2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Cooperation and Competition: A Sequential Game Model of Flocking

Chenlan Wang[1], Shane G. DuBay[2], and Mingyan Liu[1]

***Abstract*— We study a sequential game model on how groups form in time. In particular, agents make asynchronous decisions on a time of arrival; those choosing the same arrival time are considered to travel together, or belong to the same *flock*. While flocking reduces travel costs, arriving earlier allows one to obtain a higher reward. Our model is primarily motivated by commonly observed flocking behavior among migratory birds, but it can also be applied to other areas of competition and cooperation, e.g., in the case of rideshare to a common destination with a limited supply of goods. Given the model's sequential nature, the solution concept we study is the subgame perfect equilibrium (SPE). We present in detail the nature of the SPE in a 2-agent and 3-agent game, respectively, and its properties in the more general n-agent game. Of particular interest are observations on when and what types of groups emerge in an SPE.**

I. Introduction

Individuals form groups for a variety of purposes, such as to benefit from resource pooling, energy efficiency, foraging efficiency, safety, etc., [1], [2]; this is frequently observed in humans and in the broader animal world. At the same time, individuals also compete with each other for limited resources, like access to territories and opportunities. This contention is clearly seen in the example of migratory birds. On the one hand, flocking can provide protection, reduce individual predation risk [3]–[5], and increase navigation accuracy by pooling information [6]; some birds such as Canada geese and white pelicans form V-shaped flocks during migration to increase energy efficiency during flight [7], [8]. On the other hand, migratory birds often compete with each other for territories upon arrival at a breeding ground to increase their reproductive success [9]–[12].

How stable groups form among (strategic) agents has been studied in many fields, including economics, computer networks, social science, political science, and biology; see e.g., [13]–[16]. A variety of models have been developed to study the process of group formation, including coalition formation games [9], [17]–[21], clustering [22], [23], and agent-based modeling [24], [25]. In the context of game theory, these problems are most typically studied as strategic, one-shot games, including all those cited above.

By contrast, in this paper we examine a *sequential* game model to study how groups form *in time*. In particular, agents make decisions on a *time of arrival* and they do so asynchronously; those choosing the same arrival time are considered to travel together, or belong to the same *flock* (we will use the terms *group* and *flock* interchangeably throughout the paper). An agent's utility depends not only on others in the same group but also those outside its group, capturing the trade-off between the benefit of flocking or cooperation (e.g., savings in travel cost, like reduced predation risk in birds) and the benefit of an early arrival due to competition (for territory, seating, etc.).

Given the sequential nature, the solution concept we study is the subgame perfect equilibrium (SPE). After presenting the game model in Section II, we discuss in detail the nature of the SPE in a 2-agent (Section III) and 3-agent (Section IV) game, respectively, and discuss how they extend to the more general, n-agent scenario. Of particular interest are observations on when and what types of groups emerge in an SPE (e.g., multiple smaller groups vs. a single, grand flock), shedding light on the interplay between competition and cooperation in rational decision making. Section V then compares the sequential game model with a one-shot counterpart and examines the effect of the ordering of decision making in the sequential game.

Our primary motivation comes from commonly observed flocking behavior in migratory birds, and for this reason our model is closely related to that studied by Kokko in [26], with the main difference that the model in [26] only considers competition while our model also takes into consideration the benefit of cooperation (flocking), as we detail in the next section. However, our model can also be applied to other areas of competition and cooperation, e.g., in the case of rideshare, where individuals can choose to share a ride to lower travel costs but compete upon arriving at the same destination for better seating at a concert or in a restaurant, or for a limited number of sale items in a store, etc.

II. A Flock Formation Game (FFG)

Consider $n \geqslant 2$ agents who must travel to a destination in order to reach and compete for n territories. They may form groups to reduce travel costs (e.g., lower predation risk and increased foraging efficiency in the case of migratory birds, and increased energy efficiency and/or reduced monetary cost in the case of rideshare), but they are also interested in arriving early to obtain territories with better quality (e.g., in terms of food abundance/quality, suitability for nesting, and protection from predators). For the remainder of the paper, agents choosing to arrive at the same time will be considered to travel together in a flock, though this is clearly a simplification.

Let $\mathcal{N} = \{1, 2, ..., n\}$ denote the set of agents. Each agent is endowed with a positive *strength* value $\beta_i > 0, i \in \mathcal{N}$, which represents the natural quality of the agent (e.g., surviving skills, foraging ability, flight experience, etc.). It

[1] University of Michigan, Ann Arbor, USA. [2] University of Texas, Arlington, USA. This work is supported by the the ARO under contract W911NF1810208.

is assumed that no two agents have identical strength, and they are indexed in descending order of their strength, $\beta_1 > \beta_2 > \cdots > \beta_n$. Similarly, each territory is associated with a positive *quality* $E_k > 0, k \in \mathcal{N}$; they are also indexed in descending order, $E_1 > E_2 > \cdots > E_n$. An agent's decision is its time of arrival at the destination, denoted by t_i, with a joint action profile $\mathbf{t} = (t_1, t_2, \ldots, t_n)$. As is the convention, the action profile of all but the i-th agents is written as $\mathbf{t}_{-i}$. Accordingly, we will also often write $\mathbf{t} = (t_i, \mathbf{t}_{-i})$.

