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From Noisy Data to Consensus Control: A Localized Design Approach

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Abstract— **This paper considers a localized data-driven consensus problem for leader-follower multi-agent systems characterized by unknown linear agent dynamics, where each agent computes its local control gain using only its locally collected noise-corrupted data. Both discrete-time and continuous-time data-driven protocols are presented, which can achieve leader-follower consensus by handling the challenge of the heterogeneity in control gains caused by local data sampling. The design of these data-driven consensus protocols involves low-dimensional linear matrix inequalities. Simulation examples are provided to demonstrate the effectiveness of the proposed methods.**

I. INTRODUCTION

Over the past decades, there has been a growing interest in data-driven control due to its advantages of not requiring precise system models that often contain redundant parameters and are difficult to identify accurately through experiments. Quite a few studies have emerged on data-driven control for linear systems, adopting the paradigm of Willems's fundamental lemma proposed in [1]. For instance, a data-based architecture is developed in [2], where the control input sequences are set to be persistently exciting (PE) during the data collection procedure, to address several classic control problems including stabilization, optimality, and robust control. In [3], the definition of data informativity is first introduced to decide the necessity of the PE condition by defining a system set that is compatible with the collected data. Additionally, various control tools, including S-lemma [4], [5], linear fractional transformations [6], [7], Finsler's lemma [8], Petersen's lemma [9], and system level synthesis approach [10], have been leveraged to address data-driven control problems based on the noise-corrupted data.

More lately, considerable attention has been devoted to data-driven control of network systems, owing to their wide-ranging applications and remarkable extensibility. Nevertheless, the inherent constraint of the network system and the distinct data sampled from different agents generally bring challenges to the design of the data-based algorithms. The works in [11] address an output synchronization problem for heterogeneous linear systems using noise-corrupted data. However, this approach assumes precise knowledge of the external noise data, which is often unrealistic in cyber-physical systems. Moreover, a distributed predictive control scheme is devised in [12] utilizing sampled data to stabilize coupled network systems. Additionally, intriguing studies in [13], [14], [15] introduce data-based consensus algorithms for general linear multi-agent systems, in which an identical control gain is computed based on the data collected at one single agent and shared across all agents.

Inspired by the aforesaid discussions, in this paper we consider consensus control based on noise-corrupted data for both continuous-time and discrete-time leader-follower multi-agent systems with general linear dynamics from a fresh perspective. In contrast to existing centralized data-driven mechanisms relying on data sampled from one single agent [13], [14], we propose a localized data-based control architecture adopting the informativity approach, where each agent samples its local data and calculates its local control gain using its own collected data. It is noteworthy that the sampled data from different agents are generally distinct from each other, introducing heterogeneity into control gains. To surmount this obstacle, we develop control protocols in a distributed fashion by synchronizing the distinct control gains to reach consensus for the leader-follower system.

The main contributions of this paper are at least three-fold: i) The localized data-driven consensus algorithms presented in this paper, circumventing the centralized design approach in [13], [14] necessitating an identical data-based gain for all agents, determine local gains for each agent in a coordinated way using locally collected data, which is consistent with the essential nature of distributed control. ii) We present specific design methods for data-driven consensus control algorithms tailored to both continuous-time and discrete-time multi-agent systems, accompanied by rigorous theoretical stability analyses. iii) The data-driven consensus protocols are formulated by solving low-dimensional linear matrix inequalities (LMIs) in this paper. By contrast, the LMI conditions obtained in [14], proportional to the scale of the network, are typically of high dimensions.

The organization of this article is as follows: In Section II, we present the problem formulation. In Sections III and IV, we propose the localized data-driven consensus protocol for discrete-time and continuous-time multi-agent systems based on the noise-corrupted data, respectively. In Section V, we present simulation examples to verify the validity of the proposed algorithms. Section VI concludes this paper.

II. PROBLEM FORMULATION

Consider the multi-agent system consisting of one leader and N followers with the following linear time-invariant dynamics:

$$\delta x_i(t) = Ax_i(t) + Bu_i(t),\ i = 0, 1, \cdots, N, \tag{1}$$

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where $\delta x_i(t) = x_i(t+1)$ for the discrete-time case and $\delta x_i(t) = \dot{x}_i(t)$ for the continuous-time case, $u_i(t) \in \mathbb{R}^p$ and $x_i(t) \in \mathbb{R}^n$ represent the input and state of the i-th agent. The pair (A,B) in (1) is assumed to be controllable but *unknown*. The control input of the leader indexed by node 0 in (1) is set to $u_0 = 0$.

The network topology of the system (1) is denoted by a graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, where $\mathscr{V}$ represents the set of nodes and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ stands for the set of edges. Agent j can receive information from agent i if the ordered pair $(i,j) \in \mathscr{E}$. The adjacent matrix $\mathscr{A}$ associated with the graph $\mathscr{G}$ is defined such that $a_{ij} > 0$ if $(j,i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. The degree matrix $\mathscr{D}$ is defined as $\mathscr{D} = \mathrm{diag}(d_0, \cdots, d_N)$, where $d_i = \Sigma_{j=0}^N a_{ij}$ for $i = 0,1,2,\cdots,N$. The Laplacian matrix is defined as $L = \mathscr{D} - \mathscr{A}$.

Assumption 1: The graph $\mathscr{G}$ associated with the agents in (1) has a directed spanning tree with the leader agent as a root. The subgraph among followers is undirected.

Under the above assumption, the Laplacian matrix associated with $\mathscr{G}$ can be partitioned as $L = \begin{bmatrix} 0 & 0 \\ L_{fl} & L_{ff} \end{bmatrix}$.

