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Circumnavigation Control of Non-holonomic Vehicle System with Distance-rate Measurements

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Abstract— **This article studies the circumnavigation problem of a non-holonomic vehicle independent of the global coordinate. Specifically, the vehicle circumnavigates an unknown stationary target with anticipated distance and velocity. However, the position information (i.e., absolute position and positive position) is not available to the vehicle. Instead, the vehicle can just obtain the distance rate relative to the target. We propose a generic continuous and bounded control algorithm based on only the distance-rate measurement. It is demonstrated using Poincaré-Bendixon Criterion that the proposed control algorithm in this article ensures the asymptotically stable circumnavigation. Simulation is finally given to verify the effectiveness of the proposed control algorithm.**

I. INTRODUCTION

Recent years have witnessed rapid development in robotic technology [1]–[6]. As a special case, the circumnavigation problem has been studied a lot owing to their applications in space circumnavigation [7], [8], security and surveillance [9], [10], and target capturing [11]. In a typical circumnavigation scenario, one or more vehicles entrap/escort a given target along a prescribed orbit, in order to keep supervising the target or to protect the target from intrusion.

By postulating that the states of the target are accessible, a Lyapunov guidance vector field method was adopted in [12], [13] for the target tracking. However, in most unknown circumstances, it is impossible to obtain the target information in advance. In such a case, it is a feasible manner to make use of active relative measurements with respect to the target, such as distance-related or bearing-related measurements.

As for bearing-related measurements, the main methodology is to estimate the target position according to the knowledge obtained by the measurement of the vehicle, and then to design the control algorithm according to the estimated target position. In [14], a localization algorithm using an orthogonal projection method was studied, but the estimation error can just converge to the neighborhood around the origin. Next, the accurate target-state estimation was realized in [15], [16], where the estimation error resulting from the localization algorithm is asymptotically unbiased. However, in these works, vehicles need to know their own absolute positions. For the case where any position information is unavailable, an estimator was proposed in [17] to estimate the relative position with respec to the target in its local frame, and then a control algorithm was given to fulfill the circumnavigation objective. The method in [17] was further extended in [18]–[20] to multiagent systems with a prescribed shape in a common circle.

For distance-related measurements (e.g., distance and distance rate), there is no need to locate the unknown target. For a wheeled mobile vehicle, the algorithms with Equiangular Navigation Guidance [21] and the sliding mode control method [22], [23] were proposed, and then expanded to multiple targets in [24]. However, In these works, only local convergence can be guaranteed. To ensure the global convergence, a switching algorithm based on distance and distance rate measurement was proposed in [25], [26]. Nevertheless, the inner zero control input proposed in the switching algorithm for the sake of driving the vehicle outside the preset circular trajectory may result in overshoot. [26] proposed an unbounded algorithm based on the backstepping technique with a switching form in order to avoid singularity of the algorithm, which introduces complexity.

In this paper, we propose a simple bounded and continuous circumnavigation control algorithm for a non-holonomic vehicle based on just the distance rate measurement. The stability analysis is carried out by Poincaré-Bendixon Criterion. Specifically, we first show that the non-holonomic vehicle will never collide with the target in the process of approaching the target. Then, we show that the system trajectory converges to a bounded set containing the equilibrium point of the controlled system in finite time. Further, using Bendixson-Dulac Theorem, we show that there exists no periodic orbit in the bounded set. Based on Poincaré-Bendixon Criterion, this concludes that the trajectory asymptotically converges to the equilibrium, which indicates the achievement of the circumnavigation objective.

The remaining sections of this article are arranged as follows. Section II introduces the circumnavigation problem to be solved. Section III proposes a circumnavigation control algorithm with only the distance rate measurement, and also conducts the stability analysis. In Section IV, the simulation is given to verify the effectiveness of the proposed control algorithm. Finally, Section V is given to conclude the paper.

II. PROBLEM STATEMENT

A. System model

We consider an autonomous kinematically controlled non-holonomic vehicle. By fixing a constant linear velocity v_r,

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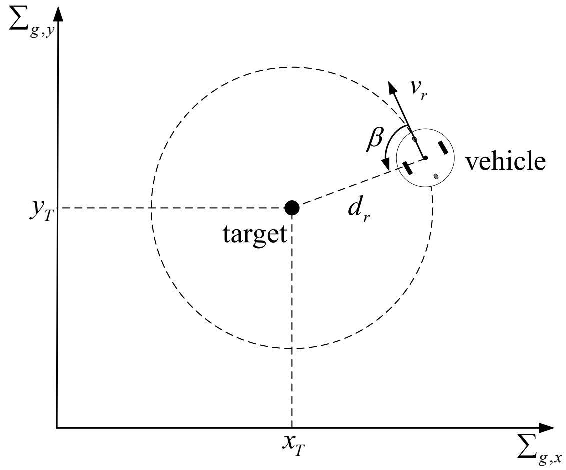