A. Assumptions

We shall adopt the following assumptions.
(1) If a territory is already occupied by an agent, it cannot be taken away by another who arrived later (though this can happen in nature, usually involving extra cost such as a fight). This can also be understood as a first-come first-serve rule.
(2) Agents arriving at the same time get territories in accordance with their strengths: the strongest of the group gets the best of the remaining territories, and so on.
(3) Agents make decisions in sequence of increasing order of their indices. This means that the strongest agent decides on time t_1, which is announced, followed by agent 2 deciding on t_2, and so on. It is assumed that all agents then take actions in accordance with these decisions, i.e., agent i will indeed arrive at the decided and announced time t_i. We do *not* assume that $t_i \leqslant t_j, j > i$; that is, a later decision maker can decide on an earlier time after observing the choices made by earlier decision makers[1]. This is thus very different from assuming that an agent i simply takes the action at t_i, having observed all earlier actions $t_j < t_i, \forall j \in \mathcal{N}$.

While the first two assumptions are rather natural, the last one deserves further elaboration. In migratory birds, stronger and more dominant individuals are often observed to leave their winter grounds earlier and arrive at their breeding ground earlier than weaker individuals. A real-world example is given in Figure 1, which depicts the arrival time of three species of bush-robins, each subdivided into adult and subadult (first-year) male [27]. This subdivision is one way to provide a (binary) proxy for the strength measure, as adult males are typically larger, stronger, and more experienced than subadult males. In each case, the mean arrival time of the adults is earlier than that of the subadults; similar observations have been made in other studies, see e.g., [27] (Fig 5 therein). One can also use the tarsus bone length as a proxy for strength and obtain similar observations [28], [29] – larger and stronger birds of the same species tend to have longer tarsi.

These data, however, at best show that, provided the proxies used are sound, stronger agents tend to arrive earlier, but these data say nothing about the ordering of decision making, which is crucial in a sequential model. For this reason, we will discuss in Section V what happens if this is modeled as a one-shot game or if we employ a different ordering of decision making.

[1]It turns out, as we will show shortly, that in an SPE an earlier decision maker will always preempt this possibility and choose a time that a later decision maker finds unprofitable to get ahead of.

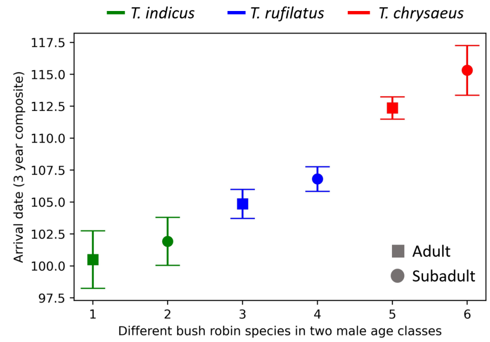

Fig. 1: Arrival times (Julian date) of adult and subadult males of three bush-robin species (genus *Tarsiger*) in three consecutive years. Mean arrival time and standard error bars are plotted.

B. The utility function

The utility of agent i is formulated as follows:

$$u_i(\mathbf{t}) = e(\mathbf{t}) - c_i(t_i) - p_i(\mathbf{t}) \ , \tag{1}$$

where $e(\mathbf{t})$ is the benefit conferred by the territory that agent i occupies, $c_i(t_i)$ the (travel) cost solely dependent on the agent's chosen time, and $p_i(\mathbf{t})$ a second cost that depends on all agents' chosen times. This last term will also be referred to as the (predation) risk term. We detail each term below.

a) Benefit: $e(\mathbf{t}) = E_k$, where k is the position of t_i in the re-shuffled vector $\mathbf{t}$ where its elements are ordered from the smallest to the largest: agent i gets the best of what remains available by its time of arrival.

b) Travel cost: $c_i(t_i) = \frac{1}{\beta_i}(t_i - t_o)^2 + c_o^i$, where t_o is an *optimal arrival time* w.r.t. this cost alone. At this optimal time, an agent's travel cost reduces to a fixed c_o^i; deviation in either direction will increase the travel cost, and weaker agents (smaller β) are more sensitive to the sub-optimality of this deviation. This model captures the cost of travel purely due to external factors (such as climate in the case of spring migration, where traveling during colder or warmer weather can be detrimental; or in the case of concert going, where arriving too early incurs excess waiting, while arriving late causes one to miss the beginning.). The fixed cost c_o^i is agent-dependent, and generally lower for a stronger agent. However, the presence of this fixed cost does not impact our subsequent analysis since the agent's decision making is entirely relative to the optimal t_o. For this reason and without loss of generality, we will set $c_o^i = 0$ for the rest of the paper.