In view of the fact that the system parameter matrices A and B are unknown, we need to design consensus control protocols based on the sampled data. It should be noted that the agent dynamics are often affected by prevalent disturbances and measurement errors [16]. Thus, during the data collection procedure, we consider the following agent dynamics corrupted by external perturbations:

$$\delta x_i(t) = Ax_i(t) + Bu_i(t) + \omega_i(t),\ i = 0,1,\cdots,N. \tag{2}$$

Specifically, we sample finite-length data from each agent and build the following data matrices:

$$\begin{aligned}
X_{i-} &= \begin{bmatrix} x_i(1) & x_i(2) & \cdots & x_i(T) \end{bmatrix}, \\
U_{i-} &= \begin{bmatrix} u_i(1) & u_i(2) & \cdots & u_i(T) \end{bmatrix}, \\
X_{i+} &= \begin{bmatrix} x_i(2) & x_i(3) & \cdots & x_i(T+1) \end{bmatrix} \text{ in discrete time}, \\
X_{i+} &= \begin{bmatrix} \dot{x}_i(1) & \dot{x}_i(2) & \cdots & \dot{x}_i(T) \end{bmatrix} \text{ in continuous time}, \\
W_i &= \begin{bmatrix} \omega_i(1) & \omega_i(2) & \cdots & \omega_i(T) \end{bmatrix},\ i = 0,1,\cdots,N,
\end{aligned} \tag{3}$$

where the noise matrix W_i is unknown and each agent only has access to its own sampled data.

Clearly, the constructed data matrices (3) satisfy

$$X_{i+} = AX_{i-} + BU_{i-} + W_i \tag{4}$$

for $i = 0,1,\cdots,N$, according to (2).

Assumption 2: The constructed data matrix $\begin{bmatrix} X_{i-} \\ U_{i-} \end{bmatrix}$ satisfies $\mathrm{rank} \begin{bmatrix} X_{i-} \\ U_{i-} \end{bmatrix} = n + p$ for $i = 1,2,\cdots,N$.

Next, the control objective is to devise discrete-time and continuous-time consensus control protocols based on the sampled data (3) for the followers in (1) to achieve leader-follower consensus, i.e., $\lim_{t\to\infty} x_i(t) - x_0(t) = 0$ for $i = 1,2,\cdots,N$.

The following lemmas will be exploited in the subsequent data-based control protocol design.

Lemma 1: (Matrix S-lemma [4]) Consider two matrices

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}, F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

with $E_{22} \leq 0$ and $F_{22} \leq 0$. Suppose that $\ker(F_{22}) \subseteq \ker(F_{12})$. If there exists a matrix $\bar{C}$ such that $\begin{bmatrix} I \\ \bar{C} \end{bmatrix}^T F \begin{bmatrix} I \\ \bar{C} \end{bmatrix} > 0$, then $\begin{bmatrix} I \\ C \end{bmatrix}^T E \begin{bmatrix} I \\ C \end{bmatrix} > 0$ holds for all C satisfying $\begin{bmatrix} I \\ C \end{bmatrix}^T F \begin{bmatrix} I \\ C \end{bmatrix} > 0$ if and only if there exist scalars $\alpha \geq 0$ and $\beta > 0$ such that

$$E - \alpha F \geq \begin{bmatrix} \beta I & 0 \\ 0 & 0 \end{bmatrix}.$$

Lemma 2: (Petersen's Lemma [9]) Consider matrices $\mathscr{Q} \in \mathbb{R}^{n\times n}$, $\mathscr{W} \in \mathbb{R}^{n\times p}$, $\bar{\mathscr{H}} \in \mathbb{R}^{q\times q}$, and $\mathscr{S} \in \mathbb{R}^{q\times n}$ with $\mathscr{Q} = \mathscr{Q}^T$ and $\bar{\mathscr{H}} = \bar{\mathscr{H}}^T \geq 0$. Define $H = \{\mathscr{H} \in \mathbb{R}^{p\times q} : \mathscr{H}^T\mathscr{H} \leq \bar{\mathscr{H}}\}$. Then, $\mathscr{Q} + \mathscr{W}\mathscr{H}\mathscr{S} + \mathscr{S}^T\mathscr{H}^T\mathscr{W}^T < 0$ for all $\mathscr{H} \in H$ if and only there exists a scalar $\varepsilon > 0$ such that $\mathscr{Q} + \varepsilon\mathscr{W}\mathscr{W}^T + \varepsilon^{-1}\mathscr{S}^T\bar{\mathscr{H}}\mathscr{S} < 0$.

III. Localized Data-driven consensus control for discrete-time systems

In this section, we provide the specific design methods of discrete-time data-driven consensus control protocols for the agents in (1).

We suppose that the additive noise matrix W_i satisfies the following assumption, which also appears in several existing results [4], [14], [17].

Assumption 3: The noise matrix W_i satisfies

$$\begin{bmatrix} I & W_i \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} I \\ W_i^T \end{bmatrix} \geq 0,\ i = 0,1,\cdots,N, \tag{5}$$

where known matrices $N_{11} > 0$, $N_{22} < 0$, and $N_{12}^T = N_{21}$ are of suitable dimensions.

