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Nonlinear Predictor-Feedback Cooperative Adaptive Cruise Control

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***Abstract*— We construct a nonlinear predictor-feedback Cooperative Adaptive Cruise Control (CACC) design for homogeneous vehicular platoons subject to actuators delays, which achieves: i) positivity of vehicles' speed and spacing states, ii) $\mathcal{L}_\infty$ string stability of the platoon, iii) stability of each individual vehicular system, and iv) tracking of a constant reference speed (dictated by the leading vehicle) and spacing. The design relies on a nominal, nonlinear control law, which guarantees i)–iv) in the absence of actuator delay, and nonlinear predictor feedback. We consider a second-order, nonlinear vehicle model with input delay. The proofs of the theoretical guarantees i)–iv) rely on derivation of explicit estimates on solutions (both during open-loop and closed-loop operation), capitalizing on the ability of predictor feedback to guarantee complete delay compensation after the dead-time interval has elapsed, and derivation of explicit conditions on initial conditions and parameters of the nominal control law. We also present consistent simulation results.**

I. Introduction

Delay compensation in vehicular platoons equipped with Adaptive Cruise Control (ACC) and CACC capabilities is a significant objective of ACC/CACC designs, in view of its potential in improvement of the safety and performance properties of platoons; see, for example, [3], [4], [5], [6], [12], [14], [15], [19], [23], [24]. Three different types of delays are, typically, evident in such systems, namely, actuation, sensing, and communication delays. Each delay type may have a negative effect in individual vehicle stability and string stability, when is left uncompensated; while each of delay type requires specific treatment for its compensation. Here we address actuation delay, which, typically, takes the largest values among the three types [24].

ACC and CACC designs, aiming at delay compensation[1] in vehicular platoons, have been developed in, for example, [1], [3], [4], [8], [13], [14], [22], [23], [24], [25], [26], [27], [28]. Almost all of these results utilize linear or linearized models for each individual vehicular system, for either control design or stability and string stability analysis, which, in certain scenarios, may not be as realistic as nonlinear vehicle models (for both control design and analysis) that may capture additional, lower-level vehicle dynamics (see, for example, [7], [17], [18], [20]). Such linear vehicle models have been successfully utilized in these works for delay-compensating ACC/CACC design, with derivation of theoretical guarantees and validation in experimental platforms. However, under actuation delay, considering nonlinear models of vehicle dynamics is important because a nominal (i.e., without a predictor structure), feedback linearizing precompensator, which may be implicitly employed in control of vehicular platoons (to subsequently enabling linear, ACC/CACC designs), may not result in a linear vehicle model, due to the presence of actuation delay. To the best of our knowledge, the only result that is related to construction of a nonlinear predictor-feedback ACC (or CACC) design can be found in [13].

In the present paper, complementing [13], we a) provide the predictors formulae as explicitly as possible, b) design a predictor-feedback CACC law relying on real-time measurements of the control input of the preceding vehicle (thus avoiding utilization of open-loop predictors, for the preceding vehicle's states, which, potentially, may be less robust), c) consider a platoon of vehicles, and d) provide explicit conditions on initial conditions and control parameters, which guarantee positivity of speed and spacing states, as well as $\mathcal{L}_\infty$ string stability. In particular, we construct a nonlinear predictor-feedback CACC law, which aims at actuation delay compensation for vehicular platoons in which each vehicle's dynamics are described by a second-order, nonlinear system with input delay[2]. The design relies on two ingredients-a nominal (for the delay-free case) nonlinear, feedback-linearizing ACC design and states' predictors. In fact, because to predict the speed of the preceding vehicle (employed in the nominal ACC design), measurements of the control input variable of the preceding vehicle are required (obtained via vehicle-to-vehicle communication), the resulting control law is of CACC type.

The feedback law constructed guarantees the primary objectives of a CACC design, namely, i) positivity of speed and spacing states, ii) $\mathcal{L}_\infty$ string stability, iii) stability of each individual vehicular system, and iv) tracking of a constant reference speed (dictated by the leading vehicle) and spacing. The proofs of guarantees i)–iv) rely on derivation of explicit estimates on solutions, capitalizing on the ability of predictor feedback to achieve complete delay compensation after the dead-time interval has elapsed, and derivation of explicit conditions on initial conditions and parameters of the nominal controller. The conditions derived on the initial states are consistent with the practical requirement that

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[1] Here we review only papers dealing with delay compensation by design and not with studying robustness to small delay values.

[2] To avoid burying the key contribution of the paper, which is delay compensation for vehicular platoons via nonlinear predictor-feedback CACC, in technical details, we consider homogeneous platoons of second-order, vehicle models. Such models may be viewed as approximation of more realistic third-order models and they have been successfully employed for ACC/CACC design and analysis under delay effects, in existing works, see, for example, [5], [13], [27]. There is no conceptual obstacle to extend the results presented to a third-order model.

