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An Optimization-based Method for Transient Stability Assessment

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Abstract— **The paper proposes an optimization-based method for the transient stability assessment of lossy multi-machine power systems. To achieve this objective, a global control Lyapunov function candidate including an auxiliary state is introduced. On this basis, a new excitation control law is proposed. This control law is well-defined provided that an 'index' matrix remains non-singular along the closed-loop trajectories. Such a matrix plays a key role in the formulation of an optimization problem, which allows calculating the so-called *critical value* associated to the introduced Lyapunov function. This permits a direct assessment of transient stability property of the considered post-fault power system. To illustrate the effectiveness of such an optimization-based method, a case study on the model of a three-machine system is presented.**

I. Introduction

Transient stability of a multi-machine power system is considered as its capability, when subject to a large disturbance, to maintain synchronism of all synchronous generators (SGs) [1]–[3]. A typical large disturbance is a short circuit fault on the transmission facilities, which drives the dynamic state of the SGs away from the pre-fault operating equilibrium [1]. After the fault is cleared, the problem of transient stability assessment (TSA) arises, *i.e.*, problem of checking whether the post-fault state trajectory converges to a desired post-fault operating equilibrium [2], [4]. Reliable TSA is crucial to power system plannings and operations [1]. Currently, time domain simulations (TDS) provide one of the most widely accepted methods for TSA [1], [2]. This is typically achieved by off-line numerical integration of the dynamic model of the considered post-fault system [1]. If the resulting state trajectory converges to a desired operating equilibrium, the post-fault system is deemed to be transiently stable; otherwise, it is deemed to be transiently unstable and corrective actions have to be undertaken [1]. However, since power systems continue to expand, the TDS methods have become increasingly computationally expensive [2]. In addition, the operating conditions of many power systems have been pushed closer to their stability limits, which makes such an off-line practice not suitable for future real-time operations [2].

Compared to the TDS methods, direct methods have a distinct advantage in that they allow assessing transient stability without time-consuming numerical integration [2]. In addition, direct methods provide quantitative information of stability margins. Such information is valuable for power system plannings and operations [2]. The concept of direct methods was originally proposed, termed as *transient-energy method*, by Magnusson in 1947 (see the seminal work [5]). The idea is firstly to construct a transient energy function (TEF), then to estimate the region of attraction (ROA) of the post-fault equilibrium by calculating the so-called *critical value* of the TEF, and finally to check whether the initial post-fault state is inside the ROA.

Historically, considerable efforts have been devoted to the design of direct methods especially for lossy multi-machine power systems, see *e.g.* [6]–[12]. The problem encountered at the early stage of this line of research was to handle the transmission losses, since such an intrinsic characteristic severely hinders the construction of a well-defined TEF or Lyapunov function. Several results have been proposed to confront this problem, see *e.g.* [7], [9], [10]. These efforts however have been partially in vain, because the proposed TEFs or Lyapunov functions are either not well-defined or such that their time-derivatives along the state trajectories are not always negative definite. In 1984, a pessimistic conclusion was drawn in [11], implying that there exists no general analytical TEF for multi-machine power systems with transmission losses. Compared to traditional TEFs, control Lyapunov functions (CLFs) can be constructed together with proper designs of control laws. Hence, it is believed by the author that CLFs are more suitable to address the problem of transient stabilization for lossy multi-machine power systems.

To the best of our knowledge, the problem of handling the transmission losses has been firstly addressed in 2005 in [4]. By *Implicit function theorem*, [4] has proved the existence of a flexible form of CLF for transient stabilization of multi-machine power systems with nontrivial transfer conductances. However, neither an explicit form of CLFs nor a feasible control law has been suggested. Later, in [13], a well-defined excitation control law for transient stabilization has been proposed. This renders the desired operating equilibrium asymptotically stable. However, no follow-up work towards the design of direct methods for

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TSA has been reported. As a step forward, in [14], a well-defined CLF with emphasis on bounded control input has been proposed. This has been shown to be applicable to the direct methods for TSA of a single machine model. However, the feasibility of extending such a CLF to the TSA for lossy multi-machine power systems has not been proven yet. Then, [15] has proposed an explicit form of CLFs by including an auxiliary state. Such a form of CLFs has been shown to be applicable to the transient stabilization of lossy multi-machine power systems with any number of SGs.

