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Delay Feedback Active Inference based on Predicted States for Uncertain Systems with Input Delay

Mingyue Ji, Yang Lyu, Kunpeng Pan, Xiaoxuan Zhang, Quan Pan, Yang Li, Chunhui Zhao, Jinwen Hu

Abstract— **Active inference (AIF) as a comprehensive theory has been proven to be promising in state estimation and adaptive control of uncertain systems. However, the input delay in the controller was ignored in the normal framework. When taking input delay into consideration in the uncertain system, the optimal estimation state in the normal AIF differs greatly from the real state of the system due to the accumulation of the delay effect. Therefore, delay feedback active inference (D-AIF) is proposed in this paper. Different from normal AIF, the predictive state based on the delay state becomes the expectation of the state in the generative model. Meanwhile, an epitaxial delayed feedback Proportional-Integral (PI) control is introduced to be the expectation of the preference controller. The variational free energy (VFE) is extended by adding a quadratic of control consumption. The model uncertainty and measurement uncertainty are approximated by the Gaussian distributions. It can be proven that the state estimation does not depend on the given target state in D-AIF. In the simulation experiment of the trajectory tracking of an unmanned aerial vehicle with input delay, the results show that delay feedback active inference control (D-AIFC) has smaller tracking error and perceptual accuracy and shows stronger robustness when dealing with sudden disturbance than active inference control (AIFC).**

Index Terms— **active inference, predicted state, input delay, PI control**

I. INTRODUCTION

Active inference (AIF), which is also known as the free energy principle, has been confirmed to be a theory that can effectively mimic the human brain's perception of external world states and make corresponding preferences actions [1], [2]. It is a theoretical framework integrating perception, learning, and control which is equivalent to the combination of a filter and a controller. The optimization objective of the filter is to minimize the variational free energy (VFE) [3], and the optimization objective of the controller is to minimize the expected free energy (EFE) [4]. The mathematical form of AIF is similar to linear quadratic Gauss (LQG) [5]. The free energy function is the quadratic form of state prediction error [6] and perception prediction error [7], [8]. The mathematical form of its controller is similar to a Proportional-Integral control (PI) controller [9].

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At present, AIF has been widely applied for state estimation and adaptive control of classical uncertain systems with model noise and measurement noise. AIF does not need to calculate the possibility of each hidden state in uncertain systems. An approximated distribution is used to approximate the observed real state of the system. AIF has advantages in fast convergence and reducing computing consumption [10]. Dynamic expectation maximum (DEM) plays a role in filter AIF [11], which has been proven to be superior to classical Kalman filter in color noise processing [12] and be used for state estimation of quadrotor Unmanned Aerial Vehicles (UAV) under unmodeled wind power [13]. The controller in AIF was shown to be superior to the model reference adaptive controller in adapting to internal and external parameter perturbation in the 7 degrees of freedom manipulator experiment [14], [15]. AIF control (AIFC) can effectively predict the future state of the system and select appropriate actions [16]. Meanwhile, the perceived prediction error of the unbiased AIF device in the free energy changes with the tracking target [17]. The hierarchical model in deep AIF can be used for optimal path selection of robots [18] which also shows a good prospect for more complex environments with partial observability and high-dimensional input summations [19], [20].

However, input delay [21], [22] is often ignored in the estimation and control of uncertain systems based on AIF. Because of the time of signal transmission, the controller is often unable to act on the system in time. A small delay has been proved to cause a large oscillation of the system [23]. It is vital that the input delay of the controller cannot be ignored, which will bring lots of challenges. In the normal AIF, the state in the generative model takes the current target state of the system as prior belief, and the controller without clear mathematical form is given by solving the optimization gradient of free energy with respect to perceptual measurement, which may be not applicable to time-delay systems

Our main contributions are given as follows

(1) We introduce a prediction state about the delay state in AIFC to approximate the current state and propose a new delay feedback active inference control (D-AIFC) for uncertain systems with input delay.

(2) We introduce the delay feedback PI control as the mean of the preference control which is modeled as a random variable about the target state and the estimated state. The normal free energy function is extended by adding a function of energy consumption which is represented by the quadratic form of control. The innovation and expansion of the theory

provide guidance for the development and application of AIF.

The main structure of this paper is given as follows

The theoretical framework for AIF is provided in Section II; The normal AIFC and convergence analysis for an uncertain system in Section III; The D-AIFC based on a prediction state is presented in Section IV; Section V compare the performance of two controllers by the simulation of an unmanned aerial vehicle; Finally, the conclusion is summarized in Section VI.