Fig. 1: Circumnavigation example of non-holonomic vehicle.

the motion equation of the vehicle is given by

$$\begin{aligned}\dot{x}(t) &= v_r \cos\psi(t),\\ \dot{y}(t) &= v_r \sin\psi(t),\\ \dot{\psi}(t) &= \omega(t),\end{aligned} \tag{1}$$

where $p(t) = [x(t)\; y(t)]^T \in \mathbb{R}^2$, $\psi(t) \in \mathbb{R}$ denote the position and azimuth of its head within the global coordinate, and $\omega(t) \in \mathbb{R}$ denotes its angular velocity and serves as a sole control input. Assume that the vehicle is not equipped with a global positioning device such that it has no information about the global coordinate. This thus leads to a plight where the vehicle has no access to its position and azimuth.

B. Control Objective

In this paper, we propose a control algorithm based on the distance rate information for the considered non-holonomic vehicle (1) such that the vehicle is capable of circumnavigating a given stationary target $T = [x_T, y_T]^T$ with a desired radius. See Fig. 1 for an example, in which $d = \|p - T\|$ represents the relative distance between the vehicle and the target, d_r represents the desired circumnavigating radius, and $\beta \in [0, 2\pi)$ denotes the angle between the heading direction of the vehicle and the direction to the target. Formally, the circumnavigation objective is described as follows.

***Definition** 1:* Consider the non-holonomic vehicle (1). The circumnavigation objective is referred to be achieved provided that

$$\lim_{t\to\infty} d(t) = d_r, \quad \lim_{t\to\infty} \dot{d}(t) = 0. \tag{2}$$

Since only the distance rate information, is available, to facilitate the subsequent analysis, we transform the motion equation (1) into a polar coordinate centered at the vehicle, i.e.,

$$\dot{d}(t) = -v_r \cos\beta(t), \tag{3a}$$

$$\dot{\beta}(t) = -\omega(t) + \frac{v_r \sin\beta(t)}{d(t)}. \tag{3b}$$

***Remark** 1:* Note from (3a) that $\beta = 3\pi/2$ is also sufficient for $\dot{d} = 0$. However, as will be shown later, only $\beta = \pi/2$ can be guaranteed by the proposed control algorithm. This means that the vehicle finally moves along the direction of the velocity v_r.

III. CONTROL ALGORITHM DEVELOPMENT

We focus on the control algorithm development in this section for achieving the circumnavigation objective of the non-holonomic vehicle (1). The detailed stability analysis associated with the resulting closed-loop system is then carried out.

A. Control Algorithm

Rather than the control inputs in [25], [26] based on distance and distance rate measurements that introduce a switching mechanism, we propose a control algorithm with just the distance rate measurement for the circumnavigation objective in a generic continuous form:

$$\omega(t) = \frac{v_r + \dot{d}(t)}{d_r}. \tag{4}$$

By virtue of (3a), $\dot{d}(t)$ is inherently bounded. This guarantees the boundedness of the control input (4) without necessarily introducing an additional saturation constraint. Specifically, when $\dot{d}(t) = 0$, the anticipated angular velocity becomes v_r/d_r. The vehicle will keep this angular velocity afterwards, which, together with the fact that the circumnavigating radius is d_r, is exactly the circumnavigation in Definition 1.

B. Stability Analysis

Substituting (4) and (3), the closed-loop system can be obtained as follows:

$$\dot{d}(t) = -v_r \cos\beta(t), \tag{5a}$$

$$\dot{\beta}(t) = -\frac{v_r}{d_r} + \frac{v_r \cos\beta(t)}{d_r} + \frac{v_r \sin\beta(t)}{d(t)}, \tag{5b}$$

which is in an interconnected form.

To begin with, we introduce a lemma to indicate that, if initialized properly, β evolves within a single interval, and d evolves away from 0.

***Lemma** 1:* Consider the closed-loop system (5b). If $\beta(0) \in (0, 2\pi)$ and $d(0) > 0$ are chosen, then

(i) $\beta(t) \in (0, 2\pi), \forall t \geq 0$; and

(ii) $d(t) > 0, \forall t \geq 0$.

