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Impulse Elimination and Synchronization in Descriptor Multi-Agent Systems

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***Abstract*— In this work, a control methodology is proposed to address the problems of impulse elimination and leader-follower state synchronization in a descriptor multi-agent system (DMAS), where each agent in the network is a descriptor system. A distributed static state feedback control protocol is proposed to achieve the control objectives. By making the closed loop DMAS impulse-free through a feedback gain matrix, it is transformed into a set of decoupled descriptor systems using the property of network graph Laplacian matrix, and then, the synchronization problem is transformed into stabilization problem of a set of ordinary state space systems. Since we have used only orthogonal matrices for system transformations, the proposed algorithm is numerically efficient. The effectiveness of the proposed methodology is demonstrated with an example.**

***Index Terms*— Multi-agent system, Descriptor system, Distributed control.**

I. Introduction

Research in the field of multi-agent system (MAS) has become popular due to its applications in a wide variety of areas, such as multi-vehicle robotic systems [1], smart power networks [2], defense and space sectors [3]. A multi-agent system (MAS) consists of multiple subsystems, referred to as *agents*, where agents communicate with their neighbors through a communication network to achieve some global control objectives, such as consensus, leader-follower synchronization, formation, stabilization and rendezvous (see [1], [4]–[11] and the references therein). In a *descriptor multi-agent system* (DMAS), the dynamics of an agent is represented by a *descriptor model*, which consists of a set of differential and algebraic equations. Some of the physical systems, which are represented by descriptor model, are: power network [12], robotic manipulators [13], biological systems [14], and cyber-physical systems [15]. Such systems, with descriptor model, are often referred to as *descriptor systems*. The state or output response of a descriptor system makes it different from an ordinary state-space system. For instance, a class of descriptor systems, having (nilpotent) index greater or equal to two, show impulsive behavior in its response due to the presence of inconsistent initial conditions and/or non-smoothness of (control) input [13], [16]–[18]. One can see this behaviour in the following circuit example.

Example 1: The dynamic behavior of an electric circuit, shown in Figure 1-A, can be described by the following equations: $\frac{dv_c}{dt} = \frac{1}{C} i_c,\ v_c = v$, which can be represented in the following descriptor model (considering i_c as output):

$$\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}\dot{i}_c\\ \dot{v}_c\end{bmatrix} = \begin{bmatrix}\frac{1}{C}&0\\0&1\end{bmatrix}\begin{bmatrix}i_c\\ v_c\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix} v,\ y = [1\ \ 0]\begin{bmatrix}i_c\\ v_c\end{bmatrix}. \quad (1)$$

Since the output $y(t) = i_c(t) = C\dot{v}(t)$, one may expect impulse in the response if the input $v(t)$ in Figure 1-A, is piece-wise continuous. Now, assume that $v(t)$, in Figure 1-A, is a constant voltage source, that is, $v(t) = V$ for $t \geq 0$, and $v(t) = 0$ for $t < 0$. Let the switch S be moved from position X to Y at $t = T > 0$, as in Figure 1-B. Then, following set of equations: $v_c = 0$ and $i_c = C\dot{v}_c$, are obtained from (1) (by setting $v = 0$). Solving them: $y(t) = i_c(t) = -Cv_{co}\delta(t)$, where $\delta(t)$ is an impulse function and v_{co} is the voltage in the capacitor just before the switching action ($t = T^-$). Hence, $y(t)$ contains impulse, which is due to the inconsistent initial condition, that is, $v_c(T^-) = v_{co} > 0$, whereas $v_c(T^+) = 0$ (since $v_c = 0$) for $t \geq T$. For a consistent initial condition, that is, $v_c(T^-) = v_c(T^+) = 0$, there will be no impulse in the output response, and $y(t) = i_c(t) = 0$.

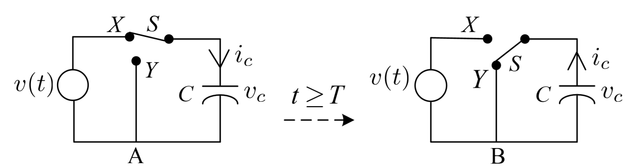

Fig. 1. In this circuit, $v(t)$ denotes the voltage source, v_c and i_c are the capacitor (C) voltage and current, respectively. S is the switch.

In a DMAS, since the agents are descriptor systems, one may expect impulse in the state response. The impulsive behaviour is undesirable in the applications, since it can destroy or saturate the physical components (such as sensors, in the feedback control action). Hence, it is important to eliminate impulse from the response of a DMAS. With the above observations, in this work, we propose a methodology to achieve the following objectives in a closed loop DMAS: i) the state response is impulse-free, and ii) the states of the follower agents synchronize with the states of a leader agent. To achieve these objectives, we propose a *distributed static state feedback control* for the DMAS, and develop algorithms to compute the underlying feedback gain matrices. The closed loop DMAS is first made impulse-free using a feedback gain matrix, and then, it is transformed into a set of decoupled descriptor systems using spectral decomposition of a network associated matrix. By performing appropriate decompositions of the transformed descriptor systems, the synchronization objective is achieved by stabilizing a set of ordinary state space systems via solving an Algebraic Riccati Equation (ARE). Since we have used only orthogonal

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matrices for system transformations and design of feedback gain matrices, the proposed algorithm is numerically stable.

In the existing literature on DMAS, broadly, the following objectives are considered: i) consensus [19]–[22], ii) bipartite consensus [23] and bipartite containment problem [24]. The bipartite consensus problem is addressed in [23] by considering the cooperative and antagonistic interaction among the agents. In [24], observer-based bipartite containment problem is addressed where follower agents converge to the convex hull spanned by multiple leaders. In [21], the consensus protocol is designed in the presence of disturbances to the agents. In [22], a combination of event-triggered control and impulsive control is proposed to achieve consensus.

Notations: $\otimes$: Kronecker product, det($\bullet$): determinant of a matrix, diag$\{\bullet\}$: a diagonal matrix, blkdiag$\{\bullet\}$: a block diagonal matrix, I_r: an identity matrix of size $(r \times r)$.

