2024 IEEE 63rd Conference on Decision and Control (CDC)
December 16-19, 2024. MiCo, Milan, Italy

Interval observer-based active fault tolerant control for discrete-time uncertain switched LPV systems

Duc To Nguyen, Saïd Mammar, Dalil Ichalal

Abstract— **This paper presents a novel method for co-designing a TNL interval observer and fault-tolerant control (FTC) for discrete-time switched linear parameter-varying (LPV) systems. These systems are subject to faults, unknown but bounded uncertainties, state disturbances, and measurement noise. By introducing weighting matrices T and N, the design gains additional flexibility in calculating the observer gain matrices, ensuring the cooperative condition of estimation errors. The fault is incorporated into an augmented state vector, allowing the TNL interval observer to jointly estimate the lower and upper bounds of both the system state and faults. The FTC is then designed to compensate for the estimated fault and stabilize the closed-loop system in the presence of faults. Sufficient conditions for the stability of the proposed methodology are formulated as Linear Matrix Inequalities (LMIs), derived using the Input-to-State Stability (ISS) property with multiple Lyapunov functions and the Average Dwell Time (ADT) technique. The approach is applied to vehicle lateral dynamics, demonstrating its effectiveness in estimating lateral speed within a tight interval between the lower and upper bounds, while successfully controlling the yaw rate.**

Index Terms— **Interval observer, TNL structure, switched LPV systems, fault-tolerant control.**

I. Introduction

Observer-based controller is an important topic in theory of modern control. It has been widely applied in various fields such as aerospace, robotics, and automotive. In real-world scenarios, it is inevitable for the system to encounter faults that may have impacts on its stability and performance. These faults can include unidentified actuator or sensor faults, component malfunctions or external perturbations from the environment that may lead to undesirable behaviors or even catastrophic failures for the system. Therefore, it is essential to design a robust and reliable approach to estimate and compensate for such failures. To deal with this challenging task, a common approach is to synthesize an observer-based FTC which employs observers to simultaneously estimate both the states and faults of the system and then design feedback FTC to compensate for the fault effects and achieve the desired performance [1].

Moreover, systems are subject to process disturbances, measurement noise and modeling uncertainties which introduce significant errors in unmeasured state estimation. Therefore, conventional observers may struggle to cope well with the challenges of state estimation issues effectively [2]. This motivated the development of robust estimation methods such as interval observer for different classes of systems.

All the authors are with the Laboratory of Computing, BioInformatics and Complex Systems (IBISC), Univ Evry, Paris-Saclay University, Evry 91000, France Email: ducto.nguyen, said.mammar, dalil.ichalal@univ-evry.fr

The concept of interval observers has been an active area of research in the last decades as they have numerous applications in control and estimation theory [3], [4]. Unlike traditional observers, interval observer synthesis requires both stability and inclusion properties. When using classical design methods, it is not straightforward to find the observer gain matrices that satisfy both Hurwitz and Metzler conditions. To deal with this difficulty, recent research, as reported in [5]. has investigated the utilization of coordinate transformation to relax the cooperative property of the interval observer. Although the obtained results can provide more flexibility for design conditions and reduce the conservatism, the interval observer performance is heavily affected by selecting the non-singular transfer matrix [6].

To overcome this limitation, a novel methodology called TNL is proposed to deal with the challenge of ensuring the cooperative property of the interval observer. The key idea, in addition to the traditional observer gain, is to introduce a weighting matrix, which gives more flexibility to make the lower and upper estimation errors positive without resorting to the change of coordinates.

Numerous achievements for TNL observers have been researched. For example, the study in [7] designed a new interval observer structure to estimate the lower and upper bounds of state for discrete-time linear systems with process disturbance and measurement noise. However, the limitation of these works is that they did not take into account uncertainties in parameters which may affect the performance of the observers in some realistic scenarios. The work in [8] extended the TNL interval observer for continuous-time LPV system with parametric uncertainties while minimizing influence and obtained a tight state interval. Authors in [9] considered TNL interval observer for joint estimation of the state and unknown input by considering the unknown input as an auxiliary state for discrete-time LPV system and for continuous-time switched system [10]. However, all the aforementioned approaches are only dedicated to the problem of state and fault estimation and did not address the issue of FTC for systems with state disturbances and measurement noise. To fill this gap, the study in [11] investigated the design problem of an active FTC for discrete-time LPV system subject to disturbance and measurement noise. The state and fault are simultaneously estimated by an augmented system. The interval observer synthesis and FTC are, however, separately designed by two sets of LMIs. It may lead to instability when put together [12].