This paper presents a continuing work towards the design of direct methods for TSA of general lossy multi-machine power systems. Its main contribution is as follows.

- A global CLF for TSA of the model of lossy multi-machine power systems is introduced. This is mainly achieved by selecting a globally positive definite function for the potential-energy-like term. Based on the CLF candidate, an explicit dynamic control law is derived. This ensures locally asymptotic stability of the desired closed-loop post-fault operating equilibrium.
- Compared to the auxiliary state in [15], the new introduced auxiliary state contributes to the derivation of a more concise form of the denominators of the so-called cross term. Hence, it simplifies one of the constraints to be used in the optimization problem.
- The problem of calculating the *critical value* of the CLF is converted into a constrained optimization problem. With the calculated *critical value* at hand, one can assess directly the transient stability property of the considered post-fault power system.

The remaining part of the paper is organized as follows. In Section II, two matrices facilitating the representations of the model and the stability analysis are introduced. Then, the flux decay model of a lossy multi-machine power system is given. Then, the problem formulation for TSA is presented. In Section III, a CLF candidate is introduced and a new excitation control law is proposed. Their properties are summarized in **Proposition 1**. In Section IV, the selection of the potential-energy-like term is discussed, followed by a constrained optimization problem. This is instrumental to solve the TSA problem. Then, **Algorithm 1** illustrating the procedure to solve the TSA is presented. The main result (*i.e.*, the proposed direct method for TSA) is summarized in **Proposition 2**. In Section V, a case study on the model of a three-machine power system to demonstrate the effectiveness of the proposed method is presented. Finally, conclusions are drawn and future work is discussed in Section VI.

Notation: All vectors are considered as column vectors. The subscripts i and j represent the index of the states or parameters of the ith and the jth SG, respectively; double-subscripts ij represent the network connection between the ith and the jth SG. The integer n represents the number of SGs in the considered multi-machine model, termed as the n-machine model. Note that $i \in \mathbb{N}$, $j \in \mathbb{N}$, $n \in \mathbb{N}$, $i \leq n$ and $j \leq n$. The operation $\mathrm{diag}\{\cdot\}$ converts a column vector of dimension $n \times 1$ into a diagonal matrix of dimension $n \times n$ with diagonal entries given by the elements of the vector. The number in bold font represents the corresponding column vector of dimension $n \times 1$, *e.g.*, $\mathbf{0} = [0, 0, \dots, 0]^\top \in \mathbb{R}^n$. Finally, the superscript * attached to a variable indicates its equilibrium value.

II. Preliminaries

To simplify the expressions of the dynamic model, we define the two $n \times n$ matrices

$$\Gamma(\delta) = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}, \tag{1}$$

where $\gamma_{ij} := Y_{ij} \sin(\delta_i - \delta_j + \alpha_{ij})$, and

$$\Pi(\delta, \omega) = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} \end{bmatrix}, \tag{2}$$

where $\pi_{ij} := Y_{ij} \cos(\delta_i - \delta_j + \alpha_{ij})(\omega_i - \omega_j)$. The parameter $Y_{ij} \in \mathbb{R}_{>0}$ represents the magnitude of the complex element, located at the ith row and the jth column of the reduced bus admittance matrix; while $\alpha_{ij} \in \mathbb{R}$ is the corresponding complementary angle, see [4, equation (3)].

Note that the non-zero diagonal entries in (1) represent the transfer conductances, *i.e.*, $\gamma_{ii} = Y_{ii} \sin \alpha_{ii} = G_{ii}$ for all i. This indicates the intrinsic property that the system is *lossy*. Note also that the diagonal entries in (2) are zeros, *i.e.*, $\pi_{ii} = 0$ for all i.