c) Predation risk: $p_i(\mathbf{t}) = \frac{r}{|\{j:t_j=t_i\}|}$, where $|\{j : t_j = t_i\}|$ is the total number of agents arriving at t_i including agent i itself, and r the (nominal) risk for a single individual. Thus, the larger the flock, the less risk each member of the flock will experience during the trip.

If we ignore the last risk term, then the individual utility function in (1) reduces to:

$$u_i(\mathbf{t}) = e(\mathbf{t}) - c_i(t_i), \tag{2}$$

which is the one used in Kokko [26] with a specific travel cost function. Thus the original model in [26] is only focused on the competition among agents for territories, while the present model captures both competition and the benefit of cooperation through flocking. As we shall see, this modeling difference results in different equilibrium properties.

For brevity, only proof sketches are provided; complete proofs can be found in an extended arXiv version.

III. The 2-Agent (Stackelberg) Game

We will start with the simplest case of $n = 2$, i.e., a Stackelberg game. The SPE is relatively easy to obtain in this case using standard backward induction; the solution sheds significant light on the more general case.

Here we have a leader, the stronger agent 1, and a follower, the weaker agent 2: $\beta_1 > \beta_2$, competing for two territories $E_1 > E_2$. Our main result is that there are only two types of pure strategy SPEs depending on the quality differential between the two territories: if they are sufficiently similar in quality, then the benefit of cooperation outweighs that of competition and the two agents will flock at the optimal time; if they are sufficiently far apart, then competition takes over and the stronger agent will advance its arrival time to secure the better territory. This is given in Theorem 1 below.

Theorem 1. *There exist two and only two types of pure strategy SPEs for the 2-agent FFG:*
(1) If $E_1 - E_2 \leqslant \frac{1}{2}r$, then $\mathbf{t}^* = (t_o, t_o)$ is the unique SPE; this will also be referred to as the cooperation SPE.
(2) If $E_1 - E_2 > \frac{1}{2}r$, then $\mathbf{t}^* = (t_o - \sqrt{\beta_2(E_1 - E_2)}, t_o)$ is the unique SPE; this will also be referred to as the competition SPE.

Proof. Assuming Agent 1's decision is t_1, Agent 2 then best responds to t_1 with t_2; this response depends on the different regions t_1 falls into. Given the best response in each case, which Agent 1 fully anticipates, Agent 1 then selects the t_1^* that yields the highest reward for itself; together with the corresponding best response t_2^*, $[t_1^*, t_2^*]$ constitutes a pure strategy SPE for this game[2]. Utilities below are written as $u_i(t_1, t_2)$, $i = 1, 2$.

Case 1: $E_1 - E_2 \leqslant \frac{1}{2}r$

1) If $t_1 = t_o$, Agent 2's best response is $t_2 = t_o$, giving

$$u_1(t_o, t_o) = E_1 - \frac{r}{2} \; ; \tag{3}$$

$$u_2(t_o, t_o) = E_2 - \frac{r}{2} \; . \tag{4}$$

This is because (a) if Agent 2 arrives earlier than t_o to get the higher quality E_1, then it has a lower utility as the increase in predation risk by not flocking with Agent 1 is higher than the territory difference; (b) if Agent 2 arrives later than t_o, its utility is even lower with the worse territory E_2 and a higher travel cost.

2) If $t_1 = t_o + \delta$ for some $\delta > 0$, and $\delta \leqslant \sqrt{\beta_2(r/2 - (E_1 - E_2))}$, then Agent 2's best response is to flock with Agent 1, resulting in

$$u_1(t_1, t_2 = t_1) = E_1 - \frac{r}{2} - \frac{\delta^2}{\beta_1} \; ; \tag{5}$$

$$u_2(t_1, t_2 = t_1) = E_2 - \frac{r}{2} - \frac{\delta^2}{\beta_2} \; . \tag{6}$$

If $\delta > \sqrt{\beta_2(r/2 - (E_1 - E_2))}$, then Agent 2 will arrive at t_o to get ahead:

$$u_1(t_1, t_2 = t_o) = E_2 - r - \frac{\delta^2}{\beta_1} \; ; \tag{7}$$

$$u_2(t_1, t_2 = t_o) = E_1 - r \; . \tag{8}$$

3) If $t_1 = t_o - \delta$ for some $\delta > 0$, and $\delta \leqslant \sqrt{\beta_2 r/2}$, then Agent 2's best response is to flock:

$$u_1(t_1, t_2 = t_1) = E_1 - \frac{r}{2} - \frac{\delta^2}{\beta_1} \; ; \tag{9}$$

$$u_2(t_1, t_2 = t_1) = E_2 - \frac{r}{2} - \frac{\delta^2}{\beta_2} \; . \tag{10}$$

