

# An adaptive observer for time-varying nonlinear systems - application to a crop irrigation model

M.G. Dadjo, D. Efimov, J. Harmand, A. Rapaport, R. Ushirobira

**Abstract**—We propose an adaptive observer for a class of nonlinear time-varying systems, for which the regressor depends not only on the known input-output signals but also on all the unmeasured states. There are state disturbances, and measurements are corrupted by noise. The Lyapunov function method is used to prove that the coupled system-observer dynamics admits a state-independent input-to-output stability property in estimation errors from unknown inputs. Numerical simulations illustrate the presented approach on a three-dimensional crop irrigation model.

## I. INTRODUCTION

Observer theory has been a powerful tool for several decades to estimate the state and the parameters of an uncertain dynamical system in real time, and many researchers have worked in this field; see, for instance, [8]. Nevertheless, there is no general method to design observers for nonlinear systems in the current literature, and usually, certain canonical model representations are considered. This is also related to the lack of a standard selection approach for Lyapunov functions, which are needed in the stability analysis.

We consider in this note the problem of simultaneously reconstructing the unmeasured state variables and the constant unknown parameters in the presence of disturbances and measurement perturbations. For this purpose, adaptive observers have been proposed in the literature in the linear and the nonlinear contexts, starting from earlier works [11], [12], where two main ways for adaptive observer design were recognized: when the unknown parameters appear in the derivative of the output, or when they appear for the first time in higher order derivatives, and additional filters are needed to compensate the high relative degree obstruction. Significant results have been formulated at the beginning of this century, e.g., [2], [17], where canonical forms of systems admitting nonlinear adaptive observers were defined, and the canonical structure of the auxiliary filters was given. It is worth highlighting that an important restriction for a majority of the results is the requirement of dependence of regressor (the gain function that multiplies the vector

of unknown parameters) on measured input and output signals only, which was resolved in [7] for the cases of linear and nonlinear parameterization. Another common drawback is the lack of analysis in the presence of state disturbances and measurement noises, where only special cases were studied [5].

This work has been motivated by a concrete problem of crop irrigation. Usually, soil humidity and biomass can be measured or estimated [1], but the estimation of nitrogen content is often described as challenging, while it is crucial for growth prediction [14]. These variables' dynamics are nonlinear, time-varying, and rely on several parameters that need to be estimated for each soil-crop system [3]. Moreover, the measurements are noisy, and the state dynamics contains uncertain external inputs. So, we aim to propose a new approach for the adaptive estimation of this model, which due to its special characteristics, does not fall within the scope of previous works on observers.

In this paper, we propose an adaptive observer for a class of non-autonomous nonlinear dynamics with unknown inputs, for which the regressor depends not only on the known input-output signals but also on the unmeasured state, and the measurements are noisy. The analysis uses the theory of input-to-state stability (ISS) and other related properties. A Lyapunov function candidate is defined to prove that the system is state-independent input-to-output stable from unknown inputs to the estimation errors, relying on a condition of persistent excitation of the control variables.

The paper is organized as follows. In Section II, the used results from ISS theory are reviewed. In Section III, we give assumptions, present our observer, and prove our main result. Section IV introduces the irrigation problem and shows how to apply our main result. Finally, Section V presents numerical simulations and compares the performances of the adaptive observer with a high-gain observer.

## Notations

- $\mathbb{R}_+$ ,  $\mathbb{R}_+^*$  denote the sets of non-negative real numbers and positive real numbers, respectively.
- $\mathbb{R}^p$  and  $\mathbb{R}^{n \times m}$  denote the real vector space of dimension  $p$ , and the set of matrices with real coefficients of dimension  $n \times m$ , respectively.
- $I_n$  denotes the identity matrix of dimension  $n \times n$ .
- $\|\cdot\|$  denotes the Euclidean norm for vectors and the induced norm for matrices.

M.G. Dadjo, D. Efimov, R. Ushirobira are with Inria, Univ. Lille, CNRS, UMR 9189 - CRISTAL, F-59000 Lille, France; M.G. Dadjo and A. Rapaport are with MISTEA, Univ. Montpellier, INRAE, Institut Agro, 34060 Montpellier, France; J. Harmand is with LBE, INRAE, Univ. Montpellier, 11100 Narbonne, France; E-mails: mahugnon.dadjo@inrae.fr, jerome.harmand@inrae.fr, alain.rapaport@inrae.fr, denis.efimov@inria.fr, rosane.ushirobira@inria.fr,