A. Dynamic Model

With these two matrices, the dynamics of all the SGs in a lossy n-machine power system can be described by the *flux decay model* in vector form (see *e.g.* [15, equation (1)], or [12], [13], [16] for the original scalar form). This is given by the equations

$$\begin{aligned} \dot{\delta} &= \omega, \\ \dot{\omega} &= -\mathrm{diag}\{D\}\omega + P - \mathrm{diag}\{E\}\Gamma E, \\ \dot{E} &= u, \end{aligned} \tag{3}$$

where the dynamic state vectors consist of the SG rotor angles $\delta(t) \in \mathbb{R}^n$, the angular speed deviations with respect to the synchronous speed $\omega(t) \in \mathbb{R}^n$, and the internal transient voltages $E(t) \in \mathbb{R}^n_{>0}$. The system parameters include the damping ratios $D \in \mathbb{R}^n_{>0}$ and the constant mechanical powers $P \in \mathbb{R}^n_{>0}$. Finally, $u(t) \in \mathbb{R}^n$ is the vector of excitation control inputs after partial feedback linearization.

To simplify the expression for the CLF, we define the vector function

$$I(\delta, E) := \Gamma E. \tag{4}$$

It is worth noting that the time-derivative of (4) along the solution of (3) is given by

$$\dot{I} = \Pi E + \Gamma \dot{E}. \tag{5}$$

Denote by $x^* = \left[x_1^{*\top}, x_2^{*\top}, \cdots, x_n^{*\top}\right]^\top \in \mathbb{R}^{3n}$ the desired operating equilibrium of (3), where $x_i^* = [\delta_i^*, 0, E_i^*]^\top \in \mathbb{R}^3$ for the ith SG.

Assumption 1: The equilibria of the rotor angular separations and of the internal transient voltages for all SGs, *i.e.*, $\delta_{ij}^* := \delta_i^* - \delta_j^*$ and E_i^*, for all i and j, are known.

Note that **Assumption 1** is standard in transient stability analysis. By considering the rotor angular separation, we remove the requirement that each rotor angle converge to a specific equilibrium.

B. Problem Formulation

Consider the model (3) and a desired post-fault operating equilibrium x^*. The objective of designing a direct method for TSA is

- to construct a well-defined control Lyapunov function, the time-derivative of which along the post-fault state trajectories is negative definite;
- to estimate the ROA of x^* by calculating the *critical value* of the Lyapunov function;
- to check whether a given initial post-fault state converges to x^*.

III. Control Lyapunov Function Design

A. A Brief Review

Recall that [4] has proved the existence of a form of CLF for transient stabilization of multi-machine power systems with nontrivial transfer conductances. The proposed CLF is given by (see [4, equation (27)])

$$\begin{aligned} H_d(\delta, \omega, E) &= \psi(\delta) + \frac{1}{2}|\omega|^2 \\ &+ \frac{1}{2}[E - \mathrm{diag}\{\lambda(\delta)\}E^*]^\top [E - \mathrm{diag}\{\lambda(\delta)\}E^*], \end{aligned} \tag{6}$$

where $\psi : \mathbb{R}^n \to \mathbb{R}$ is the so-called potential-energy-like term and $\lambda : \mathbb{R}^n \to \mathbb{R}^n$ is the so-called cross-term. Both are functions to be defined. Note that $\psi(\delta^*) = 0$ and $\lambda(\delta^*) = \mathbf{1}$.

The inclusion of the cross-term λ allows the selection of any suitable potential-energy-like term ψ in (6). However, this leads to the difficulty in the explicit computation of the cross-term.

B. CLF Candidate With an Auxiliary State

To construct a CLF for TSA, we need to compute explicitly the cross-term. To this end, we consider an auxiliary state denoted by $\zeta(t) \in \mathbb{R}^n$. Such a state is motivated by the requirement that, during the transient, it provides an approximation of the vector function in (4), *i.e.*, $\zeta(t) \approx I(t)$ for all $t \geq 0$.