If $\delta > \sqrt{\beta_2 r/2}$, then Agent 2 will advance:

$$u_1(t_1, t_2 = t_o) = E_1 - r - \frac{\delta^2}{\beta_1} \; ; \tag{11}$$

$$u_2(t_1, t_2 = t_o) = E_2 - r \; . \tag{12}$$

Agent 1's choice now lies in comparing the utilities in all these cases, i.e., (3), (9), (11), (5), and (7), with (3) yielding the highest utility. Therefore Agent 1's best choice is t_o, and the unique pure strategy SPE in **Case 1** is $t_1^* = t_2^* = t_o$.

Case 2: $E_1 - E_2 > \frac{1}{2}r$

1) If $t_1 > t_o$, Agent 2's best response is $t_2 = t_o$, as higher E_1 is now worth the risk of traveling alone at the optimal time.
2) If $t_1 = t_o$, Agent 2's can always advance its arrival by some small ϵ to get the higher quality E_1, which more than offsets the increase in predation risk by not flocking with Agent 1. The smaller the ϵ the higher its utility, so in this case there doesn't exist a pure strategy best response.
3) If $t_1 = t_o - \delta$ for some $\delta > 0$, and if $\delta \geqslant \sqrt{\beta_2(E_1 - E_2)}$, then Agent 2's best response is t_o,

$$u_1(t_1, t_2 = t_o) = E_1 - r - \frac{\delta^2}{\beta_1} \; ; \tag{13}$$

$$u_2(t_1, t_2 = t_o) = E_2 - r \; . \tag{14}$$

If $\delta < \sqrt{\beta_2(E_1 - E_2)}$, then Agent 2's best response is to arrive just before t_1, $t_1 - \epsilon$ for some small ϵ, which does not yield a pure strategy best response.

In comparing all these cases, the best option for Agent 1 is given by (13), which is further maximized when δ is at a minimum, meaning the unique pure strategy SPE in this case is $t_1^* = t_o - \sqrt{\beta_2(E_1 - E_2)}, t_2^* = t_o$. □

We have the following interpretations on the SPEs. (1) Cooperation is only sustained when the gain from the competition is mild (the quality differential between the two territories is small). Further, under such cooperation, both

[2] Typically, if there are multiple choices of t_1^* a tie-breaking rule may be introduced; this is not needed in this game.

agents get to arrive at the optimal time. This results in the cooperation SPE. (2) When the better territory can yield a much higher reward, the stronger agent will preempt the weaker agent by advancing its arrival to secure the better territory; the advance has to be sufficient to discourage the weaker agent from attempting to get ahead. Once discouraged, the weaker agent arrives at the optimal time. This results in the competition SPE.

It is worth noting that the second, competition SPE is the same as the one derived under the original model with utility function (2) by Kokko [26], which is also the only SPE under (2). This is not surprising as (2) does not provide any benefit for cooperation. To shed further light on the competition SPE in terms of how much Agent 1 needs to advance its arrival, we note that $\sqrt{\beta_2(E_1 - E_2)}$ is a tipping point:

- If Agent 1 advances less than this amount ($\delta < \sqrt{\beta_2(E_1 - E_2)}$), then Agent 2 can always move up its arrival by an infinitesimal amount ϵ to get a higher utility: this is better than flocking with Agent 1 ($u_2(t_o - \delta, t_o - \delta - \epsilon) > u_2(t_o - \delta, t_o - \delta)$); this is also better than arriving alone at t_o ($u_2(t_o - \delta, t_o - \delta - \epsilon) > u_2(t_o - \delta, t_o)$). Thus Agent 1 would not be able to secure E_1.
- Once Agent 1's advance hits $\delta = \sqrt{\beta_2(E_1 - E_2)}$, Agent 2's best option becomes t_o, i.e., $u_2(t_o - \delta, t_o) > u_2(t_o - \delta, t_o - \delta) > u_2(t_o - \delta, t_o - \delta - \epsilon)$. Agent 1 secures E_1 in this case.

The tipping point $\delta = \sqrt{\beta_2(E_1 - E_2)}$ is essentially where Agent 2 becomes ambivalent between arriving alone at $t_o - \delta$ and taking E_1 (which can only happen if Agent 1 advances $< \delta$) and arriving alone at t_o and taking E_2.