- For a Lebesgue measurable function  $u: \mathbb{R}_+ \rightarrow \mathbb{R}^m$ , define the norm  $\|u\|_{[t_1, t_2]} = \text{esssup}_{t \in [t_1, t_2]} \|u(t)\|$  for  $t_1, t_2 \in \mathbb{R}_+$ . We denote by  $\mathcal{L}_\infty^m$  the set of functions  $u$  with  $\|u\|_\infty := \|u\|_{[0, \infty)} < +\infty$ .
- A continuous function  $\sigma: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  belongs to class  $\mathcal{K}$  if it is strictly increasing and  $\sigma(0) = 0$ ; it belongs to class  $\mathcal{K}_\infty$  if it is also radially unbounded. A continuous function  $\beta: \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  belongs to class  $\mathcal{KL}$  if  $\beta(\cdot, r) \in \mathcal{K}$  and  $\beta(r, \cdot)$  is a decreasing function going to zero for any fixed  $r \in \mathbb{R}_+$ .
- $A^\top$  (resp.  $\xi^\top$ ) denotes the transpose of the matrix  $A$  (resp. the vector  $\xi$ ).
- For a matrix  $M$  of  $\mathbb{R}^{n \times n}$ ,  $M \preceq 0$  (resp.  $M \prec 0$ ) means  $\xi^\top M \xi \leq 0$  (resp.  $\xi^\top M \xi < 0$ ) for all  $\xi \in \mathbb{R}^n$ .

## II. PRELIMINARIES

Following [15] consider a class of nonlinear systems:

$$\begin{aligned} \dot{x}(t) &= f(x(t), d(t)), \quad t \geq 0, \\ y(t) &= h(x(t)), \end{aligned} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $d(t) \in \mathbb{R}^m$  is the external perturbation, with  $d \in \mathcal{L}_\infty^m$ , and  $y(t) \in \mathbb{R}^p$  is the output vector. Moreover,  $f: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$  is a locally Lipschitz continuous function,  $f(0, 0) = 0$ , and  $h: \mathbb{R}^n \rightarrow \mathbb{R}^p$  is a continuously differentiable function. For an initial state  $x_0 \in \mathbb{R}^n$  and  $d \in \mathcal{L}_\infty^m$ , we denote the corresponding solution of the system (1) by  $x(t, x_0, d)$  for the values of  $t \geq 0$  the solution exists, so the corresponding output is  $y(t, x_0, d) = h(x(t, x_0, d))$ . The system (1) is called *forward complete* if for all  $x_0 \in \mathbb{R}^n$  and  $d \in \mathcal{L}_\infty^m$ , the solution  $x(t, x_0, d)$  is uniquely defined for all  $t \geq 0$ .

*Definition 1:* A forward complete system (1) is:

- 1) *practical input-to-output stable* (pIOS) if there exist  $\beta \in \mathcal{KL}$ ,  $\gamma \in \mathcal{K}$  and  $c \in \mathbb{R}_+$  such that

$$\|y(t, x_0, d)\| \leq \beta(\|x_0\|, t) + \gamma(\|d\|_\infty) + c, \quad \forall t \geq 0$$

for any  $x_0 \in \mathbb{R}^n$  and  $d \in \mathcal{L}_\infty^m$ . The system is called *input-to-output stable* (IOS) if  $c = 0$ . In the special case when  $y = x$ , the IOS property is called *ISS*.

- 2) *state-independent input-to-output stable* (SIIOS) if there exist  $\beta \in \mathcal{KL}$  and  $\gamma \in \mathcal{K}$  such that

$$\|y(t, x_0, d)\| \leq \beta(\|h(x_0)\|, t) + \gamma(\|d\|_\infty), \quad \forall t \geq 0$$

for any  $x_0 \in \mathbb{R}^n$  and  $d \in \mathcal{L}_\infty^m$ .

As we can deduce from this definition, SIIOS is a direct extension of the ISS property to the systems demonstrating convergence only with respect to a part of the variables.

*Definition 2:* A forward complete system (1) is said to be *uniformly bounded-input-bounded-state stable* (UBIBS) if there exists  $\sigma \in \mathcal{K}$  such that

$$\|x(t, x_0, d)\| \leq \max\{\sigma(\|x_0\|), \sigma(\|d\|_\infty)\}, \quad \forall t \geq 0$$

for all  $x_0 \in \mathbb{R}^n$  and  $d \in \mathcal{L}_\infty^m$ .

*Definition 3:* For the system (1), a smooth function  $V: \mathbb{R}^n \rightarrow \mathbb{R}_+$  is:

- 1) an *IOS-Lyapunov function* if there exist  $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ ,  $\chi \in \mathcal{K}$ , and  $\alpha_3 \in \mathcal{KL}$  such that

$$\alpha_1(\|h(x)\|) \leq V(x) \leq \alpha_2(\|x\|), \quad (2)$$

$$V(x) \geq \chi(\|d\|) \Rightarrow \nabla V(x)f(x, d) \leq -\alpha_3(V(x), \|x\|)$$

for all  $x \in \mathbb{R}^n$  and  $d \in \mathbb{R}^m$ .

- 2) a *SIIOS-Lyapunov function* if there exist  $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$  and  $\chi, \alpha_3 \in \mathcal{K}$  such that

$$\alpha_1(\|h(x)\|) \leq V(x) \leq \alpha_2(\|h(x)\|),$$

$$V(x) \geq \chi(\|d\|) \Rightarrow \nabla V(x)f(x, d) \leq -\alpha_3(V(x))$$

for all  $x \in \mathbb{R}^n$  and  $d \in \mathbb{R}^m$ .