II. Mathematical Preliminaries

A. Preliminaries on Descriptor Systems

Consider a descriptor system of the following form:

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

with initial condition $x(0) = x_0 \in \mathbb{R}^n$, where $E \in \mathbb{R}^{n\times n}$ is a singular matrix, $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^p$ are the state and input vectors, respectively. The system (2) is said to be *regular* if there exists a complex number $s \in \mathbb{C}$ such that $\det(sE - A) \neq 0$. For a regular descriptor system, the roots of the polynomial $\det(sE-A)$ are referred to as *finite poles* of (2) or the pair (E, A). The set of finite poles of (2) is denoted as $\sigma(E, A)$. For a regular descriptor system (2), there exist two non-singular matrices $U \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{n\times n}$ such that $UEV = \left[\begin{smallmatrix} I_{n_1} & 0 \\ 0 & N \end{smallmatrix}\right]$, $UAV = \left[\begin{smallmatrix} J & 0 \\ 0 & I_{n_2} \end{smallmatrix}\right]$, $UB = \left[\begin{smallmatrix} B_1 \\ B_2 \end{smallmatrix}\right]$, where $n_1 + n_2 = n$, and $N \in \mathbb{R}^{n_2\times n_2}$ is a Nilpotent matrix with index α ($N^\alpha = 0$ and $N^{\alpha-1} \neq 0$) [17]. By introducing a new state variable $\bar{x} = V^{-1}x$ ($\bar{x} = \left[\bar{x}_1^T \quad \bar{x}_2^T\right]^T$), and using the above decomposition, following two subsystems are obtained from (2): $\dot{\bar{x}}_1(t) = J\bar{x}_1(t) + B_1u(t)$ and $N\dot{\bar{x}}_2(t) = \bar{x}_2(t) + B_2u(t)$. The solution of the first subsystem, for initial condition $\bar{x}_{10} := [I_{n_1}\ 0]V^{-1}x_0$, is: $\bar{x}_1(t) = e^{Jt}\bar{x}_{10} + \int_0^t e^{J(t-\tau)}B_1u(\tau)d\tau$. The solution of second subsystem, for initial condition $\bar{x}_{20} := [0\ I_{n_2}]V^{-1}x_0$, is: $\bar{x}_2(t) = -\sum_{i=1}^{\alpha-1}\delta^{(i-1)}(t)N^i\bar{x}_{20} - \sum_{i=0}^{\alpha-1}N^iB_2u^{(i)}(t)$, where $\delta(t)$ is the Dirac delta function, $u^{(i)}(t)$ is the i^{th} derivative of $u(t)$ and $N^0 = I_{n_2}$. Then, the state response (distributional solution) of system (2) is [17, Chapter 1]:

$$x(t) = V\left[\begin{smallmatrix} I_{n_1} \\ 0 \end{smallmatrix}\right]\bar{x}_1(t) + V\left[\begin{smallmatrix} 0 \\ I_{n_2} \end{smallmatrix}\right]\bar{x}_2(t). \tag{3}$$

If $N \neq 0$, the solution $\bar{x}_2(t)$ contains impulse terms (with its distribution derivatives [17, Appendix A]), which arise due to the presence of initial condition ($\bar{x}_{20}$) and/or non-smoothness of input $u(t)$. For a set of *consistent initial condition* x_{co} of (2), which is defined as follows [17]:

$$\mathcal{X}_{co} := \Big\{x_{co} \in \mathbb{R}^n \mid x_{co} = V\left[\begin{smallmatrix} I_{n_1} \\ 0 \end{smallmatrix}\right]\bar{x}_{10} - V\left[\begin{smallmatrix} 0 \\ I_{n_2} \end{smallmatrix}\right]\sum_{i=0}^{\alpha-1}N^iB_2u^{(i)}(0),\ \forall\, x_0 \in \mathbb{R}^n\Big\}, \tag{4}$$

the distributional solution (3) becomes the classical solution of (2), where the first term of $\bar{x}_2(t)$ will not appear in (3), and the other terms will remain same.

B. Preliminaries on Graph theory

An undirected graph $G_c(\mathcal{V}, \mathcal{E})$ consists of a set of vertices: $\mathcal{V} := \{v_0, v_1, \cdots, v_r\}$ and a set of undirected edges $\mathcal{E}$ between the vertices. The graph $G_c(\mathcal{V}, \mathcal{E})$ is said to be connected, if there exists a path between each pair of vertices in $\mathcal{V}$. A sub-graph $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ of $G_c(\mathcal{V}, \mathcal{E})$ is obtained by removing the vertex v_0 and the associated edges from $G_c(\mathcal{V}, \mathcal{E})$. For instance, a graph $G_c(\mathcal{V}, \mathcal{E})$ and its corresponding subgraph $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ are shown in Fig. 2(a) and Fig. 2(b), respectively. Two vertices v_i and v_j in $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ are said to be *adjacent* if an edge exists between them. For a graph $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$, we associate an *adjacency matrix* $\mathcal{A}(G) \in \mathbb{R}^{r\times r}$, whose $(i, j)^{th} = (j, i)^{th}$ element is 1, if v_i and v_j are adjacent; otherwise it is zero. The diagonal elements of $\mathcal{A}(G)$ are 0 by assuming that there are no self-loops in $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$. The neighborhood set of vertex v_i is defined as follows: $\mathcal{N}(i) := \{j \neq i \mid v_i$ and v_j are adjacent in $G_c(\mathcal{V}, \mathcal{E})\}$. The degree d_i of vertex v_i is the cardinality of $\mathcal{N}(i)$. The *degree matrix* and *Laplacian matrix* of $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ are defined as: $\mathcal{D}(G) = \text{diag}\{d_1, d_2, \cdots, d_r\}$ and $\mathcal{L}(G) = \mathcal{D}(G) - \mathcal{A}(G)$, respectively.

For a given matrix $H \in \mathbb{R}^{r\times r}$, we associate a directed graph $G_m(H)$ on the set of r vertices: $\{v_1, v_2, \cdots, v_r\}$, where v_i represents a column or row of H, whereas an edge, having direction from vertex v_j to vertex v_i, represents a non-zero entry h_{ij} of H. A *directed path*, from vertex v_k to v_l, in $G_m(H)$ is a sequence of vertices, where the following conditions hold: i) no edge enters at v_k, ii) no edge leaves from v_l and iii) only one edge enters and one edge leaves from the intermediate vertices. For instance, a digraph for matrix $H = \left[\begin{smallmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{smallmatrix}\right]$ is represented in Fig. 2 (c). The digraph $G_m(H)$ is said to be *strongly connected*, if there is a directed path between every two vertices in $G_m(H)$. A matrix H has the property of *strongly connected* if and only if the associated digraph $G_m(H)$ is strongly connected [25].

