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Asymmetric Risk-Field Based Spatio-Temporal Trajectory Planning for Autonomous Driving Considering Game Interaction

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***Abstract*— Aiming at the trajectory planning problem of autonomous vehicles, a spatio-temporal joint planning framework considering both multi-vehicle interactions through a game theoretic approach and asymmetric risk field theory was proposed in this article. Through game theoretic forward propagation, the predicted future trajectory of the surrounding vehicle is acquired and coupled into the framework of the ego vehicle decision-making and planning, so that the ego vehicle trajectory planning considering multi-vehicle interaction can be realized. The trajectory is derived from a spatio-temporal planning approach to integrate the velocity planning and path planning and the safety of the trajectory is guaranteed. Furthermore, the trajectory points generated by forward propagation can effectively consider the asymmetric risk field generated by surrounding vehicles and integrate it into the solution of the numerical optimization problem, comprehensively considering the impact of different types of surrounding vehicles, together with their states and other factors, so that the calculated route is safer and in line with human decision-making. The simulation results show that in the dense traffic flow with frequent interactions with surrounding vehicles, the autonomous ego vehicle can reasonably change lanes and achieve efficient and safe driving.**

I. INTRODUCTION

Autonomous driving industry has witnessed vigorous growth over the last decade. Within the modularized framework of autonomous driving algorithms, trajectory planning algorithms stand as one of the core components, acting as the bottleneck constraining the real-time performance and safety of the entire system. Existing trajectory planning algorithms primarily fall into four categories: graph searching based, sampling based, learning based, and optimization based. Graph-based algorithms such as Dijkstra's algorithm, mixed A* algorithm[1], suffer from rapid growth in computational complexity with increasing problem scales, making deployment in real and complex environments challenging. Sampling-based methods like RRT[2] struggle to ensure optimality and require smoothing, while deterministic sampling algorithms like Lattice, though capable of generating trajectories adhering to vehicle kinematics, are limited by the size of the sampling space. Learning-based approaches primarily employ reinforcement learning to derive trajectories[3,4], showcasing excellent performance but lacks interpretability, hindering their adoption in safety-critical industries. Optimization-based algorithms construct boundary conditions based on the ego vehicle's start and end positions, along with road information, considering vehicle dynamics equations as constraints. They formulate optimization problems with objectives such as traffic efficiency and comfort to derive safe and comfortable routes. However, the primary drawback lies in the difficulties ensuring convexity in the solution space, leading to difficulties in converging to global optimum solutions[5].

Decoupling the velocity and the path in the planning has been a common approach in the trajectory planning problem, addressing throttle and steering control separately by first determining a fixed lateral path and then considering how to control the throttle to achieve obstacle avoidance, or vice versa. While the approach simplifies the problem into convex optimization, it neglects the relationship between the longitudinal and lateral controls and contradicts with real human drivers' habits[6]. Spatio-temporal planning approaches construct the solution space in the spatio-temporal map, considering drivable areas and obstacles and derive the trajectory integrating the time and space information which is similar to the plannig of human drivers. Previous works have extensively studied the formulation of spatio-temporal planning problems. Zhang et al have constructed spatio-temporal graphs based on predicted trajectories of surrounding vehicles, employing enhanced A* algorithms to search these graphs for the main vehicle's trajectory control points, further computing Bezier curve trajectories to ensure safety. However, this method is still limited by the rapid growth of computational complexity in graph search algorithms as problem scales increase[7]. In subsequent research, the authors have introduced spatio-temporal voxels and constructed spatial and temporal edges to form spatio-temporal regions as trajectory solution spaces. However, this method inadequately considers the future trajectories of surrounding vehicles and their interactions[8]. Ding et al[9,10] proposed a unified method incorporating various road elements (e.g., traffic lights, speed limits) into constructing spatio-temporal corridors. They modeled the ego vehicle's trajectory planning problem as a POMDP, integrating predictions of surrounding vehicles into the ego vehicle's planning algorithm to consider its impact, while assuming fixed intentions for surrounding vehicles without explicitly inferring their driving intents.

Moreover, safety considerations are also critical in trajectory planning for autonomous vehicles. Many previous works on spatio-temporal planning solely focus on avoid-

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ing collisions with surrounding vehicles or penalize on the distances between ego vehicle and surrounding vehicles, neglecting various factors affecting driving risks apart from direct collisions and distances. Zhang et al combined normal distributions on surrounding vehicle trajectories as risk estimates[11], fundamentally considering the distance to surrounding vehicles without accounting for other factors such as vehicle types, mass, etc. which cannot explain the human driver's tendency to keep a distance from heavy vehicles[12]. In summary, the exploration of how to infer surrounding vehicle intents and construct convex solution spaces for efficient resolution within spatio-temporal joint planning, while comprehensively considering risks based on vehicle attributes, remains an essential direction for further research in spatio-temporal planning algorithms.