*Theorem 1 ([16]):* A UBIBS (forward complete) system (1) is IOS (SIIOS) if and only if it admits an IOS (SIIOS)-Lyapunov function.

## III. MAIN RESULT

Consider a dynamical system of the form:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + \varphi(y(t), u(t)) \\ &\quad + G(y(t), u(t), x(t))\theta + d(t), \\ y(t) &= Cx(t) + v(t), \end{aligned} \quad (3)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $y(t) \in \mathbb{R}^p$  is the output measurement vector,  $\theta \in \mathbb{R}^m$  is the vector of unknown parameters,  $u(t) \in \mathbb{R}^d$  is the known input vector,  $v(t) \in \mathbb{R}^p$  is a measurement noise,  $d(t) \in \mathbb{R}^n$  is a disturbance, the functions  $G: \mathbb{R}^{n+p+d} \rightarrow \mathbb{R}^{n \times m}$  and  $\varphi: \mathbb{R}^{p+d} \rightarrow \mathbb{R}^n$  are assumed to be continuous and guaranteeing existence of the solutions in forward time.

We require the following hypotheses.

*Assumption 1:* Let  $d \in \mathcal{L}_\infty^n$ ,  $v \in \mathcal{L}_\infty^p$ , and  $\|\theta\| \leq \theta_{\max}$  with a known bound  $\theta_{\max} > 0$ .

Hence, the unknown inputs are bounded without a known upper limit, but the set of admissible values for the vector of unknown parameters is given. The latter is not a hard restriction since the parameters usually must respect physical constraints.

*Assumption 2:* There exists  $\lambda > 0$  such that  $\|G(y, u, x) - G(y, u, \hat{x})\| \leq \lambda\|x - \hat{x}\|$  for all  $y \in \mathbb{R}^p$ ,  $u \in \mathbb{R}^d$  and  $x, \hat{x} \in \mathbb{R}^n$ .

*Assumption 3:* There is  $G_{\max} > 0$  such that  $\|G(y, u, x)\| \leq G_{\max}$  for all  $y \in \mathbb{R}^p$ ,  $u \in \mathbb{R}^d$  and  $x \in \mathbb{R}^n$ .

These restrictions imply that  $G$  is uniformly Lipschitz continuous in the third argument and globally bounded.

Following [17], we propose an observer in the form:

$$\begin{aligned} \dot{\hat{x}}(t) &= A(t)\hat{x}(t) + \varphi(y(t), u(t)) + G(y(t), u(t), \hat{x}(t))\hat{\theta}(t) \\ &\quad + L(t)(y(t) - C\hat{x}(t)) + \Omega(t)\hat{\theta}(t), \\ \dot{\Omega}(t) &= (A(t) - L(t)C)\Omega(t) + G(y(t), u(t), \hat{x}(t)), \\ \hat{\theta}(t) &= \gamma\Omega^\top(t)C^\top(y(t) - C\hat{x}(t)), \end{aligned} \quad (4)$$

where  $\hat{x}(t) \in \mathbb{R}^n$  is the estimate vector for  $x(t)$ ,  $L(t) \in \mathbb{R}^{n \times p}$  is the observer gain providing the desired stability property for the matrix  $A(t) - L(t)C$ ,  $\Omega(t) \in \mathbb{R}^{n \times m}$  is an intermediate filter state variable, and  $\gamma > 0$  is the adaptation gain. We detail other conditions below (further, if

the dependence on time of a variable is evident, it may be omitted for brevity of presentation).

Next, by introducing an auxiliary estimation error and a parameter estimation error

$$\delta = x - \hat{x} - \Omega(\theta - \hat{\theta}), \quad \tilde{\theta} = \theta - \hat{\theta},$$

after straightforward computations, we obtain:

$$\begin{aligned} \dot{\delta} &= (A - LC)\delta + (G - \hat{G})\theta - Lv + d, \\ \dot{\tilde{\theta}} &= -\gamma\Omega^\top C^\top (C\delta + C\Omega\tilde{\theta} + v), \end{aligned}$$

where, with a slight abuse of notation, we use the shorthand notations  $G = G(y, u, x)$  and  $\hat{G} = G(y, u, \hat{x})$ .

We now set  $e := x - \hat{x}$  as the state estimation error. We aim to prove that  $e$  and  $\tilde{\theta}$  converge to 0 in the absence of the disturbances  $d$  and  $v$  and demonstrate a SIOS property for bounded perturbations.

