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Inverse Optimal Control and Passivity-Based Design for Converter-Based Microgrids

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Abstract— **Passivity-based approaches have been suggested as a solution to the problem of decentralised control design in many multi-agent network control problems due to the plug-and-play functionality they provide. However, it is not clear if these controllers are optimal at a network level due to their inherently local formulation, with designers often relying on heuristics to achieve desired global performance. On the other hand, solving for an optimal controller is not guaranteed to produce a passive system. In this paper, we address these dual problems by using inverse optimal control theory to formulate a set of sufficient local conditions, which when satisfied ensure that the resulting decentralised control policies are the solution to a network optimal control problem, while at the same time satisfying appropriate passivity properties. These conditions are then reformulated into a set of linear matrix inequalities (LMIs) which can be solved to obtain such controllers for linear systems. The proposed approach is demonstrated through a DC microgrid case study. The results substantiate the feasibility and efficacy of the presented method.**

I. Introduction

Passivity-based approaches are widely used in the control of dynamic multi-agent networks as they enable the design of decentralised controllers which have desired plug-and-play characteristics, i.e. stability is ensured when subsystems are added/removed from the network [1], [2]. Such approaches have been successfully applied to problems such as network congestion control [3] and in chemical and biological systems [1]. One particular case of interest is that of power systems, specifically the control of converter-based grids [4]. Passivity-based approaches for grid-forming converter control have been proposed to guarantee stability in a decentralised way in both AC [5], [6] and DC microgrids [7], [8]. However, the performance metrics often used for control design are only local and thus do not provide network-wide performance guarantees.

Consequently, the need arises for a comprehensive metric capable of evaluating controller performance for such systems at the network level. Previous work revealed that a duality exists between passivity-based cooperative control problems and network optimisation problems [9]. However, these results only considered steady-state equilibria and did not address optimisation of system trajectories. While the passive controllers from the literature discussed above may be optimal at a local level (e.g. the mixed H_∞/passive controller from [5]), the tuning of these controllers is often based on heuristics and lacks a systematic way of achieving network-wide performance objectives.

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To address this gap, we propose an approach that applies the framework of inverse optimal control to network systems where the local subsystems are designed to be passive. Inverse optimal control is a well established branch of optimal control theory that focuses on deriving a cost functional for which a given controller is the optimal solution [10], [11], [12]. We show that passive network systems naturally fit within this theory, as the separable structure of the Lyapunov functions allows to formulate optimal control problems where the optimal controller is both decentralised and passivates the subsystem dynamics.

The contributions of this paper are as follows:

1) We introduce a set of sufficient conditions that allow passive network systems to have an inverse optimal control formulation, i.e. the corresponding decentralised controllers are solutions to a network-wide optimal control problem.
2) We show that these conditions can be transformed into linear matrix inequalities (LMIs) for linear systems, enabling the design of controllers that simultaneously achieve plug-and-play functionality and global optimality. Furthermore, we demonstrate that controllers derived using this method can be easily tuned while upholding both objectives.
3) This approach is tested and verified using a DC microgrid case study.

The paper is organised as follows. Section II introduces the network model and reviews both passivity theory and inverse optimal control. These concepts are combined in Section III to analyse a class of decentralised controllers that passiviate the local subsystems and also solve a network-wide optimal control problem. A DC microgrid case study is analysed in Section IV, before concluding in Section V.

II. Preliminaries

A. Notation and Definitions

The matrix-weighted (semi)norm of a vector $x \in \mathbb{R}^n$ is given by $\|x\|_R^2 = x^T R x$ for positive (semi-)definite $R \in \mathbb{R}^{n\times n}$. I_n denotes the n-dimensional identity matrix. The Kronecker product is denoted by $\otimes$. The direct sum of a set of indexed matrices B_k, with k an element of some ordered index set $\mathcal{A}$ is denoted by $\oplus_{k\in\mathcal{A}} B_k$. The composite vector constructed from a set of indexed vectors a_k, where k is an element of some ordered index set $\mathcal{A}$ is denoted $[a_k]_{k\in\mathcal{A}}$. For ease of presentation, we will denote $(\nabla V(x))^T$ as $\nabla^T V(x)$.