Proof: (i) The proof is completed by first showing that $\beta(t) < 2\pi$, $\forall t \geq 0$ with contradiction argument. Suppose that the result does not hold. Then, there exists a finite time $t_a > 0$ such that $\beta(t_a) = 2\pi$ and $\beta(t) < 2\pi$ for $t \in (0, t_a)$. This means that there exist a sufficiently small constant $\xi_1 > 0$ and a moment $t'_a \in (0, t_a)$ with $\beta(t) \in (2\pi - \xi_1, 2\pi)$ and $\dot{\beta}(t) > 0$ for $t \in (t'_a, t_a)$. On the other hand, in the case of $\beta(t) \in (2\pi - \xi_1, 2\pi)$, it can be obtained from (5b) that

$$\dot{\beta}(t) < -\frac{v_r}{d_r} + \frac{v_r}{d_r} + \frac{v_r \sin\beta(t)}{d(t)} < 0. \tag{6}$$

This leads to a contradiction. It therefore follows that $\beta(t) < 2\pi$, $\forall t \geq 0$.

Next, we show that $\beta(t) > 0$, $\forall t \geq 0$ by contradiction argument. Suppose that the result is invalid. Then, there exists a finite time $t_b > 0$ with $\beta(t_b) = 0$ and $\beta(t) > 0$ for $t \in (0, t_b)$. This means that there exist a sufficiently small constant ξ_2 and an instant $t'_b \in (0, t_b)$ such that $\beta(t) \in (0, \xi_2)$ and $\dot{\beta}(t) < 0$ for $t \in (t'_b, t_b)$. This implies that $\beta(t)$ is strictly decreasing for $t \in (t'_b, t_b)$ and reaches 0 at t_b. On the other hand, by virtue of (5a), we have that $\dot{d}(t) \leq |v_r|$. This implies that $d(t) < d(0) + |v_r|t_b$ for $t \in (0, t_b)$. Moreover, since $\beta(t)$ is strictly decreasing for $t \in (t'_b, t_b)$, there exists another instant $t''_b \in (t'_b, t_b)$ such that $\beta(t''_b) = \xi'_2$, where $\xi'_2 \in (0, \xi_2)$ is a sufficiently small constant such that $d_r/\tan(\xi'_2/2) > d(0) + |v_r|t_b$. This thus implies that

$$d(t) < \frac{d_r}{\tan(\xi'_2/2)} < \frac{d_r}{\tan(\beta(t)/2)}, \tag{7}$$

for $t \in (t''_b, t_b)$. In such a case, it follows from (5b) that

$$\begin{aligned}\dot{\beta}(t) &> -\frac{v_r}{d_r} + \frac{v_r \cos\beta(t)}{d_r} + \frac{v_r \sin\beta(t)}{d_r/\tan(\beta(t)/2)} \\ &= \frac{v_r\big(-1 + 1 - 2\sin^2(\beta(t)/2) + 2\sin^2(\beta(t)/2)\big)}{d_r} \\ &= 0,\end{aligned} \tag{8}$$

for $t \in (t''_b, t_b)$, which also results in a contradiction. Therefore, it can be obtained that $\beta(t) > 0$, $\forall t \geq 0$.

With the above arguments, we can conclude that $\beta(t) \in (0, 2\pi)$, $\forall t \geq 0$.

(ii) Still, we show that $d(t) > 0$, $\forall t \geq 0$ using contradiction argument. Suppose that the result does not hold. Then, there exists a limited time $t_c > 0$ with $d(t_c) = 0$ and $d(t) > 0$ for $t \in (0, t_c)$. This indicates that there exist a sufficiently small constant $\sigma_1 \in (0, d_r)$ and an instant $t'_c \in (0, t_c)$ with $d(t) \in (0, \sigma_1)$ and $\dot{d}(t) < 0$ for $t \in (t'_c, t_c)$. In such a case, $\beta(t) \in (0, \pi/2)$ or $\beta(t) \in (3\pi/2, 2\pi)$ for $t \in (t'_c, t_c)$. We next continue our discussions according to two cases.

Case I: $\beta(t) \in (0, \pi/2)$, $t \in (t'_c, t_c)$. In such a case, it follows from (5b) that

$$\dot{\beta}(t) > -\frac{v_r}{d_r} + \frac{v_r \cos\beta(t)}{d_r} + \frac{v_r \sin\beta(t)}{d_r} > 0. \tag{9}$$