In this article, we employ a potential game theoretic approach[13] to explicitly infer surrounding vehicle intentions to address the problem of mutual interactions between the ego vehicle and surrounding vehicles. By coupling the prediction of surrounding vehicles with the ego vehicle's planning through forward propagation, it efficiently considers interactive scenarios, and furthermore, based on the obtained reference waypoints, an asymmetric risk field theory[14] is utilized for a first-order approximation around these waypoints, thus considering risks based on vehicle attributes. Finally, incorporating these risk factors into the cost function of spatio-temporal joint planning, the ego vehicle's trajectory is derived which ensures safety. The research framework is depicted in Figure 1. We summarize our contributions as follows:

1) The spatio-temporal planning algorithm is incorporated with a game theory approach to explicitly estimate the driving intentions of surrounding vehicles which promotes the construction of solution spaces more accurately.
2) The risk field generated by asymmetric attributes of the surrounding vehicles and ego vehicle is utilized in the game theory module and spatio-temporal planning module to comprehensively assess risk, so as to achieve more human-like decision-making processes.

The remainder of this article is organized as follows. The relevant literature is discussed in Section II. The problem is formulated and our methodology is introduced in Section III. The experiment results are discussed in Section IV. And Section V concludes the article.

II. Related works

A. Game Theoretic Motion Planning

There have been numerous studies focusing on the application of game theory on the motion planning of autonomous vehicles, aiming to explicitly consider the interaction between vehicles. In the game theoretic problem settings, each vehicle aims to maximize their own payoff while considering other vehicles' payoff functions. Stackelberg Games is a formulation adopted by many existing works[15-17], which follows a leader-follower pattern. Assuming the leader dominates the game, the followers are responsible for avoiding collisions. As a result, aggressive driving styles are more likely to be generated and any inaccurate estimation of surrounding vehicles may lead to collision[13]. Normal-form games[18], differential games[19] and potential games[13] are also popular methods applied to autonomous driving scenario. In this article, we adopt a potential-game pattern due to its convergence property and is easier to solve.

B. Risk Field

Risk field theories are derived from APF (Artificial Potential Field), and are widely adopted in robotics and autonomous driving[20,21]. Risk field method is an efficient way to take uncertainties into account, while there are a few drawbacks. Classic risk field methods model the risk field regardless of the velocity directions of vehicles and there have been some improvements[22]. Another drawback is that conventional methods generate the same risk value for the vehicle in the similar location and they lack to consider the asymmetry between the risk producer and sufferer. For example, a truck would generate more risk to a car than to an identical truck in the same location. Therefore in this article, we consider a risk field method considering the asymmetry in the attributes of vehicles[14].

III. Methodology

A. System Overview

Our proposed planning system mainly consists of two modules, namely the game theory module and spatio-temporal planning module. The former receives outputs from perception module which contain information about surrounding vehicles and environment information such as lanes, obstacles etc. The game theory module determines the vehicles to be taken into account as players, sets up and solves the game theory problem formulation to acquire reference trajectory points of both ego vehicle and surrounding vehicles which are to be utilized to construct the solution spaces. The spatio-temporal planning module receives reference trajectory points from upstream and is responsible to plan an interactive-aware safety-guaranteed trajectory for the ego vehicle. In the meantime, the asymmetric risk field(refers to as mechanical wave risk-field for the following section) is utilized to obtain related risk metric and is applied in the payoff function for the game module and target function for the spatio-temporal planning module, in order to achieve a planning framework to jointly considering game interaction and asymmetric risk field.

B. Game Theory Module

The decision module of the ego vehicle, based on potential game theory[13], takes environmental road information, surrounding vehicle positions, and observed historical states of surrounding vehicles as inputs to generate an output decision sequence $D_t : [x_{t+1}, x_{t+2}, ..., x_{t+H}]$, where x_t is the status of all vehicles in the scene at time t, including the

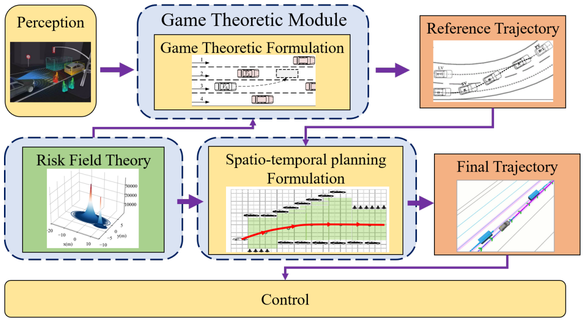

Fig. 1. The concept framework of the proposed planning method.

leading vehicle, with H denoting the predictive time horizon. The output trajectories of all vehicles are utilized within the subsequent spatio-temporal joint planning module as initial solutions to further construct the solution space (refer to Section III C). This approach is employed to transform the entire trajectory space, which possesses non-convex characteristics concerning behaviors like maneuvering before a specific surrounding vehicle in the context of the game theory module, into a convex space. This transformation allows the downstream spatio-temporal joint planning module to efficiently solve optimization problems to obtain final trajectories. Additionally, decoupling the behavior decision module based on game theory from the trajectory planning module based on optimization enables a larger temporal resolution for the behavior decision module, reducing computational costs and ensuring better real-time performance for the entire algorithm.