IV. Three or more agents

Deriving the precise SPEs for cases with > 2 agents is considerably more complicated. Below we present the result in a 3-agent game and a number of interesting properties in the more general, n-agent case, which help us construct algorithms to find the SPE.

A. The 3-agent game

We summarize the eight types of pure strategy SPEs in a 3-agent game in Table I. These are obtained using a similar backward induction approach, but whereas in Theorem 1 we only need to consider the quality differential between E_1 and E_2, in the 3-agent case we have three pairwise quality differentials. The main takeaways from the 3-agent game are as follows.

1) Consistent with observations from the 2-agent game, the territorial quality differentials completely determine what type of SPEs emerge in the 3-agent game: groups form when the differences are small, allowing the benefit of flocking to outweigh the territorial reward. Whenever there is a substantial quality gap, there is an incentive for a stronger agent to advance its arrival in order to secure a better territory.
2) Interestingly, all but the last combination result in flocking. In three extreme cases, agents form a *grand flock* (Cases 1 and 2, cooperation among all agents) or a *cascade* (Case 8, competition among all agents).
3) How much an agent advances its arrival is not arbitrary, even though the action space is continuous. There is a discrete set of acceptable time points, most of which in the form of $\sqrt{\sum_{(i,j,i<j)\in A} \beta_j(E_i - E_j)}$, for some combinatorial set A.

B. The n-agent game

Proposition 1. *No agents will arrive later than t_o in an SPE.*

Sketch: This can be shown by contradiction and considering the last arrival time in the SPE, $t_l > t_o$. If there is only one agent at t_l and if it is the weakest, agent n, then clearly it can move up its arrival time by an infinitesimal amount and improve its utility (lower travel cost); thus this cannot be an SPE. If the lone arrival at t_l is agent $l < n$, then moving up its arrival by an infinitesimal amount either results in no change in subsequent, weaker agents' arrival times (earlier than t_o) or it triggers some weaker agents to delay their arrival and join agent l to form a group. Either way this improves agent l's utility, so t_l cannot be l's best response and again this cannot be an SPE. When there are multiple arrivals at t_l, then considering the strongest of the group, agent l, a very similar argument can be applied.

The next set of results shows that the nature of the SPE is shaped by the competition and cooperation relationship among agents induced by the relative quality of the territories. We say agent $i+k$ *competes* with a stronger agent i if $E_i - E_{i+k} > \frac{k}{k+1} r$ for some $i, i+k \leqslant n$. For instance, two neighboring agents i and $i+1$ compete if $E_i - E_{i+1} > \frac{1}{2} r$, a condition we have seen repeatedly in the 2-agent analysis.

Proposition 2. *If $E_1 - E_i \leqslant \frac{i-1}{n(n-i+1)} r$, $\forall i \in \{2, \ldots, n\}$, then the unique SPE is $\mathbf{t}^* = (t_o, t_o, \ldots, t_o)$. This is called a* grand flock.

Sketch: This can be shown by noting that Agent 1 achieves maximum utility under t^*, among all possible sequences of arrival times. Similarly, given Agent 1 has chosen t_o, Agent 2 achieves maximum utility under t^* among all possible subsequences of arrival times, and so on.

This is a case where no agent has an incentive to arrive early in order to obtain a better territory because the territorial gain is less than the gain from being part of the group.

Proposition 3. *If every agent i competes with its neighboring agent $i-1$: $E_{i-1} - E_i > \frac{1}{2} r$, then the unique SPE is ${t_n}^* = t_0, {t_j}^* = t_0 - \sqrt{\sum_{i=j}^{n-1} \beta_{i+1}(E_i - E_{i+1})}$, $j = 1, 2, \cdots, n-1$. This is called a* cascade.

Sketch: This result can be established in two steps, with the first showing that given sufficient separation among the E_i's, there will be no flocking anywhere. This is then followed by a second step that consists of a relatively straightforward backward induction calculation of all the arrival times.[3]

[3] Again, this SPE is the *only* SPE attainable in the pure-competition model by Kokko (utility function (2)).

TABLE I: SPEs in a 3-agent game; which type emerges depends on the pairwise relationship between territories. $T_1^1 = t_0 - \min\left(\sqrt{\beta_2(E_1 - E_2)}, \sqrt{\beta_3(E_2 - E_3 - \frac{r}{6})}\right)$, $T_1^2 = t_0 - \max\left(\sqrt{\beta_2(E_1 - E_2 - \frac{r}{2})}, \sqrt{\beta_3(E_2 - E_3)}\right)$, $T_1^3 = t_0 - \sqrt{\beta_3(E_2 - E_3)}$, $T_2^1 = t_0 - \sqrt{\beta_3(E_1 - E_3)}$, $T_1^4 = t_0 - \sqrt{(\beta_2(E_1 - E_2) + \beta_3(E_1 - E_3))}$.