This implies that $\beta(t)$ is strictly increasing for $t \in (t'_c, t_c)$. Also, by virtue of (5), we have that

$$\begin{aligned}\dot{\beta}(t) &> -\frac{v_r}{d_r} + \frac{v_r \sin\beta(t)}{d(t)} \\ &= -\frac{v_r}{d_r} - \tan\beta(t)\frac{\dot{d}(t)}{d(t)} \\ &> -\frac{v_r}{d_r} - \tan\beta(t'_c)\frac{\dot{d}(t)}{d(t)},\end{aligned} \tag{10}$$

for $t \in (t'_c, t_c)$. By integrating this inequality on both sides over interval (t'_c, t_c), it can be obtained that

$$\begin{aligned}\beta(t_c) - \beta(t'_c) &> \int_{t'_c}^{t_c} \Big(-\frac{v_r}{d_r} - \tan\beta(t'_c)\frac{\dot{d}(\mu)}{d(\mu)}\Big)\mathrm{d}\mu \\ &= -\frac{v_r}{d_r}(t_c - t'_c) - \tan\beta(t'_c)\ln\frac{d(t_c)}{d(t'_c)} \\ &= +\infty,\end{aligned} \tag{11}$$

given the fact that $d(t_c) = 0$. This implies that $\beta(t_c) = +\infty$, contradicting with the fact $\beta(t) \in (0, 2\pi)$, $\forall t \geq 0$ that has been proven before.

Case II: $\beta(t) \in (3\pi/2, 2\pi)$, $t \in (t'_c, t_c)$. In such a case, it can be obtained from (5b) that

$$\dot{\beta}(t) < \frac{v_r \sin\beta(t)}{d(t)} < 0. \tag{12}$$

This implies that $\beta(t)$ is strictly decreasing for $t \in (t'_c, t_c)$. From (5a) and (12), we next have that

$$\begin{aligned}\dot{\beta}(t) &< -\tan\beta(t)\frac{\dot{d}(t)}{d(t)} \\ &< -\tan\beta(t'_c)\frac{\dot{d}(t)}{d(t)},\end{aligned} \tag{13}$$

for $t \in (t'_c, t_c)$. It can be obtained that $\beta(t_c) = -\infty$, contradicting with the proven fact $\beta(t) \in (0, 2\pi)$, $\forall t \geq 0$.

It follows from the above two cases that $d(t) > 0$, $\forall t \geq 0$. ∎

Note that $\beta(0) \in (0, 2\pi)$ and $d(0) > 0$ require that the vehicle is initialized not towards the target and also not in coincidence with the target. Under such an initial condition, Lemma 1 manifests that the vehicle will never collide with the target. Besides, using Lemma 1, the boundedness of the closed-loop system (5) is guaranteed. This is sufficient for the uniform continuity of $\beta(t)$ and $d(t)$. Additionally, from Lemma 1 we can obtain that $\beta(t)$ always evolves within $(0, 2\pi)$. In such a case, the equilibrium of the closed-loop system (5) is a singleton, i.e., $M = (d_r, \pi/2)$, which is exactly the target state of the circumnavigation.

Next, a theorem is presented to sum up the main result of this paper, which indicates the achievement of the circumnavigation objective using the proposed control algorithm.

Theorem 1: Consider the non-holonomic vehicle system (1). If $\beta(0) \in (0, 2\pi)$ and $d(0) > 0$ are chosen, the proposed control input (4) guarantees that the circumnavigation objective claimed in Definition 1 is achieved.

To prove Theorem 1, we just need to analyze the stabilization of the closed-loop system (5). For this purpose, we first specify a bounded set as follows:

$$S = \Big\{(d, \beta) \mid d \in \Big(0, \Big(1 + \frac{\pi}{2}\Big)d_r\Big], \beta \in (0, \pi]\Big\}. \tag{14}$$

It is trivial that the equilibrium M is within the set S. According to Poincaré-Bendixon Criterion in [27], if i) the state trajectory of (d, β) enters the set S within a finite time and does not leave it afterwards, and ii) there is no periodic orbit within the set S, then the state trajectory of (d, β) asymptotically converges to the equilibrium M. Motivated by this observation, it is thus divided into three parts for the proof of Theorem 1. In the first place, it is shown that β enters the set $(0, \pi]$ within a finite time and does not escape. Upon this basis, it is next shown that d enters the set $(0, (1+\pi/2)d_r]$ within a finite time and does not escape. Finally, it is shown that there exists no periodic orbit within the set S. We next introduce three Propositions to shed light upon these three facts, respectively.

***Proposition** 1:* Consider the closed-loop system (5). If $\beta(0) \in (0, 2\pi)$ and $d(0) > 0$ are chosen, there exists a finite time μ_1 such that $\beta(t) \in (0, \pi]$, $\forall t > \mu_1$.

Proof: Lemma 1 has shown that $\beta(t) \in (0, 2\pi)$, $\forall t \geq 0$ given $\beta(0) \in (0, 2\pi)$. Accordingly, we next discuss two cases depending on the initialization of β.