The game theoretic problem is formulated as follows: consider a driving scenario consists of N vehicles indexed by $I : \{1, 2, ..., N\}$ .Any vehicle $i \in I$ regarded as a player makes a decision on the control variable $u_k^i \in R^n$ at time step t based on all other players' states $X : \{x_k^1, x_k^2, ..., x_k^N\}$, where x_k^i is the state of ith vehicle and is constrained with vehicle dynamics $x_{k+1}^i = f_k^i(x_k^i, u_k^i)$, and X consists of all other players' state vectors. Each player's reward follows sequential reward pattern so that each player selects action in order to maximize the sum of payoffs within the horizon. The payoff function is defined as

$$P_k^i = \sum_{k=0}^{H-1} p_k^i(x_k, u_k^i, u_k^{-i}) + p_H^i(x_H). \tag{1}$$

where H is the length of predictive horizon, u_k^{-i} means the control input aside of ith vehicle, $p_H^i(x_H)$ is the terminal reward. Besides, the problem is constrained by:

$$\begin{aligned} &U^i = \{u \in R^n |\ \underline{u}^i \le u \le \overline{u}^i\}, \\ &h^i(x_k^i, u_k^i) \le 0, \\ &H^i(x_k^i, x_k^{-i}, u_k^i, u_k^{-i}) \le 0 \end{aligned} \tag{2}$$

In real scenarios, drivers might not have complete knowledge of each other's reward functions. Uncertain reward functions pose challenges in formulating game theory problems. However, the structure of the reward function should be generally applicable to all drivers. Hence, we assumes all other players' payoff functions P_k^i can be parameterized by θ_k^i , so that the uncertainties of payoff functions can be addressed by estimating θ_k^i. In general, the following discrete finite-horizon optimization problem formulation is acquired, which contains coupled constraints:

$$\begin{aligned} \underset{u_0^i, u_1^i, \ldots, u_{H-1}^i}{\text{minimize}} \ & \sum_{k=1}^{H-1} p_k^i(x_k, u_k^i, u_k^{-i}) + p_H^i(x_H) \\ s.t. \quad & x_{k+1}^i = f_k^i(x_k^i, u_k^i), \qquad \forall k \in N_{[0,H-1]} \\ & h^i(x_k^i, u_k^i) \le 0, \qquad \forall k \in N_{[0,H-1]} \\ & h_f^i(x_H^i) \le 0, \\ & H^i(x_k^i, x_k^{-i}, u_k^i, u_k^{-i}) \le 0, \ \forall k \in N_{[0,H-1]} \\ & H_f(x_N) \le 0 \end{aligned} \tag{3}$$

The ahead formulation is called a generalized Nash equilibrium problem (GNEP) and is generally difficult to solve. Therefore we further formulate the problem into a generalized potential game (GPG) which is much easier to solve[8].

Definition III.1(Generalized Potential Game).*A GNEP corresponds to a GPG if:*
(i) there exists a nonempty, closed set $U \subseteq R^n$, $n = \sum_{i=1}^{M} n_i$, such that for every $i \in I$,

$$U^i(u^{-i}) = \{u^i \in U^i |\ (u^i, u^{-i} \in U\}$$

(ii) there exists a continuous function $P : R^n \to R$, for every $i \in I$ and $u^{-i} \in \Pi_{v \in I \setminus i}$,

$$\frac{\partial P(u^i, u^{-i})}{\partial u^i} = \frac{\partial J^i(u^i, u^{-i})}{\partial u^i}$$

For the driving scenario interested in this article, definition III(i) demands that all the control input in included in a single closed set, which is already satisfied by the saturation constraint on the input $U^i = \{u \in R^n |\ \underline{u}^i \le u \le \overline{u}^i\}$. As for definition III(ii), The potential function can be designed as a combination of a common term for all vehicles, such as the sum of distances to other vehicles, and individual terms considered by each vehicle, such as penalties for certain vehicle inputs. According to [23], this approach of constructing the potential function is applicable to the driving behaviors of most human drivers. Therefore, the potential function is constructed as follows:

$$\begin{aligned} P(u) &= c(u) + \sum_{i=1}^{N} \sum_{k=1}^{H} d_k^i(u_k^i) \\ s.t. \quad & c(u) = w_c e^{-\frac{d^T A d}{2}}, \\ & d_k^i(u) = w_{a1}(d_{y,k}^i)^2 + w_{a2}(a_k^i)^2 + w_{a3}(\delta_k^i)^2 \\ & \qquad + w_{a\phi} d_k^i, \qquad \forall i \in A, k < H \\ & d_H^i(u) = w_{a4}(d_{y,H}^i)^2, \\ & d_k^i(u) = w_{s1}(d_{gap,k}^i - d_0^i)^2 + w_{s2}(a_k^i)^2, \\ & \qquad \qquad \forall i \in I \setminus A, k < H \\ & d_H^i(u) = w_{s3}(d_{gap,H}^i - d_0^i)^2 \end{aligned} \tag{4}$$