<table>
<tr><th rowspan="3">Case</th><th colspan="8">Conditions satisfied</th><th rowspan="3">SPE</th></tr>
<tr><th colspan="2">$E_1 - E_3$</th><th colspan="3">$E_1 - E_2$</th><th colspan="3">$E_2 - E_3$</th></tr>
<tr><th>$\leqslant \frac{2}{3}r$</th><th>$> \frac{2}{3}r$</th><th>$\leqslant \frac{1}{6}r$</th><th>$(\frac{1}{6}r, \frac{1}{2}r]$</th><th>$> \frac{1}{2}r$</th><th>$\leqslant \frac{1}{6}r$</th><th>$(\frac{1}{6}r, \frac{1}{2}r]$</th><th>$> \frac{1}{2}r$</th></tr>
<tr><td>1</td><td>✓</td><td></td><td>✓</td><td></td><td></td><td colspan="3">✓</td><td>(t_o, t_o, t_o)</td></tr>
<tr><td>2</td><td>✓</td><td></td><td></td><td>✓</td><td></td><td>✓</td><td></td><td></td><td>(t_o, t_o, t_o)</td></tr>
<tr><td>3</td><td>✓</td><td></td><td></td><td>✓</td><td></td><td></td><td>✓</td><td></td><td>(T_1^1, t_o, t_o)</td></tr>
<tr><td>4</td><td>✓</td><td></td><td></td><td></td><td>✓</td><td>✓</td><td></td><td></td><td>(T_1^2, t_o, t_o)</td></tr>
<tr><td>5</td><td></td><td>✓</td><td colspan="2">✓</td><td></td><td></td><td></td><td>✓</td><td>(T_2^1, T_2^1, t_o)</td></tr>
<tr><td>6</td><td></td><td>✓</td><td></td><td>✓</td><td></td><td></td><td>✓</td><td></td><td>(T_1^3, t_o, t_o)</td></tr>
<tr><td>7</td><td></td><td>✓</td><td></td><td></td><td>✓</td><td colspan="2">✓</td><td></td><td>(T_1^2, t_o, t_o)</td></tr>
<tr><td>8</td><td></td><td>✓</td><td></td><td></td><td>✓</td><td></td><td></td><td>✓</td><td>(T_1^4, T_2^1, t_o)</td></tr>
</table>

TABLE II: Pure strategy NE and SPE of the 2-agent game under two utility functions. $\epsilon \in [0, \sqrt{\beta_2 r/2}]$.

<table>
<tr><td colspan="2">Utility function</td><td colspan="2">$u_i(\mathbf{t}) = e(t_i) - c_i(t_i) - p_i(\mathbf{t})$</td><td colspan="2">$u_i(\mathbf{t}) = e(t_i) - c_i(t_i)$</td></tr>
<tr><td colspan="2">Game</td><td>One-shot</td><td>Sequential</td><td>One-shot</td><td>Sequential</td></tr>
<tr><td rowspan="2">Equilibrium</td><td>$E_1 - E_2 \in (0, \frac{1}{2}r]$</td><td>$(t_0 - \epsilon, t_0 - \epsilon)$</td><td>$(t_0, t_0)$</td><td rowspan="2">none</td><td rowspan="2">$(t_0 - \sqrt{\beta_2(E_1 - E_2)}, t_0)$</td></tr>
<tr><td>$E_1 - E_2 \in (\frac{1}{2}r, +\infty)$</td><td>none</td><td>$(t_0 - \sqrt{\beta_2(E_1 - E_2)}, t_0)$</td></tr>
</table>

C. An efficient algorithm

The complexity of directly applying backward induction to find an SPE in our game increases exponentially. However, by applying the properties shown above we can develop an efficient algorithm, which starts from the grand flock with all agents arriving at t_o and moves backwards. It first checks to see if the weakest agent, n, has an incentive to move up its arrival time in order to obtain E_1 as opposed to E_n. This is the case if $E_1 - E_n > \frac{n-1}{n} r$ (per Proposition 2), and no otherwise. If this condition is verified, then we know the grand flock cannot be an SPE, and it follows that some or all of the first $n-1$ agents will have an incentive to move up their arrival time. The algorithm then checks another set of conditions (that involve the comparison between pairs of territories such as shown in Table I) to determine whether agent $n-1$ will flock with agent n, and so on. In such a manner the algorithm progressively determines the groupings as well as the times of arrival of each group in the SPE.

Proposition 4. *A stronger agent arrives no later than a weaker agent in an SPE, and the weakest agent always arrives at the optimal time* t_0*:* $t_1^* \leqslant t_2^* \leqslant \cdots \leqslant t_n^* = t_0$.

Sketch: This is done by showing the algorithm outlined above outputs the SPE and this property holds by construction.