B. Network Model

Consider a network modelled as a directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, with $\mathcal{V}=\{\nu_1,\nu_2,\ldots,\nu_{|\mathcal{V}|}\}$ the set representing the nodes and $\mathcal{E}=\{\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{|\mathcal{E}|}\}$ is the set representing the edges of the graph. Assigning each edge in $\mathcal{E}$ an arbitrary direction, we denote $N_i^+\subset\mathcal{E}$ as the set of edges that have node i as their sink and $N_i^-\subset\mathcal{E}$ as the set of edges that have node i as their source. The edge $\varepsilon_k\in\mathcal{E}$ can equivalently be identified as the couple $\varepsilon_k\equiv(\nu_i,\nu_j)\in\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$, indicating that ε_k connects node $\nu_i\in\mathcal{V}$ to node $\nu_j\in\mathcal{V}$. Let $\mathcal{B}\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}|}$ denote the incidence matrix of the graph, with $\mathcal{B}_{jk}=1$ if node $j\in\mathcal{V}$ is the sink node of edge $k\in\mathcal{E}$, $\mathcal{B}_{jk}=-1$ if node $j\in\mathcal{V}$ is the source node of edge $k\in\mathcal{E}$, or 0 otherwise. Let $\mathcal{B}_p$ denote the matrix $\mathcal{B}\otimes I_p$.

For each node $i\in\mathcal{V}$ we associate controllable dynamics that depend non-linearly on local state $x_i\in\mathbb{R}^n$ and affinely on local control variable $u_i\in\mathbb{R}^m$ and input $w_i\in\mathbb{R}^p$ (defined in (3a) in terms of edge variables) as follows:

$$\Sigma_i:\begin{cases}\dot{x}_i &= f_i(x_i)+B_{ui}(x_i)u_i+B_i(x_i)w_i,\\ y_i &= h_i(x_i).\end{cases}\tag{1}$$

Here, $f_i:\mathbb{R}^n\to\mathbb{R}^n$, $B_{ui}:\mathbb{R}^n\to\mathbb{R}^{n\times m}$ and $B_i:\mathbb{R}^n\to\mathbb{R}^{n\times p}$ are Lipschitz continuous. In addition, the system associated with $i\in\mathcal{V}$ produces output $y_i\in\mathbb{R}^p$ and $h_i:\mathbb{R}^n\to\mathbb{R}^p$ is a continuous vector-valued function. In the paper, we will consider decentralised static state-feedback policies of the form $u_i=u_i(x_i)$, where $u_i:\mathbb{R}^n\to\mathbb{R}^m$ is Lipschitz continuous. The dependence on x_i will be henceforth omitted for simplicity in the presentation.

Similarly, with each edge $j\in\mathcal{E}$ we associate uncontrollable dynamics that depend non-linearly on the local state $x_j\in\mathbb{R}^r$ and affinely on the input $w_j\in\mathbb{R}^p$ (defined in (3b) in terms of node variables) as follows:

$$\Sigma_j:\begin{cases}\dot{x}_j &= f_j(x_j)+B_j(x_j)w_j,\\ y_j &= h_j(x_j).\end{cases}\tag{2}$$

Here, $f_j:\mathbb{R}^r\to\mathbb{R}^r$ and $B_j:\mathbb{R}^r\to\mathbb{R}^{r\times p}$ are Lipschitz continuous. In addition, the system associated with $j\in\mathcal{E}$ produces output $y_j\in\mathbb{R}^p$ and $h_j:\mathbb{R}^r\to\mathbb{R}^p$ is continuous.

Motivated by electrical circuit theory, the interconnection between the node and edge dynamics is characterised by the following relations (which respectively correspond to Kirchhoff's Current Law and Kirchhoff's Voltage Law):

$$w_i=\sum_{(l,m)\in N_i^-}y_{lm}-\sum_{(l,m)\in N_i^+}y_{lm},\quad i\in\mathcal{V}\tag{3a}$$

$$w_{lm}=-y_l+y_m,\qquad (l,m)\in\mathcal{E}.\tag{3b}$$

Here, w_{lm} and y_{lm} respectively denote the input and output of system (2) associated with the edge $(l,m)\in\mathcal{E}$.

Now, using the above relations, two composite subsystems can be created representing the node and edge dynamics. Letting $x_\mathcal{V}=[x_i]_{i\in\mathcal{V}}$, $f_\mathcal{V}(x_\mathcal{V})=[f_i(x_i)]_{i\in\mathcal{V}}$, $w_\mathcal{V}=[w_i]_{i\in\mathcal{V}}$, $\hat{u}=[u_i]_{i\in\mathcal{V}}$, $B_\mathcal{V}(x_\mathcal{V})=\oplus_{i\in\mathcal{V}}B_i(x_i)$ and $B_u(x_\mathcal{V})=\oplus_{i\in\mathcal{V}}B_{ui}(x_i)$, we have the composite block diagonal node dynamics

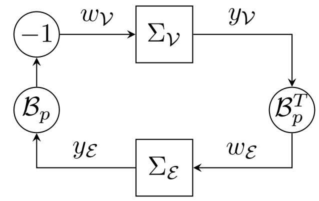

Fig. 1. Negative feedback interconnection of the node system $\Sigma_\mathcal{V}$ with state $x_\mathcal{V}$ and the edge system $\Sigma_\mathcal{E}$ with state $x_\mathcal{E}$.