Consider now the CLF candidate

$$\begin{aligned} V(\delta, \omega, E, \zeta) &= \psi(\delta) + \omega^\top \mathrm{diag}\{\frac{a}{2}\}\omega \\ &+ [E - \mathrm{diag}\{E^*\}\lambda]^\top \mathrm{diag}\{\frac{b}{2}\}[E - \mathrm{diag}\{E^*\}\lambda] \\ &+ [I - \zeta]^\top \mathrm{diag}\{\frac{c}{2}\}[I - \zeta], \end{aligned} \tag{7}$$

where $\psi : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is the potential-energy-like term, and $\lambda : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the vector of cross term, both to be selected; while $a \in \mathbb{R}^n_{>0}$, $b \in \mathbb{R}^n_{>0}$ and $c \in \mathbb{R}^n_{>0}$ are vectors of weighting coefficients. Note that $\lambda(\delta^*, \zeta^*) = \mathbf{1}$ and $\zeta^* = \mathrm{diag}\{E^*\}^{-1}P$.

C. Stability Analysis

Taking the time-derivative of V along the trajectories of the dynamic model (3) yields

$$\begin{aligned} \dot{V} &= -\omega^\top \mathrm{diag}\{a\}\mathrm{diag}\{D\}\omega \\ &+ \omega^\top \Big[\frac{\partial \psi}{\partial \delta} + \mathrm{diag}\{a\}\big[P - \mathrm{diag}\{E^*\}\mathrm{diag}\{\lambda\}\zeta\big]\Big] \\ &- \omega^\top \mathrm{diag}\{a\}\mathrm{diag}\{E^*\}\mathrm{diag}\{\lambda\}[I - \zeta] \\ &- \omega^\top \mathrm{diag}\{a\}\mathrm{diag}\{I\}[E - \mathrm{diag}\{E^*\}\lambda] \\ &+ [E - \mathrm{diag}\{E^*\}\lambda]^\top \mathrm{diag}\{b\}[\dot{E} - \mathrm{diag}\{E^*\}\dot{\lambda}] \\ &+ [I - \zeta]^\top \mathrm{diag}\{c\}[\dot{I} - \dot{\zeta}]. \end{aligned} \tag{8}$$

We now present three design selections rendering $\dot{V}$, locally around x^*, strictly negative definite.

1) Cross term design: Let the cross-term be

$$\lambda(\delta, \zeta) = \big[\mathrm{diag}\{a\}\mathrm{diag}\{E^*\}\mathrm{diag}\{\zeta\}\big]^{-1}\big[\frac{\partial \psi}{\partial \delta} + \mathrm{diag}\{a\}P\big]. \tag{9}$$

Note that λ is well-defined for all $\zeta \neq \mathbf{0}$. The time-derivative of λ in (9) is given by

$$\dot{\lambda} = \mathrm{diag}\Big\{\frac{\partial \lambda}{\partial \zeta}\Big\}\dot{\zeta} + \frac{\partial \lambda}{\partial \delta}\omega. \tag{10}$$

Assumption 2: The initial values of the auxiliary states are positive, and the auxiliary state remains positive along the closed-loop trajectories, *i.e.*, $\zeta(t) > \mathbf{0}$ for all $t \geq 0$.

Remark 1: At x^*, we have $\zeta^* = \mathrm{diag}\{E^*\}^{-1}P > \mathbf{0}$. By continuity, the auxiliary state remains positive locally around its equilibrium. Hence, **Assumption 2** holds locally. In addition, compared with the design in [15, equation (17)], the selection in (9) contains a single state in the denominator. Therefore, it simplifies the formulation of the constrained optimization problem in (19).

2) Auxiliary state design: We select the dynamics of the auxiliary state as

$$\begin{aligned} \dot{\zeta} &= \Pi E + \Gamma \dot{E} + \mathrm{diag}\{d\}[I - \zeta] \\ &- \mathrm{diag}\{c\}^{-1}\mathrm{diag}\{a\}\mathrm{diag}\{E^*\}\mathrm{diag}\{\lambda\}\omega, \end{aligned} \tag{11}$$

where $d \in \mathbb{R}^n_{>0}$ is a vector of tunable constant. Note that one could always set the initial value of the auxiliary state as its equilibrium ζ^*.

3) Control law design: Recall that $\dot{E} = u$ in (3). We select the control law in such a way that

$$\begin{aligned}\dot{E} = &\operatorname{diag}\{E^*\}\dot{\lambda} + \operatorname{diag}\{b\}^{-1}\operatorname{diag}\{a\}\operatorname{diag}\{I\}\omega \\ &- \operatorname{SAT}(E - \operatorname{diag}\{E^*\}\lambda),\end{aligned} \tag{12}$$

where $\operatorname{SAT}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is any monotonically increasing saturation function.