From (4) and (5), we can obtain that the discrete system pair (A,B) in (1) satisfies

$$\begin{bmatrix} I \\ A^T \\ B^T \end{bmatrix}^T \begin{bmatrix} I & X_{i+} \\ 0 & -X_{i-} \\ 0 & -U_{i-} \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} I & X_{i+} \\ 0 & -X_{i-} \\ 0 & -U_{i-} \end{bmatrix}^T \begin{bmatrix} I \\ A^T \\ B^T \end{bmatrix} \geq 0, \tag{6}$$

where $i = 0,1,\cdots,N$. It is worth noting that the system model (A,B) in (1) cannot be identified accurately from the sampled data (3) due to the external perturbations $\omega_i(t)$ during the data collection. In other words, there may be multiple systems that are able to generate the sampled data (3). We define a discrete system set that is compatible with the collected data (3) as $\mathbf{S}_i = \{(A,B)|(A,B) \text{ satifies } (6)\}$. Obviously, the true system $(A,B) \in \mathbf{S}_i$. Therefore, we aim to design a common control protocol for the discrete-time agents in (1) such that all the systems in $\mathbf{S}_i$ can achieve leader-follower consensus under the developed control protocol.

The localized data-driven consensus protocol for the discrete-time agents in (1) is proposed as follows:

$$\begin{aligned}
u_i(t) &= \alpha K_i(t) \sum_{j=0}^N a_{ij}[x_i(t) - x_j(t)], \\
K_i(t+1) &= K_i(t) + \sum_{j=0}^N w_{ij}[K_j(t) - K_i(t)],
\end{aligned} \tag{7}$$

for $i=1,\cdots,N$, where $\alpha=\frac{2}{\lambda_1+\lambda_N}$, $\lambda_1=\lambda_{\min}(L_{ff})$, $\lambda_N=\lambda_{\max}(L_{ff})$, $w_{ij}=\frac{a_{ij}}{1+d_i}$, and $w_{ii}=\frac{1}{1+d_i}$.

Before moving on, we extend the definition of data informativity to the case of consensus of the discrete-time multi-agent system (1), which serves as a foundational scheme to achieve leader-follower consensus.

Definition 1: Suppose that Assumptions 1-3 hold. The collected data (X_{i+},X_{i-},U_{i-}) are informative for leader-follower consensus of the discrete system (1), if there exists a common data-based feedback gain matrix $K_i(0)$ such that $I_N\otimes A+\alpha L_{ff}\otimes BK_i(0)$ is Schur stable for all $(A,B)\in\mathbf{S}_i$.

Next, we will design the initial gain matrices $K_i(0)$ in the protocol (7) for $i=0,1,\cdots,N$, based on the collected noise-corrupted data (3).

Theorem 1: Suppose that Assumptions 1-3 hold. The collected data (X_{i+},X_{i-},U_{i-}) are informative for leader-follower consensus of the discrete system (1), if there exist matrices $\Phi_i>0$, F_i and scalars $\varepsilon_i\ge 0$, $\gamma_i>0$, and $\tau_i>0$ satisfying the following LMI:

$$\begin{bmatrix}\Phi_i-\gamma_i I & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \Phi_i & 0 & 0\\ 0 & 0 & -\tau_i\nu^2 I & F_i & 0 & 0\\ 0 & \Phi_i^T & F_i^T & \Phi_i & F_i^T & 0\\ 0 & 0 & 0 & F_i & \tau_i I & 0\\ 0 & 0 & 0 & 0 & 0 & I\end{bmatrix}-\varepsilon_i\begin{bmatrix}I & X_{i+}\\ 0 & -X_{i-}\\ 0 & -U_{i-}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\times\begin{bmatrix}N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}[*]^T\ge 0, \quad (8)$$

where $\nu=\frac{\lambda_N-\lambda_1}{\lambda_N+\lambda_1}$ and $[*]^T$ denotes the matrix that can be deduced by symmetry. Then, the protocol (7) with $\alpha=\frac{2}{\lambda_1+\lambda_N}$ and $K_i(0)=F_i\Phi_i^{-1}$ for $i=0,1,\cdots,N$ achieves leader-follower consensus for the discrete-time agents in (1).

Proof: Define

$$\Psi_i=[I_N\otimes A+\alpha L_{ff}\otimes BK_i(0)]^T(I_N\otimes P_i)\times[I_N\otimes A+\alpha L_{ff}\otimes BK_i(0)]-I_N\otimes P_i,$$

where $P_i>0$. According to Definition 1, the collected data (X_{i+},X_{i-},U_{i-}) are informative for leader-follower consensus of the discrete system (1) if there exist appropriate $K_i(0)$ and P_i such that $\Psi_i<0$ for all $(A,B)\in\mathbf{S}_i$. Let $\Phi_i=P_i^{-1}$ and $F_i=K_i(0)\Phi_i$. Consequently, $\Psi_i<0$ is equivalent to

$$\Phi_i-(A\Phi_i+\alpha\lambda_k BF_i)^T\Phi_i^{-1}(A\Phi_i+\alpha\lambda_k BF_i)>0,$$

for $k=1,\cdots,N$, where λ_k denotes the k-th eigenvalue of L_{ff}. This implies that $\Psi_i<0$ can be transformed into the above N inequalities. Choose $\alpha=\frac{2}{\lambda_1+\lambda_N}$. Evidently, $-\nu\le\alpha\lambda_k-1\le\nu$ for $k=1,\cdots,N$. Motivated by [18], [19], it can be inferred that $A\Phi_i+\alpha\lambda_k BF_i$ is Schur stable for $k=1,\cdots,N$ if $A\Phi_i+(1+\Delta)BF_i$ is Schur stable for all $|\Delta|\le\nu$. Then, it follows that $\Psi_i<0$ holds, if there exists $\Phi_i>0$ such that

$$\Phi_i-(A\Phi_i+(1+\Delta)BF_i)\Phi_i^{-1}(A\Phi_i+(1+\Delta)BF_i)^T>0,$$

which is equivalent to

$$\begin{bmatrix}I\\ A^T\\ B^T\end{bmatrix}^T\begin{bmatrix}\Phi_i & 0\\ 0 & -\begin{bmatrix}\Phi_i\\ (1+\Delta)F_i\end{bmatrix}(\Phi_i)^{-1}[*]^T\end{bmatrix}\begin{bmatrix}I\\ A^T\\ B^T\end{bmatrix}>0. \quad (9)$$