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there is no finite-escape time phenomenon appearing and that the speed/spacing states remain positive during open-loop operation (i.e., during the dead-time interval). The conditions on the control law parameters are consistent with the requirements needed to guarantee $\mathcal{L}_\infty$ string stability and individual vehicle stability, in the nominal, delay-free case. No restriction on the delay size or the desired time headway are imposed, which is consistent with the fact that predictor feedback guarantees that, in closed loop, each individual vehicular system inherits the properties of the respective, nominal (for the delay-free case) closed-loop system. We also demonstrate the effectiveness of the design in a simulation example.

II. Nonlinear Predictor-Feedback CACC for Homogeneous Platoons with Actuator Delay

a) Vehicle dynamics: We consider a homogeneous platoon of vehicles each one modeled by the following second-order, nonlinear system, with delayed input

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$
$$\dot{v}_i(t) = -dv_i(t)^2 - g + cu_i(t-D), \tag{2}$$

$i = 1, \ldots, N$, where s_i is spacing between vehicles i and $i-1$, v_i is vehicle's speed, u_i is the individual vehicle's control variable, $D \geq 0$ is actuator delay, $t \geq 0$ is time, and d, g, c are positive coefficients depending on vehicle's characteristics (see, for example, [7], [17], [18], [20]). Vehicle dynamics (1), (2) are considered as sufficiently reasonable for the purpose of illustrating the delay compensation benefits of predictor feedback to individual vehicle stability and string stability, as well as to safety of CACC platoons. Such dynamic models are also utilized in, for example, [13], for design of predictor-based ACC laws. They are also viewed as extensions, of (classical) nonlinear vehicle models[3], see, for example, [7], [18], [20], to incorporate input delay, which may be more realistic in practice.

For the leading vehicle's speed dynamics, adopting the notation $v_l \equiv v_0$, $u_l \equiv u_0$, we assume that $\dot{v}_l(t) = u_l(t-D)$, where u_l is leader's acceleration, acting to the platoon as exogenous input. We consider such dynamics for the leading vehicle for simplicity as v_l is viewed here more as a reference input, rather than as a state that has to be regulated. However, there is no conceptual obstacle to re-design the predictor-feedback CACC law to account for different dynamics for the speed of the leading vehicle (in particular, being identical to (2)), since the predictor states, which rely on the vehicles' model, could be straightforwardly modified accordingly. This is the case as long as the control input of the leading vehicle is subject to an input delay D. Note that a uniform equilibrium point of systems (1), (2) is obtained when all vehicles have the same, constant speed, dictated by a constant, leader's speed, say v^* (corresponding to zero acceleration for the leading vehicle), corresponding to a constant control input value $u^* = \frac{g+v^{*2}}{c}$.

[3]Note that, in order to not distract the reader with technical details and tedious algebraic computations, we focus here on a simplified/approximated version of the nonlinear vehicle models from [7], [18], [20], which may be valid assuming a small time constant for the first-order engine dynamics.

b) String stability definition: An interconnected system of vehicles, indexed by $i = 1, \ldots, N$, following each other in single lane without passing, is $\mathcal{L}_\infty$ string stable if the following hold for $i = 1, \ldots, N$ (see also, for example, [10], [16], [21] for similar and more general definitions)

$$\begin{aligned}\|\tilde{v}_i\|_\infty \leq & \|\tilde{v}_{i-1}\|_\infty + \gamma_1\left(|\tilde{s}_{i_0}|\right) \\ & + \gamma_2\left(|\tilde{v}_{i_0}| + \sqrt{\frac{c}{d}\sup_{\theta\in[-D,0]}|\tilde{u}_{i_0}(\theta)|}\right) \\ & + \gamma_3\left(|\tilde{v}_{i-1_0}| + \sqrt{\frac{c}{d}\sup_{\theta\in[-D,0]}|\tilde{u}_{i-1_0}(\theta)|}\right),\end{aligned} \tag{3}$$

where $\tilde{v}_i(t) = v_i(t) - v^*$, $\tilde{v}_l(t) = v_l(t) - v^*$, $\tilde{s}_i(t) = s_i(t) - s^*$, $\tilde{u}_i(s) = u_i(s) - u^*$, $\tilde{u}_l(s) = u_l(s)$, $\|v_i - v^*\|_\infty = \sup_{t\geq 0}|v_i(t) - v^*|$, $s^* = hv^*$, with $h > 0$ being the desired headway, and $\gamma_1 : [0, +\infty) \to [0, +\infty)$, $\gamma_2, \gamma_3 : \left[0, \frac{1}{dD}\right) \to [0, +\infty)$ are class $\mathcal{K}$ functions (see, for example, [11]). Other definitions are also possible, in particular, which may involve studying disturbance propagation (upstream in the platoon) of spacing errors or accelerations. For simplicity we consider definition of string stability with respect to speed errors only and since this is the most commonly employed definition.