Substitute (11) into $\dot{\zeta}$ in (10) and then substitute (10) into $\dot{\lambda}$ in (12). This yields the equation for calculating $\dot{E}$, *i.e.*,

$$\dot{E} = \mathcal{A}(\delta, \zeta)\dot{E} + \mathcal{B}(\delta, \omega, E, \zeta), \tag{13}$$

where

$$\mathcal{A} = \operatorname{diag}\{E^*\}\operatorname{diag}\Big\{\frac{\partial \lambda}{\partial \zeta}\Big\}\Gamma, \tag{14}$$

and

$$\begin{aligned}\mathcal{B} = &\operatorname{diag}\{E^*\}\operatorname{diag}\Big\{\frac{\partial \lambda}{\partial \zeta}\Big\}\Big[\Pi E + \operatorname{diag}\{d\}[I - \zeta] \\ &\quad - \operatorname{diag}\{c\}^{-1}\operatorname{diag}\{a\}\operatorname{diag}\{E^*\}\operatorname{diag}\{\lambda\}\omega\Big] \\ &+ \operatorname{diag}\{E^*\}\frac{\partial \lambda}{\partial \delta}\omega + \operatorname{diag}\{b\}^{-1}\operatorname{diag}\{a\}\operatorname{diag}\{I\}\omega \\ &- \operatorname{SAT}(E - \operatorname{diag}\{E^*\}\lambda).\end{aligned} \tag{15}$$

Hence, the resulting dynamic control law is given by the explicit expression

$$u(\delta, \omega, E, \zeta) = [\mathcal{I} - \mathcal{A}]^{-1}\mathcal{B}. \tag{16}$$

Assumption 3: The *index* matrix $[\mathcal{I} - \mathcal{A}]$ is non-singular at the initial state and it remains non-singular along the trajectories of the closed-loop system, *i.e.*, $\det[\mathcal{I}-\mathcal{A}(\delta(t), \zeta(t))] \neq 0$ for all $t \geq 0$.

Remark 2: At the operating equilibrium x^*, we have $\det[\mathcal{I}-\mathcal{A}(\delta^*, \zeta^*)] > 0$. By continuity, $\det[\mathcal{I}-\mathcal{A}]$ is non-zero locally around x^*.

D. Summary of CLF Design

In summary, the three design selections (9), (11) and (16) are such that

$$\begin{aligned}\dot{V} = &-\omega^\top\operatorname{diag}\{a\}\operatorname{diag}\{D\}\omega \\ &- [E - \operatorname{diag}\{E^*\}\lambda]^\top\operatorname{diag}\{b\}\operatorname{SAT}(E - \operatorname{diag}\{E^*\}\lambda) \\ &- [I - \zeta]^\top\operatorname{diag}\{c\}\operatorname{diag}\{d\}[I - \zeta] \leq 0,\end{aligned} \tag{17}$$

which is well-defined provided that $\zeta > \mathbf{0}$ and $\det[\mathcal{I}-\mathcal{A}] \neq 0$.

From (17), we conclude that **Assumptions 3** holds locally and that the desired closed-loop operating equilibrium x^* is locally stable. Furthermore, a direct application of *LaSalle's invariance principle* shows that x^* is also attractive, hence it is locally asymptotically stable. The discussion is summarized in the following statement.

Proposition 1: Consider the lossy multi-machine system model (3) and a desired operating equilibrium x^*. Let $\psi : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ be such that $\psi \in \mathcal{C}^2$ and $\delta^* = \arg\min \psi$. Hence, $\frac{\partial \psi}{\partial \delta}|_{\delta=\delta^*} = \mathbf{0}$. Then, the dynamic control law (16) is such that x^* is a locally asymptotically stable equilibrium of the closed-loop system.