Case I: $\boldsymbol{\beta(0) \in (0, \pi]}$. In such a case, we show that $\beta(t) \in (0, \pi]$, $\forall t \geq 0$ by contradiction argument. Suppose that the result is invalid. Then, there is a finite time μ_a with $\beta(\mu_a) = \pi$ and $\dot{\beta}(\mu_a) > 0$. However, in the case of $\beta(\mu_a) = \pi$, it follows from (5b) that $\dot{\beta}(\mu_a) = -2v_r/d_r < 0$, leading to a contradiction. Therefore, it follows that $\beta(t) \in (0, \pi]$, $\forall t \geq 0$.

Case II: $\boldsymbol{\beta(0) \in (\pi, 2\pi)}$. In such a case, we show that β will enter $(0, \pi]$ within a finite time and not leave it afterwards. To show this, we make discussions in terms of three subcases, i.e., $\beta(0) \in (\pi, 3\pi/2]$, $\beta(0) \in (3\pi/2, 3\pi/2+\sigma]$, and $\beta(0) \in (3\pi/2+\sigma, 2\pi)$, where $\sigma > 0$ is a constant such that $\sin\sigma < \pi/(2+\pi)$.

Subcase I: $\boldsymbol{\beta(0) \in (\pi, 3\pi/2]}$. We assume that β is always within $(\pi, 3\pi/2]$. If so, it can be obtained from (5b) that $\dot{\beta}(t) < -v_r/d_r$. This indicates that $\beta(t) \leq \pi$, $\forall t \geq \mu_{b_1} = (\beta(0)-\pi)d_r/v_r$, which leads to a contradiction. We thus have that $\beta(t)$ will get away from $(\pi, 3\pi/2]$ and enter $(0, \pi]$ within the finite time μ_{b_1}. Next, by following the analysis in **Case I**, it further follows that $\beta(t) \in (0, \pi]$, $\forall t \geq \mu_{b_1}$.

Subcase II: $\boldsymbol{\beta(0) \in (3\pi/2, 3\pi/2+\sigma]}$. Similar to **Subcase I**, we assume that β is always within $(3\pi/2, 3\pi/2+\sigma]$. If so, it follows from (5b) that

$$\begin{aligned}\dot{\beta}(t) &< -\frac{v_r}{d_r} + \frac{v_r\cos(3\pi/2+\sigma)}{d_r} \\ &= -\frac{v_r}{d_r} + \frac{v_r\sin\sigma}{d_r} < 0.\end{aligned} \tag{15}$$

This implies that $\beta(t) \leq 3\pi/2$, $\forall t \geq \mu_{b_2} = (\beta(0) - 3\pi/2)d_r/(v_r(1-\sin\sigma))$, which results in a contradiction. Hence, we have that $\beta(t)$ will escape from $(3\pi/2, 3\pi/2+\sigma]$ and enter $(\pi, 3\pi/2]$ within the finite time μ_{b_2}. In addition, by following the analysis in **Subcase I** and **Case I**, it further follows that $\beta(t) \in (0, \pi]$, $\forall t \geq \mu'_{b_2} = \mu_{b_2} + \pi d_r/(2v_r)$.

Subcase III: $\boldsymbol{\beta(0) \in (3\pi/2+\sigma, 2\pi)}$. Likewise, we assume that β is always within $(3\pi/2+\sigma, 2\pi)$. If so, it follows from (5a) that $\dot{d}(t) < -v_r\sin\sigma$, which implies that $d(t)$ will decay to zero within a finite time $d(0)/(v_r\sin\sigma)$. This contradicts with the result $d(t) > 0$, $\forall t \geq 0$ given in Lemma 1. On the premise of Lemma 1, it thus follows that $\beta(t)$ will escape from $(3\pi/2+\sigma, 2\pi)$ and enter $(3\pi/2, 3\pi/2+\sigma)$ within a finite time $\mu_{b_3} < d(0)/(v_r\sin\sigma)$. Additionally, by following the analysis in **Subcase II**, **Subcase I**, and **Case I**, it further follows that $\beta(t) \in (0, \pi]$, $\forall t \geq \mu'_{b_3} = \mu_{b_3} + (\pi/2 + \sigma/(1-\sin\sigma))d_r/v_r$.

By combining the results in the above two cases, it can be concluded that $\beta(t) \in (0, \pi]$, $\forall t \geq \mu_1$ with $\mu_1 \geq \mu'_{b_3}$. ■

***Proposition** 2:* Consider the closed-loop system (5). If $\beta(0) \in (0, 2\pi)$ and $d(0) > 0$ are chosen, there exists a finite time $\mu_2 \geq \mu_1$ with $d(t) \in (0, (1+\pi/2)d_r]$, $\forall t \geq \mu_2$.