Note that

$$\begin{bmatrix}\Phi_i & 0\\ 0 & -\begin{bmatrix}\Phi_i\\ (1+\Delta)F_i\end{bmatrix}(\Phi_i)^{-1}[*]^T\end{bmatrix}>0 \quad (10)$$

holds if and only if

$$\begin{bmatrix}\Phi_i & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \Phi_i & 0\\ 0 & 0 & 0 & F_i & 0\\ 0 & \Phi_i & F_i^T & \Phi_i & 0\\ 0 & 0 & 0 & 0 & I\end{bmatrix}+\mathbf{F}_i^T\Delta\mathbf{I}+\mathbf{I}^T\Delta\mathbf{F}_i>0, \quad (11)$$

where $\mathbf{F}_i=\begin{bmatrix}0 & 0 & 0 & F_i & 0\end{bmatrix}$ and $\mathbf{I}=\begin{bmatrix}0 & 0 & I & 0 & 0\end{bmatrix}$. Utilizing Lemma 2, (11) holds for all $|\Delta|<\nu$ if and only if there exists a scalar $\tau_i>0$ such that

$$\begin{bmatrix}\Phi_i & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \Phi_i & 0 & 0\\ 0 & 0 & -\tau_i\nu^2 I & F_i & 0 & 0\\ 0 & \Phi_i^T & F_i^T & \Phi_i & F_i^T & 0\\ 0 & 0 & 0 & F_i & \tau_i I & 0\\ 0 & 0 & 0 & 0 & 0 & I\end{bmatrix}>0. \quad (12)$$

It is worth noting that using the Schur Complement lemma [20] and pre- and post-multiplying $\begin{bmatrix}I & A & B\end{bmatrix}$ and $\begin{bmatrix}I & A & B\end{bmatrix}^T$ on (12) directly leads to (9), implying that (12) is a sufficient condition of (9). Note that all systems (A,B) in $\mathbf{S}_i$ satisfy (6). Then, using Lemma 1 for (6) and (12) leads directly to (8). Next, we can conclude that if (8) is true, then (9) holds for all $(A,B)\in\mathbf{S}_i$ and thereby $\Psi_i<0$, implying that the data (X_{i+},X_{i-},U_{i-}) are informative for consensus and hence $I_N\otimes A+\alpha L_{ff}\otimes BK_i(0)$ is Schur stable for all $(A,B)\in\mathbf{S}_i$ with $\alpha=\frac{2}{\lambda_1+\lambda_N}$ and $K_i(0)=F_i\Phi_i^{-1}$.

Finally, we need to prove that the proposed control protocol (7) along with the feedback gain matrix $K_i(0)$ obtained by (8) can achieve consensus for the discrete-time agents in (1). Define $\tilde{x}_i(t)=x_i(t)-x_0(t)$, $\tilde{K}_i(t)=K_i(t)-K_0(0)$, and $\tilde{x}(t)=\begin{bmatrix}\tilde{x}_1^T(t) & \tilde{x}_2^T(t) & \cdots & \tilde{x}_N^T(t)\end{bmatrix}^T$. Substituting (7) into (1) gives

$$\tilde{x}(t)=[I_N\otimes A+\alpha L_{ff}\otimes BK_0(0)]\tilde{x}(t)+\begin{bmatrix}\alpha(L_{ff})_1\otimes B\tilde{K}_1(t)\\ \vdots\\ \alpha(L_{ff})_N\otimes B\tilde{K}_N(t)\end{bmatrix}\tilde{x}(t), \quad (13)$$

where $(L_{ff})_i$ represents the i-th row of L_{ff}. It can be inferred from [21] that $K_i(t)$ exponentially converges to $K_0(0)$ for $i=1,\cdots,N$. Note that $K_0(0)$ renders $I_N\otimes A+\alpha L_{ff}\otimes BK_0(0)$ Schur stable for all (A,B) in $\mathbf{S}_0$, as evidenced by the aforementioned analysis. Then, in view of Lemma 4 in [21], we can derive from (13) that $\tilde{x}(t)\to 0$. This completes the proof. ∎

Remark 1: Discrete-time data-driven consensus problems are also studied in [13], [14]. However, the control algorithms in these works require an identical data-based feedback gain for all agents, essentially demanding a centralized mechanism to collect data, compute the gain, and assign it to every agent. In contrast, our approach provides a distributed control architecture, where each follower computes its initial local gain using its own locally sampled data. To tackle the heterogeneity induced by different data-based gains, an interaction mechanism is designed to synchronize the feedback gain $K_i(t)$ in (7). Besides, the network system in [14] is transformed into a single linear system represented in compact forms, directly leading to a high-dimensional LMI that is hard to solve for large-scale networks. On the contrary, the method given in Theorem 1 allows each agent to compute its own initial gain matrix with a low-dimensional LMI, making the proposed algorithm more applicable and accessible in complex network scenarios.