where $c(u)$ is the common term corresponding to the distances to other surrounding vehicles, $d_k^i(u)$ is the individual terms. As for the lane-changing scenario that this article mainly takes interest in, for the ego vehicle, the individual term is the lateral deviation to the central line of the target lane $d^i_{y,k}$, and the input penalty to the acceleration a^i_k(and steering wheel angle δ^i_k and the risk factor $\phi(d^i_k)$ (refers section III.D). We only consider the risk between ego vehicle and the target vehicle behind the gap for simplicity. For vehicles controlled by other human drivers, to reduce computational burden, in the scenario where an autonomous vehicle merges, only their yielding/not yielding behavior is considered. Hence, the cost function solely accounts for the deviation from the ideal following distance $d^i_{gap,k} - d^i_0$ and input penalty. $w_\odot$ are weight terms and are manually selected. Specifically, for the weight terms in the cost function of surrounding human drivers, the autonomous vehicle needs to estimate based on observed behaviors. Each vehicle is represented by a rectangle, with its four sides described by linear equation $a_i x + b_i y + c_i = 0$. Therefore, The collision constraints are represented by

$$a_{i,k}x_1 + b_{i,k}y_1 + c_{i,k} \leq 0, \quad \forall k \in \{1, 2, 3, 4\} \tag{5}$$

Note that (5) only ensures that the vertices of the vehicles are not within the rectangular range of the other vehicle. It doesn't strictly guarantee the absence of collision between the two vehicles. However, as previously mentioned, the primary objective of the game theory module is to generate reference trajectory points. The final trajectory's safety is guaranteed by the downstream spatio-temporal joint planning module. Therefore, conducting strict safety checks here would impose unnecessary additional computational burden. Hence, the final mathematical expression for solving the problem is:

$$\begin{aligned} &\underset{u \in U}{\text{minimize}} \ P(u) \\ s.t. \quad & x^i_{k+1} = f^i_k(x^i_k, u^i_k), \quad \forall k \in N_{[0,H-1]} \\ & H(u) \leq 0, \\ & j(u) \leq 0 \end{aligned} \tag{6}$$

The vehicle dynamics model adopts a linear two-degree-of-freedom bicycle model, described by the following equations, where $\beta = \arctan(l_r/(l_f + l_r)\tan\sigma)$, l_f and l_r as the the distance between the centre of the mass and respectively the front and the rear axis, μ the friction coefficient and β the slip angle, ψ the heading angle and σ the sterring angle:

$$\begin{aligned} \dot{x} &= v\cos(\psi + \beta), \\ \dot{y} &= v\sin(\psi + \beta), \\ \dot{\psi} &= \frac{v}{l_r}\sin(\beta), \\ \dot{v} &= a - \mu v \end{aligned} \tag{7}$$

Since the weights in the payoff functions of surrounding drivers is not directly observable, an algorithm of parameter estimation needs to be developed. Similar to [13], we employ a lagrangian-based method to approximate the parameters:

$$\begin{aligned} & L_\Sigma(u, \lambda) = P(u) + \langle \lambda, g(u) \rangle + \frac{1}{2}\|H(u)\|^2_\Sigma \\ s.t. \quad & g(u) = (h(u), u - \overline{b}, \underline{b} - u), \end{aligned} \tag{8}$$

If other driver's behavior u_o is observed, u_o must be a local minimum of $L_\Sigma(u, \lambda)$, therefore the KKT conditions must be met. In short, the following estimation procedures are given. We refer to [13] for readers that are interested in details:

$$\begin{aligned} &\underset{\theta, u, \lambda}{\text{minimize}} \ \|\nabla_u L_\Sigma(\overline{u}, \overline{\lambda}; \theta)\|^2_2 + r(\theta, \hat{\theta}_t) \\ s.t. \quad & u_0 = \hat{u}_t, \\ & g(u;\theta) \leq 0, \\ & \lambda \geq 0, \\ & y^T g(u;\theta) = 0 \end{aligned} \tag{9}$$

where $r(\theta, \hat{\theta}_t)$ is the regularization terms to penalize large update of estimate $\hat{\theta}_t$.

C. Spatio-temporal Planning Module

The game theory module is responsible of calculating a rough estimation of how other vehicle and ego vehicle would move given the current traffic situation, so that an interaction-aware prediction of other traffic participants is acquired and enables the downstream motion planning module to be capable of generating an interaction-aware trajectory at the same time. However, solving (11) still imposes heavy computational costs when the time resolution is high enough to ensure the safety of the trajectory and to respond to any errors in prediction. Therefore we decouple the motion planning module from the game-theory based decision module in order that the latter can run at a lower frequency to generate rather coarse trajectories of ego vehicle and surrounding vehicles and the former runs a higher frequency to generate a safe and comfortable trajectory and to quickly respond to any sudden changes in the traffic environment.