II. PRELIMINARES

In this section, we mainly introduce the basic principle of AIF. AIF is similar to Bayesian inference in describing the generative model. The uncertainty in the general system often cannot be directly obtained. Bayesian inference is a set of posterior theories that estimate hidden state from a prior μ through sensor measurement $\mathbf{o}$. The generative model of an uncertain system can be described as

$$p(\mu \mid \mathbf{o}, \mathbf{a})=\frac{p(\mathbf{o}|\mu, \mathbf{a}) p(\mu, \mathbf{a})}{p(\mathbf{o}, \mathbf{a})}, \tag{1}$$

where the probability distribution $p(\mu|\mathbf{o})$ represents the posterior probability of state μ under the condition of given quantitative measurement data $\mathbf{o}$. The probability distribution $p(\mathbf{o}|\mu)$ which has a mean $g\left(\mu\right)$ is a mapping from state μ to $\mathbf{o}$ measurement . The target state is encoded in the function $f\left(\mu\right)$ which is the mean of the prior probability $p(\mu)$. Finding the belief about state μ and preference control $\mathbf{a}$ based on the observations is the goal of AIF.

It is difficult to directly estimate probability distribution $p(\mathbf{o}, \mathbf{a})$ in the generative model which describes all the probability of the hidden states. An approximate distribution $q(\mathbf{z})$ is introduced in the AIF framework. The differences between $q(\mathbf{z})$ and $p(\mathbf{x} \mid \mathbf{y})$ are measured by Kullback-Leible divergence (KL divergence) as follows

$$\begin{aligned} \mathbf{KL}[q(\mathbf{z}) p(\mu \mid \mathbf{o})] &= \textstyle\int q(\mathbf{z}) \ln \frac{q(\mathbf{z})}{p(\mu|\mathbf{o})} d\mathbf{x} \\ &= -\textstyle\int q(\mathbf{z}) \ln \frac{q(\mathbf{z})}{p(\mu,\mathbf{o})} d\mathbf{x}+\ln p(\mathbf{o}) \\ &= F+\ln p(\mathbf{o}). \end{aligned}$$

The state and behavior of the generative model can be implemented by minimizing F which is called VFE. The KL divergence is essentially the difference between approximate distribution $q(\mathbf{z})$ and joint distribution $p(\mu, \mathbf{o})$. Because KL divergence is always non-negative, VFE is also called the upper bound of 'surprise', which means $F \geq -\ln p(\mathbf{o})$ [24]. VFE is equivalent to the negative logarithm of the marginal likelihood of measure when $\mathbf{KL}[q(\mathbf{z}) p(\mu|\mathbf{o})]=0$.

III. AIFC FOR UNCERTAIN SYSTEMS

A general uncertain system can be linearized as a nominal system plus noises as follows

$$\left\{\begin{array}{l} \dot{\mu}=\mathbf{A} \mu+\mathbf{B a}(t)+\mathbf{w} \\ \mathbf{o}=\mathbf{C} \mu+\mathbf{v} \end{array}\right. . \tag{2}$$

The first equation is the state equation about μ including the process noise $\mathbf{w}$ and the second one is the measurement equation about $\mathbf{o}$ including the measurement noise $\mathbf{v}$. The generative model is the first step in AIF [25]. Fig. 1 shows the factor graph of the AIF generative model.

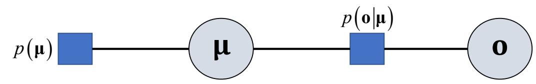

Fig. 1. Schematic diagram of AIF generative model.

Similar to the normal system equation, the belief model by AIF is also a form of the nominal system plus noise. The belief model corresponding to the system (1) is given as follow

$$\left\{\begin{array}{l} \dot{\mathbf{x}}=\hat{\mathbf{A}} \mathbf{x}+\hat{\mathbf{B}} \mathbf{u}(t)+\mathbf{w} \\ \mathbf{y}=\hat{\mathbf{C}} \mathbf{x}+\mathbf{v} \end{array}\right. , \tag{3}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the belief of state and measurement. Because the state of the uncertain system is mainly affected by the external uncertainty disturbance. $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ which are the coefficient matrix of the generative model in the nominal system can be simplified by $\hat{\mathbf{A}}=\mathbf{A}$, $\hat{\mathbf{B}}=\mathbf{B}$ and $\hat{\mathbf{C}}=\mathbf{C}$. According to the mean field theorem and Laplace approximation [26], the model uncertainty $\mathbf{w}$ and measurement uncertainty $\mathbf{v}$ are approximated by standard Gaussian distribution whose covariance matrix respectively are Σ_{μ} and $\Sigma_{\mathbf{o}}$. The VFE of the belief model (3) which can be written by the sum of the quadratic form is given by

