2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Distributed leader-followers constrained platooning control of linear homogeneous vehicles

Ramzi Gaagai, Mattia Giaccagli and Joachim Horn

Abstract— **This paper presents a feedback-distributed controller that guarantees exponential synchronization of a vehicle platoon described by linear dynamics. To achieve desired inter-vehicle spacing between two vehicles and to improve platoon cohesiveness, a leader-based bidirectional communication scheme is employed. To take into account the constraints of the system and to avoid collision between vehicles, safety features are implemented using control barrier functions via linear quadratic programming. String stability properties of the platoon are analyzed, where we show via simulations that the proposed design allows considering a gap-spacing policy for inter-vehicle distance with zero headway. To conclude, the effectiveness of the proposed controller is verified in a simulation study.**

I. Introduction

Highway capacity limits traffic systems, resulting in traffic jams. Cooperative Adaptive Cruise Control (CACC) known as platooning is deemed a promising solution to improve traffic safety, reduce fuel consumption and vehicles emission due to reduced air resistance, as well as increase traffic throughput and road capacity [11], [20]. A platoon is a group of closely spaced vehicles that drive safely and automatically by regulating the inter-vehicle distance to a desired value and employing wireless communication in addition to onboard sensors [13]. Vehicle platoons have been an area of focus for researchers and industry for decades [19], [13]. While field experiments and real-world implementations of the technology are still rare, research becomes more refined.

Due to the distributed nature of the problem, the majority of existing solutions make use of multi-agent synchronization tools, with a focus on string stability. Existing controllers usually rely on inter-vehicle distance error feedback designs. In [13], a controller for homogeneous platoons that guarantees string stability was derived using Predecessor-Following (PF) topology. The controller was tested and evaluated on a platoon of passenger cars. Leveraging the controller presented in [13], the authors of [25] presented a bidirectional distributed consensus controller for homogeneous platoons, showing that bidirectional interaction can potentially improve platoon cohesiveness. The approach was further extended in [26] to consider velocity constraints. If the former works mainly focused on vehicles described by linear dynamical models, string stability of nonlinear systems has been studied more recently for instance in [12] with L_∞ tools for heterogeneous platoons, or in [17] with integral control. At the same time, recent works started to consider more real-life oriented scenarios, by taking into consideration in the control design the presence of disturbances [12], [21], or delays in the communication lines, see e.g. [3].

In this work, we consider a linear platoon of homogeneous vehicles and we present a distributed-feedback control law for CACC. In particular, we consider a leader-connected network, where all the vehicles' communication links are bidirectional, with an exception for the first "virtual" leading vehicle. We first propose a full-state feedback distributed control law coupling the vehicles. In order to take into account input and state constraints while assuring safety requirements, we rely on the notion of Control Barrier Functions (CBFs), see e.g. [1], [14], [7], implemented by means of an online optimization problem via Linear Quadratic (LQ) programming. To the authors' knowledge, such a tool has been employed in platooning for example in [15] with a PD controller and recently in [8] with a two-layer controller. Differently for the former designs, however, we rely on a full state-feedback design rather than an error-feedback one. We show via simulations, that such a difference allows achieving string stability of the platoon while considering a gap-spacing policy for inter-vehicle distance with (possibly) a *zero headway*, contrarily from many literature results where a minimum strictly positive bound is required.

The paper is structured as follows: In section II the platoon model is presented and the challenges tackled in this paper are laid out. Section III contains some preliminaries on the main tools used. Section IV describes the proposed control design. In Section V, some simulations to illustrate the performance of the proposed approach are shown. Conclusions can be found in Section VI.

Notation: $\mathbb{N}$ is the set of (non-negative) natural numbers. $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}_{>0} = (0, +\infty)$ and $\mathbb{R}_{\geq 0} := [0, +\infty)$. We write $A = [a_{ik}]$ to indicate that A is a matrix where a_{ik} is the element in row i and column k. For $b_i \in \mathbb{R}$ with $i = \{1, \ldots, n\}$, $\text{diag}\{b_i\}$ is a $n \times n$ diagonal matrix with b_i on the main diagonal and zeros elsewhere. $||x||_2$ is the Euclidean norm of $x \in \mathbb{R}^n$. For $f : \mathbb{R}^n \mapsto \mathbb{R}^n, g : \mathbb{R}^n \mapsto \mathbb{R}^n$ and $h : \mathbb{R}^n \mapsto \mathbb{R}$ sufficiently smooth, we let $L_f h(x) = \frac{\partial h}{\partial x}(x) f(x)$ and $L_g L_f h(x) = \frac{\partial (L_f h)}{\partial x}(x) g(x)$. Given $r \in \mathbb{N}$, we let $L_f^r h(x) = \frac{\partial (L_f^{r-1} h)}{\partial x}(x) f(x)$ with $L_f^0 h(x) = h(x)$.

II. Problem Setting and Preliminaries

A. Single vehicle model

In this work, we consider an homogeneous platoon of M vehicles as shown in Fig. 1. We indicate with v_i the velocity and with p_i the position of vehicle V_i, where

The research leading to these results is partially funded by the Israel Science Foundation grant ISF 3621/21 joint with the National Science Foundation of China

$i = \{1, \dots, M\}$. Let a_i be the acceleration of vehicle V_i. Following [18], [16], each vehicle is modeled as a linear system of the form

$$\dot{x}_i = Ax_i + Bu_i\,, \tag{1}$$

where $x_i = \begin{pmatrix} p_i & v_i & a_i \end{pmatrix}^\top \in \mathbb{R}^3$ is the state of vehicle V_i, $u_i \in \mathbb{R}$ is its input and where

$$A := \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{pmatrix}, \qquad B := \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{pmatrix} \tag{2}$$

with $\tau < 1$ being the time constant of the engine. We take the initial conditions of the system such that $p_i(0) > p_{i+1}(0)$.