This paper proposes a constructive methodology to design a TNL interval observer and FTC for uncertain discrete-time

LPV switched systems represented under polytopic form with unknown but bounded uncertainties, state disturbances and measurement noise. The main contributions of this paper are listed as follows:

- An extension of the results from [8], [9] on interval observer for uncertain discrete-time switched LPV system subject to faults and unknown but bounded uncertainties, disturbances and measurement noise.
- An integrated design of the interval observer and FTC law is synthesized to stabilize the closed-loop system and to compensate the fault effects. As previously outlined, in [11] the design of the observer and controller are treated as separate tasks. Instead, in this paper, the observer and FTC are co-designed.
- Less conservative Metzler conditions compared to change of coordinates approach as discussed in [13]. In other words, the TNL interval observer synthesis procedure provides more degrees of freedom in computing the observer gains is designed to simultaneously estimate the lower and upper bounds of the system state and faults through an augmented system.
- The sufficient conditions for the existence of the proposed interval observer and associated FTC are formulated using switched ISS-Lyapunov functions under ADT. They are expressed in terms of LMIs.

The paper is organized as follows: preliminaries and problem formulation are given in section II. Section III provides the main contribution with the design procedure of a TNL interval observer-based controller for uncertain discrete-time switched LPV systems subject to faults and unknown but bounded uncertainties, state disturbances and measurement noises. Simulation results of the proposed technique are described in Section IV. Finally, the conclusion and future works are detailed in Section V.

Notation. For a vector $x \in \mathbb{R}^n$ or a matrix $M \in \mathbb{R}^{n\times n}$, one denotes $x^+ = \max\{0, x\}$, $x^- = x^+ - x$ or $M^+ = \max\{0, M\}$, $M^- = M^+ - M$, where $\max$ is element-wise maximum. For two vectors x_1 and x_2, the inequalities $x_1 \leq x_2$ $(x_1 \geq x_2)$ are interpreted element-wise. The relation $M \succ 0$ (resp. $M \prec 0$) means that the matrix M is positive (resp. negative) definite. M^T stands for the transpose of the matrix M and $\mathcal{I}_n$ is an $n \times n$ identity matrix. $\|.\|_2$ and $\|.\|_\infty$ stands to Euclidean and infinity norms, respectively. The symbol $\dagger$ denotes the pseudo inverse, $He\{M\} = M + M^T$ and $(*)$ represent the symmetric part of a matrix.

II. Preliminaries and Problem Formulation

Consider the uncertain discrete-time switched LPV system:

$$\begin{cases} x_{k+1} = \big(\mathcal{A}_\sigma(\rho_k) + \Delta\mathcal{A}_\sigma(\rho_k)\big)x_k + \mathcal{B}_\sigma(\rho_k)u_k \\ \qquad\quad + F_\sigma f_k + \mathcal{D}^w_\sigma w_k \\ y_k = \mathcal{C}x_k + D^v_\sigma v_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the system state, $u_k \in \mathbb{R}^{n_u}$ is control input, $f_k \in \mathbb{R}^{n_f}$ is fault vectors, $w_k \in \mathbb{R}^{n_w}$ is disturbance vector, $v_k \in \mathbb{R}^{n_v}$ is measurement noise, $y_k \in \mathbb{R}^{n_y}$ is system output. Switching signal $\sigma_k \in \mathcal{N} = \{1, \ldots, N\}$ with N is the number of subsystems.