c) Delay-free control design: Without actuator delay, the following control strategy is employed, which combines a nonlinear, feedback linearization-based lower level control law and a constant time-headway (CTH), linear ACC law (see, for example, [7])

$$\begin{aligned}u_i(t) = & \frac{1}{c}\left(g + dv_i(t)^2 + \alpha\left(\frac{s_i(t)}{h} - v_i(t)\right)\right. \\ & \left. + b\left(v_{i-1}(t) - v_i(t)\right)\right),\end{aligned} \tag{4}$$

where α and b are positive design parameters.

d) Predictor-feedback CACC design: The predictor-based control laws for system (1), (2) are given by

$$\begin{aligned}u_i(t) = & \frac{\alpha}{ch}q_{i,1}(t) - \frac{\alpha+b}{c}q_{i,2}(t) + \frac{d}{c}q_{i,2}(t)^2 + \frac{b}{c}q_{i,3}(t) \\ & + \frac{g}{c},\end{aligned} \tag{5}$$

where for $i = 1, \ldots, N$

$$q_{i,1}(t) = s_i(t) + \int_{t-D}^{t}\left(q_{i,3}(s) - q_{i,2}(s)\right)ds, \tag{6}$$

$$q_{i,2}(t) = v_i(t) + \int_{t-D}^{t}\left(-dq_{i,2}(s)^2 - g + cu_i(s)\right)ds, \tag{7}$$

and

$$\begin{aligned}q_{i,3}(t) = & v_{i-1}(t) + \int_{t-D}^{t}\left(-dq_{i,3}(s)^2\right. \\ & \left. - g + cu_{i-1}(s)\right)ds, \quad i = 2, \ldots, N,\end{aligned} \tag{8}$$

$$q_{1,3}(t) = v_l(t) + \int_{t-D}^{t}u_l(s)ds, \tag{9}$$

with $q_{i,1}$ and $q_{i,2}$ being initialized for $\theta \in [-D, 0)$ as

$$q_{i,1}(\theta)=s_i(0) + \int_{-D}^{\theta} (q_{i,3}(s) - q_{i,2}(s))\, ds, \tag{10}$$

$$q_{i,2}(\theta)=v_i(0) + \int_{-D}^{\theta} \left(-dq_{i,2}(s)^2 - g + cu_{i_0}(s)\right) ds, \tag{11}$$

for $i = 1, \ldots, N$, and $q_{i,3}$ being initialized as

$$q_{i,3}(\theta)=v_{i-1}(0) + \int_{-D}^{\theta} \left(-dq_{i,3}(s)^2 - g + cu_{i-1_0}(s)\right) ds, \quad i = 2, \ldots, N, \tag{12}$$

$$q_{1,3}(\theta)=v_l(0) + \int_{-D}^{\theta} u_l(s) ds. \tag{13}$$

Control law (5) employs the D-time units ahead predictor states of $\bar{x}_i = \begin{bmatrix} s_i & v_i & v_{i-1} \end{bmatrix}^{\mathrm{T}}$, namely, $q_i = \begin{bmatrix} q_{i,1} & q_{i,2} & q_{i,3} \end{bmatrix}^{\mathrm{T}}$ (see, for example, [2]). The predictor states (6)–(13) involved in control design (5) can be numerically computed via a numerical approximation scheme (see, for example, [9]). Note that the nominal, delay-free control design (4) is of ACC type. However, for constructing the predictor state for the preceding vehicle's speed, measurements of the control input u_{i-1} of the preceding vehicle have to be available. This is possible through V2V communication. The rest of the measurements required for implementation of (5), i.e., s_i, v_i, u_i, v_{i-1}, are obtained from on-board sensors.

III. Positivity of Speed/Spacing States and String Stability Under Nonlinear Predictor-Feedback CACC

Theorem 1: Consider a platoon of vehicles with dynamics modeled by (1), (2), under the control laws (5) with (6)–(13). Assume that the leading vehicle satisfies $v_{l_0} + D \min \left\{0, \inf_{\theta\in[-D,0]} u_l(\theta)\right\} > 0$ and $v_{l_0} + \int_0^t u_l(s-D) ds > 0$, for all $t \geq 0$, with $v_{l_0} + \int_0^t u_l(s-D) ds \in \mathcal{L}_\infty$ and $u_l \in C[-D, +\infty)$. There exist parameters p_1, p_2, satisfying

$$p_2 < p_1 < 0, \tag{14}$$

$$0 < -hp_1p_2 - p_1 - p_2 < -p_2, \tag{15}$$

such that with the choice of control gains

$$\alpha = hp_1p_2, \tag{16}$$

$$b = -hp_1p_2 - p_1 - p_2, \tag{17}$$

and for any $D \geq 0$, $h > 0$, the closed-loop systems' solutions satisfy for $i = 1, 2, \ldots, N$

$$s_i(t) > 0, \quad v_i(t) > 0, \quad \text{for all } t \geq 0, \tag{18}$$

provided that the initial conditions s_{i_0}, $v_{i_0} \in \mathbb{R}_+$ and $u_{i_0} \in C[-D, 0]$[4], $i = 1, 2, \ldots, N$, satisfy