We utilize spatio-temporal semantic corridor(SSC)[4,5] method to construct the solution spaces. However, there are two key differences between our proposed method and SSC method. First, we explicitly formulate the decision process of other traffic participants through game theory. And second, we additionally consider the risk generated by different surrounding vehicles, taking into account their velocity states and vehicle types because merely considering distances between vehicles is not enough to evaluate safety in real driving scenarios.The trajectory generation primarily consists of two steps: corridor generation and optimization-based formulation. Corridor generation involves generating inflated cubes according to the reference points from upstream, until undrivable areas are reached such as other vehicles' trajectories at the time or road boundaries, as depicted in fig 2.

Then an optimization-based formulation is derived by setting an objective function, in which we mainly consider jerk, differences from reference trajectory and linearized risk factor (Section III D). Reducing jerk helps to derive a smooth and comfortable trajectory, and the reason to consider the differences from reference trajectory is to avoid possible discrepancy between the decision generated by the game-theory module and motion planning module. And deviating

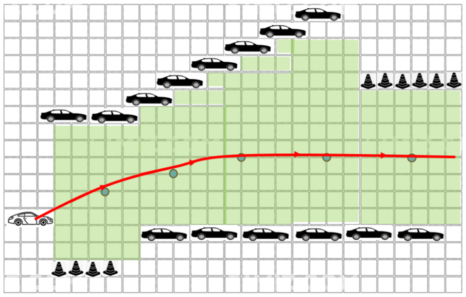

Fig. 2. The construction of spatio-temporal semantic corridor where the upper and lower cars represent the spatio-temporal trajectories of two different cars in different time steps. The resulting trajectory is obtained through an optimization problem and lies in the corridor which can guarantee safety.

from reference points may lead to inaccurate risk value due to the error in the linearization. The cost function formulation can be written as

$$\begin{aligned} J_k =& w_s \cdot \int_{t_{j-1}}^{t_j} (\frac{d^3 f_k(t)}{dt^3})^2 dt \\ &+ w_d \cdot \frac{1}{n_k} \sum_{k=0}^{n_k} (f_k(t_k) - r_k(t_k))^2 \\ &+ w_r \cdot \phi_k(f_k(t_k) - r_k(t_k)) \end{aligned} \quad (10)$$

where the first term corresponding to smoothness, the second term corresponding to the difference from reference trajectory point r_k at time step t_k and the last term corresponding to risk(Section III D) . $w_\odot$ refers weight terms. And the above formulation could be further formulated as follows:

$$\begin{aligned} J_k &= p_k^T (w_s Q_S + w_d Q_d + w_r Q_r) p_k + w_d c^T p_k \\ &= \frac{1}{2} p_k^T \hat{Q} p_k + \hat{c}^T p_k \end{aligned} \quad (11)$$

where a quadratic programming formulation is derived and can be solved using OOQP solver[25]. Finally a piecewise Bèzier curve is obtained as the trajectory whose convex hull property and hodograph property can guarantee the safety of the whole trajectory.

D. Risk Field Module

One common drawback of the optimization scheme is that the objective function only consists of "simple" terms such as the distances to surrounding vehicles or jerks in order to promote safety and smoothness. Although it is due to the property of optimization problems so that it can be efficiently solved, it is certain that some factors which humans will pay attention to when he/she makes decisions are missing in the original formulation. Risk field is a common technique to introduce additional factors into the construction of the distribution so that the "risk field" can represent the comprehensive idea of how a person likes/dislikes a certain area. Among the various researches, we introduce the mechanical-wave asymmetrical risk field proposed in [14] because it has a unified formulation and considers the asymmetrical property of social interactions. The risk field formulation is

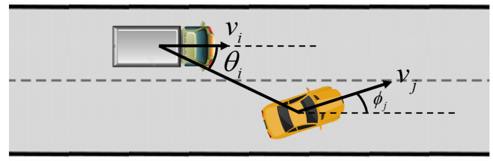

Fig. 3. Illustration of the symbols used in the risk field calculation where the truck is regarded as the vehicle exerting risk (AGV) and the car suffering risk (SFV).