B. Platoon interconnection

Interconnection between vehicles is modeled using graph theory. In a general framework, the information exchange between vehicles in the platoon is represented by a directed graph (digraph) $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{V_0, V_1, V_2, ..., V_M\}$ is a set of nodes (vehicles) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges representing the information exchange between vehicles. The edge set $\mathcal{E}$ can be described by an adjacency matrix $\bar{A} = [\bar{a}_{ik}]$, with weights $\bar{a}_{ik} = 1$ if $(V_k, V_i) \in \mathcal{E}$ and $\bar{a}_{ik} = 0$ otherwise. Define the weighted in-degree of vehicle V_i as the i-th row sum of $\bar{A}$, that is, $d_i = \sum_{k=1}^{M+1} \bar{a}_{ik}$ The diagonal in-degree matrix is defined as $D = \mathrm{diag}\{d_i\}$. The Laplacian matrix is defined as the difference between the in-degree matrix D and the adjacency matrix $\bar{A}$, i.e. $\mathcal{L} = D - \bar{A} = [\ell_{ik}]$. In particular, in this work, we consider a leader-followers communication topology with bidirectional links. Vehicle V_i is allowed to collect and share information with both its preceding vehicle V_{i-1} and the following one V_{i+1}, as shown in Fig. 1. The only exception is V_0, which is a "virtual" vehicle (i.e. it is not a real vehicle but part of the algorithm) that shares the same dynamical model of V_i but *does not* receives any information from V_1, i.e. it acts as a *leader* of the network. Thus, the Laplacian $\mathcal{L} \in \mathbb{R}^{(M+1)\times(M+1)}$ is

$$\mathcal{L} := \begin{pmatrix} 0 & 0 \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -1 & 2 & -1 \\ 0 & \dots & 0 & -1 & 1 \end{pmatrix} \tag{3}$$

with $\mathcal{L}_{22} \in \mathbb{R}^{M\times M}$ and $\mathcal{L}_{21} \in \mathbb{R}^M$. Such an interconnection topology is summarized in the following.

Assumption 1. *The graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is leader connected with V_0 being the leader, i.e. it contains a spanning three with V_0 as the root. Moreover, the Laplacian $\mathcal{L}$ is defined as in* (3).

The choice of the communication graph has two main reasons. On one hand, bidirectional links have been considered to provide additional safety and avoid collisions [25]. On the other hand, a leader-connected topology is considered in order to improve performances. The role of the virtual leading vehicle is only to follow the desired reference profile, "imposing" the steady-state behavior of the platoon, while all the rest of the vehicles have to follow it. By considering such a single-leader network, the heading vehicle is not affected by the rest of the platoon, but rather it acts as an autonomous system.

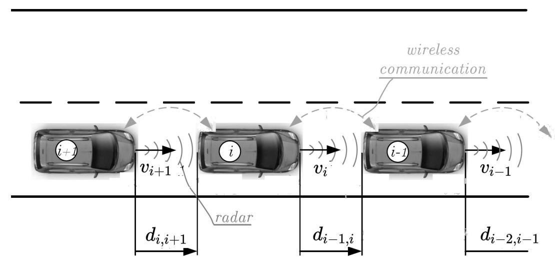

Fig. 1. CACC-equipped homogeneous platoon of vehicles with bidirectional communication (edited from [24]).

C. The control problem

In this work, we aim to solve a reference tracking platooning control, namely, the state of each of the vehicles has to follow a desired reference profile $r^\star(t) = \begin{pmatrix} p^\star(t) & v^\star(t) & a^\star(t) \end{pmatrix}^\top$, while maintaining a spacing distance in between neighbor vehicles. We consider a gap spacing policy for the desired inter-vehicle distance defined as

$$d_i^\star\,(v_i(t)) = r + hv_i(t)\,, \tag{4}$$

where r is the constant distance at standstill, and h is the constant headway. Therefore, the inter-vehicle spacing error e_i is defined as:

$$e_i = (p_{i-1} - p_i - L) - (r + hv_i) \tag{5}$$

with L being the length of each vehicle.

To make the problem more challenging, we consider a scenario in which constraints in the vehicle actuators and in its state are taken into account. In other words, we assume that, for all i, there exist $\underline{u}_i, \overline{u}_i, \underline{a}_i, \overline{a}_i, \underline{v}_i, \overline{v}_i \in \mathbb{R}$ such that, for all i and for all t, the control problem has to be solved, while satisfying

$$\underline{u}_i \le u_i(t) \le \overline{u}_i\,,\ \underline{a}_i \le a_i(t) \le \overline{a}_i\,,\ \underline{v}_i \le v_i(t) \le \overline{v}_i\,. \tag{6}$$

In order to assure the feasibility of the problem, we take a reference profile that satisfies the bounds (6) and the desired spacing distance with respect to the system's dynamics. The problem is to ensure that, for any constant cruise speed, the inter-vehicle distance errors asymptotically tend to zero, while satisfying the constraints and avoiding collision between vehicles.

III. Preliminaries

A. Synchronization of linear leader-connected networks

In this section we briefly recall the main tools that will be used in the following. In particular, how to design a state-feedback distributed controller achieving leader-synchronization for a network of linear systems is a well-known result. A general overlook of the problem and of the tools can be found for instance in [10, Section V]. A standard assumption for networks of linear systems is the graph to be *connected* in the sense of [10, Definition 5.1] (some authors also call this "weak connectivity"), namely the fact that there exists one node $V^\star$ in the network such that there exists a path from $V^\star$ to any other node or that, equivalently, the Laplacian matrix has all the non-zero eigenvalue with strictly positive real part. In our leader-followers framework, such a condition holds through the leader-connectivity Assumption 1 where $V^\star = V_0$. From [10], [5], the following holds.

Proposition 1. *Consider a homogeneous network whose agents' dynamics is defined by* (1) *and let Assumption 1 hold. If there exists a symmetric positive matrix* $P = P^\top > 0$ *and* $\rho, \lambda \in \mathbb{R}_{>0}$ *such that*

$$PA + A^\top P - \rho PBB^\top P \leq -2\lambda P\,, \tag{7}$$

then there exists $\kappa^\star \in \mathbb{R}_{>0}$ *such that, for any* $\kappa \geq \kappa^\star$*, there exists* $k \in \mathbb{R}_{>0}$ *such that the network in closed loop with* $u_i = -\kappa \sum_{j=1}^{M+1} \ell_{(i+1)j} B^\top P x_{j-1}$ *satisfies* $|x_i(t) - x_j(t)| \leq k|x_i(0) - x_j(0)| \exp(-\lambda t)$ *for all initial conditions* $x_i(0), x_j(0) \in \mathbb{R}^n$*, for all* $t \geq 0$*, and all* $i, j \in \{1, \dots, M\}$*.*

Remark 1. We recall that for the Riccati inequality (7) to be solvable is equivalent to ask for the pair (A, B) to be stabilizable. The resulting distributed control law is of infinite gain margin [6], where the value of $\kappa^\star$ depends on the constant ρ in (7) and on the smallest nonzero eigenvalue of the Laplacian matrix $\mathcal{L}$, while the overshoot gain k depends on the ratio between the biggest and smallest eigenvalues of the matrix P.