*Theorem 2:* Let assumptions 1, 2 and 3 hold. If there are symmetric matrix functions  $P_1 : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$ ,  $P_2 : \mathbb{R}_+ \rightarrow \mathbb{R}^{m \times m}$  satisfying the following conditions for all  $t \geq 0$ :

- (i)  $0 < \underline{a}I_n < P_1(t) < \bar{a}I_n$ ,  $0 < \underline{a}I_m < P_2(t) < \bar{a}I_m$ , for some  $\underline{a}, \bar{a} \in \mathbb{R}_+$ ;
- (ii)  $\dot{P}_1(t) + P_1(t)(A(t) - L(t)C) + (A(t) - L(t)C)^\top P_1(t) < -Q_1$ , for some  $Q_1^\top = Q_1 \succeq 0$ ;
- (iii) for some  $Q_2^\top = Q_2 \succeq 0$ ,  $\dot{P}_2(t) - \gamma P_2(t)\Omega^\top(t)C^\top C\Omega(t) - \gamma\Omega^\top(t)C^\top C\Omega(t)P_2(t) \preceq -Q_2$ ;
- (iv) there exist  $\beta_1, \beta_2, \gamma_1, \gamma_2 > 0$  such that

$$\Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{21}^\top & P_1 & -P_1L & P_1 \\ \Gamma_{21} & \Gamma_{22} & 0 & \Gamma_{24} & 0 \\ P_1 & 0 & -\gamma_1 I_n & 0 & 0 \\ -L^\top P_1 & \Gamma_{24}^\top & 0 & -\gamma_2 I_p & 0 \\ P_1 & 0 & 0 & 0 & -I_n \end{pmatrix} < 0$$

with  $\Gamma_{11} = -Q_1 + (\beta_1 + 2\lambda^2\theta_{\max}^2)I_n$ ,  $\Gamma_{22} = -Q_2 + (\beta_2 + 2\lambda^2\theta_{\max}^2\|\Omega\|^2)I_m$ ,  $\Gamma_{21} = \Gamma_{24}C$ , and  $\Gamma_{24} = -\gamma P_2\Omega^\top C^\top$ ,

then the system (3)-(4) is SIOS in errors  $x - \hat{x}$  and  $\theta - \hat{\theta}$  from the inputs  $d$  and  $v$ , and the variable  $\Omega$  is bounded.

*Remark 1:* In this theorem, it is assumed that the gain function  $L(t)$  is chosen in a way to ensure the stability of the time-varying matrix  $A(t) - L(t)C$ , which is equivalent to the existence of a positive definite matrix  $P_1(t)$ . It is also assumed that the control  $u$  and the perturbations  $d, v$  are persistently exciting, providing the stability of a symmetric time-varying matrix  $\Omega^\top(t)C^\top C\Omega(t)$ . Then the existence of a positive definite matrix  $P_2(t)$  follows.

Note that the matrix  $\Gamma$  used in the formulation of this theorem is time-dependent. So, to get a linear matrix inequality, additional constraints can be imposed:

*Corollary 1:* Let all conditions of Theorem 2 be satisfied except (iv), and consider the matrix

$$\Delta = \begin{pmatrix} \Delta_{11} & 0 & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ 0 & \Delta_{22} & 0 & 0 & 0 \\ \Gamma_{31} & 0 & \Gamma_{33} & 0 & 0 \\ \Gamma_{41} & 0 & 0 & \Delta_{44} & 0 \\ \Gamma_{51} & 0 & 0 & 0 & \Gamma_{55} \end{pmatrix}$$

where  $\Delta_{11} = -Q_1 + \left(\beta_1 + \frac{\|\Gamma_{21}\|^2}{\epsilon} + 2\lambda^2\theta_{\max}^2\right)I_n$ ,  $\Delta_{22} = -Q_2 + (\beta_2 + \epsilon + \epsilon' + 2\lambda^2\theta_{\max}^2\Omega_{\max}^2)I_m$ ,  $\Delta_{44} = \left(\frac{\|\Gamma_{42}\|^2}{\epsilon'} - \gamma_2\right)I_p$  and  $\|\Omega\| \leq \Omega_{\max} < \infty$  (such a bound  $\Omega_{\max}$  exists in the conditions of Theorem 2), with some  $\epsilon, \epsilon' > 0$ . If  $\Delta \preceq 0$ , then (4) is SIOS in the errors  $x - \hat{x}$  and  $\theta - \hat{\theta}$  for the inputs  $d$  and  $v$ , and the variable  $\Omega$  stays bounded.

The proofs are omitted for the sake of brevity.

#### IV. APPLICATION TO A CROP IRRIGATION MODEL

A fundamental concern in agriculture is related with the state of the soil ensuring the good conditions for crop growth (such as tomatoes, lettuce, etc.), especially under climate change and drought events. Having visibility on the water and nutrient needs of crops is crucial for reliable predictions and stimulation of the production. Humidity is a crucial state variable in most of the crop models, and therefore humidity sensors are widely used to evaluate the behavior of this quantity. However other crucial information, necessary to obtain an accurate prediction, is contained in the parameters as a well as in other state variables such as the nitrogen concentration [10], [6]. Most of the time, sensors of chemical composition are not available on the field, or not accurate, or too costly, etc. This is where the software sensors enter into the picture as an expected cheap alternative.