$$\Sigma_\mathcal{V}:\quad \dot{x}_\mathcal{V}=f_\mathcal{V}(x_\mathcal{V})+B_\mathcal{V}(x_\mathcal{V})w_\mathcal{V}+B_u(x_\mathcal{V})\hat{u}.\tag{4}$$

Similarly, the composite block diagonal edge dynamics can be described using $x_\mathcal{E}=[x_j]_{j\in\mathcal{E}}, f_\mathcal{E}(x_\mathcal{E})=[f_j(x_j)]_{j\in\mathcal{E}}, w_\mathcal{E}=[w_j]_{j\in\mathcal{E}}$ and $B_\mathcal{E}(x_\mathcal{E})=\oplus_{j\in\mathcal{E}}B_j(x_j)$, giving

$$\Sigma_\mathcal{E}:\qquad \dot{x}_\mathcal{E}=f_\mathcal{E}(x_\mathcal{E})+B_\mathcal{E}(x_\mathcal{E})w_\mathcal{E}.\tag{5}$$

The relations (3) then reduce to $w_\mathcal{V}=-\mathcal{B}_p y_\mathcal{E}$ and $w_\mathcal{E}=\mathcal{B}_p^T y_\mathcal{V}$, where $y_\mathcal{V}=[y_i]_{i\in\mathcal{V}}$ and $y_\mathcal{E}=[y_j]_{j\in\mathcal{E}}$. Therefore, the whole network can be described by $\dot{\hat{x}}=\hat{f}(\hat{x})+\hat{B}(\hat{x})\hat{u}$, with $\hat{x}=[x_\mathcal{V}^T\ x_\mathcal{E}^T]^T$ and $\hat{f}(\hat{x})$ and $\hat{B}(\hat{x})$ given by the following expression:

$$\begin{bmatrix}\dot{x}_\mathcal{V}\\ \dot{x}_\mathcal{E}\end{bmatrix}=\begin{bmatrix}f_\mathcal{V}(x_\mathcal{V})-B_\mathcal{V}(x_\mathcal{V})\mathcal{B}_p y_\mathcal{E}\\ f_\mathcal{E}(x_\mathcal{E})+B_\mathcal{E}(x_\mathcal{E})\mathcal{B}_p^T y_\mathcal{V}\end{bmatrix}+\begin{bmatrix}B_u(x_\mathcal{V})\\ 0\end{bmatrix}\hat{u}.\tag{6}$$

We therefore see that the system (6) in closed-loop is a negative feedback interconnection of two aggregate systems as demonstrated in Figure 1: the node subsystem $\Sigma_\mathcal{V}$ with state $x_\mathcal{V}$ and the edge subsystem $\Sigma_\mathcal{E}$ with state $x_\mathcal{E}$.

C. Review of Passivity Theory

As described in e.g. [1], a local state feedback controller can be designed to achieve certain passivity properties of the closed-loop node subsystem.

A system (1) or (2), with input w_k and output y_k, $k\in\mathcal{V}\cup\mathcal{E}$, is said to be *strictly passive* if there exists a positive semi-definite, continuously differentiable function $V_k:\mathbb{R}^{n_k}\to\mathbb{R}_{\geq 0}$ (called the *storage function*) such that

$$w_k^T y_k\geq \dot{V}_k(x_k)+\psi_k(x_k),\qquad \forall w_k,x_k\tag{7}$$

where $\psi_k:\mathbb{R}^{n_k}\to\mathbb{R}_{>0}$ is a positive definite function [13].

For linear systems $i\in\mathcal{V}$ of the form

$$\begin{aligned}\dot{x}_i&=A_i x_i+B_{ui}u_i+B_i w_i\\ y_i&=C_i x_i,\end{aligned}\tag{8}$$

with linear controllers $u_i=K_i x_i$, $K_i\in\mathbb{R}^{m\times n}$ we can take $V_i(x_i)=\frac{1}{2}x_i^T P_i x_i$ and $\psi_i(x_i)=\frac{1}{2}x_i^T\Gamma_i^{-1}x_i$ for some matrices satisfying $P_i=P_i^T>0$ and $\Gamma_i=\Gamma_i^T>0$. Then, (7) can equivalently be written as the following bilinear matrix inequality (BMI) in P_i, K_i and Γ_i [5], [13]:

$$\begin{bmatrix}\tilde{A}^T P_i+P_i\tilde{A}+\Gamma_i^{-1} & P_iB_i-C_i^T\\ B_i^TP_i-C_i & 0\end{bmatrix}\leq 0,\tag{9}$$

where $\tilde{A} = A_i + B_{ui}K_i$. This BMI can then be converted to an LMI using simple transformations.