Proof: According to Lemma 1, we have that $d(t) > 0$, $\forall t \geq 0$. We next analyze the behavior of $d(t)$ for $t \geq \mu_1$ in terms of two cases depending on the value of $d(\mu_1)$.

Case I: $\boldsymbol{d(\mu_1) > (1+\pi/2)d_r}$. In such a case, we show that $d(t)$ will enter $(0, (1+\pi/2)d_r]$ within a finite time and not escape from it afterwards. For this purpose, we first show that d will decrease below $(1+\pi/2)d_r$ within a finite time by contradiction argument. Suppose that the result is invalid. This means that $d(t) > (1+\pi/2)d_r$, $\forall t \geq \mu_1$. We continue our discussions based on three subcases, i.e., $\beta(\mu_1) \in (0, \pi/2-\sigma]$, $\beta(\mu_1) \in (\pi/2-\sigma, \pi/2)$, and $\beta(\mu_1) \in [\pi/2, \pi)$, where the constant σ is defined identically with that in the proof of Proposition 1.

Subcase I: $\boldsymbol{\beta(\mu_1) \in (0, \pi/2-\sigma]}$. First, we illustrate that $\beta(t) \in (0, \pi/2-\sigma]$, $\forall t \geq \mu_1$ by contradiction argument. Assume that the result is ineffective. Then, there exists a limited instant $\zeta_a > \mu_1$ with $\beta(\zeta_a) = \pi/2-\sigma$ and $\dot{\beta}(\zeta_a) > 0$. However, in the case of $\beta(\zeta_a) = \pi/2-\sigma$, it follows from (5b) that

$$\begin{aligned}\dot{\beta}(\zeta_a) &= -\frac{v_r}{d_r} + \frac{v_r\sin\sigma}{d_r} + \frac{v_r\cos\sigma}{d} \\ &< -\frac{v_r}{(1+\pi/2)d_r} + \frac{v_r\cos\sigma}{(1+\pi/2)d_r} < 0,\end{aligned} \tag{16}$$

given $\sin\sigma < \pi/(2+\pi)$. This brings in a contradiction. It therefore follows that $\beta(t) \in (0, \pi/2-\sigma]$, $\forall t \geq \mu_1$. Using this result and by virtue of (5a), it next follows that $\dot{d}(t) \leq -v_r\sin\sigma < 0$ for $t \geq \mu_1$. This implies that $d(t) < (1+\pi/2)d_r$, $\forall t \geq \zeta'_a + \mu_1$ with $\zeta'_a = (d(\mu_1) - (1+\pi/2)d_r)/(v_r\sin\sigma)$, which introduces a contradiction with the fact that $d(t) > (1+\pi/2)d_r$, $\forall t \geq \mu_1$.

Subcase II: $\boldsymbol{\beta(\mu_1) \in (\pi/2-\sigma, \pi/2]}$. We assume that β is always within $(\pi/2-\sigma, \pi/2]$ after μ_1. If so, it follows from (5b) and (16) that

$$\dot{\beta}(t) < -\frac{v_r}{d_r} + \frac{v_r\sin\sigma}{d_r} + \frac{v_r\cos\sigma}{d} < 0, \tag{17}$$

for $t \geq \mu_1$. This indicates that $\beta(t) \in (0, \pi/2-\sigma]$, $\forall t \geq \zeta_b + \mu_1$ with $\zeta_b < d_r\sigma(1+\pi/2)/(v_r(1-\cos\sigma))$, which leads to a contradiction. We thus have that $\beta(t)$ will escape from $(\pi/2-\sigma, \pi/2]$ and enter $(0, \pi/2-\sigma]$ within the finite time $\zeta_b + \mu_1$. In addition, by following the analysis in **Subcase I**, it further follows that $d(t) < (1+\pi/2)d_r$, $\forall t \geq \zeta'_b + \mu_1$ with $\zeta'_b = \max\{(d(\zeta_b+\mu_1) - (1+\pi/2)d_r)/(v_r\sin\sigma), 0\} + \zeta_b$, which also introduces a contradiction with the fact that $d(t) > (1+\pi/2)d_r$, $\forall t \geq \mu_1$.