as follows:

$$\begin{cases} \Omega_{i\to j} &= \frac{m_i |v_i| exp(\mu_1 v d_{ij} + \mu_2 v_j d_{ji} e^{-\sigma m_i^{-1} R_{ij}})}{2\delta m_j}, \\ &= \frac{m_i |v_i|}{2\delta m_j} e^{\xi_1 + \xi_2} \\ & \xi_1 = \mu_1 |v_i| \cos\theta_i,\ \xi_2 = -\sigma m_i^{-1} R_{ij} \end{cases} \quad (12)$$

$$\begin{gathered} R_{ij} = \sqrt{\begin{aligned} &((x_j - x_i)\cos\phi_i - (y_j - y_i)\sin\phi_i)^2/\rho^2 \\ &+((x_j - x_i)\sin\phi_i + (y_j - y_i)\cos\phi_i)^2/\eta^2 \end{aligned}} \\ \rho = \tau e^{\beta(|v_i| + \cos\theta_i a_i t_0)}, \eta = \tau \end{gathered} \quad (13)$$

where $\Omega_{i\to j}$ means the risk imposed by ith vehicle(AGV) to jth vehicle(SFV), $(x_\odot, y_\odot)$ is the coordinates of the vehicles, $m_\odot$ is the vehicle mass, $v_\odot$ is the vehicle velocity, $\phi_\odot$ is the angle between velocity vector and the positive direction of x axis and refers to the lane direction in this study, θ_i is the angle between the velocity vector of ith vehicle(AGV) and the line direction connecting ith vehicle(AGV) and jth vehicle(SFV), β is the shape coefficient, τ is the safe distance, μ_1, μ_2 are velocity coefficients, σ, δ are decay coefficient and proportional coefficient, respectively.

As Fig.4 shows, the risk field formulation captures the asymmetrical property of social interactions between vehicles as well as the velocity directions, for example, a car driver would stay away from a heavy truck while the truck driver would not stay away from a car, which implies that different types or masses of the vehicles(AGV) generate different risks for the same car. Although the exact masses are not available to the ego car without V2V communications, an estimate can be made based on the perceptions just as humans do.

As is shown in the equation, the risk field formulation is highly non-linear which brings difficulty to the solution of the optimization problem if applied to the objective function directly, leading to excess solution time or convergence to local minimum. Note that the game theory module outputs reference waypoints for the ego vehicle and the spatio-temporal planning module penalizes deviating from the reference waypoints, based on which we can reasonably assume that our actual waypoints taking the risk field into account will be relatively close to reference points. Therefore we utilize the first-order derivative of the risk field relative to position to preserve linearity without jeopardizing too much

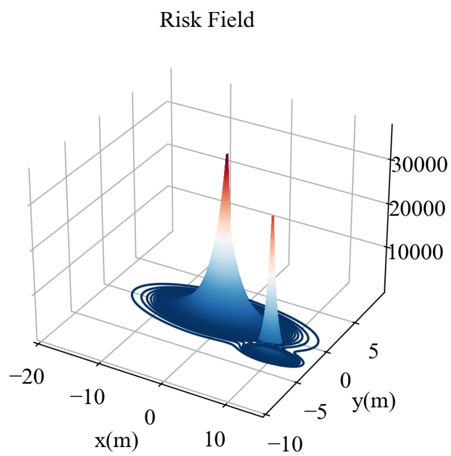

Fig. 4. The risk field adopted in this study. The higher peak represents a heavier vehicle of 1500kg, and the lower peak represents a lighter vehicle of 300kg in comparison. Both vehicle has a velocity of 30km/h. The Z axis represents the value of risk.

accuracy of the risk estimation:

$$\begin{aligned}\frac{\partial \Omega}{\partial x_j} &= \Omega_{i\to j} \cdot \left[\xi_{Cx} - \frac{\sigma}{m_i} \cdot \frac{\xi_{Rx\_1} + \xi_{Rx\_2}}{R_{ij}|_{p_{ref}}}\right] \\ \frac{\partial \Omega}{\partial y_j} &= \Omega_{i\to j} \cdot \left[\xi_{Cy} - \frac{\sigma}{m_i} \cdot \frac{\xi_{Ry\_1} + \xi_{Ry\_2}}{R_{ij}|_{p_{ref}}}\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\xi_{Cx} &= \frac{(\mu_1 |v_i| - \mu_2 |v_j|)(y_i - y_j)^2}{\left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\right)^3} \\ \xi_{Cy} &= \frac{(-\mu_1 |v_i| + \mu_2 |v_j|)(x_i - x_j)(y_i - y_j)}{\left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\right)^3} \\ \xi_{Rx\_1} &= \frac{\cos\phi_i}{\rho^2}\left[(x_j - x_i)\cos\phi_i - (y_j - y_i)\sin\phi_i\right] \\ \xi_{Rx\_2} &= \frac{\sin\phi_i}{\eta^2}\left[(x_j - x_i)\sin\phi_i - (y_j - y_i)\cos\phi_i\right] \\ \xi_{Ry\_1} &= \frac{-\sin\phi_i}{\rho^2}\left[(x_j - x_i)\cos\phi_i - (y_j - y_i)\sin\phi_i\right] \\ \xi_{Ry\_1} &= \frac{-\cos\phi_i}{\eta^2}\left[(x_j - x_i)\sin\phi_i - (y_j - y_i)\cos\phi_i\right]\end{aligned} \tag{15}$$

where $R_{ij}|_{p_{ref}}$ means R_{ij} obtained according to the reference waypoint. In this manner, we could use the term as $\phi_k(r_k(t_k))$ in the spatio-temporal planning formulation to take the risk factor into account in the motion planning algorithm. As for the game theory module, we use the original $\Omega_{i\to j}$ because there are no reference points and the solver we use can handle nonlinear situation.

