24	52	103	189
EOTB-TSP	32	65	134	252

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