$$F=\frac{1}{2}\left(\sum_{i} \boldsymbol{\varepsilon}_{i}^{\top} \Sigma_{i}^{-1} \boldsymbol{\varepsilon}_{i}-\ln \left|\Sigma_{i}^{-1}\right|+(m+n) \ln 2 \pi\right),$$

where $i \in\{\mu, \mathbf{o}\}$, and $\boldsymbol{\varepsilon}_{\mu}=\dot{\mathbf{x}}-\hat{\mathbf{A}} \mathbf{x}-\hat{\mathbf{B}} \mathbf{u}$ is state prediction error and $\boldsymbol{\varepsilon}_{\mathrm{o}}=\mathbf{y}-\hat{\mathbf{C}} \mathbf{x}$ is sense prediction error. Both the state and the measured probability density can be assumed as the following multivariate Gaussian distribution:

$$p(\mu)=\frac{1}{\sqrt{(2 \pi)^{m}\left|\Sigma_{\mu}\right|}} \exp \left(-\frac{1}{2} {\varepsilon_{\mu}}^{T} \Sigma_{\mu}^{-1} \varepsilon_{\mu}\right),$$

$$p(\mathbf{o} \mid \mu)=\frac{1}{\sqrt{(2 \pi)^{n}\left|\Sigma_{\mathbf{o}}\right|}} \exp \left(-\frac{1}{2} {\varepsilon_{\mathbf{o}}}^{T} \Sigma_{\mathbf{o}}^{-1} \varepsilon_{\mathbf{o}}\right),$$

where m, n is the dimension of $\mathbf{x}$ and $\mathbf{y}$. State estimation in minimization VFE is divided into perceptual state $\mathbf{x}$ with respect to F gradient descent scheme

$$\dot{\mathbf{x}}=\frac{d}{d t} \mathbf{x}-\kappa_{\mathbf{x}} \frac{\partial F}{\partial \mathbf{x}}=\frac{d}{d t} \mathbf{x}+\kappa_{\mathbf{x}}\left(\hat{\mathbf{A}}^{T} \Sigma_{\mu}^{-1} \boldsymbol{\varepsilon}_{\mu}+\hat{\mathbf{C}}^{T} \Sigma_{\mathbf{o}}^{-1} \varepsilon_{\mathbf{o}}\right), \tag{4}$$

where $\kappa_{\mathbf{x}}$ is the learning parameter. AIFC does not depend on estimated states but on measurement. The form of the controller is a linear optimization iteration format as follows

$$\dot{\mathbf{u}}=-\kappa_{\mathbf{u}} \frac{\partial F}{\partial \mathbf{u}}=-\kappa_{\mathbf{u}} \frac{\partial F}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{u}}=-\kappa_{\mathbf{u}} \hat{\mathbf{C}}^{T} \Sigma_{\mathbf{o}}^{-1} \varepsilon_{\mathbf{o}} \frac{\partial \mathbf{y}}{\partial \mathbf{u}}, \tag{5}$$

where $\kappa_{\mathbf{u}}$ is the learning parameter. For a motion system with displacement $\mathbf{y}_{\mathbf{p}}$ and velocity $\mathbf{y}_{\dot{\mathbf{p}}}$ included in general measurements, AIFC is given by an approximate PI control form as follows

$$\dot{\mathbf{u}}=-\kappa_{\mathbf{p}}\left[\Sigma_{y_{\mathbf{p}}}^{-1} \varepsilon_{\mathbf{p}}+\Sigma_{y_{\dot{\mathbf{p}}}}^{-1} \frac{d}{d t} \varepsilon_{\mathbf{p}}\right], \tag{6}$$

where $\kappa_{\mathbf{p}}$ is the learning parameter. Assuming that the system is a simple first-order linear uncertain system, the mean of the probability distribution $p(o \mid \mu)$ is $g(x) = x$, the mean of the prior probability $p(\mu)$ is $f(x) = \bar{x} - x$, where $\bar{x}$ is the target state. The VFE can be transformed to the function as follows

$$F = \tfrac{1}{2}\left[\Sigma_{\mu}^{-1}(\dot{x} - \bar{x} + x)^2 + \Sigma_{o}^{-1}(y - x)^2 \right. \\ \left. - \ln\left|\Sigma_{\mu}^{-1}\right| - \ln\left|\Sigma_{o}^{-1}\right| + 2\ln 2\pi\right].$$