We consider here a simplified crop irrigation model, inspired from [13], [4], where we explicitly include the dynamics of nitrogen, in addition to humidity one.

$$\dot{S} = k_1(-\varphi(t)K_S(S) - (1 - \varphi(t))K_R(S) + k_2u(t))$$

$$\dot{B} = \varphi(t)K_S(S)f\left(\frac{N}{S}\right)$$

$$\dot{N} = -k_3\varphi(t)K_S(S)f\left(\frac{N}{S}\right) + k_4C_N^{\text{in}}u(t)$$

where  $S$  denotes the soil humidity level (between 0 and 1) and  $B, N \in \mathbb{R}_+$  are biomass and nitrogen content per unit of soil surface. As often met in crop modelling, terms  $\varphi(t)K_S(S)$ ,  $(1 - \varphi(t))K_R(S)$  represent crop transpiration and soil evaporation. Functions  $K_S, K_R$  are usually piecewise linear non-decreasing from  $[0, 1]$  to  $[0, 1]$ . The time function  $\varphi$  is the crop radiation interception efficiency, which is usually  $C^1$  and increasing, while  $f$  is a piecewise  $C^1$  non-decreasing function, which regulates the growth depending on the nitrogen concentration. The input variable  $u$  is the irrigation flow rate, and  $C_N^{\text{in}}$  the nitrogen concentration of the irrigation water. Typical instances of the functions  $K_S, K_R, f$  are:

$$K_S(S) = \begin{cases} 0 & \text{if } S \in [0, S_w] \\ \frac{S - S_w}{S^* - S_w} & \text{if } S \in (S_w, S^*), \\ 1 & \text{if } S > S^* \end{cases}, \quad 0 < S_w < S^*,$$

$$K_R(S) = \begin{cases} 0 & \text{if } S \in [0, S_h] \\ \frac{S - S_h}{1 - S_h} & \text{if } S > S_h \end{cases} \quad S_h > 0,$$

$$f\left(\frac{N}{S}\right) = \begin{cases} \frac{N}{\eta_c S} & \text{if } \frac{N}{S} \in [0, \eta_c) \\ 1 & \text{if } \frac{N}{S} \in [\eta_c, 1] \end{cases} \quad \eta_c > 0,$$

where  $S^*$  is the water stress threshold,  $S_w$  is the humidity threshold above which the plant wilts,  $S_h$  is the hydroscopic point, and  $\eta_c$  is the nitrogen saturation coefficient.

Online measurements are the humidity  $y_1 = S + v_1(t)$  and the biomass  $y_2 = B + v_2(t)$ , where  $v_1, v_2$  are unknown measurement noises. We shall consider the beginning of the season ( $t = 0$ ) for which the initial humidity level  $S(0)$  can be assumed to be high, i.e., larger than the threshold  $S^*$  and the unknown initial nitrogen  $N(0)$  can be small, i.e., such that  $N(0)/S(0) < \eta_c$ . We then look for two estimation problems on a time window for which  $S$  stays above  $S^*$  and  $N/S$  below  $\eta_c$ . In this sub-domain of the state space, the dynamics can be written, using the expressions of  $K_S, K_R, f$  given above, as follows:

$$\dot{S} = k_1(-\varphi(t) - (1 - \varphi(t))K_R(S) + k_2u(t)) \quad (5)$$

$$\dot{B} = \varphi(t)\frac{N}{\eta_c S} \quad (6)$$

$$\dot{N} = -k_3\varphi(t)\frac{N}{\eta_c S} + k_4C_N^{\text{in}}u(t) \quad (7)$$

One can notice that the system has a cascade structure, i.e., the dynamics of the first variable  $S$  is independent of  $B$  and  $N$ . We consider two estimation problems:

- reconstruct parameters  $k_1, k_2, S_h$  related to soil characteristics and the humidity dynamics (5) with the single measurement  $y_1$ , assuming that the functions  $\varphi$  and  $u$  are known;
- reconstruct the nitrogen variable  $N$  and parameter  $k_3$  (related to soil-crop system) with both measurements  $y_1, y_2$ , assuming that parameters  $\eta_c, k_4C_N^{\text{in}}$  and functions  $\varphi$  and  $u$  are known.

#### A. Estimation of $k_1, k_2$ and $S_h$

Equation (5) can be written as

$$\dot{S} = k_1 \left( -1 - \frac{1}{1 - S_h} (1 - \varphi(t))(S - 1) + k_2u(t) \right) \eta(t)^\top \theta$$

where we posit

$$\eta(t) = \begin{pmatrix} -1 \\ -(1 - \varphi(t))(S(t) - 1) \\ u(t) \end{pmatrix}, \quad \theta = \begin{pmatrix} k_1 \\ \frac{k_1}{1 - S_h} \\ k_1 k_2 \end{pmatrix}.$$

Then, one can consider the sub-system

$$\dot{S} = \eta(t)^\top \theta \quad (8)$$

$$y_1 = S + v_1 \quad (9)$$

where the parameter vector  $\theta$  is unknown, i.e., here we consider that  $k_1, k_2$  and  $S_h$  are unknown. It is then possible to reconstruct these parameters with a classical adaptive observer with the single measurement  $y_1$ :

*Proposition 1:* Consider the system

$$\begin{aligned} \dot{\hat{\xi}} &= \omega(t)^\top \hat{\theta} + L(y - \hat{\xi}) \\ \dot{\hat{\theta}} &= \Gamma \omega(t)(y - \hat{\xi}) \end{aligned}$$

where  $\omega(t) = [-1 - (1 - \varphi(t))(y_1(t) - 1)u(t)]^\top$  and  $L > 0, \Gamma > 0$ . If  $\omega$  is persistently exciting, then  $\hat{\theta}$  is an asymptotic estimator of  $\theta$  for system (8) when  $v_1 = 0$ .