Subcase III: $\boldsymbol{\beta(\mu_1) \in (\pi/2, \pi)}$. We assume that β is always within $(\pi/2, \pi)$ after μ_1. If so, from (5b), it follows that

$$\dot{\beta}(t) \leq -\frac{v_r}{d_r} + \frac{v_r}{d} < -\frac{v_r}{d_r} + \frac{v_r}{(1+\pi/2)d_r} < 0, \quad (18)$$

for $t \geq \mu_1$. This implies that $\beta(t) < \pi/2$, $\forall t \geq \zeta_c + \mu_1$ with $\zeta_c < (1+\pi/2)d_r/v_r$, which results in a contradiction. We thus have that $\beta(t)$ will escape from $(\pi/2, \pi)$ and enter $(\pi/2-\sigma, \pi/2]$ within the finite time $\zeta_c+\mu_1$. By following the analysis in **Subcase I** and **Subcase II**, it further follows that $d(t) < (1+\pi/2)d_r, \forall t \geq \zeta_c'+\mu_1$ with $\zeta_c' = \max\{(d(\zeta_b+\zeta_c+\mu_1)-(1+\pi/2)d_r)/(v_r \sin\sigma), 0\}+\zeta_b$, which also brings in a contradiction with the fact that $d(t) > (1+\pi/2)d_r, \forall t \geq \mu_1$.

By combining the results in the above three subcases, it can be concluded that there exists a finite time $\mu_2' \geq \zeta_c' + \mu_1$ such that $d(\mu_2') \in (0, (1+\pi/2)d_r]$. Besides, from the above arguments, we also obtain that $\beta(\mu_2') \in (0, \pi/2)$.

Next, we focus on showing that $d(t) \in (0, (1+\pi/2)d_r]$, $\forall t \geq \mu_2'$. By contradiction argument, we assume that the result is ineffective. It implies that there exists a finite time $\zeta_d > \mu_2'$ such that $d(\zeta_d) = (1+\pi/2)d_r$, $\dot{d}(\zeta_d) > 0$, and $d(t) \in (0, (1+\pi/2)d_r)$ for $t \in (\mu_2', \zeta_d)$. According to (5a), this implies that $\beta(\zeta_d) \in (\pi/2, \pi]$. It then follows that there exists an instant $\zeta_d' \in (\mu_2', \zeta_d)$ such that $\beta(\zeta_d') = \pi/2$, $\dot{\beta}(\zeta_d') > 0$, and $\beta(t) \in (\pi/2, \pi]$ for $t \in (\zeta_d', \zeta_d]$. This, from (5a), implies that $\dot{d}(t) > 0$ for $t \in (\zeta_d', \zeta_d]$. Besides, in the case of $\beta(\zeta_d') = \pi/2$, it follows from (5b) that $\dot{\beta}(\zeta_d') = -v_r/d_r + v_r/d(\zeta_d') > 0$ and further that $d(\zeta_d') < d_r$. It thus follows that there exists another instant $\zeta_d'' \in (\zeta_d', \zeta_d)$ such that $d(\zeta_d'') = d_r$, and further that $d(t) > d_r$ for $t \in (\zeta_d'', \zeta_d]$.

Moreover, by virtue of (5), it follows that

$$\begin{aligned}\dot{\beta}(t) &< -\frac{v_r}{d_r} + \frac{v_r \cos\beta(t)}{d_r} + \frac{v_r}{d_r} \\ &= -\frac{\dot{d}(t)}{d_r},\end{aligned} \quad (19)$$

for $t \in (\zeta_d'', \zeta_d]$. Integrating both sides of this inequality over interval $(\zeta_d'', \zeta_d]$ yields

$$\beta(\zeta_d) - \beta(\zeta_d'') < -\frac{(d_r(\zeta_d) - d_r)}{d_r}. \quad (20)$$

given the fact that $d(\zeta_d'') = d_r$. This thus implies that

$$\begin{aligned}d(\zeta_d) &< d_r + d_r(\beta(\zeta_d'') - \beta(\zeta_d)) \\ &< (1+\pi/2)d_r,\end{aligned} \quad (21)$$

which contradicts with the fact that $d(\zeta_d) = (1+\pi/2)d_r$. Therefore, it can be concluded that $d(t) \in (0, (1+\pi/2)d_r]$, $\forall t \geq \mu_2'$.

Case II: $d(\mu_1) \in (0, (1+\pi/2)d_r]$. We show that $d(t) \in (0, (1+\pi/2)d_r]$ after a finite time and the result is always valid. For one thing, if $d(t) \in (0, (1+\pi/2)d_r]$, $\forall t \geq \mu_1$, then the result holds naturally. For another thing, if $d(t) > (1+\pi/2)d_r$ within a finite time $\xi_a > \mu_1$, it can follows from **Case I** that $d(t) \in (0, (1+\pi/2)d_r]$, $\forall t > \mu_2'' > \xi_a$.