IV. Localized data-driven consensus control for continuous-time systems

In this section, we consider the design of a data-driven consensus control algorithm for the continuous-time multi-agent system (1). We need the following assumptions for the solvability of this problem

Assumption 4: The input matrix B satisfies $BB^T \leq \Omega\Omega^T$, where Ω is a known matrix with full row rank.

Assumption 5: The noise matrix W_i satisfies $W_iW_i^T \leq \Delta\Delta^T$ for $i=0,1,\cdots,N$, where Δ is a known matrix with full row rank.

Similar to the analysis in Section III, there may be infinite systems that can generate the collected data. Therefore, we define the continuous system set that is consistent with the sampled data (3) as $\Sigma_i = \{\begin{bmatrix} A & B\end{bmatrix} : X_{i+} = AX_{i-} + BU_{i-} + W_i, W_iW_i^T \leq \Delta\Delta^T\}$. Substituting $W_i = X_{i+} - AX_{i-} - BU_{i-}$ into the noise constraint, we can reformulate the system set Σ_i as

$$\begin{aligned}\Sigma_i &= \{\begin{bmatrix} A & B\end{bmatrix} : \begin{bmatrix} I & A & B\end{bmatrix}\begin{bmatrix}\mathbf{Z}_i & \mathbf{X}_i^T\\ \mathbf{X}_i & \mathbf{V}_i\end{bmatrix}\begin{bmatrix} * \end{bmatrix}^T \leq 0\}\\ &= \{\begin{bmatrix} A & B\end{bmatrix} = C^T : \mathbf{Z}_i + C^T\mathbf{X}_i + \mathbf{X}_i^TC + C^T\mathbf{V}_iC \leq 0\},\end{aligned}$$

where $\mathbf{Z}_i = X_{i+}X_{i+}^T - \Delta\Delta^T$, $\mathbf{X}_i = -\begin{bmatrix}X_{i-}\\ U_{i-}\end{bmatrix}X_{i+}^T$, and $\mathbf{V}_i = \begin{bmatrix}X_{i-}\\ U_{i-}\end{bmatrix}\begin{bmatrix}X_{i-}\\ U_{i-}\end{bmatrix}^T$. It is straightforward to note that the system set Σ_i is a matrix ellipsoid, which is equivalent to

$$\Sigma_i = \{\begin{bmatrix} A & B\end{bmatrix} = C^T : (C-\xi_i)^T\mathbf{V}_i(C-\xi_i) \leq \mathbf{A}_i\},$$

where $\xi_i = -\mathbf{V}_i^{-1}\mathbf{X}_i$ and $\mathbf{A}_i = \mathbf{X}_i^T\mathbf{V}_i^{-1}\mathbf{X}_i - \mathbf{Z}_i$. Define $\mathscr{T}_i = \{(\xi_i + \mathbf{V}_i^{-1/2}\mathscr{R}_i\mathbf{A}_i^{1/2})^T : \|\mathscr{R}_i\| \leq 1\}$. It is proved in [9] that $\Sigma_i = \mathscr{T}_i$ and the unknown system matrices $\begin{bmatrix} A & B\end{bmatrix} = C^T$ in Σ_i can be parameterized by $C = \xi_i + \mathbf{V}_i^{-1/2}\mathscr{R}_i\mathbf{A}_i^{1/2}$, which is a combination of the constructed data matrices $\xi_i, \mathbf{V}_i, \mathbf{A}_i$, and the uncertainty term $\mathscr{R}_i$.

The goal of this section is to devise a common control protocol for all the continuous systems in Σ_i to achieve leader-follower consensus. In this case, the localized control protocol is devised as follows:

$$\begin{aligned} u_i(t) &= \beta K_i(t)\sum_{j=0}^{N} a_{ij}[x_i(t) - x_j(t)],\\ \dot{K}_i(t) &= h\sum_{j=0}^{N} a_{ij}[K_j(t) - K_i(t)], i = 1,2,\cdots,N,\end{aligned} \tag{14}$$

where $h > 0$ is a scalar and $\beta = \frac{2}{\lambda_1 + \lambda_N}$.

Next, we will design the original gain matrix $K_i(0)$ in (7) for $i = 0,1,\cdots,N$ based on their locally collected data (3). Toward this, we first introduce the definition of data informativity for leader-follower consensus of the continuous linear multi-agent system (1).

Definition 2: The collected data (X_{i+}, X_{i-}, U_{i-}) are informative for leader-follower consensus of the continuous system (1), if there exists a common data-based feedback gain matrix $K_i(0)$ such that $I_N \otimes A + \beta L_{ff} \otimes BK_i(0)$ is Hurwitz for all $(A,B) \in \Sigma_i$.

We present the main result of this section as below.

Theorem 2: Let Assumptions 1, 2, 4, and 5 hold. If there exist matrices Q_i, Y_i, and a scalar $\varepsilon_i > 0$ satisfying the following LMI:

$$\begin{bmatrix} -\mathbf{Z}_i^T + \varepsilon_i\Omega\Omega^T & Y_i^T & \mathbf{X}_i^T - \begin{bmatrix}Q_i\\ Y_i\end{bmatrix}^T\\ Y_i & -\frac{\varepsilon_i}{\nu^2} & 0\\ \mathbf{X}_i - \begin{bmatrix}Q_i\\ Y_i\end{bmatrix} & 0 & -\mathbf{V}_i\end{bmatrix} < 0, \tag{15}$$

in which $\nu = \frac{\lambda_N - \lambda_1}{\lambda_N + \lambda_1}$, then the sampled data (X_{i+}, X_{i-}, U_{i-}) are informative for leader-follower consensus of the continuous system (1). Moreover, the proposed control protocol (7) with $K_i(0) = Y_iQ_i^{-1}$ for $i = 0,1,\cdots,N$ achieves leader-follower consensus for the continuous-time agents in (1).