The optimal state can be obtained by solving the derivative of the free energy with respect to the estimated state as follows

$$\begin{aligned} \frac{\partial F}{\partial x} &= -\Sigma_{\mu}^{-1}(\dot{x} - \bar{x} + x) + \Sigma_{o}^{-1}(y - x) = 0, \\ x &= \frac{\Sigma_{o}^{-1}y + \Sigma_{\mu}^{-1}\bar{x} - \Sigma_{\mu}^{-1}\dot{x}}{\Sigma_{\mu}^{-1} + \Sigma_{o}^{-1}}. \end{aligned} \tag{7}$$

It can be seen from the formula (7) that the optimal state depends on the priors of the measurement and target states when minimizing VFE. Meanwhile, state estimation by AIFC depends on measurement error due to it is often affected when sensor failure occurs. Taking the input delay of control into consideration, only the delayed state can be obtained at the current moment, but the real-time state. In this case, the optimal solution obtained by AIFC will be biased. Therefore, extending AIF to time-delay uncertain systems will be studied in the next section.

IV. D-AIFC FOR UNCERTAIN SYSTEMS WITH INPUT DELAY

The input delay in an uncertain system is caused by signal transmission and only the delayed signal can be obtained by the sensor. As a result, the measure-based controllers found in conventional AIF are no longer suitable for such a system with input delay τ as follows

$$\left\{ \begin{array}{l} \dot{\mu} = \mathbf{A}\mu + \mathbf{B}\mathbf{a}(t-\tau) + \mathbf{w} \\ \mathbf{o} = \mathbf{C}\mu + \mathbf{v} \end{array} \right. . \tag{8}$$

Therefore, the delayed state information is used to obtain the approximate predicted state at the current time which becomes the expectation of the state in the D-AIF generative model. At the same time, the delayed feedback PI control based on the predicted state is introduced as the expectation of the preference control, and the probability model is approximated to the multivariate Gaussian distribution by the mean field approximation and Laplace transform. Finally, the control consumption which is described by the quadratic term of control is added to the free energy function to implement state estimation and optimal control by minimizing the free energy. The structure of delay feedback active inference is shown in Fig. 2.

As shown in Fig. 3, the prior of the instant state is encoded in the control distribution. The posterior of the state and the preference control can be obtained by minimizing the free energy. The joint variational distribution of state and control $p(\mathbf{a}_{t-\tau}, \mu_t)$ can also be approximated by multivariate gaussian distribution according to the mean-field approximation

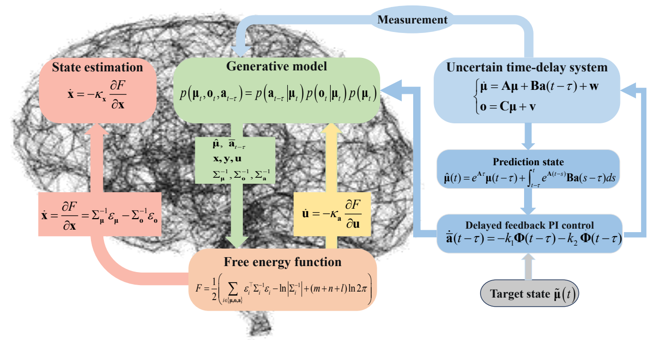

Fig. 2. Diagram of the structure of delay feedback active inference.

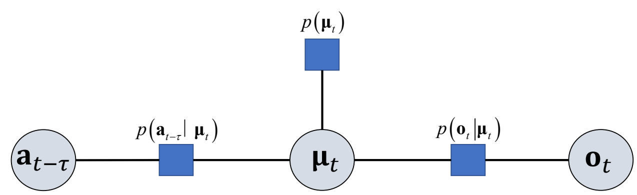

Fig. 3. Schematic diagram of D-AIF generative model.

and Laplace approximation. The D-AIF generative model of the system (8) is given by

$$p(\mu_t, \mathbf{o}_t, \mathbf{a}_{t-\tau}) = p(\mathbf{a}_{t-\tau} \mid \mu_t)\, p(\mathbf{o}_t \mid \mu_t)\, p(\mu_t). \tag{9}$$

The belief of state μ_t ,measurement $\mathbf{o}_t$ and control $\mathbf{a}_{t-\tau}$ are respectively $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{u}$. The prediction belief about the current state is obtained based on the delayed state as follows

$$\hat{\mu}(t) = e^{\mathbf{A}\tau}\mu(t-\tau) + \int_{t-\tau}^{t} e^{\mathbf{A}(t-s)}\mathbf{B}\mathbf{a}(s-\tau)ds. \tag{10}$$