IV. EXPERIMENT

The simulation was implemented on Ubuntu 18.04 and we use ROS Melodic to enable different modules to correspond. We use the original spatio-temporal planning method EPSILON[10] as baseline. The optimization problem is formulated in CasADi[26] and solved using PANOC[24] for the game theoretic module and OOQP[25] for the spatio-temporal planning module. We implement our method in a ramp merging scenario where the ego vehicle has to merge into the main highway within a predetermined distance as is depicted in fig. 6. We choose this scenario setup because interactions are especially important when the traffic are dense in the main highway. The merging procedure starts by selecting the target gap, namely the gap beside the ego vehicle in the current time step, then the ego vehicle constantly observes the behavior of the target vehicle behind the gap to update the corresponding weights in the payoff function. We should note that due to the heading directions of the lanes, we assume that the vehicle behind the gap is well aware of the merging intention of the ego vehicle. Then the game theoretic problem is formulated and solved and the control input is applied directly to the target vehicle as its behavior. When the target vehicle refuses to yield and the merging attempt failed, the ego vehicle must decelerate and turn to the next gap. The weights to be select beforehand are shown in Table I.

In the scenario setup, there are 8 vehicles in total and we set the first 4 vehicles to be aggressive and the last 4 vehicles to be courteous so that the ego vehicle is able to do a successful merging maneuver potentially, where being ”aggressive” is represented as larger weights on the deviation from ideal following gap. We set the initial gap between surrounding vehicles to be 7 meters so that forcing a merge-in without speculating surrounding vehicles' intentions is dangerous and always denied by the motion planning module. With the help of our integrated game theoretic module, it can be seen that the ego vehicle successfully merges ahead of the 5th vehicle which is courteous and willing to give way to the merging ego vehicle, whereas the ego vehicle without the module hesitates to a stop and waits for all vehicle to pass before merging because the gap is too narrow to achieve a successful and safe merging if the ego vehicle fails to notice that the gap will be widened when the ego vehicle makes the attempt. As the velocity and acceleration profiles show, the vehicle equipped with game theoretic planning is able to derive a more efficient trajectory in the interactive traffic flow.

Then we illustrate the effects of the risk field. When we replace the 5th car with a heavy truck, the ego vehicle will eventually give up to merging before it as previously done and merges before the 7th vehicle because the truck is assumed to cast greater risk around it which matches human drivers' preferences. It is worth noting that if we tune up the weight factor and set the 5th vehicle to be courteous, the ego vehicle will accelerate to overtake the 5th vehicle to merge in as the regularization term is relatively less influential. As the constraints in the game theory module basically cannot guarantee the safety of the whole trajectory obtained, therefore a collision can be generated in some cases which indicates the necessity of spatio-temporal planning module. And the computational time is 20.0 milliseconds for the