Robustness to noise can be established similarly to Theorem 2, see [5].

#### B. Estimation of $N, k_1$ and $k_3$

We write dynamics of the system(5)-(6)-(7) as follows

$$\dot{S} = k_1(-\varphi(t) - (1 - \varphi(t))K_R(S) + k_2u(t)),$$

$$\dot{B} = \frac{\varphi(t)}{\eta_c S} N$$

$$\dot{N} = -\kappa \frac{\varphi(t)}{\eta_c S} N + (\kappa - k_3) \frac{\varphi N}{\eta_c S} + k_4 C_N^{\text{in}} u(t)$$

where  $\kappa$  is any positive number. In the noise-free case it takes the form of (3) with

$$x = \begin{bmatrix} S \\ B \\ N \end{bmatrix}, \quad A(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\varphi(t)}{\eta_c y_1(t)} \\ 0 & 0 & -\kappa \frac{\varphi(t)}{\eta_c y_1(t)} \end{bmatrix},$$

$$\theta = \begin{bmatrix} k_1 \\ \kappa - k_3 \end{bmatrix}, \quad G(t, x, y, u) = \begin{bmatrix} G_1(t, x, u) & 0 \\ 0 & 0 \\ 0 & \frac{\varphi(t)N}{\eta_c y_1} \end{bmatrix}$$

$$\phi(u) = \begin{bmatrix} 0 \\ 0 \\ k_4 C_N^{\text{in}} u \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

with  $G_1(t, x, u) = -\varphi(t) - (1 - \varphi(t))K_R(S) + k_2u$ . In the presence of the noise  $v$  the discrepancy can be hidden in the disturbance term  $d$ . We then consider an observer of the form:

$$\dot{\hat{x}} = A(t)\hat{x} + G(t, \hat{x}, y, u(t))\hat{\theta} + L(t)(y - C\hat{x}) + \Omega\dot{\hat{\theta}} + \phi(u(t))$$

$$\dot{\hat{\Omega}} = (A(t) - L(t)C)\hat{\Omega} + G(t, \hat{x}, y, u(t))$$

$$\dot{\hat{\theta}} = \gamma \hat{\Omega}^\top C^\top (y - C\hat{x})$$

where  $\Omega = (\Omega_{i,j})_{1 \leq i \leq 3, 1 \leq j \leq 2}$

and  $L(t) = (L_{i,j})_{1 \leq i \leq 3, 1 \leq j \leq 2}$  with  $L_{12} = L_{21} = L_{31} = 0$

Note that  $A(t) = \frac{\varphi(t)}{y_1(t)} \tilde{A}$ ,  $\tilde{A}_{23} = \frac{1}{\eta_c}$  and  $\tilde{A}_{33} = -\frac{\kappa}{\eta_c}$ .

We can choose  $L(t) = \frac{\varphi(t)}{y_1(t)} \tilde{L}$ , where the constant matrix  $\tilde{L}$  is such that  $\tilde{A} - \tilde{L}C$  is Hurwitz. Finally, the