The complex integrals $\Gamma(t) = \int_{t-\tau}^{t} e^{\mathbf{A}(t-s)}\mathbf{B}\mathbf{a}(s-\tau)ds$ can be calculated according to [28]

$$\begin{aligned} \Gamma(t) &= \textstyle\int_{t-\tau}^{t} e^{\mathbf{A}(t-s)}\mathbf{B}\mathbf{a}(s-\tau)ds \\ &= \textstyle\int_{-\tau}^{0} e^{-\mathbf{A}s}\mathbf{B}\mathbf{a}(s+t-\tau)ds \\ &= \textstyle\int_{-(NT-\eta)}^{-(N-1)T} e^{-\mathbf{A}s}\mathbf{B}\mathbf{a}(s+t-\tau)ds \\ &\quad + \ldots + \textstyle\int_{-T}^{0} e^{-\mathbf{A}s}\mathbf{B}\mathbf{a}(s+t-\tau)ds \\ &= e^{\mathbf{A}(NT-\eta)} \textstyle\int_{0}^{T-\eta} e^{-\mathbf{A}s}ds\mathbf{B}\mathbf{a}(t-2NT+\eta) \\ &\quad + \ldots + e^{\mathbf{A}T} \textstyle\int_{0}^{T} e^{-\mathbf{A}s}ds\mathbf{B}\mathbf{a}(t-(N+1)T+\eta). \end{aligned}$$

Let $\varphi_1(\zeta) = e^{\mathbf{A}\zeta}$ $\varphi_2(\zeta) = \int_0^{\zeta} e^{-\mathbf{A}s}ds\mathbf{B}$, then:

$$\begin{aligned} \Gamma(t) &= \varphi_1(NT-\eta)\varphi_2(T-\eta)\mathbf{a}(t-2NT+\eta) \\ &\quad + \varphi_1(NT-1)\varphi_2(T)\mathbf{a}(t-2NT+T+\eta) \\ &\quad + \ldots + \varphi_1(T)\varphi_2(T)\mathbf{a}(t-NT-T). \end{aligned}$$

If $\eta = 0$, $\Gamma(t)$ can be simplified as:

$$\begin{aligned} \Gamma(t) &= \varphi_1(NT)\varphi_2(T)\mathbf{a}(t-2NT) \\ &\quad + \varphi_1(NT-1)\varphi_2(T)\mathbf{a}(t-2NT+T) \\ &\quad + \ldots + \varphi_1(T)\varphi_2(T)\mathbf{a}(t-NT-T). \end{aligned}$$

The mean of the control distribution is $\mathbf{u}(t)$ which can be selected as the PI control. With the target state $\tilde{\mu}(t)$ as the input, the tracking error is $\psi(t) = \mu(t) - \tilde{\mu}(t)$. The state equation in the generative model can be transformed into a new error equation as follows

$$\dot{\psi}(t) = \mathbf{A}\psi(t) + \mathbf{B}\mathbf{a}(t-\tau) + \omega(t) + \sigma(t), \qquad (11)$$

where $\sigma(t) = \mathbf{A}\tilde{\mu} - \dot{\tilde{\mu}}$. For the system with time delay, it is necessary to transform the time-delay system into a delay-free one through the following integral transformation

$$\mathbf{\Phi}(t)=\psi(t) + \int_{t-\tau}^{t} \mathrm{e}^{-\mathbf{A}(s-t+\tau)}[\mathbf{B}\mathbf{a}(s) + \sigma(s+\tau)]ds,$$

where $\mathbf{\Phi}(\mathbf{t})$ and $\psi(\mathbf{t})$ satisfy $\psi(t+\tau) = \mathrm{e}^{\mathbf{A}\tau}\mathbf{\Phi}(t)$. A new delay-free system is given as follows

$$\dot{\mathbf{\Phi}}(t) = \mathbf{A}\mathbf{\Phi}(t) + \mathbf{B}_0\mathbf{a}(t) + \mathrm{e}^{-\mathbf{A}\tau}\sigma(t+\tau), \qquad (12)$$

where $\mathbf{B}_0 = \mathrm{e}^{-\mathbf{A}\tau}\mathbf{B}$. The PI control law for tracking error is given by

$$\dot{\tilde{\mathbf{a}}}(t) = -k_1\mathbf{\Phi}(t) - k_2\dot{\mathbf{\Phi}}(t). \qquad (13)$$

The corresponding delay feedback PI control is

$$\dot{\tilde{\mathbf{a}}}(t-\tau) = -k_1\mathbf{\Phi}(t-\tau) - k_2\dot{\mathbf{\Phi}}(t-\tau). \qquad (14)$$