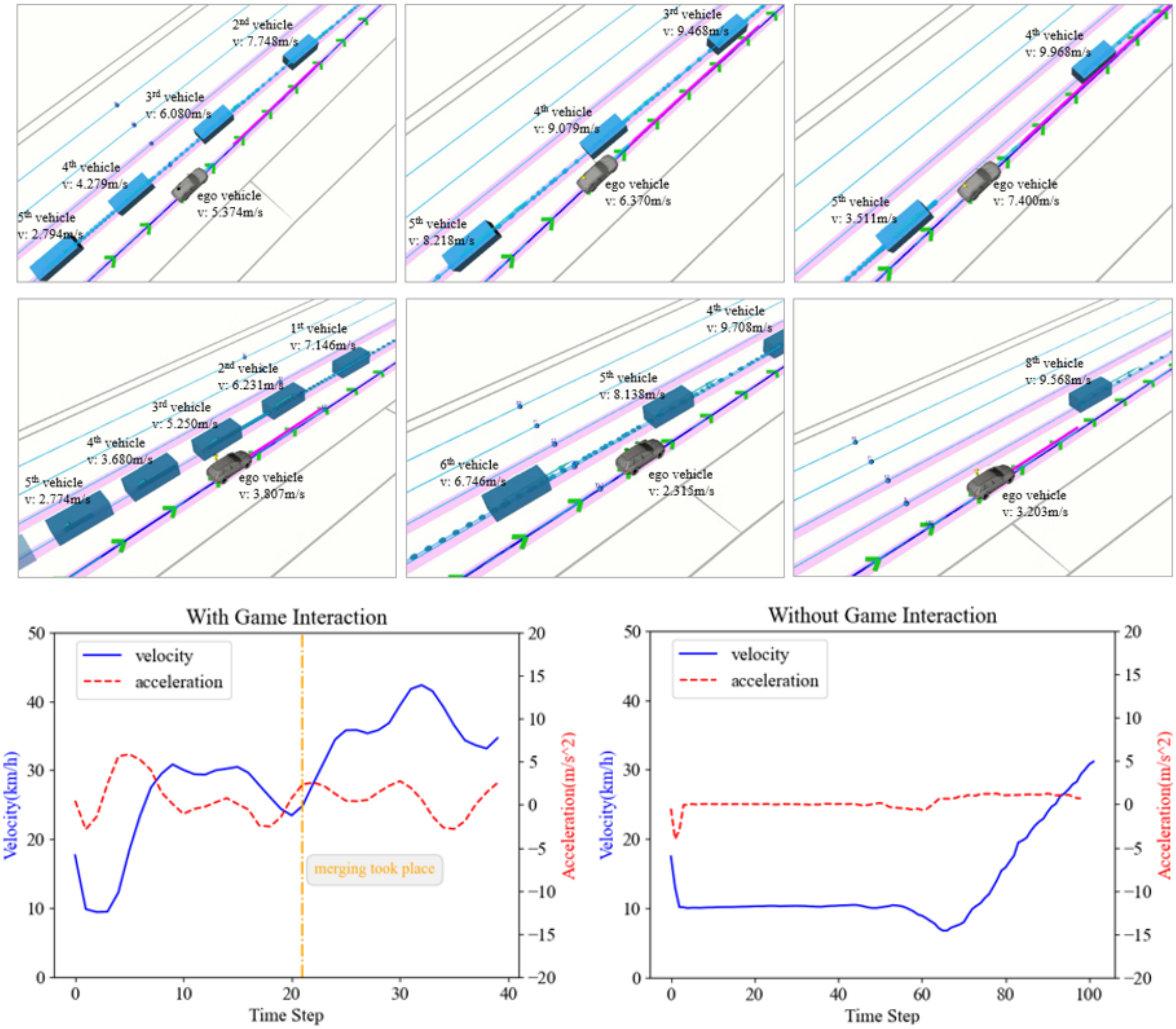

Fig. 5. The first row: When the scenario begins, the ego vehicle let the 4th vehicle by because it is observed that the vehicle will not yield and then merges before the 5th vehicle which is courteous. The second row: When the game interaction is not considered, the ego vehicle will let all vehicles by because forcing a merge-in is predicted to incur a collision. The third row: The velocity and acceleration profiles in two settings respectively, where the right profile shows that the ego vehicle maintains a low speed in order to let all surrounding vehicles pass before merging and accelerating.

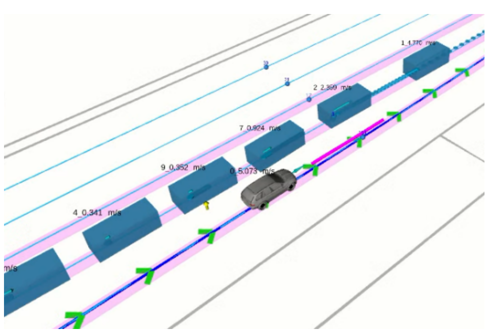

Fig. 6. The ramp merging scenario adopted in this study.

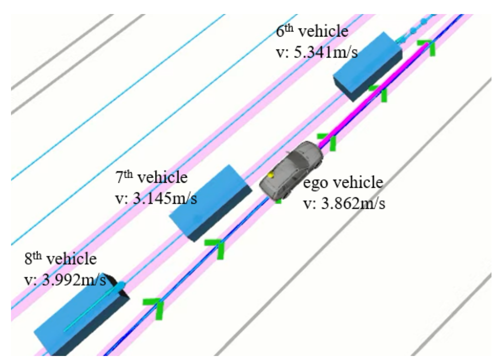

Fig. 7. The ego vehicle merges before the 7th vehicle when the 5th vehicle is replaced with a truck.

spatio-temporal planning module which is applicable onto real vehicles and 101.2ms for the game-theoretic module which is acceptable because it is not executed as often as the spatio-temporal planning module.

TABLE I
RELATED VARIABLE VALUES

Variables	Values
w_c	4
w_a	[0.05 0.1 0.5 1e-5 0.05]
$w_s(aggressive)$	[10 0.1 10]
$w_a(courteous)$	[0.02 0.1 0.02]
A	**diag**(4 2.25)
α	0.9827
β	0.9827
λ	1.2
σ	600
δ	5e-4
μ_1, μ_2	0.15, 0.16
t_0	2
τ	0.2

V. CONCLUSIONS

In this paper, we proposed a joint planning method for autonomous driving considering game theoretic interaction and asymmetric risk field theory. We introduce the game theoretic approach into the spatio-temporal planning framework and integrate the mechanical-wave formulation based risk field into the aforementioned algorithm to consider more comprehensive factors so as to achieve a more human-like decision process. Future works include making the algorithm more robust and introducing learning-based algorithm such as reinforcement learning to enhance flexibility as well as the speed of the method.

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