Now, assuming the system associated with each edge $j \in \mathcal{E}$ is strictly passive from w_j to y_j with storage function $V_j(x_j)$, and the system associated with each node $i \in \mathcal{V}$ can be rendered strictly passive from w_i to y_i with storage function $V_i(x_i)$ by control action u_i, we can then construct a storage function for the entire network $\hat{V}(\hat{x}) = \sum_{i\in\mathcal{V}} V_i(x_i) + \sum_{j\in\mathcal{E}} V_j(x_j)$, which can serve as a network Lyapunov function. If all systems are linear, this can be written in matrix form as $\hat{V}(\hat{x}) = \frac{1}{2}\hat{x}^T\hat{P}\hat{x}$. The existence of this separable Lyapunov function ensures stability of the network under changes in network topology [5].

However, controllers designed using passivity-based methods do not necessarily guarantee desirable or optimal performance due to their inherent conservatism. We show here that inverse optimal control theory can be used to address this problem.

D. Review of Inverse Optimal Control

The aim of inverse optimal control is to find a performance metric for which a given controller is the optimal solution. This problem has a long history, and various formulations have been presented in the literature ([10], [11], [12]). The approach taken in [12], [14] for nonlinear systems is reproduced here:

Theorem 1: Consider the optimal control problem

$$\begin{aligned} \min_u \quad & \int_0^\infty q(x,R) + \|u\|_R^2 \; dt \\ \text{s.t.} \quad & \dot{x} = f(x) + G^T(x)u, \; x(0) = x_0. \end{aligned} \tag{10}$$

Here $x \in \mathbb{R}^n$ is the state vector, $x(0) = x_0$ is the initial condition and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear vector field with $f(0) = 0$. The input matrix is given by the continuous matrix-valued function $G : \mathbb{R}^n \to \mathbb{R}^{m\times n}$. We have $R = R^T > 0 \in \mathbb{R}^{m\times m}$ as a design matrix, and the function $q : \mathbb{R}^n \times \mathbb{R}^{m\times m} \to \mathbb{R}_{>0}$ has the condition $q(0,\cdot) = 0$.

Now, let $V : \mathbb{R}^n \to \mathbb{R}_{>0}$ be a continuously differentiable function associated with a stabilising feedback control law

$$u^\star = -\tfrac{1}{2}R^{-1}G(x)\nabla V(x) \tag{11}$$

where

$$\nabla^T V(x)\left(f(x) + G^T(x)\,u^\star\right) < -\|u^\star\|_R^2. \tag{12}$$

Define

$$q(x,R) = -\nabla^T V(x)\left(f(x) + G^T(x)\,u^\star\right) - \|u^\star\|_R^2. \tag{13}$$

Then the following statements hold:

1) The unique optimal control law is given by (11).
2) The problem (10) has the optimal value $V(x_0)$.

Therefore, (10) serves as a performance metric for the system. In addition, as described in [14], $V(x)$ can be used as a Lyapunov function to prove asymptotic stability, though additional system structure needs to be exploited when the right-hand-side of (12) is not positive definite in x.

For linear systems, (10) reduces to a linear quadratic regulator (LQR) problem with $q(x,R) = x^TQ(R)x$, where $Q(R) = Q(R)^T \geq 0$ is the state cost matrix. Applying Theorem 1 requires the optimal controller to have an associated control Lyapunov function $V(x) = \frac{1}{2}x^TPx$ with $P = P^T > 0$ and be of the form $u^\star = -\frac{1}{2}R^{-1}B^TPx$. Equation (13) gives an expression for the state cost matrix $Q(R)$ and (12) gives the condition $Q(R) > 0$.

Remark 1: Note that Theorem 1 requires the control law to be of the form (11) in order to be the solution to an optimisation problem of the form (10). For linear systems, a set of necessary and sufficient conditions for this to be the case and for inverse optimality to hold are specified in [15].

For networks of interconnected systems, using a decentralised controller at each node does not necessarily lead to an inverse optimal control interpretation. At the same time, solving a network-wide optimal control problem is not guaranteed to produce a decentralised set of controllers, nor is it guaranteed to produce controllers that lead to passivity properties for the subsystems, and thus a plug-and-play operation. In the sequel, we will discuss under what conditions a decentralised controller can solve both these problems simultaneously.

III. Global Optimality from Decentralised Passive Controllers

We observe that a passivity-based control design leads to a Lyapunov function for the entire interconnection. At the same time inverse optimal control theory relies on the existence of a Lyapunov function which satisfies certain additional conditions. We will see in this section that by combining these two approaches we can derive a set of sufficient local conditions under which a set of decentralised controllers simultaneously yield the optimal solution to a network-wide optimal control problem, while also facilitating plug-and-play operation when the network is modified. Proofs to the results in this section are provided in [16].