The VFE of D-AIF is

$$F = \frac{1}{2}\left(\sum_i \varepsilon_i^\top \Sigma_i^{-1}\varepsilon_i - \ln\left|\Sigma_i^{-1}\right| + (m+n+l)\ln 2\pi\right),$$

where $i \in \{\mu, \mathbf{o}, \mathbf{a}\}$, $\varepsilon_\mu = \mathbf{x} - \hat{\mu}$, $\varepsilon_\mathbf{o} = \mathbf{y} - C\mathbf{x}$ and $\varepsilon_\mathbf{a} = \mathbf{u} - \bar{\mathbf{a}}$. The probability density of state, measurement, and control are also assumed as the following multivariate Gaussian distribution

$$p(\mu) = \frac{1}{\sqrt{(2\pi)^m|\Sigma_\mu|}}\exp\left(-\frac{1}{2}{\varepsilon_\mu}^T\Sigma_\mu^{-1}\varepsilon_\mu\right),$$

$$p(\mathbf{o}\mid\mu) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_\mathbf{o}|}}\exp\left(-\frac{1}{2}{\varepsilon_\mathbf{o}}^T\Sigma_\mathbf{o}^{-1}\varepsilon_\mathbf{o}\right),$$

$$p(\mathbf{a}\mid\mu) = \frac{1}{\sqrt{(2\pi)^l|\Sigma_\mathbf{a}|}}\exp\left(-\frac{1}{2}{\varepsilon_\mathbf{a}}^T\Sigma_\mathbf{a}^{-1}\varepsilon_\mathbf{a}\right),$$

where m, n and l are the dimension of vector $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{u}$. For a first-order linear uncertain system, the VFE can be transformed to the function as follows

$$F = \tfrac{1}{2}\Big[\Sigma_\mu^{-1}(x-\hat{\mu})^2 + \Sigma_o^{-1}(y-x)^2 + \Sigma_u^{-1}(u-\bar{a})^2 \\ -\ln\left|\Sigma_\mu^{-1}\Sigma_o^{-1}\Sigma_a^{-1}\right| + 3\ln 2\pi\Big].$$

The optimal prediction belief and the optimal control belief of the state can be obtained by minimizing the free energy as follows

$$\dot{\mathbf{x}} = -\kappa_\mathbf{x}\frac{\partial F}{\partial\mathbf{x}} = -\kappa_\mathbf{x}\left(\Sigma_\mu^{-1}(x-\hat{\mu}) - \Sigma_o^{-1}(y-x)\right),$$

$$\dot{\mathbf{u}} = -\kappa_\mathbf{a}\frac{\partial F}{\partial\mathbf{u}} = -\kappa_\mathbf{a}\Sigma_\mathbf{a}^{-1}(\mathbf{u}-\bar{\mathbf{a}}). \qquad (15)$$

The optimal state can be obtained by solving the derivative of the free energy with respect to the estimated state as follows

$$\frac{\partial F}{\partial\mathbf{x}} = \Sigma_\mu^{-1}(x-\hat{\mu}) - \Sigma_o^{-1}(y-x) = 0,$$

$$x = \frac{\Sigma_0^{-1}y + \Sigma_\mu^{-1}\hat{\mu}}{\Sigma_\mu^{-1} + \Sigma_o^{-1}}. \qquad (16)$$

In addition, the formula (16) shows that the expectation of the system state depends on the predicted state but has nothing to do with the target state.

V. SIMULATION

The diagram of an unmanned aerial vehicle following a path under external disturbance is shown in Fig. 4. The vehicle starts from the initial position $(-3,-3)$. The position of it at time t is (X, Y). The model uncertainty W_X and measurement uncertainty W_Y of the system equation cannot be ignored because the vehicle may be disturbed by wind and electromagnetic during flight. By differentiating it, the following velocity equation can be obtained

$$\begin{cases} \dot{X} = V\cos(\varphi) + W_X \\ \dot{Y} = V\sin(\varphi) + W_Y \end{cases}. \qquad (17)$$

The lateral velocity V and longitudinal velocity φ of the unmanned aerial vehicle can be calculated from the forward velocity and yaw angle. It is supposed that the path of the tracked target is a straight line: $a_lX + b_lY + c_l = 0$. The immediate tracking error is

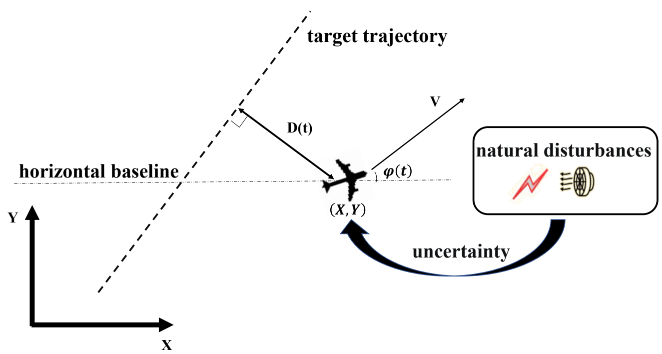

Fig. 4. Trajectory tracking diagram of unmanned aerial vehicle.