B. Control barrier functions

In many applications, to have the state of the system and the control input to be confined within pre-defined bounds, is usually a necessary requirement. This is asked in order to keep into account the physical limitations of actuators and of the system's properties. In this context, Control Barrier Functions (CBFs) (see for instance [1], [14], [7], [23], [22] and references therein) have been found to be an efficient tool. We recall in this section their main properties. Consider a system of the form

$$\dot{x} = f(x) + g(x)u \tag{8}$$

with $x \in \mathbb{R}^{n_x}, u \in \mathbb{R}$ and f, g to be sufficiently smooth of suitable dimension. Let $q : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ be a C^1 function and let $\mathbf{r} \in \mathbb{N}$. We say that the function $y = q(x)$ has a *relative degree*[1] $\mathbf{r}$ with respect to system (8) and input u in the sense of [9, Section 4] if i) $L_g L_f^k q(x) = 0$ for all $k < \mathbf{r} - 1$ and ii) $L_g L_f^{\mathbf{r}-1} q(x) \neq 0$ for all $x \in \mathbb{R}^{n_x}$. Consider now the set

$$\mathcal{C} := \{x \in \mathbb{R}^{n_x} : q(x) \geq 0\}\ . \tag{9}$$

In the following, we assume $\mathcal{C}$ to be non-empty and with no isolated points (i.e. $\mathcal{C}$ coincides with its interior). We have the following definition.

[1]For the sake of simplicity, in the following we will always assume the relative degree to be globally defined.

Definition 1. Consider the system (8) and consider a function $q : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ with relative degree $\mathbf{r}$. We say that q is a Control Barrier Function (CBF) if there exists a column vector $b = \begin{pmatrix} b_1 & \dots & b_r \end{pmatrix}^\top \in \mathbb{R}^r$ such that

$$\sup_{u \in \mathbb{R}} \left[L_g L_f^{\mathbf{r}-1} q(x) u + L_f^{\mathbf{r}} q(x) + b^\top \omega(x) \right] \geq 0 \quad \forall\, x \in \mathbb{R}^{n_x}\,, \tag{10}$$

where $\omega(x) := \begin{pmatrix} q(x) & L_f q(x) & \dots & L_f^{\mathbf{r}-1} q(x) \end{pmatrix}^\top \in \mathbb{R}^r$, and the roots of the polynomial $\mathcal{P}(\epsilon) = \epsilon^{\mathbf{r}} + b_{\mathbf{r}} \epsilon^{\mathbf{r}-1} + \dots + b_2 \epsilon + b_1$ are all negative.

In such a framework, q represents a function describing the constraints to be satisfied while $\mathcal{C}$ in (9) is the set of x satisfying such constraints. Suppose that the aim is to design a control law $u = u^C := \alpha(x)$ that solves a certain control task while, at the same time, is such that if $x(0) \in \mathcal{C}$, then $x(t) \in \mathcal{C}$ for all $t \geq 0$. Define $\mathcal{K}$ as the set of Lipschitz continuous functions $\alpha : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ such that

$$\mathcal{K} := \left\{ \alpha \mid L_g L_f^{\mathbf{r}-1} q(x) \alpha(x) + L_f^{\mathbf{r}} q(x) + b^\top \omega(x) \geq 0 \right\}\ . \tag{11}$$

The following result holds (see [22, Theorem 1]).

Proposition 2. *Consider system* (8) *and let* $q : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ *be a CBF of relative degree* $\mathbf{r} \in \mathbb{N}$*. Then for any* $u = \alpha(x)$ *with* $\alpha \in \mathcal{K}$*, there exists a subset* $\underline{\mathcal{C}} \subseteq \mathcal{C}$ *such that* $\underline{\mathcal{C}}$ *is asymptotically stable for the closed-loop system.*

Similarly to [1], [14], the implementation of CBFs can be performed through Linear Quadratic (LQ) programming by means of an optimization problem. Indeed, for a known control action $u^D = \bar{\alpha}(x)$ solving a certain control task, system (8) can be fed with the control $u = u^C$ where

$$u^C := \mathrm{argmin}_{\tilde{u}} \left\| \tilde{u} - u^D \right\|_2 \tag{12a}$$

$$\text{s.t. } \sigma(x)\tilde{u} \leq \beta(x) \tag{12b}$$

where $\sigma(x) = -L_g L_f^{\mathbf{r}-1} q(x)$ and $\beta(x) = L_f^{\mathbf{r}} q(x) + b^\top \omega(x)$. In such a way, the control input that is fed to the system is $u = u^C = u^D = \bar{\alpha}(x)$ whenever $\bar{\alpha} \in \mathcal{K}$ while, in case $\bar{\alpha} \notin \mathcal{K}$, then $u = u^C$ is the control action that minimizes the Euclidean norm (12a) while satisfying (12b), that is, while making the set $\mathcal{C}$ asymptotically stable for the closed-loop.

Remark 2. The optimization problem (12) takes into account a single constraint since the function q in (9) maps in $\mathbb{R}$. In case $m > 1$ constraints have to be taken into account, the problem can be formulated following similar lines, by treating each constraint individually. In such a case, σ, β become mapping in $\mathbb{R}^m$.

IV. Proposed control design

In this section, we show the proposed control design. We first split the analysis into the leader control action (whose role is to follow the desired reference profile) and the followers one (whose role is to synchronize to the leader). In short, each vehicle control law (besides the leading vehicle) is composed of two main blocks. The first block is a distributed control law that follows the design proposed in Section III-A. Vehicle V_i embeds in its control action the knowledge of the preceding and of the following vehicle state. The role of this block is to synchronize the state of all the vehicles. The resulting control u_i^D enters the second main block, which corresponds to an online optimization algorithm. The role of this second block is to guarantee the feasibility of the constraints while assuring safety requirements. This is done by relying on CBFs as in Subsection III-B, implemented via LQ programming. An overview of the resulting control action can be found in Fig. 2, where Δp_i and Δv_i are the inter-vehicle distance and the relative velocity between vehicle V_i and vehicle V_{i-1} and where Δp_{i+1} and Δv_{i+1} are the inter-vehicle distance and the relative velocity between vehicle V_{i+1} and vehicle V_i defined as

$$\begin{aligned} \begin{pmatrix} \Delta p_i \\ \Delta v_i \end{pmatrix} &:= \begin{pmatrix} p_{i-1} - p_i - L \\ v_{i-1} - v_i \end{pmatrix}, \\ \begin{pmatrix} \Delta p_{i+1} \\ \Delta v_{i+1} \end{pmatrix} &:= \begin{pmatrix} p_i - p_{i+1} - L \\ v_i - v_{i+1} \end{pmatrix}. \end{aligned} \tag{13}$$

A. The leader (nominal) control action

Consider the virtual leading vehicle V_0 with state x_0. The virtual leader shares the same linear dynamics of the vehicles of the platoon and receives an exogenous input to track a reference profile $r^\star(t)$. The chosen control law for the leader is a proportional controller of the form

$$u_0 = \bar{K}^\top \left(r^\star(t) - x_0 \right) \tag{14}$$

with $\bar{K}$ being a matrix such that $A - B\bar{K}^\top$ is stable. Note indeed that the pair (A, B) is controllable.