Theorem 2: Consider the negative feedback interconnection network model (6). Assume that for each edge $j \in \mathcal{E}$ the system with input w_j and output y_j is strictly passive with storage function $V_j(x_j)$. For each node $i \in \mathcal{V}$, assume dynamics (1) with

$$u_i = -\tfrac{1}{2}R_i^{-1}B_{ui}^T(x_i)\nabla V_i(x_i), \tag{14}$$

with some matrix $R_i = R_i^T > 0$ and some positive definite function $V_i(x_i)$. Assume that using (14) in (1) generates a local closed-loop system such that the node dynamics are strictly passive from w_i to y_i with storage function $V_i(x_i)$, i.e. (7) is satisfied.

If in addition at each node $i \in \mathcal{V}$, the following local condition is satisfied:

$$\begin{aligned} &-\nabla^T V_i(x_i) f_i(x_i) + \\ &\quad \tfrac{1}{4}\nabla^T V_i(x_i) B_{ui}(x_i) R_i^{-1} B_{ui}^T \nabla V_i(x_i) > 0. \end{aligned} \tag{15}$$

Then $\hat{u}^\star = [u_i]_{i\in\mathcal{V}}$ is the globally optimal control input with respect to the optimisation problem (10) for system (6) with control cost matrix given by $R = \oplus_{i\in\mathcal{V}} R_i$ and state cost function

$$q(\hat{x}, R) = \sum_{k\in\mathcal{V}\cup\mathcal{E}} \left[-\nabla^T V_k(x_k) f_k(x_k)\right] + \sum_{i\in\mathcal{V}} \tfrac{1}{4}\nabla^T V_i(x_i) B_{ui}(x_i) R_i^{-1} B_{ui}^T \nabla V_i(x_i). \quad (16)$$

In addition, using (14) at each node ensures asymptotic stability of the interconnected system independent of its topology.

Remark 2: It should be noted that modifying the network may lead to a change in the equilibrium point. For linear systems, if the passivity property is satisfied about an equilibrium point, then it is satisfied at any equilibrium point [13]. In this case, Theorem 2 guarantees stability after network modifications, thus providing plug-and-play functionality. For nonlinear systems, satisfying the passivity property at any equilibrium point is linked to incremental passivity [17]. In addition, the approach outlined here can also be useful in practical designs where a change in an equilibrium point does not significantly change the linearisation of the system at that point, thus leading to plug-and-play functionality when operating close to this equilibrium point.

It can be seen (as demonstrated in the proof of Theorem 2 in [16]) that as a result of the local passivity property of each node and edge, the state cost function (16) is the sum of terms involving local states only. This means that the cost functional scales with the size of the network in question and can easily be adjusted to account for new nodes.

Next, a method to find such controllers for linear systems is presented. Here, the model (1) is linearised so that it can be written in the form (8). Similar relationships hold for (2), leading to a linearised version of the network system (6).

Lemma 3: Consider system (6) with linear node and edge dynamics as in (8). For each $i \in \mathcal{V}$ with linearised dynamics (8), consider matrices $Y_i \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{m\times m}$ and $\Gamma_i \in \mathbb{R}^{n\times n}$ that satisfy the following set of linear matrix inequalities (LMIs):

$$\begin{bmatrix} Y_i A_i^T + A_i Y_i + 2B_{ui} S_i B_{ui}^T & Y_i & B_i - Y_i C_i^T \\ Y_i & -\Gamma_i & 0 \\ B_i^T - C_i Y_i & 0 & 0 \end{bmatrix} \leq 0, \quad (17a)$$

$$-\tfrac{1}{2} B_{ui} S_i B_{ui}^T - \tfrac{1}{2}\left(A_i Y_i + Y_i A_i^T\right) > 0, \quad (17b)$$

$$Y_i = Y_i^T > 0, \quad \Gamma_i > 0, \quad S_i = S_i^T > 0. \quad (17c)$$

Considering $P_i = Y_i^{-1}$ and $R_i = -\frac{1}{2} S_i^{-1}$ yields the following distributed local controller for each $i \in \mathcal{V}$:

$$u_i^\star = -\tfrac{1}{2} R_i^{-1} B_{ui}^T P_i x_i. \quad (18)$$

Then, together with the assumption that the edge subsystems (2) for $j \in \mathcal{E}$, are strictly passive from input w_j to output y_j, we have that the local controller (18) satisfies the conditions specified in Theorem 2 and therefore:

1) It generates a strictly passive local closed-loop node subsystem (8) for $i \in \mathcal{V}$, with input w_i and output y_i.
2) The global network-wide controller

$$\hat{u}^\star = \oplus_{i\in\mathcal{V}} \left(-\tfrac{1}{2} R_i^{-1} B_{ui}^T P_i\right) x_{\mathcal{V}} \quad (19)$$

composed of the distributed controllers (18) is the optimal solution to problem (10) for the linearised system (6) with cost matrices

$$R = \oplus_{i\in\mathcal{V}} R_i, \quad (20a)$$

$$Q(R) = \text{blockdiag}\left[Q_{\mathcal{V}}(R) \quad Q_{\mathcal{E}}\right], \quad (20b)$$

where

$$\begin{aligned} Q_{\mathcal{V}}(R) = \oplus_{i\in\mathcal{V}} \big(&\tfrac{1}{4} P_i B_{ui} R_i^{-1} B_{ui}^T P_i \\ &- \tfrac{1}{2}\left(P_i A_i + A_i^T P_i\right)\big), \\ Q_{\mathcal{E}} = - \oplus_{j\in\mathcal{E}} &\tfrac{1}{2}\left(P_{jk} A_{jk} + A_{jk}^T P_{jk}\right). \end{aligned} \quad (21)$$

3) The linearised system (6) is asymptotically stable independent of network topology.

Remark 3: Solving (17a) produces a local controller that generates a strictly passive closed-loop node subsystem, while (17b) ensures that condition (15) is satisfied. Therefore, satisfying all these constraints simultaneously produces a controller that fulfils the passivity property and is also a component of the global optimal network controller.

Remark 4: If desired, additional LMI constraints can be added to (17) to impose desired dynamic behaviour on the local system. For example, the constraint

$$A_i^T P_i + P_i A_i + 2 P_i B_{ui} S_i B_{ui}^T P_i < \lambda_i P_i \quad (22)$$

has the effect of imposing a minimum decay rate of λ_i on the storage function $V_i(x_i) = \frac{1}{2} x_i^T P_i x_i$, which can improve performance by decreasing settling time for the local system [18]. This then has a direct impact on global performance in cases where the node dynamics dominate and there are weak node-edge interactions.

In [14], it was noted that the control cost matrix R can be used as a tuning parameter, whereby optimality is preserved for any $\bar{R} \leq R$. We show below that when this property is applied to the controllers designed in Lemma 3, decentralization and passivity are also retained.

In particular, once suitable P_i have been found for each $i \in \mathcal{V}$ using (17), application of Theorem 1 means that new controllers $\bar{u}_k$ can be found via $\bar{u} = -\frac{1}{2}\bar{R}^{-1}\hat{B}^T \hat{P}\hat{x}$ for any $\bar{R} = \bar{R}^T > 0$ with $\bar{R} \leq R$. For a decentralised controller, we require a block diagonal $\bar{R}$. Taking $\bar{R} = \oplus_{i\in\mathcal{V}} \bar{R}_i$ with all $\bar{R}_i = \bar{R}_i^T > 0$, gives the local controller $\bar{u}_i = -\frac{1}{2}\bar{R}_i^{-1} B_{ui}^T P_i x_i$ and $\bar{u}$ satisfies the global optimality condition with control cost $\bar{R}$, provided $Q\left(\bar{R}\right) \geq 0$.

Thus, the local control cost matrices $\bar{R}_i$ can be used to tune the control matrix gains, while maintaining an optimal controller that solves (10). The lemma below shows that the conditions in Lemma 3 ensure that the passivity property is also satisfied when this class of control policies is considered.

Lemma 4: Let R_i and P_i denote particular solutions found by solving (17). Let $\bar{R}_i \in \mathbb{R}^{m\times m}, \bar{R}_i = \bar{R}_i^T > 0$ be a new tuning matrix that gives rise to local controller $\bar{u}_i = -\frac{1}{2}\bar{R}_i^{-1} B_{ui}^T P_i x_i$ and global controller $\bar{u}$ via $\bar{u} = [\bar{u}_i]_{i\in\mathcal{V}}$. Then, if $\bar{R}_i \leq R_i$,

1) The local closed-loop node subsystem (8) with $i \in \mathcal{V}$ is strictly passive from input w_i to output y_i.
2) $\bar{u}$ is the optimal solution to the global network LQR problem (10) with cost matrices $\bar{R} = \oplus_{i\in\mathcal{V}}\bar{R}_i$ and $Q(\bar{R})$ given by (20b).

Remark 5: Lemma 4 suggests it may be useful to find a maximal R_i that fulfils both the passivity and optimality objectives when solving (17). The control cost can then be lowered by choosing $\bar{R}_i < R_i$. We can achieve this by setting $S_i = -s_i I_m$ in (17) with $s_i \in \mathbb{R}, s_i > 0$ and minimising over s_i, turning (17) into an optimisation problem in s_i, Y_i and Γ_i. Therefore, we have that $\bar{R}_i > R_i$ cannot satisfy both the passivity and optimality conditions simultaneously when R_i is the maximal value considered here.