IV. The Optimization Problem for Calculating the Critical Value

A. Selection of the Potential-energy-like Term

The potential-energy-like term offers a degree of freedom in the construction of a CLF, as long as it satisfies the conditions stated in **Proposition 1**. In what follows, we select

$$\psi(\delta) := \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\sigma}{2}(\delta_i - \delta_j - \delta_{ij}^*)^2, \tag{18}$$

where $\sigma \in \mathbb{R}_{>0}$ is a tunable weighting coefficient.

B. Formulation of the Optimization Problem

With the globally positive definite ψ in (18), we now discuss how an estimation of the ROA of x^* can be calculated. To this end, we consider the problem of calculating the so-called *critical value* of V. Note in (17) that the negative definiteness of $\dot{V}$ is guaranteed provided that $\zeta > \mathbf{0}$ and $\det[\mathcal{I} - \mathcal{A}] \neq 0$.

Hence, the *critical value* can be obtained by solving the constrained optimization problem

$$\begin{aligned}&\min V(\delta, \omega, E, \zeta), \\ &s.t.\ \zeta > \mathbf{0}, \\ &\qquad \det[\mathcal{I} - \mathcal{A}(\delta, \zeta)] = 0.\end{aligned} \tag{19}$$

Remark 3: The objective of (19) is to determine the smallest level line of V which is contained in the set $\zeta > 0$ and is "tangent" to the set of points such that $\det[\mathcal{I}-\mathcal{A}(\delta, \zeta)] = 0$. The global solution of (19) yields the *critical value* of V, denoted by V_{cr}.

On its basis, the estimated ROA of the desired post-fault operating equilibrium x^* is given by the set

$$A(x^*) = \big\{x \in \mathbb{R}^{3n} : V(\delta, \omega, E, \zeta^*) < V_{cr}\big\}. \tag{20}$$

As a result, the transient stability property of any given initial post-fault state can be assessed by **Algorithm 1**.

Algorithm 1

Require: Let the state, at the instant the fault is cleared, be x_c, and evaluate $V(x_c, \zeta^*)$
if $V(x_c, \zeta^*) < V_{cr}$ **then**
the post-fault trajectory starting from x_c converges to the desired operating equilibrium x^*
else
no conclusion is drawn, and TDS has to be conducted for deriving the post-fault trajectory
end if

The result of the optimization-based method for TSA is summarized as follows.

Proposition 2: Consider the closed-loop post-fault system (3) controlled with (16), and a desired post-fault operating equilibrium x^*. Select the CLF (7) with the potential-energy-like term in (18). Let V_{cr} be given. Then, if $V(x_c, \zeta^*) < V_{cr}$, the state x_c is "transiently" stable.

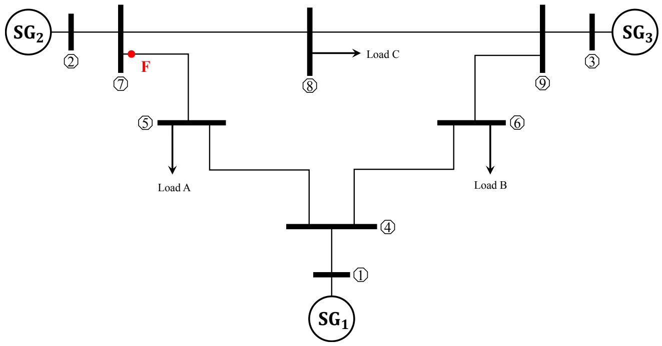

Fig. 1. Diagram of the three-machine system model. The label **F** identifies the location where the fault is applied. After the fault is cleared by opening the line 5-7, the system is switched into the post-fault mode.