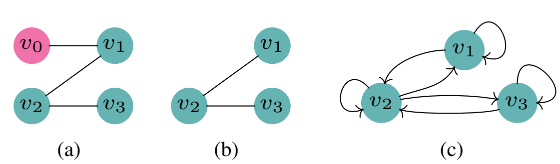

Fig. 2. (a) Graph $G_c(\mathcal{V}, \mathcal{E})$ on four vertices, (b) a subgraph of $G_c(\mathcal{V}, \mathcal{E})$, (c) the digraph $G_m(H)$ corresponding to matrix H.

III. Main Results

Consider a DMAS system, which consists of a group of $r+1$ identical descriptor systems with dynamics as follows:

$$E\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \{0, 1, \cdots, r\}, \tag{5}$$

with initial condition $x_i(0) = x_0 \in \mathbb{R}^n$, where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^p$ are the state and input vectors of the i^{th}

agent. Out of the $r+1$ number of agents in (5), let us consider one of the agents as *leader*, which is represented as

$$E\dot{x}_0(t) = Ax_0(t) + Bu_0(t), \tag{6}$$

and remaining r agents as *follower*, represented as:

$$E\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad \text{for } i \in \{1, 2, \cdots, r\}. \tag{7}$$

For all $i \in \{0, 1, \cdots, r\}$, we consider the following assumptions on (5): i) E is singular and rank$(E) = n_0 < n$, ii) the pair (E, A) is regular, and iii) (E, A, B) is C-controllable (I-controllable and R-controllable) [13, Chapter 4].

By defining vectors: $\mathbf{x} := [x_0^T \ x_1^T \ \cdots \ x_r^T]^T$ and $\mathbf{u} = [u_0^T \ u_1^T \ \cdots \ u_r^T]^T$, the dynamics (5) can be represented as:

$$(I_{r+1} \otimes E)\,\dot{\mathbf{x}} = (I_{r+1} \otimes A)\,\mathbf{x} + (I_{r+1} \otimes B)\,\mathbf{u}. \tag{8}$$

We assume that the information exchange between agents is bidirectional. Then, the considered DMAS is represented by an undirected graph $G_c(\mathcal{V}, \mathcal{E})$, where the agents and the communication links between the agents are represented by the vertices and edges of $G_c(\mathcal{V}, \mathcal{E})$, respectively. Two agents i and j in the network are said to be *neighbor*, if v_i and v_j are adjacent in $G_c(\mathcal{V}, \mathcal{E})$. By denoting the set of neighborhood agents of i^{th} agent as $\mathcal{N}(i)$, define a signal $\zeta_i(t)$ as follows:

$$\zeta_i(t) = \sum_{j \in \mathcal{N}(i)} [x_i(t) - x_j(t)]\,, \ \forall\, i \in \{1, 2, \cdots, r\}. \tag{9}$$

Then, we propose the following control protocol for each agent $i \in \{0, 1, \cdots, r\}$:

$$u_i(t) = \bar{u}_i(t) + \tilde{u}_i(t), \tag{10}$$

where the control signals $\bar{u}_i(t)$ and $\tilde{u}_i(t)$ are:

$$\bar{u}_i(t) = Kx_i(t), \qquad \forall\, i \in \{0, 1, \cdots, r\}, \tag{11a}$$

$$\tilde{u}_0(t) = 0, \ \tilde{u}_i(t) = \eta \widetilde{K} \zeta_i(t), \qquad \forall\, i \in \{1, 2, \cdots, r\}, \tag{11b}$$

where $K \in \mathbb{R}^{p \times n}$ and $\widetilde{K} \in \mathbb{R}^{p \times n}$ are the feedback gain matrices, and $\eta \in \mathbb{R}$ is a scalar gain that need to be designed. According to the control law proposed in (11a), each agent utilizes only the local state information of the agents in the network, and control law in (11b) uses relative neighborhood state information. Then, we are interested in addressing the following problem.

Problem 1: Design the feedback gain matrices K, $\widetilde{K}$, and scalar gain η such that the closed-loop DMAS is impulse-free, and the states of all the follower agents asymptotically synchronize with the states of leader agent, that is, $\lim_{t \to \infty}(x_i - x_0) = 0$ for all $i \in \{1, 2, \cdots, r\}$.

We first design K to make the closed-loop system impulse-free, and then $\widetilde{K}$ for synchronization. Defining vectors: $\bar{\mathbf{u}} = [\bar{u}_0^T \ \bar{u}_1^T \ \cdots \ \bar{u}_r^T]^T$ and $\tilde{\mathbf{u}} = [\tilde{u}_0^T \ \tilde{u}_1^T \ \cdots \ \tilde{u}_r^T]^T$, we have the following relation, which follows from (10) and (11a):

$$\mathbf{u} = \bar{\mathbf{u}} + \tilde{\mathbf{u}} = (I_{r+1} \otimes K)\mathbf{x} + \tilde{\mathbf{u}}. \tag{12}$$

Then, by defining the matrix $A_c := I_{r+1} \otimes (A + BK)$, and using (12) in (8), the closed loop DMAS is:

$$(I_{r+1} \otimes E)\,\dot{\mathbf{x}} = A_c \mathbf{x} + (I_{r+1} \otimes B)\tilde{\mathbf{u}}. \tag{13}$$

Since the matrix E is singular, there exists two orthogonal matrices W_1 and W_2 such that E has the following singular value decomposition (SVD):

$$\bar{E} := W_1 E W_2 = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, \tag{14}$$

where Σ is a diagonal matrix with diagonal entries are the non-zero singular values of E. Using W_1 and W_2, let:

$$\bar{A} := W_1 A W_2 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \ \bar{B} := W_1 B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$
$$\bar{K} := K W_2 = \begin{bmatrix} K_1 & K_2 \end{bmatrix}, \tag{15}$$

where A_{ij}, B_i, and K_i are the block matrices, and their sizes are conformal with $\left[\begin{smallmatrix} \Sigma & 0 \\ 0 & 0 \end{smallmatrix}\right]$. Note that the agents dynamics (5) could be impulsive. Then, it follows from [13, Theorem 7.3] that the matrix A_{22} in (15) is a singular matrix (it is non-singular, when (5) are impulse-free). Consider another two orthogonal matrices T_1 and T_2 such that the SVD of A_{22} is:

$$T_1 A_{22} T_2 = \begin{bmatrix} \Sigma_a & 0 \\ 0 & 0 \end{bmatrix}, \tag{16}$$

where Σ_a is a diagonal matrix, and its entries are the non-zero singular values of A_{22}. Using T_1 and T_2, let the matrices B_2 and K_2 in (15) are decomposed as:

$$T_1 B_2 = \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix}, \ K_2 T_2 = \begin{bmatrix} K_{21} & K_{22} \end{bmatrix}. \tag{17}$$

Then, we have the following result.