Intuitively, the key observation here is that advancing travel time, provided all travel times are no later than t_o, is more costly for a weaker agent (j) than for a stronger agent ($i < j$). Therefore if it were beneficial for j to arrive at $t_j < t_i$, then the benefit to arrive at t_j for i is only greater, and therefore agent t_i cannot be i's best response. Indeed, agent i will always preempt j, since i makes its decision first, and prevent this from happening.

V. Discussion

We first discuss what happens if the game is modeled as a simultaneous move one. With all game parameters being the same but letting all agents make simultaneous decisions results in a one-shot game, whose solution concept is the pure strategy Nash equilibrium (NE).

Proposition 5. *In the simultaneous-move game, if* $E_1 - E_n \leqslant \frac{n-1}{n} r$*, then there exist infinitely many pure strategy NE:* $\mathbf{t}^* = (t_o - \epsilon, t_o - \epsilon, \ldots, t_o - \epsilon)$ *where* $\epsilon \in [0, \sqrt{\frac{n-1}{n} r \beta_n}]$.

Sketch: Since flocking is more beneficial due to the small territorial quality difference, arriving as a group at any time close enough to t_o will prevent unilateral deviation from an individual agent.

Proposition 6. *There does not exist any pure strategy NE in the one-shot game with the utility function* (2).

Sketch: Since there is no benefit in forming groups, for any given set of arrival times, one can always find one of the following types of agents with a profitable unilateral move: (1) a lone arrival, who can improve its utility by delaying its arrival by an infinitesimal amount to lower its travel cost without losing its territory, or (2) a weaker agent in a group, who can advance its arrival by an infinitesimal amount to gain a better territory with negligible increase in travel cost.

The comparison among these different games and solution concepts is summarized in Table II for the 2-agent case.

We next examine the effect of the ordering of decision making in the sequential game, which has been assumed to be in decreasing order of the agents' strengths. A natural question arises as to what happens if this is not the case. First, it's worth noting that the decreasing order of strengths appears (per Proposition 4) to match the decreasing order of arrivals in an SPE; in other words, the sequencing in decision making appears to match the sequencing in decisions made or actions taken. This is also consistent with observations in migratory birds as discussed earlier. This matching between decision-making and decisions made does not easily emerge under a different model.

Consider now the special, 2-agent game with a flipped decision order: Agent 2 the leader and Agent 1 the follower. The only SPE, in this case, is $\mathbf{t}^* = (t_0, t_0)$, which can be shown using backward induction similar to that used in Theorem 1. It is also straightforward to interpret: when the weaker agent chooses its action first, the stronger agent can always get the better territory by choosing to arrive at the same time, eliminating any benefit to the weaker agent by advancing. Thus, the only pure SPE is that both agents arrive at the same, optimal time. This result can be extended to the n-agent game, with the opposite ordering of decision making, whereby agent n decides first, followed by agent $n-1$, and so on, with agent 1 last. The only pure strategy SPE, in this case, is the grand flock of everyone arriving at the optimal time t_o, with the same interpretation as given above. When the ordering is arbitrary, the resulting SPE becomes harder to characterize. Interestingly, under the pure-competition model (utility function (2)), if the weaker agent is the leader, then the only SPE of a 2-agent game is also $\mathbf{t}^* = (t_0, t_0)$, which is easily verified.

VI. Conclusion

We introduced a sequential game of how agents form groups in time. Properties of the resulting SPE are analyzed in the 2-agent and 3-agent cases, and characterized in the more general, n-agent game. The result of the sequential game is also contrasted with one obtained under a one-shot game as well as under an alternative utility function when group formation does not bring benefits.

References

[1] TJ Pitcher, AE Magurran, and IJ Winfield. Fish in larger shoals find food faster. *Behavioral Ecology and Sociobiology*, 10(2):149–151, 1982.

[2] Todd Sandler and John T. Tschirhart. The economic theory of clubs: An evaluative survey. *Journal of Economic Literature*, 18(4):1481–1521, 1980.

[3] Guy Beauchamp. Reduced flocking by birds on islands with relaxed predation. *Proceedings of the Royal Society of London. Series B: Biological Sciences*, 271(1543):1039–1042, 2004.

[4] Guy Beauchamp. *Social predation: how group living benefits predators and prey*. Elsevier, 2013.

[5] Guy Beauchamp. Flocking in birds increases annual adult survival in a global analysis. *Oecologia*, 197(2):387–394, 2021.

[6] Guy Beauchamp. Long-distance migrating species of birds travel in larger groups. *Biology Letters*, 7(5):692–694, 2011.

[7] Amir Mirzaeinia and Mostafa Hassanalian. Energy conservation of v-shaped flocking canada geese through leader and tail switching. In *AIAA Propulsion and Energy 2019 Forum*, page 4152, 2019.