IV. DC Microgrid Case Study

We illustrate our results using a DC microgrid example. This was chosen as passivity-based approaches in DC microgrids have been frequently suggested in the literature where they are used to provide a decentralised means for control design without requiring knowledge of the microgrid structure [19]. However, the tuning of such control schemes is often based on heuristics due to the lack of a network-wide performance metric.

In such a microgrid, we will assume that every bus contains a DC source distributed generation unit (DGU) (such as a solar PV source or a battery), or a load. The DGU units interface with the grid via a DC-DC converter to regulate the DC voltage [7], [8]. We will assume that the node set $\mathcal{V}$ will be split between a collection of buses containing DGUs indexed by the set $\mathcal{D}$ and a collection of load buses indexed by the set $\mathcal{L}$, such that $\mathcal{V} = \{\mathcal{D}, \mathcal{L}\}$.

All DGU buses will be modelled with a buck converter producing a controllable DC voltage. At the outlet of each converter is an RLC filter which is connected to the lines of the microgrid, as demonstrated in Figure 2. Using Kirchhoff's Laws, this leads to the following equations for a DGU bus $i \in \mathcal{D}$:

$$l_i \dot{i}_i = -r_i i_i - v_i + u_i, \quad (23a)$$
$$c_i \dot{v}_i = i_i - i_i^o - g_i v_i, \quad (23b)$$

where l_i, r_i and c_i are the filter inductance, resistance and capacitance, g_i is a parasitic conductance, i_i is the filter current, v_i is the voltage at the output of the filter, u_i is the voltage output of the buck converter (the control variable) and i_i^o is the current injection from the neighbouring lines given by an expression analogous to (3a).

In order to track a voltage setpoint (which is broadcast by a centralised microgrid controller at semi-regular intervals), each converter also contains an integrator of the form [20]

$$\dot{\zeta}_i = v_i - v_i^{\text{set}} + z_i i_i^o, \quad (24)$$

where ζ_i is the integrator state, v_i^{set} is the setpoint for converter $i \in \mathcal{D}$ and z_i is a virtual impedance used to enhance stability [21] and allow for a passive controller of the form (18) to be feasible in the case study considered.

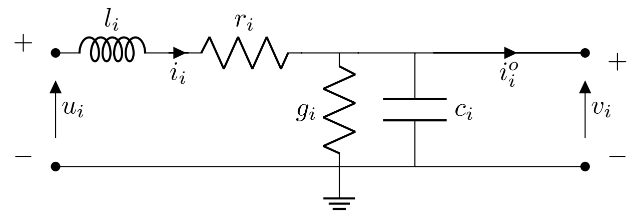

Fig. 2. Circuit diagram of the buck converter filter.

The dynamics (23) and (24) together form the subsystem $i \in \mathcal{D}$, where the input from the line subsystem is $-i_i^o$ and the output is v_i, which can be written in the form (8).

Meanwhile, we assume constant impedance loads at every $l \in \mathcal{L}$ with the following dynamics:

$$c_l \dot{v}_l = -g_l v_l - i_l^o, \quad (25)$$

where c_l is the load capacitance, g_l is the load conductance and the rest of the notation has analogous meanings to those in (23). We note that each $l \in \mathcal{L}$ is strictly passive from $-i_l^o$ to v_l with storage function $V_l(v_l) = \frac{1}{2}c_l v_l^2$.

Finally each line $(i, j) \in \mathcal{E}$ has the following dynamics:

$$l_{ij} \dot{i}_{ij} = -r_{ij} i_{ij} + v_{ij}, \quad (26)$$

where l_{ij} and r_{ij} are the line inductance and resistance, i_{ij} is the line current, and v_{ij} is the voltage difference between buses i and j, given by an expression analogous to (3b). We note that the dynamics associated with each $(i, j) \in \mathcal{E}$ are strictly passive from v_{ij} to i_{ij} with storage function $V_{ij}(i_{ij}) = \frac{1}{2} l_{ij} i_{ij}^2$.

The systems associated with $\{\mathcal{D}, \mathcal{L}\} = \mathcal{V}$ and $\mathcal{E}$ can easily be combined into a closed-loop system of the form depicted in Figure 1 and described by (6). However, here $\hat{u}$ is restricted to $\mathcal{D}$ rather than all nodes in $\mathcal{V}$, so $\hat{u} = [u_i]_{i\in\mathcal{D}}$.