By combining the results given in **Cases I** and **Case II**, it can be concluded that $d(t) \in (0, (1+\pi/2)d_r], \forall t > \mu_2 = \max\{\mu_2', \mu_2''\}$. ■

***Proposition** 3:* Consider the closed-loop (5). There is no periodic orbit within the bounded set S.

Proof: It is trivial to examine that the set S is a simply connected region. Accordingly, we can choose a continuously differentiable function $D(d\ \beta) = d$ such that

$$\frac{\partial(D\dot{d})}{\partial d} + \frac{\partial(D\dot{\beta})}{\partial \beta} = \frac{-v_r d \sin\beta}{d_r} \leq 0, \quad (22)$$

along the closed-loop system (5) within the set S. Note that the equality in (22) holds if and only if $\beta = \pi$. However, if $\beta = \pi$, by virtue of (5b), $\dot{\beta}(t) = -2v_r/d_r < 0$. This implies that there exists no sub-region within S such that (22) is identical to zero therein. Therefore, according to Bendixson-Dulac Theorem [27], there is no periodic orbit within the set S. ■

Proof of Theorem 1. According to Propositions 1-3, we know that the state trajectory of (d, β) enters the bounded set S within a finite time and does not escape afterwards, while there is no periodic orbit within S. Therefore, based upon Poincaré-Bendixon Criterion, it can be concluded that the state trajectory of (d, β) asymptotically converges to the equilibrium M. That is, the circumnavigation objective claimed in Definition 1 is achieved.

IV. SIMULATION

In this section, we make a comparison among the proposed control algorithm (4) and those in [25], [26] to highlight the advantages of the proposed one. Given a target $T = [0, 0]^T$m, the desired circumnavigating radius $d_r = 2$m and the constant linear velocity $v_r = 0.1$m/s. Further, the vehicle is initially placed at $p(0) = [0.5, -0.5]^T$m, $\psi(0) = \pi/3$ rad.

The simulation results are shown in Fig.2 and Fig.3. It can be observed from Fig.2 that under the proposed control algorithm (4) and those in [25] and [26], the vehicle converges to the desired circle. However, compared with the control algorithms in [25] and [26], our proposed one shows better performance in the process of convergence, and generates a smoother trajectory. It is shown in Fig.3 that the state d asymptotically converge to d_r and the angular velocity, serving as the input, asymptotically converges to the desired one. However, there are some differences in the process of convergence. We can observe that the proposed algorithm (4) can ensure the state d converges with a faster rate and there is no overshoot. However, the control input with the switching algorithm proposed in [25] remains zero at the beginning, which results in discontinuity and unsmoothness. Further, in [26], the control input is large at around 2 second. That is because the control algorithm is not saturated. In contrast, the proposed control algorithm (4) is bounded and continuous. According to the above comparison, it is confirmed that the control algorithm proposed in this paper has better convergence performance.

V. CONCLUSION

In this paper, we design a generic bounded and continuous control algorithm to drive a non-holonomic vehicle to circumnavigate the stationary target with only the distance rate

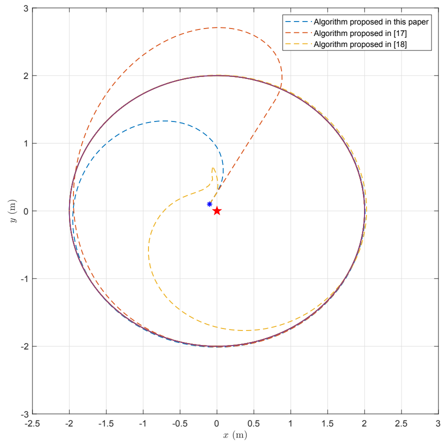

Fig. 2: Trajectory comparison using methods in this paper, [25], and [26].

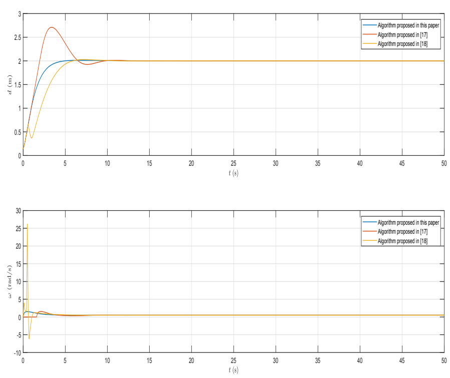

Fig. 3: Distance and angular velocity comparison using methods in this paper, [25], and [26].

measurement. Under a mild initial condition, the proposed control algorithm can ensure the asymptotic convergence without colliding with the target. Resorting to Poincaré-Bendixon Criterion, we analyze the stability in detail. The effectiveness of the proposed control algorithm is finally verified by simulation.

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