$$\frac{g-\delta}{c} \leq u_{i_0}(\theta) \leq \frac{\zeta + g}{c}, \quad \text{for all } \theta \in [-D, 0], \tag{19}$$

$$v_{i_0} > \tan\left(\sqrt{d\delta} D\right) \frac{\sqrt{\delta}}{\sqrt{d}}, \tag{20}$$

$$s_{i_0} > \frac{M_i(v_{i_0})}{-p_2} - D \min\left\{0, m_{i-1}(v_{i-1_0}) - M_i(v_{i_0})\right\}, \tag{21}$$

for some $\delta, \zeta \geq 0$, with $\delta < \frac{\pi^2}{4D^2 d}$[5], where

$$m_{i-1}(v_{i-1_0})=-\frac{\sqrt{\delta}}{\sqrt{d}} \tan\left(\sqrt{d\delta} D - \tan^{-1}\left(v_{i-1_0} \frac{\sqrt{d}}{\sqrt{\delta}}\right)\right), \quad i = 2, \ldots, N, \tag{22}$$

$$m_0(v_{l_0})=v_{l_0} + D \min\left\{0, \inf_{\theta\in[-D,0]} u_l(\theta)\right\}, \tag{23}$$

$$M_i(v_{i_0})=\max\left\{\frac{v_{i_0} + \frac{\sqrt{\zeta}}{\sqrt{d}} + \left(v_{i_0} - \frac{\sqrt{\zeta}}{\sqrt{d}}\right) \mathrm{e}^{-2\sqrt{\zeta d} D}}{v_{i_0} + \frac{\sqrt{\zeta}}{\sqrt{d}} - \left(v_{i_0} - \frac{\sqrt{\zeta}}{\sqrt{d}}\right) \mathrm{e}^{-2\sqrt{\zeta d} D}} \times \frac{\sqrt{\zeta}}{\sqrt{d}}, v_{i_0}\right\}, \quad i = 1, \ldots, N. \tag{24}$$

Furthermore, if, in addition,

$$|\tilde{v}_{i_0}| + \sqrt{\frac{c}{d} \sup_{\theta\in[-D,0]} |\tilde{u}_{i_0}(\theta)|} < \frac{1}{Dd}, \quad i = 0, \ldots, N, \tag{25}$$

then the platoon is $\mathcal{L}_\infty$ string stable and, for constant leader's speed v^*, each individual vehicular system is asymptotically stable and zero steady-state tracking errors are achieved.

Proof: The proof can be found in Appendix A. ■

Requirements (19)–(21), (25) on initial conditions may be, in practice, restrictive. Nevertheless, they are necessary to theoretically establish positivity and boundedness of speed and spacing states, in view of the nonlinear vehicle model (1), (2) and the presence of input delay considered. In particular, since within open-loop operation, during the dead-time interval, each individual control input does not affect the respective vehicular system, one should necessarily impose conditions on initial conditions to guarantee both positivity of speed/spacing states and that there is no finite-escape time phenomenon arising (that may be viewed more as a theoretical property, in view of the practical aspect of the vehicular systems considered). We further illustrate in simulation both the theoretical guarantees derived and the practical significance of the CACC design.

As it is the case with the linear, predictor-feedback CACC design in [1], the delay value in Theorem 1 is not restricted, for any given, desired headway h; whereas the control gains α, b have to be properly restricted, to guarantee stability and string stability, as well as positivity of speed and spacing states after the dead-time interval has elapsed (this is inherent to the nominal, delay-free ACC design, which is of CTH type). In fact, because in the nominal, delay-free case (i.e., when $D = 0$) the closed-loop system is linear, the same holds true for the case with input delay for $t \geq D$, under predictor-feedback employment. This allows one, for establishing positivity of speed and spacing states for $t \geq D$, as well as stability and string stability, to utilize the results from [1] (Theorem 1), which, in particular, results in derivation of the same conditions on parameters p_1, p_2, satisfying (14), (15).

[4]In fact, $u_{i_0} \in C[-D, 0]$ being compatible with the feedback laws (5).

[5]Choose, e.g., $\delta = \max_i \max\left\{0, g - c \inf_{\theta\in[-D,0]} u_{i_0}(\theta)\right\}$ and $\zeta = \max_i \max\left\{0, c \sup_{\theta\in[-D,0]} u_{i_0}(\theta) - g\right\}$, if $g - c \inf_{\theta\in[-D,0]} u_{i_0}(\theta) < \frac{\pi^2}{4D^2 d}$, $i = 1, \ldots, N$ (note that the limit of (22) as $\delta \to 0$ is $\frac{v_{i-1_0}}{1+dDv_{i-1_0}}$).