The initial state condition x_{k_0}, the state disturbance w_k and measurement noise v_k are assumed to be unknown but bounded by $\overline{x}_{k_0}$ and $\underline{x}_{k_0}$, $\underline{w}_k$ and $\overline{w}_k$, $\underline{v}_k$ and $\overline{v}_k$.

$$\begin{aligned} &\underline{x}_{k_0} \leq x_{k_0} \leq \overline{x}_{k_0} \\ &\underline{w}_k \leq w_k \leq \overline{w}_k \\ &\underline{v}_k \leq v_k \leq \overline{v}_k \end{aligned} \tag{2}$$

Matrices $\mathcal{A}_\sigma(\rho_k) \in \mathbb{R}^{n_x\times n_x}$, $\mathcal{B}_\sigma(\rho_k) \in \mathbb{R}^{n_x\times n_u}$ are parameter varying matrices of appropriate dimension. Matrices $F_\sigma \in \mathbb{R}^{n_x\times n_f}$, $\mathcal{D}^w_\sigma \in \mathbb{R}^{n_x\times n_w}$, $D^v_\sigma \in \mathbb{R}^{n_y\times n_v}$ are switching constant matrices and $\mathcal{C}$ is constant matrix.

Time-varying parameters $\rho = [\rho_1, \rho_2, ..., \rho_P]$ are assumed to be measurable vectors with P is the number of scheduling parameters and $\rho_k \in [\underline{\rho}_k, \overline{\rho}_k]$ for all $k = 1 \cdots P$ then it evolves inside a polytope represented by 2^P vertices. The switched weighting function $\mu^j_\sigma(\rho)$ guarantees convex sum property for all $\sigma \in \mathcal{N}$ and $j \in \mathcal{P} = \{1, \cdots, 2^P\}$

$$0 \leq \mu^j_\sigma(\rho) \leq 1, \quad \sum_{j=1}^{2^P} \mu^j_\sigma(\rho) = 1 \tag{3}$$

The matrices of system (1) are exactly represented under a polytopic form:

$$\mathcal{M}_\sigma(\rho_k) = \sum_{j=1}^{2^P} \mu^j_\sigma(\rho_k)\mathcal{M}^j_\sigma \tag{4}$$

where $\mathcal{M} \in \{\mathcal{A}, \mathcal{B}, \Delta\mathcal{A}\}$.

The uncertain terms $\Delta\mathcal{A}_\sigma(\rho_k)$ are bounded by known matrices $\underline{\Delta\mathcal{A}}_\sigma$, $\overline{\Delta\mathcal{A}}_\sigma$ with $\forall\sigma \in \mathcal{N}$, such that

$$\underline{\Delta\mathcal{A}}_\sigma \leq \Delta\mathcal{A}_\sigma(\rho_k) \leq \overline{\Delta\mathcal{A}}_\sigma \tag{5}$$

We now introduce lemmas which are necessary to prove the main results.

Lemma 1. *[14] The switched system $x_{k+1} = A_{\sigma_k}x_k + \delta_{\sigma_k}$ is Input-to-State-Stable with respect to disturbance δ_i if the following inequalities*

$$\begin{aligned} \alpha\|x_k\|^2 \leq V_i(x_k) \leq \beta\|x_k\|^2 \\ \Delta V_i(x_k) < -\varepsilon V_i(x_k) + \gamma_i\|\delta_i\| \end{aligned} \tag{6}$$

hold for any $\sigma_k = i \in \mathcal{N}$ with $0 < \alpha < \beta$, $0 < \varepsilon < 1$, $\gamma_i > 0$ and σ_k is switching signal with average dwell time

$$\tau_a \geq \tau^*_a = -\frac{\ln(\frac{\beta}{\alpha})}{\ln(1-\varepsilon)} \tag{7}$$

III. TNL Interval Observer-Based Fault Tolerant Control

In this section, a new co-design approach of observer and controller for a class of discrete-time switched LPV systems is presented. The TNL interval observer is proposed to simultaneously estimate the lower and upper bounds of state x_k and faults f_k and a FTC is then synthesized to stabilize the closed-loop system in the presence of faults.