Theorem 1: For a given DMAS with agent dynamics (5) and feedback control (10), let the associated system matrices be decomposed as in (14), (15), (16) and (17). Let K_{21} in (17) be chosen as: $K_{21} = 0$. Then, the closed-loop system (13) is impulse free if and only if: $\det(B_{22} K_{22}) \neq 0$.

Proof: Consider the orthogonal matrices W_1 and W_2, used in (14), and define the following matrices: $\mathbf{W_1} = \left[\begin{smallmatrix} I_r \otimes W_1 & 0 \\ 0 & I_r \otimes W_1 \end{smallmatrix}\right]$ and $\mathbf{W_2} = \left[\begin{smallmatrix} I_r \otimes W_2 & 0 \\ 0 & I_r \otimes W_2 \end{smallmatrix}\right]$. Then, the following relation holds:

$$\begin{aligned} &\text{rank} \begin{bmatrix} I_{r+1} \otimes E & 0 \\ A_c & I_{r+1} \otimes E \end{bmatrix} \\ &= \text{rank } \mathbf{W_1} \begin{bmatrix} I_{r+1} \otimes E & 0 \\ I_{r+1} \otimes (A + BK) & I_{r+1} \otimes E \end{bmatrix} \mathbf{W_2} \\ &= \text{rank } \begin{bmatrix} I_{r+1} \otimes \bar{E} & 0 \\ I_{r+1} \otimes (\bar{A} + \bar{B}\bar{K}) & I_{r+1} \otimes \bar{E} \end{bmatrix}. \end{aligned} \tag{18}$$

Denote $\bar{E}_c = I_{r+1} \otimes \bar{E}$, $\bar{A}_c = I_{r+1} \otimes (\bar{A} + \bar{B}\bar{K})$, and:

$$\bar{\Sigma} = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{A}_{c_r} = \begin{bmatrix} A_{11} + B_1 K_1 & A_{12} + B_1 K_2 \\ A_{21} + B_2 K_1 & A_{22} + B_2 K_2 \end{bmatrix}.$$

Then, using the relations (14) and (15), we can write the following matrices: $\bar{E}_c = \text{blkdiag}\{\bar{\Sigma}, \bar{\Sigma}, \cdots, \bar{\Sigma}\}$ and $\bar{A}_c = \text{blkdiag}\{\bar{A}_{c_r}, \bar{A}_{c_r}, \cdots, \bar{A}_{c_r}\}$. Recall that Σ is a diagonal matrix with n_0 positive diagonal entries. Hence, it follows from (18) that:

$$\begin{aligned} &\text{rank} \begin{bmatrix} I_{r+1} \otimes E & 0 \\ A_c & I_{r+1} \otimes E \end{bmatrix} \\ &= (r+1)\left(\text{rank}(\Sigma) + \text{rank}(\Sigma) + \text{rank}(A_{22} + B_2 K_2)\right) \\ &= (r+1)(n_0 + n_0 + \text{rank}(A_{22} + B_2 K_2)). \end{aligned}$$

Since $A_{22}+B_2K_2$ is a matrix of size $(n-n_0)\times(n-n_0)$, it then follows that rank $\begin{bmatrix} I_{r+1}\otimes E & 0 \\ A_c & I_{r+1}\otimes E \end{bmatrix} = (r+1)(n+n_0)$ if and only if rank$(A_{22}+B_2K_2) = n-n_0$. Hence, according to [13, Theorem 7.2], the closed-loop system (13) is impulse-free if and only if $A_{22}+B_2K_2$ is non-singular. Now, using the decompositions in (16) and (17), and since $K_{21}=0$, we have: $T_1\left(A_{22}+B_2K_2\right)T_2 = \begin{bmatrix} \Sigma_a & B_{12}K_{22} \\ 0 & B_{22}K_{22} \end{bmatrix}$. Further, since Σ_a is invertible, $A_{22}+B_2K_2$ is non-singular if and only if $B_{22}K_{22}$ is non-singular, hence, the result holds. Since it is assumed that the agent dynamics (5) is impulse controllable, one can always find K_{22} such that $B_{22}K_{22}$ is non-singular, which follows from [13, Theorem 7.6] and its proof. ■

Note that Theorem 1 gives a condition to design the gain matrix K such that the closed loop system (13) become impulse-free. We will now design $\tilde{\mathbf{u}}$ for (13) such that the state synchronization objective can be achieved. Define a matrix $\mathcal{L}_g := \mathcal{L}(G)+\mathcal{M}$, where $\mathcal{M} = \text{diag}\{m_1, m_2, \cdots, m_r\}$, and $m_i = 1$ if the vertex v_0 is connected to v_i, for $i \in \{1, 2, \cdots, r\}$, otherwise it is zero. Further, define the vectors: $\zeta := [\zeta_1^T \ \zeta_2^T \ \cdots \ \zeta_r^T]^T$, $\mathbf{x}_f = [x_1^T \ x_2^T \ \cdots x_r^T]^T$, $\mathbf{x}_0 = [x_0^T \ x_0^T \ \cdots x_0^T]^T$, and $\tilde{\mathbf{u}}_f = [\tilde{u}_1^T \ \tilde{u}_2^T \ \cdots \ \tilde{u}_r^T]^T$. Then, we obtain the following relation from (9):

$$\zeta = \left(\mathcal{L}_g \otimes I_n\right)\left(\mathbf{x}_f - \mathbf{x}_0\right), \tag{19}$$

and the following relation from (11b) and (19):

$$\tilde{\mathbf{u}}_f = \eta(I_r \otimes \widetilde{K})\zeta = \eta(\mathcal{L}_g \otimes \widetilde{K})(\mathbf{x}_f - \mathbf{x}_0). \tag{20}$$

Since we have assumed $\tilde{u}_0(t) = 0$ in (11b), following dynamics are obtained from (13), by denoting $A_R = A + BK$:

$$(I_r \otimes E)\dot{\mathbf{x}}_0 = (I_r \otimes A_R)\mathbf{x}_0, \tag{21}$$

$$(I_r \otimes E)\dot{\mathbf{x}}_f = (I_r \otimes A_R)\mathbf{x}_f + (I_r \otimes B)\tilde{\mathbf{u}}_f. \tag{22}$$