In string stability definition (3), the last term appears due to the presence of input delay (this becomes clear within the proof of Theorem 1 in Appendix A; see relations (A.21), (A.22)). This is explained by the fact that, during the dead-time interval, according to (1), (2) the speed dynamics of the ego vehicle depend on the initial conditions of its own speed and actuator state, whereas the respective spacing dynamics are also affected by the preceding vehicle's speed within an interval of D time units, which, in turn, depends on the initial conditions of the preceding vehicle's speed and respective actuator state. This gives rise to the last term in (3), which would not appear in the string stability definition if $D = 0$. In fact, string stability should be viewed more as a property related to the platoon only during closed-loop operation (i.e., for $t \geq D$) of each individual vehicular system (that is also consistent with the delay-free case), because during the dead-time interval each individual vehicle operates in open loop, and thus, its dynamics are affected only by initial conditions.

IV. Simulation Results

We illustrate here the safety, stability, and string stability properties of the nonlinear, predictor-feedback CACC with a platoon of five vehicles. We consider a scenario in which $D = 0.5$, $h = 0.75$, while for the vehicles we set $d = 0.00025$, $c = 0.0005$, and $g = 0.002$, which are realistic values for vehicles (see, for example, [7]). We choose $p_1 = -0.1$, $p_2 = -1.5$, which satisfy conditions (14), (15). We consider a case in which all initial accelerations are set to zero, i.e., $u_{i_0} \equiv 0$, for $i = 0, 1, \ldots, 4$, for simplicity. We set $v_{\mathrm{l}_0} = \frac{2v_{1_0}}{3} = 13 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$, $v_{i_0} = 15 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$, $i = 1, 2, 4$, and $v_{3_0} = 13 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$; while we further choose $s_{i_0} = hv_{i_0}$ m, $i = 1, 2, 3, 4$. We consider a scenario in which the leading vehicle performs a deceleration/acceleration maneuver. Thus, this scenario illustrates the effectiveness of the proposed design with respect to both initial conditions deviations from equilibrium and leading vehicle's maneuvers. As it is shown in Fig. 1, positivity of speed and spacing states is achieved, while the responses to the leading vehicle's maneuvers feature no overshoot, as result of the achieved $\mathcal{L}_\infty$ string stability. Furthermore, regulation of speed and spacing states at the desired, reference values is also achieved, as a result of the achieved asymptotic stability.

V. Conclusions

We presented a nonlinear predictor-feedback CACC design for homogeneous vehicular platoons in which each individual vehicular system is described by a second-order, nonlinear system with input delay. The delay-compensating property of the design results in positivity of speed and spacing states, as well as asymptotic stability of each individual vehicular system and string stability of the platoon; all of which are important requirements for safe and efficient operation of vehicular platoons. The guarantees are proved deriving explicit estimates on solutions and utilizing the delay-compensating property of predictor feedback. We currently work on extending the results presented to third-order vehicle dynamics, to account for, e.g., engine dynamics.