Using Lemma 3, we can then construct matrices P_i and R_i for each $i \in \mathcal{D}$ such that each DGU subsystem is strictly passive from $-i_i^o$ to v_i with storage function $V_i(x_i) = \frac{1}{2}x_i^T P_i x_i$ when the decentralised controller $u_i = -\frac{1}{2}R_i^{-1}B_{ui}^T P_i x_i$ is used to close the DGU subsystem control loop. This gives rise to a global storage function $\hat{V}(\hat{x}) = \frac{1}{2}\hat{x}^T \hat{P}\hat{x}$ with

$$\hat{P} = \text{blockdiag}\left[\oplus_{i\in\mathcal{D}} P_i \quad \oplus_{l\in\mathcal{L}} c_l \quad \oplus_{(i,j)\in\mathcal{E}} l_{ij}\right]. \quad (27)$$

In addition, we have that the composite controller $\hat{u}$ is the optimal network-wide controller with respect to the LQR problem (10) with cost matrices $R = \oplus_{i\in\mathcal{V}} R_i$ and (using $Q_{\mathcal{D}}(R_i)$ with an expression analogous to $Q_{\mathcal{V}}(R_i)$ in (21))

$$Q(R) = \text{blockdiag}\left[Q_{\mathcal{V}}(R_i) \quad \oplus_{l\in\mathcal{L}} g_l \quad \oplus_{(i,j)\in\mathcal{E}} r_{ij}\right]. \quad (28)$$

Simulation: The above results have been tested numerically on a simple microgrid composed of three DGU buses and two load buses (see Figure 3). The three DGU buses have identical converters, defined by parameters in [16]. The LMIs (17) were solved as an optimisation problem as outlined in Remark 5, and included the additional performance constraint (22) (with $\lambda_i = -8$). The resulting R_i was 1.55,

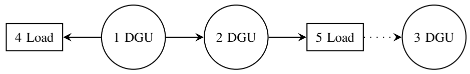

Fig. 3. Graph structure of the DC microgrid used in simulation. The line between DGU 3 and Load 5 is only connected at $t = 3$ seconds.

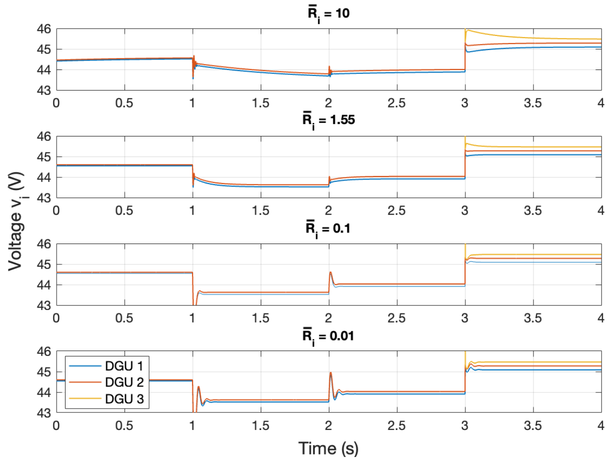

Fig. 4. The voltage v^i generated by each converter in a five-bus system containing three DGU buses and two load buses for various values for $\bar{R}_i$. Here, g_4 was altered at $t = 1\,\text{s}$ and g_5 at $t = 2\,\text{s}$ (see [16]). The link between DGU 3 and Load 5 was established at $t = 3\,\text{s}$ to test the plug-and-play property.

enabling calculation of R and $Q(R)$ using (28). The network storage function $\hat{P}$ was also found using (27).

Following this, tuning of $\bar{R}_i$ was performed, with values $\bar{R}_i < 1.55$ simultaneously generating a strictly passive bus and an optimal controller as guaranteed from Lemma 4. Values $\bar{R}_i > 1.55$ could not simultaneously satisfy the passivity and optimality conditions (but may still lead to a stable network). The controllers were tested in simulations to evaluate performance and the results can be found in Figure 4.

The results demonstrate that the controller quickly returns the microgrid voltages to equilibrium after each disturbance and easily handles the change in microgrid topology at $t = 3\,\text{s}$. As expected, lower values of $\bar{R}_i$ establish equilibrium more quickly by lowering the cost of control and increasing the corresponding cost of non-equilibrium state values. However, lower $\bar{R}_i$ values yield controllers with higher gains, potentially resulting in overly aggressive action as in the case of $\bar{R}_i = 0.1$ and $\bar{R}_i = 0.01$. Nevertheless, the framework described here offers a practical strategy for designing controllers for DC microgrids, enabling plug-and-play capabilities and quantifiable performance via an optimal control problem for the network.

V. Conclusion

In this paper, we have introduced a set of sufficient conditions that allow decentralised passivity-based controllers to be optimal with respect to a global network performance metric. A method for designing such decentralised control systems via an LMI framework was provided. This approach has the dual benefit of simultaneously rendering the closed-loop system at each node strictly passive, thus ensuring stability under network modifications, while also ensuring that the synthesised controller is the solution to a network-wide optimal control problem. The approach has also been verified via a DC microgrid case study.

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