An augmented state $z_k = \begin{bmatrix} x_k \\ f_{k-1} \end{bmatrix}$ incorporating the original state x_k and fault f_k into a single vector to facilitate simultaneous estimation is introduced. The system (1) is then rewritten under the augmented form:

$$\begin{cases} E_\sigma z_{k+1} = \big(A_\sigma(\rho_k) + \Delta A_\sigma(\rho_k)\big) z_k + B_\sigma(\rho_k) u_k + D_\sigma^w w_k \\ \qquad y_k = C z_k + D_\sigma^v v_k \end{cases} \tag{8}$$

where $E_\sigma = \begin{bmatrix} I_{n_x} & -F_\sigma \\ 0 & 0 \end{bmatrix}$, $A_\sigma(\rho_k) = \begin{bmatrix} \mathcal{A}_\sigma(\rho_k) & 0 \\ 0 & 0 \end{bmatrix}$, $B_\sigma(\rho_k) = \begin{bmatrix} \mathcal{B}_\sigma(\rho_k) \\ 0 \end{bmatrix}$, $D_\sigma^w = \begin{bmatrix} \mathcal{D}_\sigma^w \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} \mathcal{C} & 0 \end{bmatrix}$, $\Delta A_\sigma(\rho_k) = \begin{bmatrix} \Delta\mathcal{A}_\sigma(\rho_k) & 0 \\ 0 & 0 \end{bmatrix}$.

Note that the interval observer design procedure aimed at estimating both the lower and upper bounds of the state x_k and fault f_k for system (1) is turned into the problem of interval estimation of state z_k for the system (8).

To proceed, the following TNL interval observer structure for the augmented system (8) is proposed:

$$\begin{cases} \overline{\xi}_{k+1} = \overline{T}_\sigma A_\sigma(\rho_k)\overline{z}_k + \overline{T}_\sigma B_\sigma(\rho_k) u_k + \overline{\nabla}_\sigma + \overline{L}_\sigma(\rho_k)(y_k - C\overline{z}_k) \\ \quad \overline{z}_k = \overline{\xi}_k + \overline{N}_\sigma y_k \\ \underline{\xi}_{k+1} = \underline{T}_\sigma A_\sigma(\rho_k)\underline{z}_k + \underline{T}_\sigma B_\sigma(\rho_k) u_k + \underline{\nabla}_\sigma + \underline{L}_\sigma(\rho_k)(y_k - C\underline{z}_k) \\ \quad \underline{z}_k = \underline{\xi}_k + \underline{N}_\sigma y_k \end{cases} \tag{9}$$

where $\underline{\xi}_k$, $\overline{\xi}_k$ and $\underline{z}_k$, $\overline{z}_k$ are the lower and upper bounds of state vectors of the observer and augmented state z_k. Matrices $\underline{T}_\sigma, \overline{T}_\sigma \in \mathbb{R}^{(n_x+n_f)\times(n_x+n_f)}$, $\underline{L}_\sigma(\rho_k), \overline{L}_\sigma(\rho_k) \in \mathbb{R}^{(n_x+n_f)\times n_y}$, $\underline{N}_\sigma, \overline{N}_\sigma \in \mathbb{R}^{(n_x+n_f)\times n_y}$ are unknown matrices of compatible dimension to be designed to satisfy the following relations.

$$\underline{T}_\sigma E_\sigma + \underline{N}_\sigma C = I_{n_x+n_f}, \quad \overline{T}_\sigma E_\sigma + \overline{N}_\sigma C = I_{n_x+n_f} \tag{10}$$

The terms $\overline{\nabla}_\sigma$ and $\underline{\nabla}_\sigma$ appearing (9) are chosen as

$$\begin{cases} \overline{\nabla}_\sigma = \overline{T}_\sigma^+ \overline{\delta}_\sigma - \big(\overline{L}_\sigma(\rho_k) D_\sigma^v\big)^+ \underline{v}_k + \big(\overline{L}_\sigma(\rho_k) D_\sigma^v\big)^- \overline{v}_k \\ \qquad - \overline{T}_\sigma^- \underline{\delta}_\sigma - \big(\overline{N}_\sigma D_\sigma^v\big)^+ \underline{v}_{k+1} + \big(\overline{N}_\sigma D_\sigma^w\big)^- \overline{v}_{k+1} \\ \underline{\nabla}_\sigma = \underline{T}_\sigma^+ \underline{\delta}_\sigma - \big(\underline{L}_\sigma(\rho_k) D_\sigma^v\big)^+ \overline{v}_k + \big(\underline{L}_\sigma(\rho_k) D_\sigma^v\big)^- \underline{v}_k \\ \qquad - \underline{T}_\sigma^- \overline{\delta}_\sigma - \big(\underline{N}_\sigma D_\sigma^v\big)^+ \overline{v}_{k+1} + \big(\underline{N}_\sigma D_\sigma^v\big)^- \underline{v}_{k+1} \end{cases} \tag{11}$$