B. The followers (nominal) control action

The control law for the vehicles V_i with $i = 1, \dots, M$ is done following Proposition 1. Inequality (7) can be solved with computational tools such as Linear Matrix Inequalities by noticing that the pair (A, B) is controllable and therefore stabilizable. Nonetheless, in the following we provide an analytical solution that is dependent only on the constant τ. While such a choice may seem too conservative in some sense, it allows us to pursue a more detailed string stability analysis as shown in the following section. In particular, we select P, ρ, λ solving (7) as $\rho = 1$, $\lambda = \frac{1}{2}$ and matrix $P = P^\top > 0$ given by

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{pmatrix}$$

where in particular, we selected

$$\begin{aligned} P_{11} &= -\frac{(3\tau^3 - 13\tau^2 + 18\tau - 8)^2}{\tau^3(5\tau - 6)}, \\ P_{12} &= \frac{(\tau - 2)^2(6\tau^3 - 23\tau^2 + 29\tau - 12)}{\tau^2(5\tau - 6)}, \\ P_{13} &= -\frac{(\tau - 2)^2(3\tau^2 - 7\tau + 4)}{\tau(5\tau - 6)}, \\ P_{22} &= -\frac{(\tau - 2)^2(7\tau^2 - 20\tau + 14)}{\tau(5\tau - 6)}, \\ P_{23} &= (\tau - 2)^2, \quad P_{33} = -\tau^2 + 2\tau. \end{aligned}$$

where the positivity of P can be verified recalling that $\tau < 1$. Therefore, following Proposition 1, the distributed synchronization-based control law is given by :

$$u_i^D = -\kappa \sum_{j=1}^{M+1} \ell_{(i+1)j} K x_{j-1} \tag{15}$$

with

$$K = \begin{pmatrix} -\frac{(\tau-2)^2(3\tau^2-7\tau+4)}{\tau^2(5\tau-6)} & \frac{(\tau-2)^2}{\tau} & (2-\tau) \end{pmatrix}.$$

C. Implementation of CBFs

In the CBFs' implementation, we keep in consideration the constraints (6) related to input u_i, acceleration a_i, velocity v_i, and position p_i. Since the vehicle platoon is homogeneous, we have that both the upper and lower bounds in (6) for all the state and input variables do not depend on the index i, i.e. each vehicle has to satisfy the same constraints. For this reason, in the following, we drop the i-subscript when referring to the constants in (6). Each vehicle implements an online optimization problem of the form (12) as described in Section III-B, where the vector fields f, g in (8) are defined as $f(x_i) = Ax_i, g(x_i) = B$ with A, B given in (2). We define $e_i = (p_{i-1} - p_i - L) - (r + hv_i)$ as the distance error between vehicle V_i and its predecessor V_{i-1}. Moreover, the x-variable in (12) will be referred as $x = (x_{i-1}, x_i)$ since the constraints of each vehicle are related to its own input, acceleration, velocity, and position error with respect to the preceding vehicle. We treat each constraint individually (see Remark 2). Thus we split $\beta(x)$ in (12) as $\beta(x) = \beta(x_{i-1}, x_i) = \begin{pmatrix}(\beta^u(x)^\top & (\beta^a(x))^\top & (\beta^v(x))^\top & (\beta^d(x))^\top\end{pmatrix}^\top$, to indicate that β^u is related to the constraints on the input, β^a on the acceleration, β^v on the velocity and β^d on the distance between consecutive vehicles. Similarly, we split σ as $\sigma(x) = \begin{pmatrix}(\sigma^u)^\top & (\sigma^a)^\top & (\sigma^v)^\top & (\sigma^d)^\top\end{pmatrix}^\top$. The constraints will be represented by the function $q(x) := \begin{pmatrix}\overline{q}_u & \underline{q}_u & \overline{q}_a & \underline{q}_a & \overline{q}_v & \underline{q}_v & q_p\end{pmatrix}^\top$.

Actuator constraints – The nominal controller u_i of the i^{th}-vehicle V_i must not exceed the feasible range (6). Therefore, we have that $\sigma^u = \begin{pmatrix}1 & 1\end{pmatrix}^\top$ and $\beta^u = \begin{pmatrix}\underline{u} & \overline{u}\end{pmatrix}^\top$

Acceleration constraints – We define $\overline{q}_a(x_i) := \overline{a} - a_i$. Since $L_g\overline{q}_a(x_i) \neq 0$, it implies that acceleration constraints have unitary relative degree. Thus, inequality (10) for the (maximum) acceleration constraint with $q = \overline{q}_a$ translates as

$$\frac{1}{\tau}\tilde{u}_i \leq \frac{1}{\tau}a_i + b_1^{\bar{a}}\overline{q}_a(x_i) \tag{16}$$

where $b_1^{\bar{a}} \in \mathbb{R}_{>0}$. Similarly, about the minimum acceleration constraint, we can define $\underline{q}_a(x_i) = a_i - \underline{a}$ and follow similar steps to get $\beta^a(x) = \begin{pmatrix}\frac{1}{\tau}a_i + b_1^{\bar{a}}\overline{q}_a(x_i) & -\frac{1}{\tau}a_i + b_1^{\underline{a}}\underline{q}_a(x_i)\end{pmatrix}^\top$ and $\sigma^a = \begin{pmatrix}\frac{1}{\tau} & -\frac{1}{\tau}\end{pmatrix}^\top$, where $b_1^{\underline{a}} \in \mathbb{R}_{>0}$.

Velocity Constraints – Define $\overline{q}_v(x_i) = \overline{v} - v_i$. The velocity constraints have relative degree of two since $L_g\overline{q}_v(x_i) = 0$ but $L_gL_f\overline{q}_v = -\frac{1}{\tau}$. Therefore, the maximum velocity constraint in (10) is given by:

$$\frac{1}{\tau}\tilde{u}_i \leq \frac{1}{\tau}a_i + (B^{\bar{v}})^\top \begin{pmatrix} \overline{v} - v_i \\ -a_i \end{pmatrix} \tag{17}$$