equations of the observer (4) are

$$\begin{aligned} \dot{\hat{S}} &= \hat{k}_1 \left( -\varphi(t) - (1 - \varphi(t))K_R(\hat{S}) + k_2 u(t) \right) \\ &\quad + \frac{\varphi(t)}{y_1(t)} \tilde{L}_1 (y_1(t) - \hat{S}) + \Omega_{11} \hat{k}_1 - \Omega_{12} \hat{k}_3 \\ \dot{\hat{B}} &= \frac{\varphi(t)}{\eta_c y_1(t)} \hat{N} + \frac{\varphi(t)}{y_1(t)} \tilde{L}_2 (y_2(t) - \hat{B}) + \Omega_{21} \hat{k}_1 - \Omega_{22} \hat{k}_3 \\ \dot{\hat{N}} &= -\hat{k}_3 \frac{\varphi(t)}{\eta_c y_1} \hat{N} + \frac{\varphi(t)}{y_1(t)} \tilde{L}_3 (y_2(t) - \hat{B}) + \Omega_{31} \hat{k}_1 - \Omega_{32} \hat{k}_3 \\ \dot{\Omega}_{11} &= -\frac{\varphi(t)}{y_1(t)} \tilde{L}_1 \Omega_{11} - \varphi(t) - (1 - \varphi(t))K_R(\hat{S}) + k_2 u(t) \\ \dot{\Omega}_{12} &= -\frac{\varphi(t)}{y_1(t)} \tilde{L}_1 \Omega_{12} \\ \dot{\Omega}_{21} &= \frac{\varphi(t)}{y_1(t)} \left( -\tilde{L}_2 \Omega_{21} + \frac{\Omega_{31}}{\eta_c} \right) \\ \dot{\Omega}_{22} &= \frac{\varphi(t)}{y_1(t)} \left( -\tilde{L}_2 \Omega_{22} + \frac{\Omega_{32}}{\eta_c} \right) \\ \dot{\Omega}_{31} &= \frac{\varphi(t)}{y_1(t)} \left( -\tilde{L}_3 \Omega_{21} - \frac{\kappa \Omega_{31}}{\eta_c} \right) \\ \dot{\Omega}_{32} &= \frac{\varphi(t)}{y_1(t)} \left( -\tilde{L}_3 \Omega_{22} - \frac{\kappa \Omega_{32}}{\eta_c} \right) + \frac{\varphi(t)}{\eta_c y_1(t)} \hat{N} \\ \dot{\hat{k}}_1 &= \gamma \left( \Omega_{11} (y_1(t) - \hat{S}) + \Omega_{21} (y_2(t) - \hat{B}) \right) \\ \dot{\hat{k}}_3 &= -\gamma \left( \Omega_{12} (y_1(t) - \hat{S}) + \Omega_{22} (y_2(t) - \hat{B}) \right) \end{aligned}$$

Let us underline that this observer allows to reconstruct the parameter  $k_1$  as well, without the need of the former observer.

## V. NUMERICAL SIMULATIONS

For the simulations, we have considered the following set of parameters

$k_1$	$k_2$	$k_3$	$k_4$	$\Gamma$	$S^*$	$S_w$	$S_h$	$L$	$\hat{S}_0$
1.2	5.5	2.5	1.7	$3.10^5$	0.5	0.2	0.1	1.2	1

with the function  $\varphi(t) = \frac{2t}{2t+3}$  and the control

$$u(t) = \max \left( 0, \frac{t}{t^2+1} + \frac{\cos(t/2) + \sin(\pi t)}{2} + \frac{\cos(3e^t)}{4} \right)$$

### A. Estimation of $k_1$ , $k_2$ and $S_h$

Figure 1 illustrates the adaptive observer given by Proposition 1, with noise.

### B. Estimation of $N$ , $k_1$ and $k_3$

For the adaptive observer (4), the following tuning has been chosen

$C_N^{in}$	$\gamma$	$\eta_c$	$\tilde{L}_1$	$\tilde{L}_2$	$\tilde{L}_3$	$\Omega_0$
1.5	150	0.8	$L_1$	$0.5L_1$	$0.5L_2$	$0_{3 \times 2}$
			$\hat{S}_0$	$\hat{B}_0$	$\hat{N}_0$	
			$0.5S_0$	$0.5B_0$	$0.5N_0$	

Figures 2, 3 show the adaptive observer (4) without and with noise.

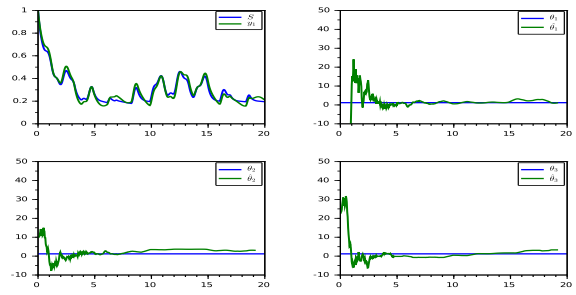


Fig. 1. Adaptive observer for problem A with a 5% noise

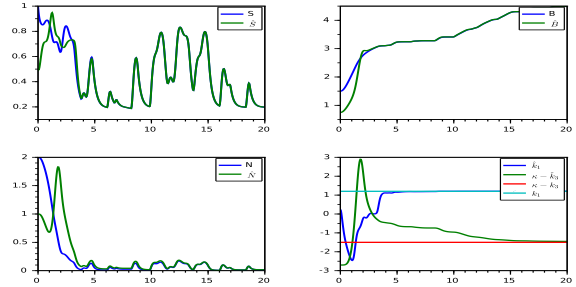


Fig. 2. Adaptive observer for problem B without noise

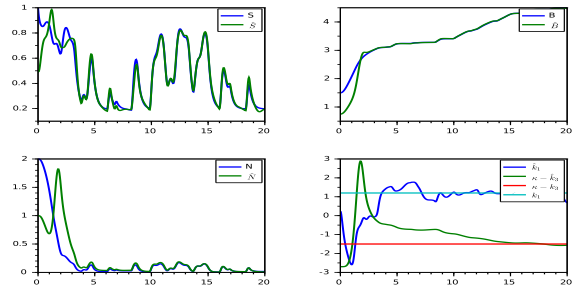


Fig. 3. Adaptive observer for problem B with 5% noise

### C. Comparison with other approaches

Problem A is a pure identification problem. One can then use a (recursive) least square method to estimate parameters  $k_1$ ,  $k_2$  and  $S_h$  from the measurement  $y_1$ . We use the `lsqrsolve` function of `scilab` based on the Levenberg-Marquardt algorithm. We shows that this method works well without noise but is not robust with measurement noise.