where $\overline{\delta}_\sigma$ and $\underline{\delta}_\sigma$ are the upper and lower bounds of $\delta_\sigma = \Delta A_\sigma(\rho_k) z_k + D_\sigma^w w_k$ determined under conditions 2, 5 and interval lemma [15].

First of all, let's define $\overline{e}_k = \overline{z}_k - z_k$ and $\underline{e}_k = z_k - \underline{z}_k$ are the lower and upper estimation errors. From (8), (9), their dynamics are given as:

$$\begin{cases} \overline{e}_{k+1} = \Big(\overline{T}_\sigma A_\sigma(\rho_k) - \overline{L}_\sigma(\rho_k) C\Big)\overline{e}_k + \overline{\nabla}_\sigma + \overline{N}_\sigma D_\sigma^v v_{k+1} \\ \qquad - \overline{T}_\sigma\big(\Delta A_\sigma(\rho_k) z_k + D_\sigma^w w_k\big) + \overline{L}_\sigma(\rho_k) D_\sigma^v v_k \\ \underline{e}_{k+1} = \Big(\underline{T}_\sigma A_\sigma(\rho_k) - \underline{L}_\sigma(\rho_k) C\Big)\underline{e}_k - \underline{\nabla}_\sigma - \underline{N}_\sigma D_\sigma^v v_{k+1} \\ \qquad + \underline{T}_\sigma\big(\Delta A_\sigma(\rho_k) z_k + D_\sigma^w w_k\big) - \underline{L}_\sigma(\rho_k) D_\sigma^v v_k \end{cases} \tag{12}$$

The aim is to prove that $\underline{e}_k$ and $\overline{e}_k$ are non-negative. Based on interval lemma [15] and the terms $\underline{\nabla}_\sigma$ and $\overline{\nabla}_\sigma$ in (11), the following terms are non-negative.

$$\begin{cases} \overline{\nabla}_\sigma + \overline{N}_\sigma D_\sigma^v v_{k+1} - \overline{T}_\sigma \delta_\sigma + \overline{L}_\sigma(\rho_k) D_\sigma^v v_k \geq 0 \\ -\underline{\nabla}_\sigma - \underline{N}_\sigma D_\sigma^v v_{k+1} + \underline{T}_\sigma \delta_\sigma - \underline{L}_\sigma(\rho_k) D_\sigma^v v_k \geq 0 \end{cases} \tag{13}$$

Moreover, the initial augmented state conditions are non-negative due to condition 2 and construction of augmented states. Therefore, the errors (12) are a positive system according to positive lemma [16] if $\underline{T}_\sigma A_\sigma(\rho_k) - \underline{L}_\sigma(\rho_k) C$ and $\overline{T}_\sigma A_\sigma(\rho_k) - \overline{L}_\sigma(\rho_k) C$ are Metzler matrices for all $\sigma \in \mathcal{N}$.

In addition, it is essential for both $\underline{T}_\sigma A_\sigma(\rho_k) - \underline{L}_\sigma(\rho_k) C$ and $\overline{T}_\sigma A_\sigma(\rho_k) - \overline{L}_\sigma(\rho_k) C$ to be Schur stable. This condition ensures that the interval error, defined as $e_k = \overline{e}_k - \underline{e}_k$ approaches zero asymptotically in the nominal case and remains in a tight interval in other cases.