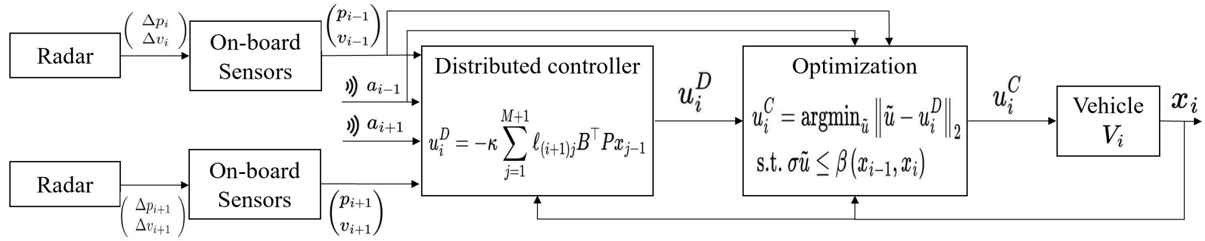

Fig. 2. Resulting control block-scheme

where $B^{\bar{v}} = \begin{pmatrix} b_1^{\bar{v}} & b_2^{\bar{v}} \end{pmatrix}^\top \in \mathbb{R}^2$ is chosen such that the roots of $\epsilon^2 + b_2^{\bar{v}}\epsilon + b_1^{\bar{v}}$ are negative definite. Similarly, the minimum velocity constraint is given by $\underline{q}_v(x_i) = v_i - \underline{v}$. Therefore, following similar steps, we define $\beta^v(x_i) = \left(\frac{1}{\tau}a_i + (B^{\bar{v}})^\top \begin{pmatrix} \bar{v} - v_i \\ -a_i \end{pmatrix} \quad -\frac{1}{\tau}a_i + (B^{\underline{v}})^\top \begin{pmatrix} v_i - \underline{v} \\ a_i \end{pmatrix} \right)^\top$ and $\sigma^v = \begin{pmatrix} \frac{1}{\tau} & -\frac{1}{\tau} \end{pmatrix}^\top$. Where $B^{\underline{v}} = \begin{pmatrix} b_1^{\underline{v}} & b_2^{\underline{v}} \end{pmatrix}^\top \in \mathbb{R}^2$ satisfies similar properties as its maximum velocity counterpart.

Spacing Constraints – We define the spacing constraint in terms of the error (5) as:

$$q_p(x_{i-1}, x_i) = e_i = p_{i-1} - p_i - L - r - hv_i \tag{18}$$

The spacing constraints have relative degree of two since $L_g q_p(x_i) = 0$ but $L_g L_f q_p(x_i) = -\frac{h}{\tau}$. Therefore, the spacing constraint in (10) is given by:

$$\frac{h}{\tau}\tilde{u}_i \leq a_{i-1} - \left(\frac{h}{\tau} + 1\right) a_i + (B^p)^\top \begin{pmatrix} p_{i-1} - p_i - L - r - hv_i \\ v_{i-1} - v_i - ha_i \end{pmatrix} \tag{19}$$

where $B^p = \begin{pmatrix} b_1^p & b_2^p \end{pmatrix}^\top \in \mathbb{R}^2$ is such that the polynomial $\epsilon^2 + b_2^p\epsilon + b_1^p$ has negative roots. Therefore, we define $\beta^d(x_{i-1}, x_i) = a_{i-1} - (\frac{h}{\tau} + 1)a_i + (B^p)^\top \begin{pmatrix} p_{i-1} - p_i - L - r - hv_i \\ v_{i-1} - v_i - ha_i \end{pmatrix}$ and $\sigma^d = \frac{h}{\tau}$. Note that we only have one spacing constraint i.e. each vehicle tries not to collide with its predecessor.

D. String stability notion

String stability refers to the attenuation of oscillations in upstream direction [4]. If the platoon is string unstable, any disturbance from the leader is amplified along the string of vehicles. For platoons with bidirectional communication, a notion of string stability is proposed in [2].

In order to analyze string stability of the homogeneous platoon, the closed-loop platoon dynamics are derived. To simplify the analysis, we consider the reference signal to impose only a desired acceleration profile $r^\star(t) = \begin{pmatrix} p^\star(t) & v^\star(t) & a^\star(t) \end{pmatrix}$, with leader control law (14) having gain matrix partitioned as $\bar{K} = \begin{pmatrix} 0 & 0 & \bar{K}_3 \end{pmatrix}$. For $1 \leq i < M$, that is, all vehicles in the platoon excluding the virtual leader and the last vehicle, the closed-loop dynamics (without the optimization algorithm) can be described by:

$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \\ \dot{a}_i \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2\phi(\tau,\kappa) & \frac{-2\kappa(\tau-2)^2}{\tau^2} & \frac{-1-2\kappa(2-\tau)}{\tau} \end{bmatrix}}_{F_0} \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\phi(\tau,\kappa) & \frac{\kappa(\tau-2)^2}{\tau^2} & \frac{\kappa(2-\tau)}{\tau} \end{bmatrix}}_{F_1} \begin{bmatrix} p_{i-1} \\ v_{i-1} \\ a_{i-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\phi(\tau,\kappa) & \frac{\kappa(\tau-2)^2}{\tau^2} & \frac{\kappa(2-\tau)}{\tau} \end{bmatrix} \begin{bmatrix} p_{i+1} \\ v_{i+1} \\ a_{i+1} \end{bmatrix} \tag{20}$$

with

$$\phi(\tau,\kappa) = \frac{\kappa(\tau-2)(3\tau^2 - 7\tau + 4)}{\tau^3(5\tau - 6)}. \tag{21}$$

The dynamics of the last vehicle ($i = M$) are described by:

$$\begin{bmatrix} \dot{p}_M \\ \dot{v}_M \\ \dot{a}_M \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \phi(\tau,\kappa) & \frac{-\kappa(\tau-2)^2}{\tau^2} & \frac{-1-\kappa(2-\tau)}{\tau} \end{bmatrix}}_{E_0} \begin{bmatrix} p_M \\ v_M \\ a_M \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\phi(\tau) & \frac{\kappa(\tau-2)^2}{\tau^2} & \frac{\kappa(2-\tau)}{\tau} \end{bmatrix} \begin{bmatrix} p_{M-1} \\ v_{M-1} \\ a_{M-1} \end{bmatrix} \tag{22}$$

Finally, the dynamics of the virtual leading vehicle are written as:

$$\begin{bmatrix} \dot{p}_0 \\ \dot{v}_0 \\ \dot{a}_0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-1-\bar{K}_3}{\tau} \end{bmatrix}}_{W_0} \begin{bmatrix} p_0 \\ v_0 \\ a_0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{\bar{K}_3}{\tau} \end{bmatrix}}_{B_r} a^\star(t). \tag{23}$$

Define the platoon state $X_{\text{pl}} = [x_0^\top \; x_1^\top \; ... \; x_M^\top]^\top$ and the platoon output $Y_{\text{pl}} = [a_0 \; a_1 \; ... \; a_M]^\top$, then write (23), (20) and (22) in the following form:

$$\dot{X}_{\text{pl}} = A_{\text{pl}} X_{\text{pl}} + B_{\text{pl}} a^\star(t) \tag{24}$$

with matrices:

$$A_{\mathrm{pl}} = \begin{bmatrix} W_0 & 0 & 0 & \dots & 0 & 0 \\ F_1 & F_0 & F_1 & \dots & 0 & 0 \\ 0 & F_1 & F_0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & F_1 & E_0 \end{bmatrix}, B_{\mathrm{pl}} = \begin{bmatrix} B_r \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{25}$$

Finally, the output is given by:

$$Y_{\mathrm{pl}} = C_{\mathrm{pl}} X_{\mathrm{pl}} \tag{26}$$

with the matrix

$$C_{\mathrm{pl}} = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix} \tag{27}$$

with $C = [0 \quad 0 \quad 1]$. Denote the elements of $\big(H_{0,\mathrm{r}}(s) \quad H_{1,\mathrm{r}}(s) \quad \dots \quad H_{M,\mathrm{r}}(s)\big)^\top = C_{\mathrm{pl}}(sI - A_{\mathrm{pl}})^{-1}B_{\mathrm{pl}}$, as the transfer functions from $a^\star$ to a_i, $0 \leq i \leq M$. The authors of [2] proposed the following definition of string stability for platoons with bidirectional communication, which will be used in the following.