$$D(t) = \frac{a_lX(t) + b_lY(t) + c_l}{\sqrt{{a_l}^2 + {b_l}^2}}.$$

When the natural disturbances are ignored, a nominal equation about acceleration can be obtained by differentiating the equation (17)

$$\begin{cases} \ddot{X} = \dot{V}\cos\varphi - V\sin\varphi\dot{\varphi} = \dot{V}\cos\varphi - \dot{Y}\dot{\varphi} \\ \ddot{Y} = \dot{V}\sin\varphi + V\cos\varphi\dot{\varphi} = \dot{V}\sin\varphi + \dot{X}\dot{\varphi} \end{cases}. \qquad (18)$$

Because the motion of unmanned aerial vehicles such as fixed-wing drones mainly depends on the forward driving force, the yaw angle changes very slowly in general. Therefore, in order to simplify the model, it is assumed that yaw angular velocity changes with constant acceleration $0.005rad/s^2$, and the term containing forward acceleration $\dot{V}$ is the control with input delay τ.

The target trajectory is selected which satisfies $-X+Y+1 = 0$. It is assumed that the target states of horizontal and longitudinal displacement a $\left(\bar{X}, \bar{Y}\right) = \left(\frac{\sqrt{2}}{2}t, \frac{\sqrt{2}}{2}t - 1\right)$.

The input delay of the D-AIFC is $0.1s$ and the total time is $30s$. The iterative parameters of state estimation and control update are $\kappa_\mu = 100$ and $\kappa_{\mathbf{u}} = 200$ respectively, and the coefficients of PI controller in D-AIFC are $k_1 = 200$ and $k_2 = 500$. The model uncertainty and measurement uncertainty are assumed to be a standard Gaussian distribution with 0.001 variances. It is assumed that the vehicle is subjected to a large disturbance with the size of [40, 50] in the direction of against from 5s to 7s which may come from strong airflow in the wind or sudden electromagnetic signal interference. In this case, we respectively compare the control performance of AIFC and D-AIFC.

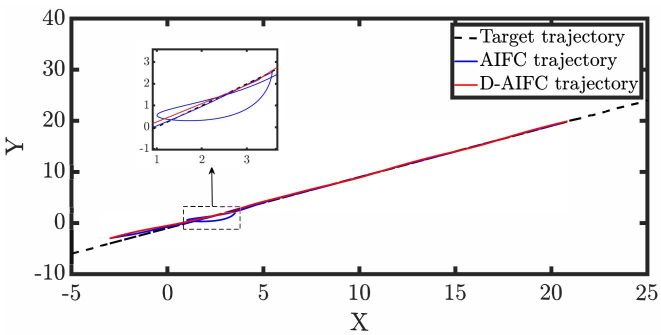

Fig. 5. The diagram of motion trajectory.

Fig. 5 shows that the motion of the system based on the AIFC has an obvious devious phenomenon caused by the untimely power input, which is nonnegligible consumption. Because of the delay of the control input, only the delay state can be obtained to be the expectation of the state in the AIF generative model. With the accumulation of delay effect, once the system is subjected to sudden external disturbance, the system is prone to instability. However, the prediction state about the current moment is taken as the expectation of the state, and the delayed feedback PI control as the expectation of the control input in the D-AIF generative model, so the system has shown stronger robustness.

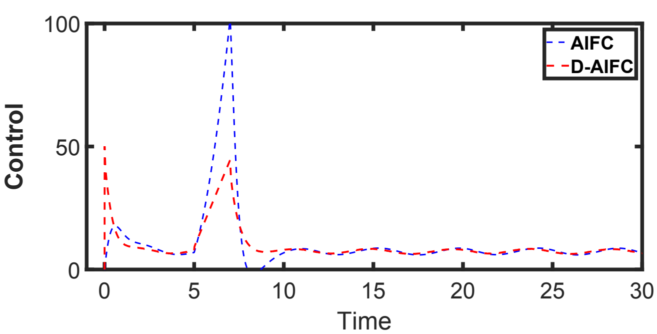

Fig. 6. Time history diagram of control.

As can be seen in Fig. 6 due to input delay, the sensor fails to obtain the state of the current moment in time which may result in the delay of the sensor data by AIFC. Therefore, when the system is subjected to a sudden disturbance, the controller gradually becomes larger so that it needs to compensate not only for the external disturbance but also for the delayed loss of the previous time in order to maintain its own stability. However, the system requires less energy input because delay losses are compensated by D-AIFC.