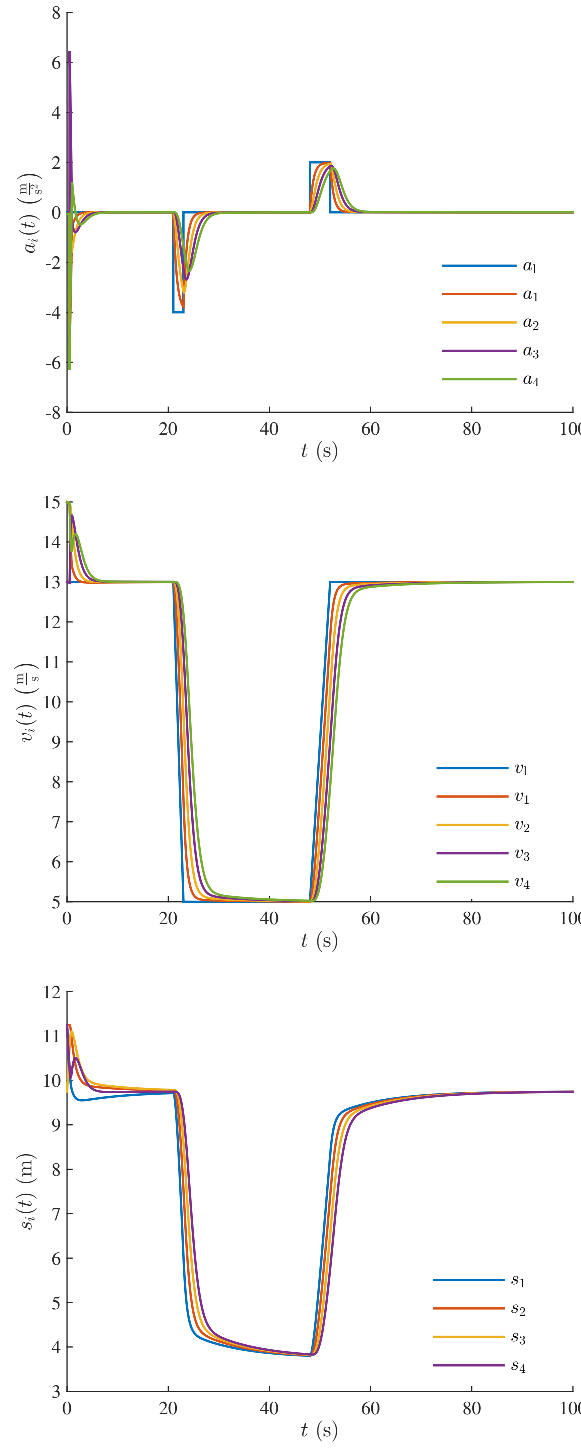

Fig. 1. Acceleration (top), speed (middle), and spacing (bottom) of four vehicles, with dynamics described by (1), (2), following a leader that performs an acceleration/deceleration maneuver, under the nonlinear predictor-feedback CACC laws (5). Initial conditions are $u_{i_0}(s) = 0$, $-D \leq s < 0$, for $i = 0, \ldots, 4$; $v_{\mathrm{l}_0} = 13$, $v_{i_0} = 15 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$, $i = 1, 2, 4$, and $v_{3_0} = 13 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$; $s_{i_0} = hv_{i_0}$ (m), $i = 1, 2, 3, 4$.

Appendix A

Proof of Theorem 1

a) Proof of positivity of speed and spacing states: The signals $q_i = \begin{bmatrix} q_{i,1} & q_{i,2} & q_{i,3} \end{bmatrix}^{\mathrm{T}}$ in (6)–(9), initialized according to (10)–(13), satisfy $q_i(t) = \bar{x}_i(t + D)$ for all $t \geq 0$; see, for example, [2]. Therefore, under the feedback laws (5), for $t \geq D$ it holds that

$$\begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ \frac{\alpha}{h} & -(a+b) \end{bmatrix} \begin{bmatrix} s_i(t) \\ v_i(t) \end{bmatrix} + \begin{bmatrix} 1 \\ b \end{bmatrix} v_{i-1}(t). \quad \text{(A.1)}$$

The fact that the closed-loop system for $t \geq D$ is linear and satisfies (A.1), enables one to utilize the results from [1] (Theorem 1) to establish positivity of speed and spacing states for $t \geq D$. Under the conditions (14), (15) for parameters p_1, p_2, this is achieved if the following condition holds

$$0 < v_i(D) < -p_2 s_i(D), \quad \text{(A.2)}$$

together with the condition that speed and spacing states remain positive for $t \in [0, D]$. We first establish positivity during the dead-time interval and subsequently we establish condition (A.2). During the dead-time interval $0 \leq t \leq D$ we have from (1), (2) that

$$\dot{v}_i(t) = -dv_i(t)^2 - g + cu_{i_0}(t - D), \quad \text{(A.3)}$$

and hence, $\dot{v}_i(t) \geq -dv_i(t)^2 - g + c \inf_{-D \leq s \leq 0} u_{i_0}(s)$. Defining $v_i = -y_i$, we obtain that y_i satisfies $\dot{y}_i(t) \leq dy_i(t)^2 + g - c \inf_{-D \leq s \leq 0} u_{i_0}(s)$. Under assumption (19) it holds that

$$\dot{y}_i(t) \leq dy_i(t)^2 + \delta. \quad \text{(A.4)}$$

Thus, under assumption (20) and the fact that $\sqrt{d\delta} D < \frac{\pi}{2}$, using the comparison principle (see, for example, Lemma 3.4 in [11]) we obtain

$$v_i(t) \geq -\frac{\sqrt{\delta}}{\sqrt{d}} \tan\left(\sqrt{d\delta} t - \tan^{-1}\left(v_{i_0} \frac{\sqrt{d}}{\sqrt{\delta}}\right)\right). \quad \text{(A.5)}$$

Note that under assumption (20) and since $\delta < \frac{\pi^2}{4D^2 d}$ we obtain that $v_i(t) > 0$, for all $0 \leq t \leq D$, because $-\frac{\pi}{2} < \sqrt{d\delta} t - \tan^{-1}\left(v_{i_0} \frac{\sqrt{d}}{\sqrt{\delta}}\right) < 0$, for all $0 \leq t \leq D$. Similarly, under (19) we get from (A.3) that $\dot{v}_i(t) \leq -dv_i(t)^2 + \zeta$, and hence, from the comparison principle we get for all $t \in [0, D]$

$$v_i(t) \leq \frac{\sqrt{\zeta}}{\sqrt{d}} \frac{v_{i_0} + \frac{\sqrt{\zeta}}{\sqrt{d}} + \left(v_{i_0} - \frac{\sqrt{\zeta}}{\sqrt{d}}\right) \mathrm{e}^{-2\sqrt{\zeta d} t}}{v_{i_0} + \frac{\sqrt{\zeta}}{\sqrt{d}} - \left(v_{i_0} - \frac{\sqrt{\zeta}}{\sqrt{d}}\right) \mathrm{e}^{-2\sqrt{\zeta d} t}}. \quad \text{(A.6)}$$