Definition 2. The platoon described by the system (24) is string stable if $H_{i,\mathrm{r}}(s)$ is stable and

$$|H_{i+1,\mathrm{r}}(\mathrm{j}\omega)| \leq |H_{i,\mathrm{r}}(\mathrm{j}\omega)|, \quad \forall \omega, 1 \leq i < M \tag{28}$$

with $|\cdot|$ indicating the magnitude of the transfer function.

Remark 3. It should be pointed out that the common definition of string stability [13], that is the transfer function from V_{i-1} to V_i is not applicable, since the response of each vehicle depends on the response of both the preceding and the following vehicle.

V. Simulations

In this section, the control approach is demonstrated on a homogeneous platoon of $M = 3$ vehicles[2] with parameters[3]: $\tau = 0.25$ [s], $h = 0.3$ [s], $r = 3$ [m], and $L = 5$ [m]. The maximum acceleration is selected as $\bar{a} = 2$ [m/s^2], while the maximum deceleration is $\underline{a} = -6$ [m/s^2]. The maximum control input is selected as $\bar{u} = 2$ [m/s^2], while the minimum control input is $\underline{u} = -6$ [m/s^2]. The maximum velocity is $\bar{v} = 40$ [m/s] and the minimum velocity $\underline{v} = 0$ [m/s]. The controller gains are chosen as $\bar{K} = 10^2 \times \big(1 \quad 2 \quad 1\big)$ and $\kappa = 15$. The parameters of the CBFs are taken as $b_1^{\bar{a}} = 5$, $b_1^{\underline{a}} = 15$, $(B^{\bar{v}})^\top = (B^{\underline{v}})^\top = \big(1 \quad 2\big)$ (roots at $\epsilon = -1$), and $(B^p)^\top = \big(0.36 \quad 1.2\big)$ (roots at $\epsilon = -\frac{3}{5}$).

The comparison between the synchronization-based (nominal) controller (with no CBFs) and the constrained controller (with CBFs) will be done using three scenarios of collision avoidance, emergency braking, and platoon forming.

[2]The choice of the value of M has been done as a good trade-off between a sufficiently large value to show the effectiveness of the control law and a sufficiently low value for clarity of the plots.

[3]In the following, the brackets $[\cdot]$ are used to indicate measurement units.

TABLE I
Initial Conditions for Collision Avoidance Scenario

Initial Condition	Value	Unit
$p_0(t_0)$	81.66	[m]
$p_1(t_0)$	54.44	[m]
$p_2(t_0)$	27.22	[m]
$p_3(t_0)$	0	[m]
$v_0(t_0)$	80/3.6	[m/s]
$v_1(t_0)$	100/3.6	[m/s]
$v_2(t_0)$	120/3.6	[m/s]
$v_3(t_0)$	140/3.6	[m/s]

A. Collision Avoidance

The collision avoidance feature is illustrated by the following scenario: The virtual leader is keeping a constant speed, while the following vehicles approach the leading vehicle with high speeds. Table I lists the initial conditions for this scenario. Fig. 3 shows that the proposed controller keeps safe distances to the preceding vehicles. It is observed that the velocity of every vehicle follows the changes of that of its preceding vehicle and the velocities converge exponentially

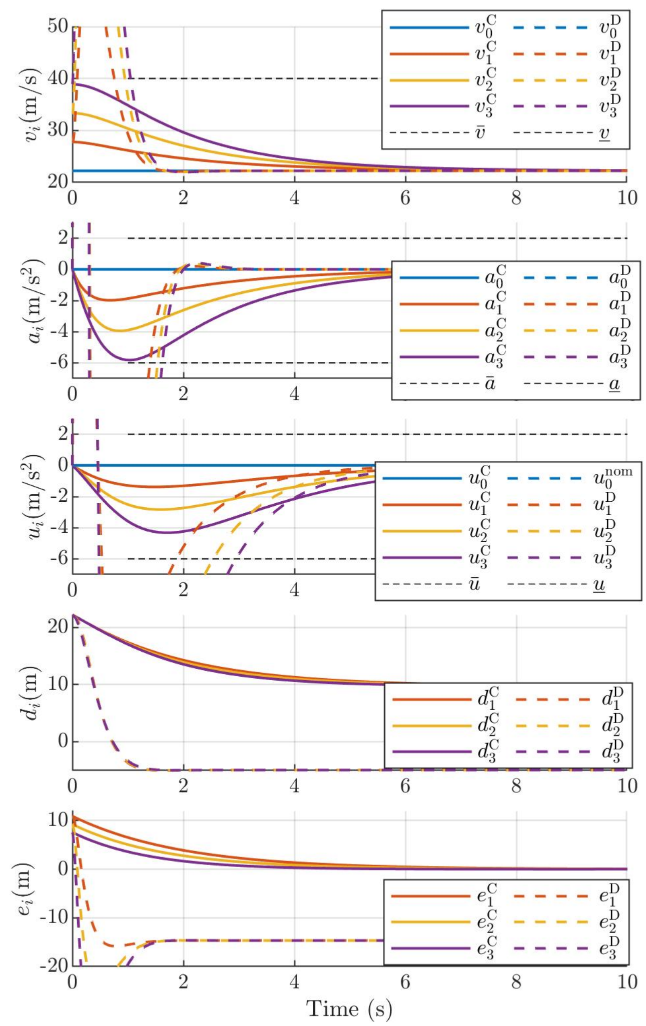

Fig. 3. Collision Avoidance Scenario

to the desired reference velocity of $v_0 = 80/3.6$ [m/s]. It shows string stable behavior and the constraints are not violated. The errors are regulated exponentially fast to zero while keeping a velocity-dependent inter-vehicle distance. Notice that the states under the synchronization-based controller converge faster. However, the constraints are violated.