TABLE I
PARAMETERS, COEFFICIENTS AND EQUILIBRIA

D	$[0.1973, 0.5655, 0.8016]^\top$
P	$[14.1381, 92.1743, 68.1339]^\top$
σ	$[0.1367, 0.1263, 0.0603]^\top$
a	$[0.0507, 0.0177, 0.0125]^\top$
b	$[0.4054, 0.1415, 0.0998]^\top$
c	$[0.2423, 0.8488, 0.5988]^\top$
d	$[20, 20, 20]^\top$
δ^*	$[0, 0.6788, 0.4221]^\top$
ω^*	$[0, 0, 0]^\top$
E^*	$[1.0970, 1.0552, 1.0248]^\top$
ζ^*	$[12.8878, 87.3512, 66.4828]^\top$

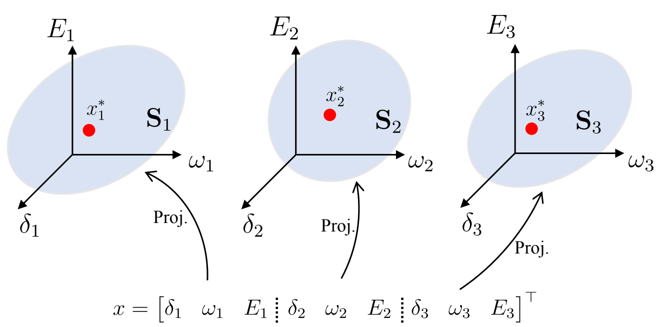

Fig. 2. Illustration of the projections of the estimated ROAs, where x_i^* is the post-fault operating equilibrium point of the ith SG.

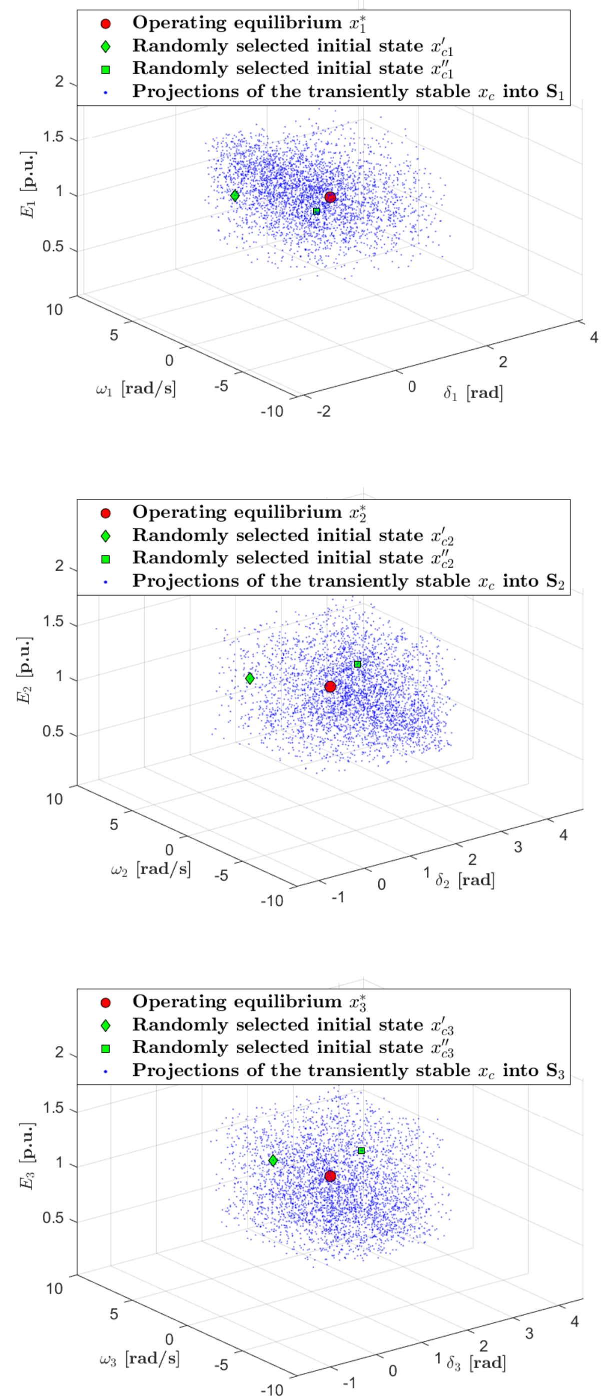

Fig. 3. The candidates x_c that are assessed to be transiently stable, and the projections of them into $\mathbf{S}_1$ (top), $\mathbf{S}_2$ (middle) and $\mathbf{S}_3$ (bottom), respectively.