Note that, for all $0 \leq t \leq D$, the right-hand side of (A.6) is always positive and finite. Using (1) we obtain for $0 \leq t \leq D$

$$s_i(t) = s_i(0) + \int_0^t (v_{i-1}(s) - v_i(s))\, ds. \quad \text{(A.7)}$$

Thus, as $v_1(t) \geq v_{1_0} + t \inf_{\theta \in [-D,0]} u_1(\theta) > 0$, $t \in [0, D]$, using (A.5), (A.6) we get that for all $0 \leq t \leq D$ the following holds

$$s_i(t) \geq s_{i_0} + t\left(m_{i-1}\left(v_{i-1_0}\right) - M_i\left(v_{i_0}\right)\right), \quad \text{(A.8)}$$

where m_{i-1} and M_i are defined in (22), (23) and (24), respectively. Thus, since $s_{i_0} + t\left(m_{i-1}\left(v_{i-1_0}\right) - M_i\left(v_{i_0}\right)\right) \geq s_{i_0} + D \min\left\{0, m_{i-1}\left(v_{i-1_0}\right) - M_i\left(v_{i_0}\right)\right\}$, for all $t \in [0, D]$, we have, under condition (21), both that $s_i(t) > 0$ for all $t \in [0, D]$, as well as that condition (A.2) holds (because from (A.6), condition (21) also implies that $-p_2 s_i(D) > M_i(v_{i_0}) \geq v_i(D)$).

b) Proof of stability and string stability: To prove $\mathcal{L}_\infty$ string stability and asymptotic stability, we capitalize on the specific form (which is a result of predictor-feedback employment) of the closed-loop system given in (A.1). We recall the differences of the speed and spacing states from their respective equilibrium values, which are obtained for a constant equilibrium for speed states, say v^*, as $\tilde{s}_i = s_i - s_i^*$ and $\tilde{v}_i = v_i - v^*$, with $s_i^* = hv^*$, corresponding to an equilibrium control input $u^* = \frac{dv^{*2} + g}{c}$ for vehicles $i = 1, \ldots, N$, with a zero acceleration for the leading vehicle. It follows from (A.1) that

$$\begin{aligned} \tilde{s}_i(t) =& \frac{1}{p_1 - p_2}\Big(-e^{p_1(t-D)}\left(\tilde{v}_i(D) + p_2 \tilde{s}_i(D)\right) \\ &+ e^{p_2(t-D)}\left(\tilde{v}_i(D) + p_1 \tilde{s}_i(D)\right)\Big) + r_{1,i}(t), \quad \text{(A.9)} \\ \tilde{v}_i(t) =& \frac{1}{p_1 - p_2}\Big(p_1 e^{p_1(t-D)}\left(\tilde{v}_i(D) + p_2 \tilde{s}_i(D)\right) - p_2 \\ &\times e^{p_2(t-D)}\left(\tilde{v}_i(D) + p_1 \tilde{s}_i(D)\right)\Big) + r_{2,i}(t), \quad \text{(A.10)} \end{aligned}$$

where

$$\begin{aligned} r_{1,i}(t) &= -\frac{1}{p_1 - p_2} \int_D^t \Big(e^{p_1(t-s)}(b + p_2) \\ &\quad - e^{p_2(t-s)}(b + p_1)\Big) \tilde{v}_{i-1}(s) ds, \quad \text{(A.11)} \\ r_{2,i}(t) &= \frac{1}{p_1 - p_2} \int_D^t \Big(p_1 e^{p_1(t-s)}(b + p_2) \\ &\quad - p_2 e^{p_2(t-s)}(b + p_1)\Big) \tilde{v}_{i-1}(s) ds. \quad \text{(A.12)} \end{aligned}$$

Using (14), (15), and (17) it is shown (see relation (A.11) in [1]) that $p_1 e^{p_1(t-s)}(b + p_2) - p_2 e^{p_2(t-s)}(b + p_1) > 0$, $t \geq s \geq D$, and hence, $\frac{1}{p_1 - p_2} \int_D^t \left|p_1 e^{p_1(t-s)}(b + p_2) - p_2 e^{p_2(t-s)}(b + p_1)\right| ds = \frac{1}{p_1 - p_2} \int_D^t \left(p_1 e^{p_1(t-s)}(b + p_2) - p_2 e^{p_2(t-s)}(b + p_1)\right) ds = \frac{b + p_2}{p_1 - p_2}\left(e^{p_1(t-D)} - 1\right) - \frac{b + p_1}{p_1 - p_2}\left(e^{p_2(t-D)} - 1\right)$. The function

$$\begin{aligned} w(t) &= \frac{b + p_2}{p_1 - p_2}\left(e^{p_1(t-D)} - 1\right) \\ &\quad - \frac{b + p_1}{p_1 - p_2}\left(e^{p_2(t-D)} - 1\right), \quad \text{(A.13)} \end{aligned}$$

is increasing for $t \geq D$ (since $\dot{w}(t) > 0$, for $t \geq D$), and hence, $\frac{b + p_2}{p_1 - p_2}\left(e^{p_1(t-D)} - 1\right) - \frac{b + p_1}{p_1 - p_2}\left(e^{p_2(t-D)} - 1\right) \leq 1$, for all $t \geq D$. Therefore, from (A.12) we obtain

$$\sup_{t \in [D, +\infty)} |r_{2,i}(t)| \leq \sup_{t \in [D, +\infty)} |\tilde{v}_{i-1}(t)|. \quad \text{(A.14)}$$