B. Emergency Braking

The emergency braking feature is illustrated by the following scenario: After initial transients, the platoon is traveling at a constant speed of $v_0 = 80/3.6$ [m/s], when at $t = 40$ [s], the virtual leader performs an emergency braking, at its maximum deceleration, until a full stop. Fig. 4 illustrates a

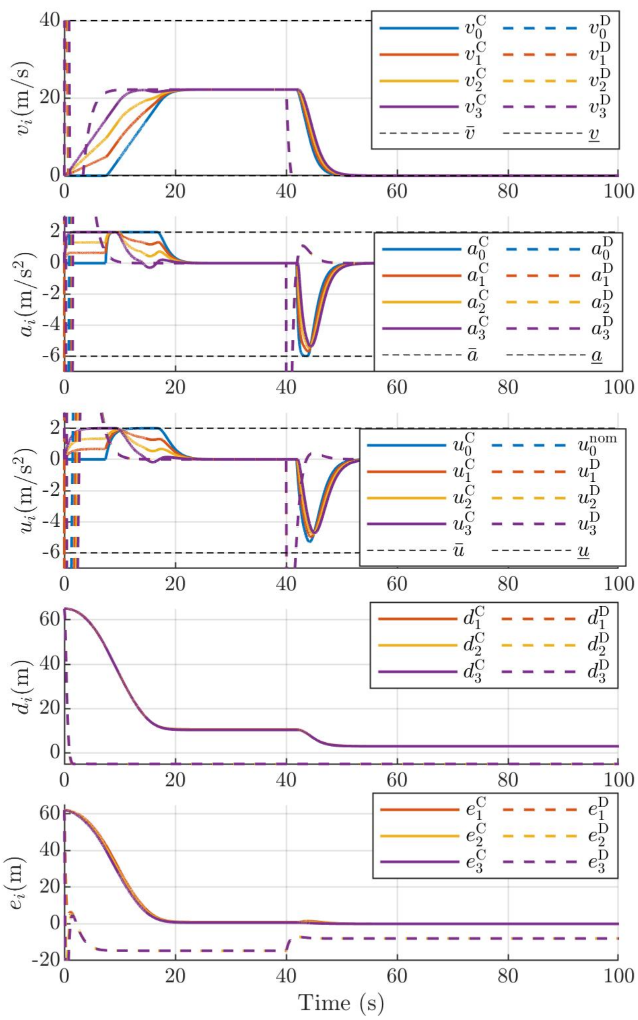

Fig. 4. Emergency Braking Scenario

good response of the vehicles to the emergency situation. The proposed controller guarantees that each vehicle within the platoon safely brakes from $v_0 = 80/3.6$ [m/s] to a full stop while avoiding collision with the preceding vehicle and satisfying the constraints.

TABLE II

INITIAL CONDITIONS FOR PLATOON FORMING SCENARIO

Initial Condition	Value	Unit
$p_1(t_0)$	150	[m]
$p_1(t_0)$	100	[m]
$p_2(t_0)$	70	[m]
$p_3(t_0)$	20	[m]
$v_0(t_0)$	15	[m/s]
$v_1(t_0)$	20	[m/s]
$v_2(t_0)$	25	[m/s]
$v_3(t_0)$	30	[m/s]
$a_0(t_0)$	1	[m/s^2]
$a_1(t_0)$	−6	[m/s^2]
$a_2(t_0)$	2	[m/s^2]
$a_3(t_0)$	−3	[m/s^2]

C. Platoon Forming

Finally, the platoon forming feature is illustrated by the following scenario: The vehicles are traveling at different speeds and different accelerations, when at $t = 0$ [s], the platoon is formed and the virtual leader receives a constant velocity reference profile of $v_0 = 30$ [m/s] . Table II lists the initial conditions for this scenario. Fig. 5 shows that the proposed controller ensures the forming of the platoon from rather challenging initial conditions and the vehicles reach the common reference velocity of $v_0 = 30$ [m/s], while satisfying the constraints. The errors are regulated exponentially fast to zero.

Fig. 6 demonstrates that for the set of parameters: $\kappa = 15$, $\bar{K}_3 = 100$, and $\tau = 0.25$ [s] , the effect of the exogenous input $a^\star(t)$ is attenuated over the homogeneous platoon (i. e. $|H_{i+1,\mathrm{r}}(\mathrm{j}\omega)| \leq |H_{i,\mathrm{r}}(\mathrm{j}\omega)| \quad \forall\omega, 1 \leq i < M$). Notice that the transfer functions do not depend on the headway h (i. e. the platoon is string stable for any headway $h \geq 0$) .

VI. CONCLUSION

In this work, a controller for CACC of linear homogeneous vehicle platoons with bidirectional communication involving the presence of a leader is presented. The controller is based on two terms. First, a distributed control law is implemented. The resulting control action is fed to an online optimization problem derived from control barrier functions in order to satisfy the constraints of the system. The effectiveness of the control law is tested in various simulations where in particular, it was shown that string stable behavior can be achieved for any nonnegative headway. Future works will focus on using similar tools for platoons described by nonlinear dynamics, analyzing the robustness of the proposed control scheme, and deriving formal proof of stability.

REFERENCES

[1] A. D. Ames, J. W. Grizzle, and P. Tabuada. Control barrier function based quadratic programs with application to adaptive cruise control. In *53rd IEEE Conference on Decision and Control*, pages 6271–6278. IEEE, 2014.

[2] S. Baldi, D. Liu, V. Jain, and W. Yu. Establishing platoons of bidirectional cooperative vehicles with engine limits and uncertain dynamics. *IEEE Transactions on Intelligent Transportation Systems*, 22(5):2679–2691, 2020.