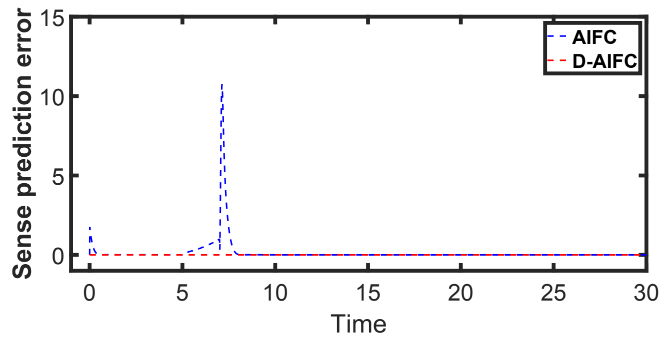

Fig. 7. Time history diagram of SPE.

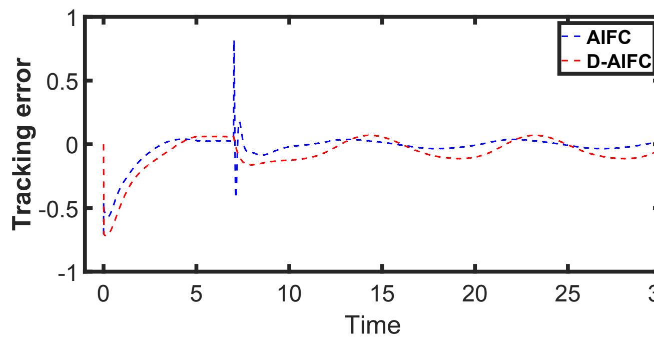

Fig. 8. Time history diagram of tracking error

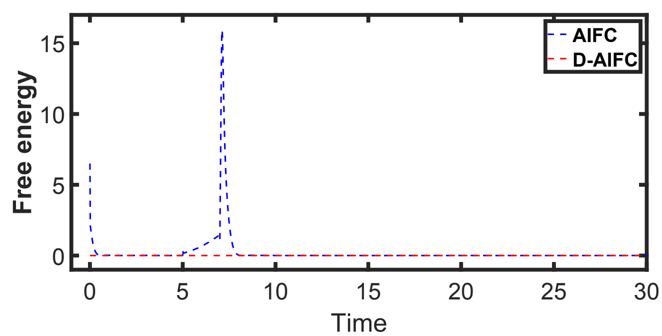

Fig. 9. Time history diagram of free energy.

Because no timely state prediction is made up for the input delay, it can be seen from Fig. 7 and 8 that the perceived prediction error and tracking error of AIFC are not very smooth compared with D-AIFC. In the fifth second, due to the effect of large error accumulation in the square, Fig. 9 shows that the free energy function of AIF also has a large fluctuation since it's the sum of the perception prediction error and the state prediction error.

At the same time, the changes in sense prediction error of unmanned aerial vehicles under the D-AIF framework with different input delays are shown in Fig. 10. When the input delay increases from 0.1s to 0.5s, the sense prediction error barely changes. It may lie in that no matter how large the delay is, the predicted state can well approximate the current state, so as to make up for the influence of input delay in the controller.

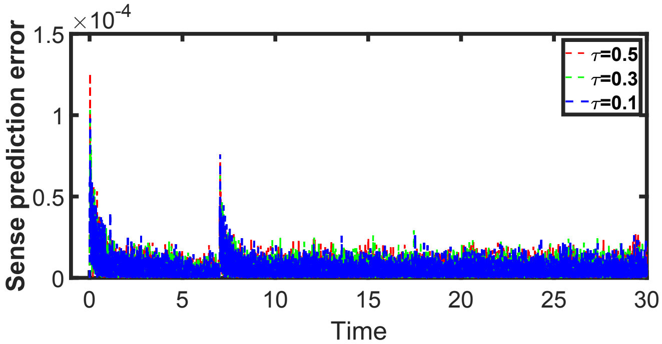

Fig. 10. Time history diagram of SPE for different delay.

VI. Conclusion

In this paper, due to the rarely considered input delay in the normal AIFC, the predicted state based on the delay state is introduced to be the expectation of the state prior. At the same time, the delay feedback PI control about the predicted state is designed to be the expectation of the preference control. Finally, the D-AIFC for delay uncertain system is proposed. The AIFC and D-AIFC are respectively used for the trajectory tracking problem of an unmanned aerial vehicle. The simulation results show that compared with the normal AIFC, the D-AIFC has smaller tracking error and perceptual accuracy and shows stronger robustness when dealing with sudden disturbance

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