For Problem B, let us show how to use a classical high-gains observer, assuming that the unknown parameter  $k_3$  belongs to a given interval  $[k_3^{min}, k_3^{max}]$ . We first note from equation (7) that  $N$  cannot reach 0 in finite time. One can then define the variable  $Z = -k_3 N$ , and write the dynamics of the system  $(B, N, Z)$  as follows

$$\begin{aligned} \dot{B} &= g(t)N \\ \dot{N} &= g(t)Z + k_4 C_N^{in} u(t) \\ \dot{Z} &= g(t) \left( \frac{Z}{N} \right) Z - \left( \frac{Z}{N} \right) k_4 C_N^{in} u(t) \end{aligned}$$

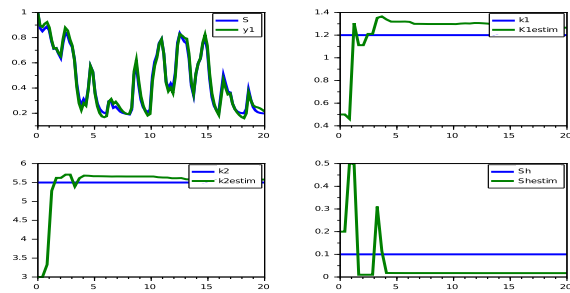


Fig. 4. Least-square estimation for problem  $B$  with 5% noise

where we posit  $g(t) = \frac{\varphi(t)n_C}{K_S(y_1(t))y_1(t)}$ . Note that this last function is zero only at  $t = 0$ . Then, for any control  $u(\cdot)$ , we can define  $\tilde{u}(\cdot)$  such that  $k_4 C_N^{in} u(t) = g(t)\tilde{u}(t)$  for almost all  $t \geq 0$ . Note also that along any trajectory one can replace  $\frac{Z}{N}$  by a globally bounded function on  $\mathbb{R}^2$ :

$$\rho(N, Z) = -\max\left(k_3^{\min}, \min\left(\frac{Z}{\max(N, \varepsilon)}, k_3^{\max}\right)\right)$$

where  $\varepsilon > 0$  is an arbitrary small number to ensure  $N > \varepsilon$  on the time window. Then, the system takes the form

$$\begin{bmatrix} \dot{B} \\ \dot{N} \\ \dot{Z} \end{bmatrix} = g(t) \begin{bmatrix} N \\ Z \\ \rho(N, Z)Z \end{bmatrix} + g(t)\tilde{u}(t) \begin{bmatrix} 0 \\ 1 \\ \rho(N, Z) \end{bmatrix}$$

which is exactly in the nonlinear canonical form in time  $\tau = \int_0^t g(s)ds$  (see [9]). Therefore, the observer

$$\begin{aligned} \dot{\hat{B}} &= g(t)\hat{N} - 2\chi g(t)(y_2(t) - \hat{B}) \\ \dot{\hat{N}} &= g(t)\hat{Z} - k_4 C_N^{in} u(t) - 3\chi^2 g(t)G_2(y_2(t) - \hat{B}) \\ \dot{\hat{Z}} &= \rho(\hat{N}, \hat{Z}) \left( g(t)\hat{Z} - k_4 C_N^{in} u(t) \right) - 3\chi^3 g(t)G_3(y_2(t) - \hat{B}) \\ \dot{\hat{k}}_3 &= -\rho(\hat{N}, \hat{Z}) \end{aligned}$$

ensures an exponential convergence provided that the tuning coefficient  $\chi > 0$  is sufficiently large. We shows good performance of this observer without noise, comparable to the adaptive observer for  $\chi = 3$ , but unfortunately it is poorly robust with respect to noise (Figure 5), which makes it unreliable in practice. For both problems, we

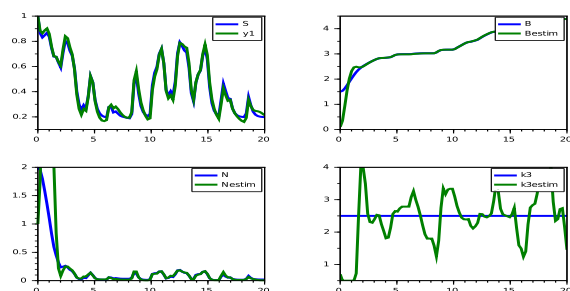


Fig. 5. High gains observer for problem  $B$  with 5% noise

conclude about the superiority of the adaptive observers.

## VI. CONCLUSION

This paper proposed an adaptive observer for a class of nonlinear time-varying systems with disturbances and measurement noises, extending existing results by considering a generic and the most complex scenario. The analysis of convergence and robustness of this type of adaptive observer used a SIOS-Lyapunov function method. A crop irrigation model from agronomy was used for illustration of the efficiency of the observer by a numerical simulation.

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