[8] Henri Weimerskirch, Julien Martin, Yannick Clerquin, Peggy Alexandre, and Sarka Jiraskova. Energy saving in flight formation. *Nature*, 413(6857):697–698, 2001.

[9] Tomasz Janiszewski, Piotr Minias, and Zbigniew Wojciechowski. Timing of arrival at breeding grounds determines spatial patterns of productivity within the population of white stork (ciconia ciconia). *Population Ecology*, 56(1):217–225, 2014.

[10] Jacob Johansson and Niclas Jonzén. Effects of territory competition and climate change on timing of arrival to breeding grounds: a game-theory approach. *The American Naturalist*, 179(4):463–474, 2012.

[11] Catriona A Morrison, José A Alves, Tómas G Gunnarsson, Böđhvar Thórisson, and Jennifer A Gill. Why do earlier-arriving migratory birds have better breeding success? *Ecology and Evolution*, 9(15):8856–8864, 2019.

[12] Etienne Sirot and Frédéric Touzalin. Temporal patterns of arrival from migration as a response to competition for breeding space. *Journal of avian biology*, 45(1):109–112, 2014.

[13] Iain D Couzin. Behavioral ecology: social organization in fission–fusion societies. *Current Biology*, 16(5):R169–R171, 2006.

[14] Lorenz Goette, David Huffman, and Stephan Meier. The impact of group membership on cooperation and norm enforcement: Evidence using random assignment to real social groups. *American Economic Review*, 96(2):212–216, 2006.

[15] Guillaume Hollard. On the existence of a pure strategy nash equilibrium in group formation games. *Economics Letters*, 66(3):283–287, 2000.

[16] Igal Milchtaich and Eyal Winter. Stability and segregation in group formation. *Games and Economic Behavior*, 38(2):318–346, 2002.

[17] Suryapratim Banerjee, Hideo Konishi, and Tayfun Sönmez. Core in a simple coalition formation game. *Social Choice and Welfare*, 18(1):135–153, 2001.

[18] Anna Bogomolnaia and Matthew O. Jackson. The stability of hedonic coalition structures. *Games and Economic Behavior*, 38(2):201–230, 2002.

[19] Debraj Ray and Rajiv Vohra. Coalition formation. In *Handbook of game theory with economic applications*, volume 4, pages 239–326. Elsevier, 2015.

[20] Marco Slikker. Coalition formation and potential games. *Games and Economic Behavior*, 37(2):436–448, 2001.

[21] Chenlan Wang, Mehrdad Moharrami, Kun Jin, David Kempe, P. Jeffrey Brantingham, and Mingyan Liu. Structural stability of a family of group formation games. In *2021 60th IEEE Conference on Decision and Control (CDC)*, pages 3080–3085, 2021.

[22] Yves van Gennip, Blake Hunter, Raymond Ahn, Peter Elliott, Kyle Luh, Megan Halvorson, Shannon Reid, Matthew Valasik, James Wo, George E Tita, et al. Community detection using spectral clustering on sparse geosocial data. *SIAM Journal on Applied Mathematics*, 73(1):67–83, 2013.

[23] George Wittemyer, Iain Douglas-Hamilton, and Wayne Marcus Getz. The socioecology of elephants: analysis of the processes creating multitiered social structures. *Animal behaviour*, 69(6):1357–1371, 2005.

[24] Andrew J. Collins, Caitlin V.M. Cornelius, and John A. Sokolowski. Agent-based model of criminal gang formation. In *Proceedings of the Agent-Directed Simulation Symposium*, pages 1–10, 2017.

[25] Andrew J. Collins and Erika Frydenlund. Strategic group formation in agent-based simulation. *Simulation*, 94(3):179–193, 2018.

[26] Hanna Kokko. Competition for early arrival in migratory birds. *Journal of Animal Ecology*, 68(5):940–950, 1999.

[27] Shane G DuBay, Yongjie Wu, Graham R Scott, Yanhua Qu, Qiao Liu, Joel H Smith, Chao Xin, Andrew Hart Reeve, Chen Juncheng, Dylan Meyer, et al. Life history predicts flight muscle phenotype and function in birds. *Journal of Animal Ecology*, 89(5):1262–1276, 2020.

[28] JC Senar and J Pascual. Keel and tarsus length may provide a good predictor of avian body size. *ARDEA-WAGENINGEN-*, 85:269–274, 1997.

[29] Jerred J Seveyka. The effects of body size and morphology on the flight behavior and escape flight performance of birds. 1999.

[30] Brian C Weeks, David E Willard, Marketa Zimova, Aspen A Ellis, Max L Witynski, Mary Hennen, and Benjamin M Winger. Shared morphological consequences of global warming in north american migratory birds. *Ecology Letters*, 23(2):316–325, 2020.