Proof: Define

$$\begin{aligned}\mathbf{R}_i = &[I_N \otimes A + \beta L_{ff} \otimes BK_i(0)]\mathscr{Q}_i\\ &+ \mathscr{Q}_i[I_N \otimes A + \beta L_{ff} \otimes BK_i(0)]^T,\end{aligned}$$

where $\mathscr{Q}_i = I_N \otimes Q_i$. The collected data (X_{i+}, X_{i-}, U_{i-}) are informative for consensus of the continuous system (1) if there exists a constant matrix $Q_i > 0$ such that $\mathbf{R}_i < 0$ for all $(A,B) \in \Sigma_i$. Note that $\mathbf{R}_i < 0$ is equivalent to

$$[A + \beta\lambda_k BK_i(0)]Q_i + Q_i[A + \beta\lambda_k BK_i(0)]^T < 0 \tag{16}$$

for $k = 1,2,\cdots,N$, where λ_k represents the k-th eigenvalue of the matrix L_{ff}. Rewrite (16) into

$$\begin{aligned}&\begin{bmatrix} A & B\end{bmatrix}\begin{bmatrix} I\\ K_i(0)\end{bmatrix}Q_i + Q_i\begin{bmatrix} I\\ K_i(0)\end{bmatrix}^T\begin{bmatrix} A & B\end{bmatrix}^T\\ &+(\beta\lambda_k - 1)BK_i(0)Q_i + (\beta\lambda_k - 1)QK_i^T(0)B^T < 0.\end{aligned} \tag{17}$$

Let $Y_i = K_i(0)Q_i$. Note that $-\nu \leq \beta\lambda_k - 1 \leq \nu$ for $k = 1,2,\cdots,N$. It then follows that (17) is true for $k = 1,2,\cdots,N$, if

$$\begin{bmatrix} A & B\end{bmatrix}\begin{bmatrix}Q_i\\ Y_i\end{bmatrix} + \begin{bmatrix}Q_i\\ Y_i\end{bmatrix}^T\begin{bmatrix} A & B\end{bmatrix}^T + \rho BY_i + \rho Y_i^TB^T < 0 \tag{18}$$

holds for all $|\rho| \leq \nu$. Utilizing Lemma 2, we can deduce that (18) holds, if there exists $\tau > 0$ such that

$$\begin{bmatrix} A & B\end{bmatrix}\begin{bmatrix}Q_i\\ Y_i\end{bmatrix} + \begin{bmatrix}Q_i\\ Y_i\end{bmatrix}^T\begin{bmatrix} A & B\end{bmatrix}^T + \tau\nu^2Y_i^TY_i + \frac{1}{\tau}BB^T < 0. \tag{19}$$

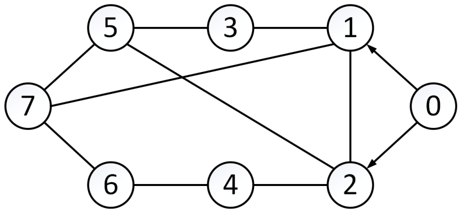

Fig. 1. The network topology

Multiple both sides of (19) by τ and "absorb" it in Q_i and Y_i, so that (19) is equivalent to

$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \begin{bmatrix} A & B \end{bmatrix}^T + \nu^2 Y_i^T Y_i + BB^T < 0. \quad (20)$$

In view of Assumption 4, it is obtained that (20) holds, if

$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \begin{bmatrix} A & B \end{bmatrix}^T + \nu^2 Y_i^T Y_i + \Omega\Omega^T < 0. \quad (21)$$

Substituting the parametrization of $\begin{bmatrix} A & B \end{bmatrix}^T = \xi_i + \mathbf{V}_i^{-1/2} \mathscr{R}_i \mathbf{A}_i^{1/2}$ into (21) yields

$$\begin{aligned} &\xi_i^T \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \xi_i + \mathbf{A}_i^{1/2} \mathscr{R}_i^T \mathbf{V}_i^{-1/2} \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} \\ &+ \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \mathbf{V}_i^{-1/2} \mathscr{R}_i \mathbf{A}_i^{1/2} + \nu^2 Y_i^T Y_i + \Omega\Omega^T < 0. \end{aligned} \quad (22)$$

Utilizing Lemma 2 again, we have (22) holds if and only if there exists $\varepsilon_i > 0$ such that

$$\begin{aligned} &\xi_i^T \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \xi_i + \frac{1}{\varepsilon_i} \mathbf{A}_i + \varepsilon_i \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \mathbf{V}_i^{-1} \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} \\ &+ \nu^2 Y_i^T Y_i + \Omega\Omega^T < 0. \end{aligned} \quad (23)$$

Multiplying both sides of (23) by ε_i again and "absorbing" it in P_i, Y_i, we can deduce that (23) is true, if

$$\begin{aligned} &\xi_i^T \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \xi_i + \mathbf{A}_i + \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}^T \mathbf{V}_i^{-1} \begin{bmatrix} Q_i \\ Y_i \end{bmatrix} \\ &+ \frac{\nu^2}{\varepsilon_i} Y_i^T Y_i + \varepsilon_i \Omega\Omega^T < 0. \end{aligned} \quad (24)$$

Then substituting $\xi_i = -\mathbf{V}_i^{-1}\mathbf{X}_i$ and $\mathbf{A}_i = \mathbf{X}_i^T \mathbf{V}_i^{-1} \mathbf{X}_i - \mathbf{Z}_i$ into (24) and exploiting the well-known Schur complement lemma, it is obtained that (24) is equivalent to (15), implying that if (15) holds, then $\mathbf{R}_i < 0$ and thereby the collected data (X_{i+}, X_{i-}, U_{i-}) are informative for consensus of the continuous system (1).