We next estimate $\|\tilde{v}_i\|_\infty$ using the fact that $\sup_{t \in [0, +\infty)} |\tilde{v}_i(t)| \leq \sup_{t \in [0, D]} |\tilde{v}_i(t)| + \sup_{t \in [D, +\infty)} |\tilde{v}_i(t)|$. To estimate $\tilde{v}_i(t)$ for $t \in [0, D]$ we re-write (A.3) as

$$\dot{\tilde{v}}_i(t) = -d\tilde{v}_i(t)^2 - 2d\tilde{v}_i(t) v^* + c\tilde{u}_{i_0}(t - D), \quad \text{(A.15)}$$

where $\tilde{u}_{i_0}(t - D) = u_{i_0}(t - D) - u^*$. Defining $r_i(t) = |\tilde{v}_i(t)| + \sqrt{\frac{c}{d} \sup_{\theta \in [-D,0]} |\tilde{u}_{i_0}(\theta)|}$, we obtain from (A.15) that

$$\frac{1}{d}\dot{r}_i(t) \leq \tilde{v}_i(t)^2 - 2\left|\tilde{v}_i(t)\right| v^* + \frac{c}{d} \sup_{\theta \in [-D,0]} \left|\tilde{u}_{i_0}(\theta)\right|. \quad \text{(A.16)}$$

Hence, with $r_i(t)=|\tilde{v}_i(t)|+\sqrt{\frac{c}{d}\sup_{\theta\in[-D,0]}|\tilde{u}_{i_0}(\theta)|}$ we get

$$\dot{r}_i(t)\le dr_i(t)^2. \tag{A.17}$$

Thus, under (25), from the comparison principle we get

$$|\tilde{v}_i(t)|\le\frac{r_{i_0}}{1-dr_{i_0}t}, \tag{A.18}$$

where

$$r_{i_0}=|\tilde{v}_{i_0}|+\sqrt{\frac{c}{d}\sup_{\theta\in[-D,0]}|\tilde{u}_{i_0}(\theta)|}. \tag{A.19}$$

Hence,

$$\sup_{t\in[0,D]}|\tilde{v}_i(t)|\le\frac{r_{i_0}}{1-dr_{i_0}D}. \tag{A.20}$$

It remains, to be able to employ relation (A.10), to estimate $\tilde{s}_i(D)$. Using the fact that $\dot{\tilde{s}}_i(t)=\tilde{v}_{i-1}(t)-\tilde{v}_i(t)$ we get from (A.20) that for $t\in[0,D]$ and $i=2,\ldots,N$ it holds

$$\sup_{t\in[0,D]}|\tilde{s}_i(t)|\le|\tilde{s}_{i_0}|+\frac{Dr_{i_0}}{1-dr_{i_0}D}+\frac{Dr_{i-1_0}}{1-dr_{i-1_0}D}, \tag{A.21}$$

whereas for $i=1$ we get

$$\sup_{t\in[0,D]}|\tilde{s}_i(t)|\le|\tilde{s}_{i_0}|+D\frac{r_{i_0}}{1-dr_{i_0}D}+D|\tilde{v}_{1_0}|+D^2\sup_{\theta\in[-D,0]}|u_1(\theta)|, \tag{A.22}$$

since the leading vehicle dynamics are $\dot{v}_1(t)=u_1(t-D)$. Using (A.20) we arrive at

$$\sup_{t\in[0,+\infty)}|\tilde{v}_i(t)|\le\frac{r_{i_0}}{1-dr_{i_0}D}+\sup_{t\in[D,+\infty)}|\tilde{v}_i(t)|, \tag{A.23}$$

and hence, combining (A.10) with (A.20)–(A.22) and (A.14) for estimating term $\sup_{t\in[D,+\infty)}|\tilde{v}_i(t)|$, the proof is completed with $\gamma_1(y)=\frac{2p_1p_2}{p_1-p_2}y$, $\gamma_2(y)=\left(\frac{|p_1|+|p_2|}{p_1-p_2}+D\frac{2p_1p_2}{p_1-p_2}\right)\frac{y}{1-dyD}$, and $\gamma_3(y)=D\frac{2p_1p_2}{p_1-p_2}\left(\frac{y}{1-dyD}+y+\frac{Dd}{c}y^2\right)$. Asymptotic stability (and zero steady-state tracking errors) follows from (A.9)–(A.12), given estimates (A.20)–(A.22), in a similar manner to the proof of Theorem 1 in [1] (Appendix A).

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