Define an error vector: $\xi := \mathbf{x}_f - \mathbf{x}_0$. Then, by using (20) in (22), we have:

$$\begin{aligned}(I_r \otimes E)\dot{\mathbf{x}}_f &= (I_r \otimes A_R)\mathbf{x}_f + \eta(I_r \otimes B)(\mathcal{L}_g \otimes \widetilde{K})\xi \\ &= (I_r \otimes A_R)\mathbf{x}_f + \eta(\mathcal{L}_g \otimes B\widetilde{K})\xi. \end{aligned} \tag{23}$$

Pre-multiplying $(I_r \otimes E)$ to the both sides of $\xi = \mathbf{x}_f - \mathbf{x}_0$, and then, taking the derivative, following relation is obtained:

$$(I_r \otimes E)\dot{\xi} = (I_r \otimes E)\dot{\mathbf{x}}_f - (I_r \otimes E)\dot{\mathbf{x}}_0. \tag{24}$$

Then, using (21) and (23) in (24), we have the following synchronization error dynamics:

$$(I_r \otimes E)\dot{\xi} = \left[(I_r \otimes A_R) + \eta(\mathcal{L}_g \otimes B\widetilde{K})\right]\xi. \tag{25}$$

Define $E_\xi := (I_r \otimes E)$, $A_\xi := (I_r \otimes A_R) + \eta(\mathcal{L}_g \otimes B\widetilde{K})$. Then we have the following result.

Proposition 1: Let $\tilde{\lambda}_i$ be an eigenvalue of $\mathcal{L}_g$. Then, the set of finite poles of pair (E_ξ, A_ξ) is equal to the union of finite poles of pairs $(E, A_R + \eta\tilde{\lambda}_i B\widetilde{K})$, for $i = 1, 2, \cdots, r$.

Proof: Since $\mathcal{L}_g$ is symmetric, there exists an orthogonal matrix S $(S^TS = I_r)$ such that $S^T\mathcal{L}_gS = \mathcal{L}_\Lambda$, where $\mathcal{L}_\Lambda = \text{diag}\{\tilde{\lambda}_1, \tilde{\lambda}_2, \cdots, \tilde{\lambda}_r\}$ [25]. By defining: $\psi(t) := (S^T \otimes I_n)\xi(t)$, the error dynamics in (25) can be transformed to: $(I_r \otimes E)\dot{\psi}(t) = \left[(I_r \otimes A_R) + \eta(\mathcal{L}_\Lambda \otimes B\widetilde{K})\right]\psi(t)$, which produces following set of descriptor systems:

$$E\dot{\psi}_i(t) = (A_R + \eta\tilde{\lambda}_i B\widetilde{K})\psi_i(t), \text{ for } i \in \{1, 2, \cdots, r\}. \tag{26}$$

Since the error dynamics (25) and the transformed error dynamics are equivalent, they have the same set of finite poles, and hence, the proposition holds. ■

Proposition 2: The matrix $\mathcal{L}_g = \mathcal{L}(G) + \mathcal{M}$ is non-singular, and hence, $\tilde{\lambda}_i \neq 0$, for all $i \in \{1, 2, \cdots r\}$.

Proof: Let $G_m(\mathcal{L}_g)$ be the digraph, associated with matrix $\mathcal{L}_g$ (refer to Section II for construction of such digraph). Notice that the digraph $G_m(\mathcal{L}_g)$ and the undirected network graph $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ have an equal number of vertices. Moreover, one can obtain $G_m(\mathcal{L}_g)$ from $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ by replacing an undirected edge of $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ with two directed edges, having opposite directions, in $G_m(\mathcal{L}_g)$. For instance, if there is an undirected edge between vertices v_i and v_j in $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$, then there are two directed edges between v_i and v_j in $G_m(\mathcal{L}_g)$: i) one is from vertex v_i to v_j and ii) another is from vertex v_j to v_i. It is then easy to notice that the directed graph $G_m(\mathcal{L}_g)$ is strongly connected if and only if the undirected graph $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ is connected. Since it is assumed that $G(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ is connected, the digraph $G_m(\mathcal{L}_g)$ is strongly connected, and hence, the matrix $\mathcal{L}_g$ has strongly connected property [25]. Further, $\mathcal{L}_g$ is a diagonally dominant matrix, that is, for all $j \in \{1, 2, \cdots r\}$, $|\bar{l}_{jj}| \geq \sum_{k=1}^{r}|\bar{l}_{jk}|$ $(k \neq j)$, where $\bar{l}_{jk}$ is an element of $\mathcal{L}_g$. Further, for i^{th} element of $\mathcal{L}_g$, following relation holds: $|\bar{l}_{ii}| > \sum_{k=1}^{r}|\bar{l}_{ik}|$ $(k \neq i)$, which is due to the presence of $m_i \neq 0$ in the i^{th} row of $\mathcal{L}_g$. Hence, it directly follows from [25, Corollary 6.2.9] that $\mathcal{L}_g$ is non-singular. ■

Note that the gain matrix K, designed according to Theorem 1, also ensures that the pair (E, A_R) is impulse-free. Hence, there exists an orthogonal matrix W_1 (consider the decomposition in (14)) such that: $W_1E = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}$, $W_1A_R = \begin{bmatrix} A_{R_1} \\ A_{R_2} \end{bmatrix}$, $W_1B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, where $E_1 \in \mathbb{R}^{n_0 \times n}$, $A_{R_1} \in \mathbb{R}^{n_0 \times n}$, $A_{R_2} \in \mathbb{R}^{(n-n_0)\times n}$, $B_1 \in \mathbb{R}^{n_0 \times p}$ and $B_2 \in \mathbb{R}^{(n-n_0)\times p}$. Since the pair (E, A_R) is impulse-free, the matrices E_1 and A_{R_1} are full rank matrices [26]. Further, consider another two orthogonal matrices P and Q such that the matrix $\begin{bmatrix} E_1 \\ 0 \end{bmatrix}$ has the following SVD: $\begin{bmatrix} E_1 \\ 0 \end{bmatrix} = Q\begin{bmatrix} \Sigma_{n_0} & 0 \\ 0 & 0 \end{bmatrix}P^T$. Let the orthogonal matrices Q and P be partitioned as follows: $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$ and $P = \begin{bmatrix} P_1 & P_2 \end{bmatrix}$, respectively. Then, by defining the following matrices:

$$\begin{aligned} A_{z_{11}} &:= \Sigma_{n_0}^{-1}Q_{11}^T\left(A_{R_1}P_1 - A_{R_1}P_2(A_{R_2}P_2)^{-1}A_{R_2}P_1\right), \\ B_{z_1} &:= \Sigma_{n_0}^{-1}Q_{11}^T\left(B_1 - A_{R_1}P_2(A_{R_2}P_2)^{-1}B_2\right), \\ \widetilde{K} &:= \begin{bmatrix} \widetilde{K}_1 & 0 \end{bmatrix}P^T, \end{aligned} \tag{27}$$

where $A_{z_{11}} \in \mathbb{R}^{n_0 \times n_0}$, $B_{z_1} \in \mathbb{R}^{n_0 \times p}$ and $\tilde{K}_1 \in \mathbb{R}^{p \times n_0}$, we have the following results.

Lemma 1: For a set of descriptor systems as in (26), we have: $\bigcup_{i=1}^{r} \sigma(E, A_R + \eta\tilde{\lambda}_i B\widetilde{K}) = \bigcup_{i=1}^{r} \Lambda(A_{z_{11}} + \eta\tilde{\lambda}_i B_{z_1}\widetilde{K}_1)$, where $\Lambda(.)$ refers to the set of eigenvalues.

Proof: Similar to the proof of [26, Theorem 3]. ■

It can be observed from Proposition 1 that the finite pole assignment problem of (25) is transformed into the finite pole assignment problem of a set of descriptor systems (26), which is then transformed to the eigenvalues assignment problem of a set of ordinary state space systems through Lemma 1. We will now give a procedure to design gain matrix $\widetilde{K}_1$ and scalar gain η, through the following result, such that eigenvalues of the set of matrices: $A_{z_{11}} + \eta\tilde{\lambda}_i B_{z_1}\widetilde{K}_1$, for $i \in \{1, 2, \cdots, r\}$, belong to the open left half of complex plane. Furthermore, we will show that the impulse-freeness property of the closed loop system will not get affected by the implementation of $\widetilde{K}_1$ and η in the network.

Theorem 2: Let $A_{z_{11}}$, B_{z_1} and $\widetilde{K}$ be defined as in (27). Let $\widetilde{K}_1 = -R_z^{-1}B_{z_1}^T P_z$, where P_z is the solution of ARE: $A_{z_{11}}^T P_z + P_z A_{z_{11}} - P_z B_{z_1} R_z^{-1} B_{z_1}^T P_z + Q_z = 0$, for some given symmetric positive definite matrices R_z and Q_z. Further, let the gain η be chosen as: $\eta \geq \frac{1}{2\tilde{\lambda}_1}$, where $\tilde{\lambda}_1$ is the smallest eigenvalue of $\mathcal{L}_g$. Then, the following statements hold: i) $\lim_{t\to\infty}(x_i - x_0) = 0$ for all $i \in \{1, 2, \cdots, r\}$, and ii) the closed loop DMAS is impulse-free.

Proof: According to the choice of: $\widetilde{K}_1 = -R_z^{-1}B_{z_1}^T P_z$ and $\eta \geq \frac{1}{2\tilde{\lambda}_1}$, it directly follows from [27, Theorem 1] that: $\Lambda(A_{z_{11}} + \eta\tilde{\lambda}_i B_{z_1}\widetilde{K}_1)$, for $i \in \{1, 2, \cdots, r\}$, belong to the open left half of complex plane. Hence, according to Lemma 1 and Proposition 1, all the finite poles of synchronization error dynamics (25) also belong to the open left half of complex plane. Hence, it follows that $\lim_{t\to\infty}(\mathbf{x}_f - \mathbf{x}_0) = 0$, since $\xi = \mathbf{x}_f - \mathbf{x}_0$. This implies: $\lim_{t\to\infty}(x_i - x_0) = 0$, for all $i \in \{1, 2, \cdots, r\}$. Since we have assumed that each agent is C-controllable, the pair $(A_{z_{11}}, B_{z_1})$ is controllable [26], and hence, one can always obtain $\widetilde{K}_1$.

Let us define a vector $\chi = \begin{bmatrix}\mathbf{x}_0^T & \mathbf{x}_f^T\end{bmatrix}^T$. Then, using (21) and (23), the overall dynamics of closed-loop system is:

$$E_{cl}\dot{\chi} = A_{cl}\chi, \tag{28}$$

where the matrix: $E_{cl} = \begin{bmatrix} I_r\otimes E & 0 \\ 0 & I_r\otimes E \end{bmatrix}$ and the matrix $A_{cl} = \begin{bmatrix} I_r\otimes A_R & 0 \\ -\eta(\mathcal{L}_g\otimes B\widetilde{K}) & (I_r\otimes A_R)+\eta(\mathcal{L}_g\otimes B\widetilde{K}) \end{bmatrix}$. Consider the orthogonal matrix S such that $S^T\mathcal{L}_g S = \mathcal{L}_\Lambda$, and define a new matrix: $\bar{S} = \text{blkdiag}\{S\otimes I_n, S\otimes I_n, S\otimes I_n, S\otimes I_n\}$. Then, using the definitions of E_{cl} and A_{cl}, and pre-multiplication of $\bar{S}^T$ and post-multiplication of $\bar{S}$ to the matrix $\begin{bmatrix} E_{cl} & 0 \\ A_{cl} & E_{cl} \end{bmatrix}$ yield following relation:

$$\begin{aligned}
&\text{rank}\begin{bmatrix} E_{cl} & 0 \\ A_{cl} & E_{cl} \end{bmatrix} \\
&= \text{rank}\begin{bmatrix} I_r\otimes E & 0 & 0 & 0 \\ 0 & I_r\otimes E & 0 & 0 \\ I_r\otimes A_R & 0 & I_r\otimes E & 0 \\ -\eta(\mathcal{L}_\Lambda\otimes B\widetilde{K}) & (I_r\otimes A_R)+\eta(\mathcal{L}_\Lambda\otimes B\widetilde{K}) & 0 & I_r\otimes E \end{bmatrix} \\
&= r\ \text{rank}\begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ A_R & 0 & E & 0 \\ -\eta\tilde{\lambda}_i B\widetilde{K} & A_R+\eta\tilde{\lambda}_i B\widetilde{K} & 0 & E \end{bmatrix}.
\end{aligned} \tag{29}$$