Next, it remains to ensure that the developed control law (14) achieves leader-follower consensus for the continuous-time agents in (1) with $K_i(0) = Y_i Q_i^{-1}$. Define $e_i(t) = x_i(t) - x_0(t)$, $\tilde{K}_i(t) = K_i(t) - K_0(0)$, and $e(t) = \begin{bmatrix} e_1^T(t) & e_2^T(t) & \cdots & e_N^T(t) \end{bmatrix}^T$. Following similar lines in the proof of Theorem 1 and in view of Lemma 1 in [22], it is obtained that $\tilde{K}_i \to 0$ exponentially for $i = 1, 2, \cdots, N$ and $\lim_{t\to\infty} e(t) = 0$. This completes the proof. ∎

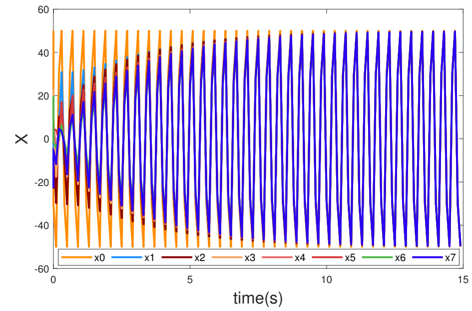

Fig. 2. The trajectories of the discrete-time multi-agent system (1) on the X-axis under the topology in Fig. 1

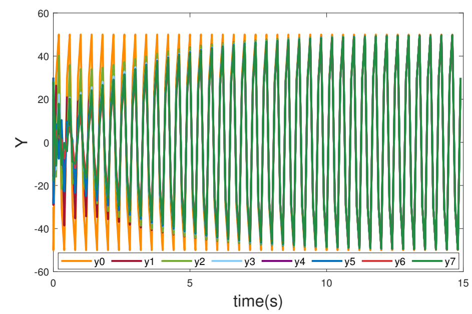

Fig. 3. The trajectories of the discrete-time multi-agent system (1) on the Y-axis under the topology in Fig. 1

Remark 2: In Theorems 1 and 2, the collected data are sampled from both leader and follower agents, while an interaction mechanism is exploited to tackle the heterogeneity in control gains. It is worth mentioning that the collected data matrices (X_{i+}, X_{i-}, U_{i-}) are informative for consensus for $i = 0, 1, \cdots, N$ in both Theorems 1 and 2, which implies that $K_i(0)$ renders $I_N \otimes A + \alpha L_{ff} \otimes BK_i(0)$ Schur stable in Section III and $I_N \otimes A + \beta L_{ff} \otimes BK_i(0)$ Hurwitz in Section IV for $i = 0, 1, 2, \cdots, N$. This enhances the reliability of the proposed control protocols, as the network system (1) can still achieve consensus even if the leader agent becomes nonfunctional and is unable to transmit its feedback gain and state information.

V. Illustrative Examples

In this section, we will demonstrate the effectiveness of the proposed algorithms via some simulation examples. The communication topology is described in Fig. 1. The dynamics of the discrete-time and continuous-time agents are both characterized by (1), with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}.$$

For the discrete-time multi-agent system (1), we add energy-bounded noises, drawn randomly from a Gaussian distribution with zero mean and unit variance, to the measurements of the agents' dynamics. The noise signals, denoted as W_i, adhere to the constraint in (5), where $N_{11} = 0.1I$, $N_{22} = -I$, and $N_{12} = N_{21} = 0$. Solving (8) via the CVX

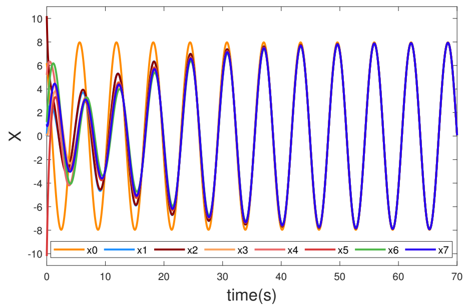

Fig. 4. The trajectories of the continuous-time multi-agent system (1) on the X-axis under the topology in Fig. 1

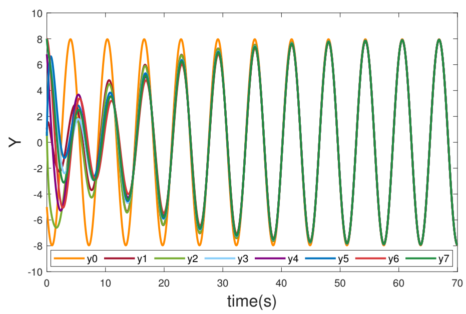

Fig. 5. The trajectories of the continuous-time multi-agent system (1) on the Y-axis under the topology in Fig. 1

toolbox [23] yields distinct initial feedback gain matrices $K_i(0)$. Subsequently, we obtain the state trajectories of agents as depicted in Figs. 2-3, from which it is manifest that the discrete-time agents in (1) reach consensus under the proposed protocol (7) based on the noise-corrupted data (3).