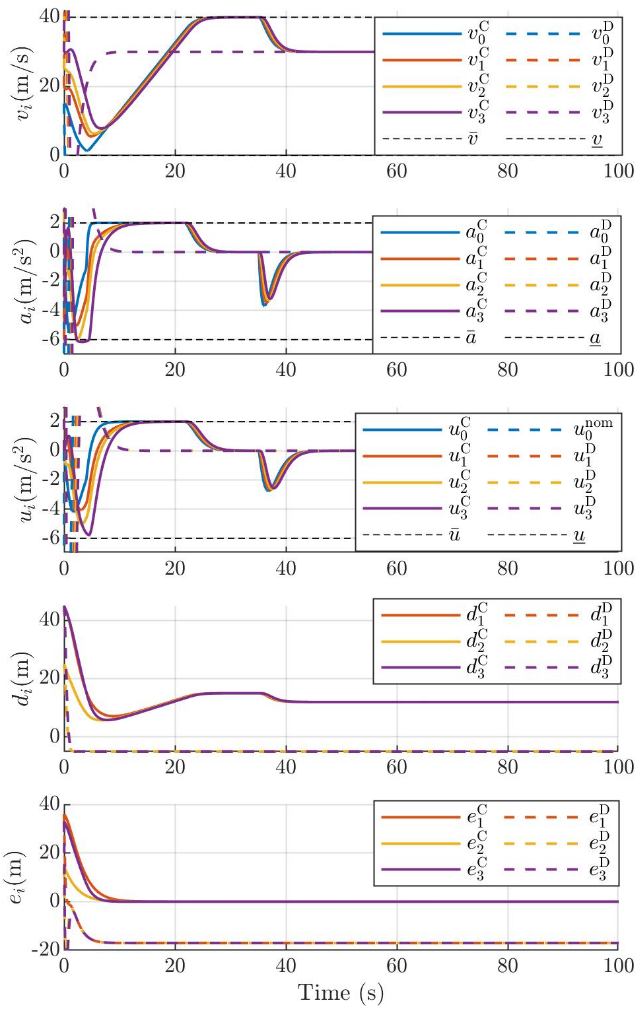

Fig. 5. Platoon Forming Scenario

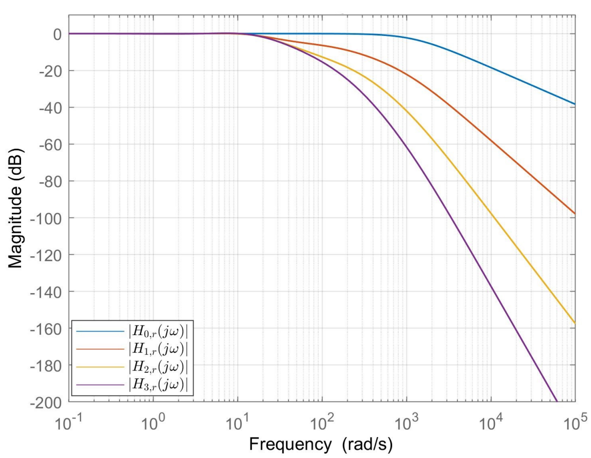

Fig. 6. String stability check

[3] B. Besselink and K. Johansson. String stability and a delay-based spacing policy for vehicle platoons subject to disturbances. *IEEE Transactions on Automatic Control*, 62(9):4376–4391, 2017.

[4] S. Feng, Y. Zhang, S. E. Li, Z. Cao, H. X. Liu, and L. Li. String stability for vehicular platoon control: Definitions and analysis methods. *Annual Reviews in Control*, 47:81–97, 2019.

[5] M. Giaccagli, V. Andrieu, D. Astolfi, and G. Casadei. Sufficient metric conditions for synchronization of leader-connected homogeneous nonlinear multi-agent systems. *IFAC-PapersOnLine*, 54(14):412–417, 2021.

[6] M. Giaccagli, V. Andrieu, S. Tarbouriech, and D. Astolfi. Infinite gain margin, contraction and optimality: an LMI-based design. *European Journal of Control*, 68:100685, 2022.

[7] S.-C. Hsu, X. Xu, and A. D. Ames. Control barrier function based quadratic programs with application to bipedal robotic walking. pages 4542–4548. IEEE, 2015.

[8] F. Hu and H. Yu. Safety-critical lane-change control for cav platoons in mixed autonomy traffic using control barrier functions. *arXiv preprint arXiv:2302.00424*, 2023.

[9] A. Isidori. *Nonlinear control systems II*. Springer, 2013.

[10] A. Isidori. *Lectures in feedback design for multivariable systems*. Springer, 2017.

[11] K. Ma, H. Wang, and T. Ruan. Analysis of road capacity and pollutant emissions: Impacts of connected and automated vehicle platoons on traffic flow. *Physica A: Statistical Mechanics and its Applications*, 583:126301, 2021.

[12] J. Monteil, G. Russo, and R. Shorten. On $\mathcal{L}_\infty$ string stability of nonlinear bidirectional asymmetric heterogeneous platoon systems. *Automatica*, 105:198–205, 2019.

[13] J. Ploeg, B. T. Scheepers, E. Van Nunen, N. Van de Wouw, and H. Nijmeijer. Design and experimental evaluation of cooperative adaptive cruise control. In *2011 14th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, pages 260–265. IEEE, 2011.

[14] M. Rauscher, M. Kimmel, and S. Hirche. Constrained robot control using control barrier functions. In *2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 279–285. IEEE, 2016.

[15] F. Seeland, F. Miekautsch, A. Fay, and J. Horn. Constrained control for heterogeneous vehicle platoons. In *2022 26th International Conference on Methods and Models in Automation and Robotics (MMAR)*, pages 164–169. IEEE, 2022.

[16] S. Sheikholeslam and C. A. Desoer. Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study. *IEEE Transactions on vehicular technology*, 42(4):546–554, 1993.

[17] G. Silva, A. Donaire, A. McFadyen, and J. Ford. String stable integral control design for vehicle platoons with disturbances. *Automatica*, 127:109542, 2021.

[18] S. S. Stankovic, M. J. Stanojevic, and D. D. Siljak. Decentralized overlapping control of a platoon of vehicles. *IEEE Transactions on Control Systems Technology*, 8(5):816–832, 2000.

[19] D. Swaroop, J. K. Hedrick, and S. B. Choi. Direct adaptive longitudinal control of vehicle platoons. *IEEE transactions on vehicular technology*, 50(1):150–161, 2001.

[20] A. Vahidi and A. Eskandarian. Research advances in intelligent collision avoidance and adaptive cruise control. *IEEE transactions on intelligent transportation systems*, 4(3):143–153, 2003.

[21] C. Wu, M. Zhang, and D. Dimarogonas. On disturbance propagation in vehicular platoons with different communication ranges. *arXiv preprint arXiv:2212.07927*, 2022.

[22] X. Xu. Control sharing barrier functions with application to constrained control. In *2016 IEEE 55th Conference on Decision and Control (CDC)*, pages 4880–4885. IEEE, 2016.

[23] X. Xu, P. Tabuada, J. W. Grizzle, and A. D. Ames. Robustness of control barrier functions for safety critical control. *IFAC-PapersOnLine*, 48(27):54–61, 2015.

[24] J. Zegers. Distributed consensus control in vehicle platooning. Master's thesis, Eindhoven University of Technology, 2015.

[25] J. C. Zegers, E. Semsar-Kazerooni, J. Ploeg, N. Van De Wouw, and H. Nijmeijer. Consensus-based bi-directional CACC for vehicular platooning. In *2016 American Control Conference (ACC)*, pages 2578–2584. IEEE, 2016.

[26] J. C. Zegers, E. Semsar-Kazerooni, J. Ploeg, N. van de Wouw, and H. Nijmeijer. Consensus control for vehicular platooning with velocity constraints. *IEEE Transactions on Control Systems Technology*, 26(5):1592–1605, 2017.