Now, consider the orthogonal matrices W_1 and W_2 as in (14), and define new matrices: $\bar{W}_1 = \text{blkdiag}\{W_1, W_1, W_1, W_1\}$, $\bar{W}_2 = \text{blkdiag}\{W_2, W_2, W_2, W_2\}$. Then, by pre-multiplying $\bar{W}_1$ and post-multiplying $\bar{W}_2$ to the matrix that appears in the right hand side of (29) produce the following relation:

$$\begin{aligned}
&\text{rank}\begin{bmatrix} E_{cl} & 0 \\ A_{cl} & E_{cl} \end{bmatrix} \\
&= r\ \text{rank}\begin{bmatrix} \Sigma & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & \Sigma & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ A_{11}+B_1K_1 & A_{12}+B_1K_2 & 0 & 0 & \Sigma\ 0\ 0\ 0 \\ A_{21}+B_2K_1 & A_{22}+B_2K_2 & 0 & 0 & 0\ 0\ 0\ 0 \\ -\eta\tilde{\lambda}_i B_{K_{11}} & -\eta\tilde{\lambda}_i B_{K_{12}} & A_{K_{11}} & A_{K_{12}} & 0\ 0\ \Sigma\ 0 \\ -\eta\tilde{\lambda}_i B_{K_{21}} & -\eta\tilde{\lambda}_i B_{K_{22}} & A_{K_{21}} & A_{K_{22}} & 0\ 0\ 0\ 0 \end{bmatrix} \\
&= r\ \text{rank}\begin{bmatrix} \Sigma & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & \Sigma & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & \Sigma\ 0\ 0\ 0 \\ 0 & A_{22}+B_2K_2 & 0 & 0 & 0\ 0\ 0\ 0 \\ 0 & 0 & 0 & 0 & 0\ 0\ \Sigma\ 0 \\ 0 & 0 & 0 & A_{K_{22}} & 0\ 0\ 0\ 0 \end{bmatrix} \\
&= r(4n_0 + \text{rank}(A_{22} + B_2K_2) + \text{rank}(A_{K_{22}}).
\end{aligned} \tag{30}$$

By using the definition of $\widetilde{K}$ as in (27), and writing $P^T W_2 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, following matrix definitions are used in (30):

$$\begin{aligned}
B_{K_{11}} &= B_1\widetilde{K}_1 Z_{11}, A_{K_{11}} = A_{11} + B_1K_1 + \eta\tilde{\lambda}_i B_1\widetilde{K}_1 Z_{11}, \\
B_{K_{12}} &= B_1\widetilde{K}_1 Z_{12}, A_{K_{12}} = A_{12} + B_1K_2 + \eta\tilde{\lambda}_i B_1\widetilde{K}_1 Z_{12}, \\
B_{K_{21}} &= B_2\widetilde{K}_1 Z_{11}, A_{K_{21}} = A_{21} + B_2K_1 + \eta\tilde{\lambda}_i B_2\widetilde{K}_1 Z_{11}, \\
B_{K_{22}} &= B_2\widetilde{K}_1 Z_{12}, A_{K_{22}} = A_{22} + B_2K_2 + \eta\tilde{\lambda}_i B_2\widetilde{K}_1 Z_{12}.
\end{aligned}$$

Since the pair (E, A_R) is impulse-free (refer to Theorem 1), we have: $\text{rank}(A_{22} + B_2K_2) = n - n_0$. Further, the gain matrix $\widetilde{K}$ and scalar gain η are designed to assign n_0 number of finite poles of the descriptor systems (26), and hence, they are impulse-free. This implies: $\text{rank}(A_{K_{22}}) = n - n_0$. Hence, it follows from (30) that $\text{rank}\begin{bmatrix} E_{cl} & 0 \\ A_{cl} & E_{cl} \end{bmatrix} = r(4n_0 + (n - n_0) + (n - n_0)) = 2r(n + n_0)$, and hence, the second statement also holds according to [13, Theorem 7.2]. This completes the proof. ■

In the next section, we present a numerical example to show the applicability of the proposed control algorithm.

IV. Numerical Example

Example 2: In this example, we consider a DMAS, consisting of four agents, where the dynamics of each agent is of the form (5) with system matrices as follows:

$$E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -5 & 0 & 0 \\ 0 & 3 & 1 & 3 \\ 0 & 1 & 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

The communication topology among the agents is as per the undirected graph $G_c(\mathcal{V}, \mathcal{E})$, shown in Fig. 2(a). In the considered system matrices, we have: $n = 4$ and $n_0 = 2$. Further, the number of follower agents is $r = 3$. Then, considering the open loop DMAS (8), we have: $\text{rank}\begin{bmatrix} I_4\otimes E & 0 \\ I_4\otimes A & I_4\otimes E \end{bmatrix} = 20$, which is strictly less than $(r + 1)(n + n_0) = 24$. Hence, it follows from [13, Theorem 7.2] that the response of open-loop DMAS is impulsive. To make the closed-loop DMAS (13) impulse-free, we have designed the following gain matrix, according to Theorem 1: $K = \begin{bmatrix} -2 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 \end{bmatrix}$. We now proceed to design the feedback gain matrix $\widetilde{K}$ and scalar gain η to achieve synchronization among the leader and follower agents in the network. According to Theorem 2 and using the relations in (27), we computed the following gain matrix: $\widetilde{K} = \begin{bmatrix} 0 & 0.1333 & 0 & -0.4328 \\ 0 & -0.8055 & 0 & 0.1415 \end{bmatrix}$. For the considered communication topology, the eigenvalue of $\mathcal{L}_g$ are: $\tilde{\lambda}_1 = 0.1981$, $\tilde{\lambda}_2 =$

1.555 and $\tilde{\lambda}_3 = 3.247$. Hence, the scalar gain $\eta \geq \frac{1}{2\tilde{\lambda}_1}$ is chosen as 6. To verify the synchronization of leader-follower network, the closed loop DMAS (21) and (23) is simulated in MATLAB (Version R2022a) using *ode15s* solver for a set of consistent initial condition. Since the pair (E, A_R) is impulse free, the set of consistent initial conditions for leader agent is chosen from the following set, which is obtained from (4): $\mathcal{X}_{co} := \left\{x_0(0) \mid x_0(0) = V \left[\begin{smallmatrix} I_{n_1} \\ 0 \end{smallmatrix}\right] \kappa, \ \forall\ \kappa \in \mathbb{R}^{n_1}\right\}$. Further, since the pair $(E, A_R + \eta\tilde{\lambda}_i BK)$ is also impulse-free in (26), we obtained $\psi_i(0)$, for all $i \in \mathcal{M}_1$, similar to $x_0(0)$. By constructing $\mathbf{x}_0(0) = [x_0(0)^T \cdots x_0(0)^T]^T$ and $\psi(0) = [\psi_1(0)^T \cdots \psi_r(0)^T]^T$, we obtained the consistent initial conditions for the follower agents as follows: $\mathbf{x}_f(0) = \xi(0) + \mathbf{x}_0(0)$, where $\xi(0) = (S \otimes I_n)\psi(0)$. The simulation results are presented in Figure 3, which shows that the states of all the follower agents synchronize with the states of leader agent.