For the continuous-time case, we suppose that the energy-bounded noise signals W_i adhere to Assumption 4 for $i = 0, 1, \cdots, N$. The upper bound of noise is set to $\Delta = 10^{-3}$. Solving (15) via CVX also gives distinct original feedback gain matrices. We demonstrate the state trajectories of the agents in Figs. 4-5. It is clear from these figures the developed data-driven consensus control law (14) guarantees leader-follower consensus for the continuous agents in (1) in the presence of noise signals.

VI. CONCLUSIONS

In this paper, we have studied the localized data-driven consensus control problem for leader-follower multi-agent systems, allowing each agent to compute its local control gain with its locally collected data. Both the discrete-time and continuous-time data-driven consensus control problems have been addressed by solving low-dimensional LMIs derived from the matrix S-lemma and Petersen's lemma, respectively. Potential future research includes data-driven consensus control with output-feedback design and the integration of data-driven and event-triggered mechanisms.

REFERENCES

[1] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. De Moor, "A note on persistency of excitation," *System and Control Letters*, vol. 54, no. 5, pp. 325-329, 2005.

[2] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909-924, 2020.

[3] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel, "Data informativity: A new perspective on data-driven analysis and control," *IEEE Transactions on Automatic Control*, vol. 65, no. 11, pp. 4753-4768, 2020.

[4] H. J. van Waarde, M. K. Camlibel, and M. Mesbahi, "From noisy data to feedback controllers: Nonconservative design via a matrix S-lemma," *IEEE Transactions on Automatic Control*, vol. 67, no. 1, pp. 162-175, 2022.

[5] H. J. van Waarde, M. K. Camlibel, P. Rapisarda, and H. L. Trentelman, "Data-driven dissipativity analysis: Application of the matrix S-lemma," *IEEE Control Systems Magazine*, vol. 42, no. 3, pp. 140-149, 2022.

[6] S. Sinha, D. Muniraj, and M. Farhood, "LFT representation of a class of nonlinear systems: A data-driven approach," In *2021 19th European Control Conference (ECC)*, 2021, pp. 866-871.

[7] J. Berberich, C. W. Scherer, and F. Allgöwer, "Combining prior knowledge and data for robust controller design," *IEEE Transactions on Automatic Control*, vol. 68, no. 8, pp. 4618-4633, 2023.

[8] H. J. van Waarde and M. K. Camlibel, "A matrix Finsler's lemma with applications to data-driven control," In *2021 60th IEEE Conference on Decision and Control (CDC)*, pp. 5777-5782, 2021.

[9] A. Bisoffi, C. De Persis, and P. Tesi, "Data-driven control via Petersen's lemma," *Automatica*, vol. 145, p. 110537, 2022.

[10] A. Xue and N. Matni, "Data-driven system level synthesis," 2020, [Online]. Available: https://arxiv.org/abs/2011.10674.

[11] J. Jiao, H. J. van Waarde, H. L. Trentelman, M. K. Camlibel, and S. Hirche, "Data-driven output synchronization of heterogeneous leader-follower multi-agent systems," In *2021 60th IEEE Conference on Decision and Control (CDC)*, 2021, pp. 466-471.

[12] A. Allibhoy and J. Cortés, "Data-based receding horizon control of linear network systems," *IEEE Control Systems Letters*, vol. 5, no. 4, pp. 1207-1212, 2021.

[13] X. Zhang, G. Wang, and J. Sun, "Data-driven control of consensus tracking for discrete-time multi-agent systems," *Journal of the Franklin Institute*, vol. 360, no. 7, pp. 4661-4674, 2023.

[14] Y. Li, X. Wang, J. Sun, G. Wang, and J. Chen, "Self-triggered consensus control of multi-agent systems from data," *IEEE Transactions on Automatic Control*, doi: 10.1109/TAC.2024.3351865.

[15] Y. Li, X. Wang, J. Sun, G. Wang, and J. Chen, "Data-driven consensus control of fully distributed event-triggered multi-agent systems," *Science China Information Sciences*, vol. 66, p. 152202, 2023.

[16] L. Furieri, B. Guo, A. Martin, and G. Ferrari-Trecate, "Near-optimal design of safe output-feedback controllers from noisy data," *IEEE Transactions on Automatic Control*, vol. 68, no. 5, pp. 2699-2714, 2023.

[17] Z. Chang, J. Jiao, and Z. Li, "Localized data-driven consensus control," [Online]. Available: https://arxiv.org/abs/2401.12707.

[18] Z. Li and J. Chen, "Robust consensus of linear feedback protocols over uncertain network graphs," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4251-4258, 2017.

[19] Z. Li, J. Jiao and X. Chen, "Distributed optimal control with robustness guarantee for uncertain network systems: A complementary design approach," *IEEE Transactions on Automatic Control*, vol. 69, no. 4, pp. 2484-2491, 2024.

[20] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*. Philadelphia, PA, USA: SIAM, 1994.

[21] T. Liu and J. Huang, "Adaptive cooperative output regulation of discrete-time linear multi-agent systems by a distributed feedback control law," *IEEE Transactions on Automatic Control*, vol. 63, no. 12, pp. 4383-4390, 2018.

[22] H. Cai, F. L. Lewis, G. Hu, and J. Huang, "Cooperative output regulation of linear multi-agent systems by the adaptive distributed observer," In *2015 54th IEEE Conference on Decision and Control (CDC)*, 2015, pp. 5432-5437.

[23] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.0 beta," 2013, [Online]. Available: http://cvxr.com/cvx.