V. CASE STUDY

To demonstrate the effectiveness of the proposed method, we consider the model of a three-machine system as shown in Fig. 1 (refer to [17, page 38]). The values of the parameters, of the coefficients, and of the post-fault equilibrium x^* are listed in Table I. Based on the aforementioned specifications, we obtain[1] the *critical value* of the CLF as

$$V_{cr} \approx 439.0574. \tag{21}$$

The visualization of the ROA for TSA is not a trivial task. To confront this problem, we consider a large number of randomly generated[2] initial post-fault state candidates x_c

[1]The optimization problem (19) is solved with the function 'fmincon' in MATLAB. Note that different selection of ψ or different specification on the values of the parameters result in different V_{cr}.

[2]In this case study, 10^5 candidates x_c are generated with the function 'rand' in MATLAB.

located around x^*. Then, we obtain the estimated ROA by combining all the transiently stable x_c such that $V(x_c, \zeta^*) < V_{cr}$ in (21). To visualize the estimated ROA, we project these transiently stable x_c into three subspaces, each of which corresponds to the state space of one SG, *i.e.*, $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$, respectively. The expected estimation of the ROAs is shown in Fig. 2.

The resulting projections obtained by *Monte Carlo simulations* are shown in Fig. 3. In each plot of projections, the red circle represents the operating equilibrium x_i^* of ith SG, while the blue dots represent the initial post-fault state candidates x_c that are assessed to be transiently stable. These provide the estimated ROAs.

A. Verification for TSA

We select randomly two initial post-fault states marked by the green diamond (denoted by x') and by the green square (denoted by x''), respectively, as shown in Fig. 3. Then, we verify through time-domain simulation that both are transiently stable. As shown in Fig. 4, the post-fault state trajectories starting from x' and from x'' converge to the post-fault operating equilibrium x^*.

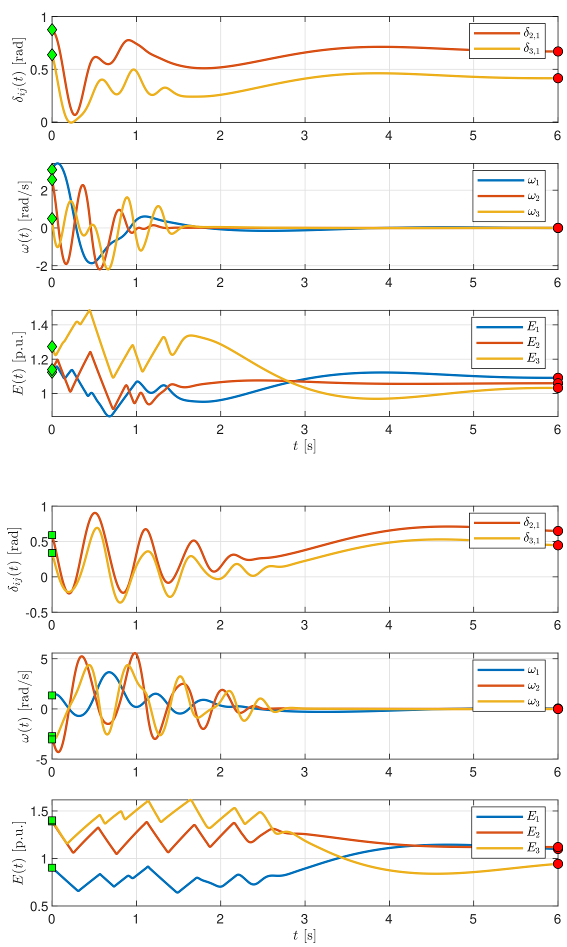

Fig. 4. Time histories of the post-fault states starting from x_c' (top) and from x_c'' (bottom), respectively.

VI. Conclusion

This paper has presented an optimization-based method for the direct TSA of lossy multi-machine power systems. A global CLF with a positive definite potential-energy-like term is proposed. On the basis of this CLF, a constrained optimization problem for calculating the *critical value* has been formulated. This allows assessing directly the transient stability property of a given initial post-fault state.

One of the potential directions for future work is to investigate the selection of the potential-energy-like term such that the estimation of the ROAs becomes less conservative.

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