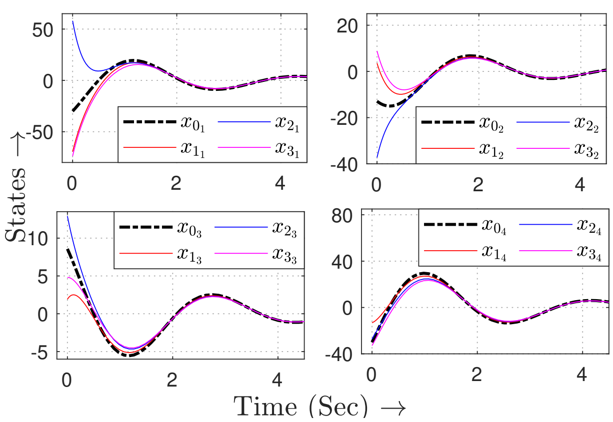

Fig. 3. Synchronization of follower states x_{i_j} (solid lines) with leader state x_{0_j} (dotted lines).

V. Conclusion

A distributed static state feedback control strategy has been proposed for eliminating the impulse response and achieving the synchronization of the leader-follower network of a DMAS. The underlying communication network among the agents is considered of fixed topology where the local controllers for the agents need state information of their neighbors and self-state information of agents in the network. We have obtained satisfactory results from the proposed algorithm while verifying with a numerical example.

References

[1] W. Ren and R. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*. Communications and Control Engineering, Springer London, 2008.

[2] A. Bidram, F. L. Lewis, and A. Davoudi, "Distributed control systems for small-scale power networks: Using multiagent cooperative control theory," *IEEE Control Systems Magazine*, vol. 34, no. 6, pp. 56–77, 2014.

[3] X. Dong, B. Yu, Z. Shi, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles: Theories and applications," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 340–348, 2015.

[4] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, 2004.

[5] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2010.

[6] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[7] K. K. Oh, M. C. Park, and H. S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[8] J. Wang, J. Qiao, G. Wen, Z. Duan, and T. Huang, "Rendezvous of heterogeneous multiagent systems with nonuniform time-varying information delays: An adaptive approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 8, pp. 4848–4857, 2021.

[9] L. Lin, J. Can, S. Zhuhai, and P. She, "Minimum-time and minimum-triggering impulsive stabilization for multi-agent systems over finite fields," *Systems & Control Letters*, vol. 155, no. 104991, pp. 1–12, 2021.

[10] S. Datta, "Well-posedness, internal stability and input-output stability in networked multi-agent systems," *IEEE Control Systems Letters*, vol. 6, pp. 1543–1548, 2022.

[11] S. Datta, "Consensus in multi-agent systems: A transfer function-based controller design approach," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 11, pp. 4384–4388, 2022.

[12] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Upper Saddle River, NJ: Prentice Hall, 1998.

[13] G. R. Duan, *Analysis and Design of Descriptor Linear Systems*. New York, NY: Springer-Verlag, 2010.

[14] P. Liu, Q. Zhang, X. Yang, and L. Yang, "Passivity and optimal control of descriptor biological complex systems," *IEEE Transactions on Automatic Control*, vol. 53, no. Special Issue, pp. 122–125, 2008.

[15] F. Pasqualetti, F. Dorfler, and F. Bullo, "Attack detection and identification in cyber-physical systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 11, pp. 2715–2729, 2013.

[16] G. C. Verghese, B. C. Lévy, and T. Kailath, "A generalized state-space for singular systems," *IEEE Transactions on Automatic Control*, vol. 26, no. 4, pp. 811–831, 1981.

[17] L. Dai, *Singular Control Systems*. Berlin, Germany: Springer-Verlag, 1989.

[18] P. Kunkel and V. Mehrmann, *Differential-Algebraic Equations, Analysis and Numerical Solution*. Zürich, Switzerland: European Mathematical Society Publishing House, 2006.

[19] X. Yang and G. Liu, "Necessary and sufficient consensus conditions of descriptor multi-agent systems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 11, p. 2669–2677, 2012.

[20] X. Yang and G. Liu, "Consensus of descriptor multi-agent systems via dynamic compensators," *IET Control Theory Applications*, vol. 8, no. 6, pp. 389–398, 2014.

[21] X. Liu, Y. Xie, F. Li, and W. Gui, "Admissible consensus for homogenous descriptor multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 2, pp. 965–974, 2021.

[22] X. Lu and H. Li, "Consensus of singular linear multiagent systems via hybrid control," *IEEE Transactions on Control of Network Systems*, vol. 9, no. 2, pp. 647–656, 2022.

[23] L. Zhang and G. Zhang, "Bipartite consensus for descriptor multiagent systems with antagonistic interactions," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 11, pp. 2602–2606, 2020.

[24] Z.-H. Zhu, B. Hu, Z.-H. Guan, D.-X. Zhang, and T. Li, "Observer-based bipartite containment control for singular multi-agent systems over signed digraphs," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 1, pp. 444–457, 2021.

[25] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[26] S. Datta, "Feedback controller norm optimization for linear time invariant descriptor systems with pole region constraint," *IEEE Transactions on Automatic Control*, vol. 62, no. 6, pp. 2794–2806, 2017.

[27] H. Zhang, F. L. Lewis, and A. Das, "Optimal design for synchronization of cooperative systems: State feedback, observer and output feedback," *IEEE Transactions on Automatic Control*, vol. 56, no. 8, pp. 1948–1952, 2011.