A. Observer matrices determination

Setting $\Omega_\sigma = \begin{bmatrix} E_\sigma \\ C \end{bmatrix}$, the generalized solution of (10) is given as follows:

$$\begin{cases} \begin{bmatrix} \underline{T}_\sigma & \underline{N}_\sigma \end{bmatrix} = \Omega_\sigma^\dagger + \underline{Z}_\sigma\big(I - \Omega_\sigma \Omega_\sigma^\dagger\big) \\ \begin{bmatrix} \overline{T}_\sigma & \overline{N}_\sigma \end{bmatrix} = \Omega_\sigma^\dagger + \overline{Z}_\sigma\big(I - \Omega_\sigma \Omega_\sigma^\dagger\big) \end{cases} \tag{14}$$

where $\underline{Z}_\sigma$ and $\overline{Z}_\sigma$ are arbitrary matrices of proper dimension. By setting $\alpha = \begin{bmatrix} I_{n_x+n_f} \\ 0 \end{bmatrix}$, $\beta = \begin{bmatrix} 0 \\ I_{n_y} \end{bmatrix}$ and $\Theta_\sigma = I - \Omega_\sigma \Omega_\sigma^\dagger$, matrices in (14) are then expressed as:

$$\begin{cases} \underline{T}_\sigma = \Omega_\sigma^\dagger \alpha + \underline{Z}_\sigma \Theta_\sigma \alpha, & \overline{T}_\sigma = \Omega_\sigma^\dagger \alpha + \overline{Z}_\sigma \Theta_\sigma \alpha \\ \underline{N}_\sigma = \Omega_\sigma^\dagger \beta + \underline{Z}_\sigma \Theta_\sigma \beta, & \overline{N}_\sigma = \Omega_\sigma^\dagger \beta + \overline{Z}_\sigma \Theta_\sigma \beta \end{cases} \tag{15}$$

At this point, the observer design problem can now be solved by computing matrices $\underline{Z}_\sigma, \overline{Z}_\sigma$ and $\underline{L}_\sigma(\rho_k), \overline{L}_\sigma(\rho_k)$ to ensure that matrices $\underline{T}_\sigma A_\sigma(\rho_k) - \underline{L}_\sigma(\rho_k) C$ and $\overline{T}_\sigma A_\sigma(\rho_k) - \overline{L}_\sigma(\rho_k) C$ are both Metzler and Schur stable for all $\sigma \in \mathcal{N}$.

B. Integrated observer and FTC synthesis

In this subsection, an integrated design of the TNL structure interval observer and a FTC is proposed to ensure the stability of the closed-loop system in the presence of faults.

According to the construction of the augmented state vector z_k, the estimation interval of state and fault are determined by:

$$\begin{cases} \underline{x}_k = \begin{bmatrix} I_{n_x} & 0 \end{bmatrix} \underline{z}_k, & \underline{f}_{k-1} = \begin{bmatrix} 0 & I_{n_f} \end{bmatrix} \underline{z}_k \\ \overline{x}_k = \begin{bmatrix} I_{n_x} & 0 \end{bmatrix} \overline{z}_k, & \overline{f}_{k-1} = \begin{bmatrix} 0 & I_{n_f} \end{bmatrix} \overline{z}_k \end{cases} \tag{16}$$

where $\underline{x}_k$, $\overline{x}_k$ and $\underline{f}_{k-1}$, $\overline{f}_{k-1}$ are the lower and upper bounds of the state and fault estimation.

The following control law is then synthesized as follows:

$$u_k = -\overline{K}_\sigma \overline{x}_k - \underline{K}_\sigma \underline{x}_k - \overline{K}_{\sigma,f} \overline{f}_k - \underline{K}_{\sigma,f} \underline{f}_k \tag{17}$$

where $\underline{K}_\sigma$, $\overline{K}_\sigma$ and $\overline{K}_{\sigma,f}$, $\underline{K}_{\sigma,f}$ are the lower and upper state feedback and accommodation gain matrices, respectively. Their values are given by $\overline{K}_{\sigma,f} = \underline{K}_{\sigma,f} = \frac{1}{2} B_\sigma^\dagger(\rho_k) F_\sigma$ with $B_\sigma^\dagger(\rho_k) = \sum_{j=1}^{2^P} \mu_\sigma^j(\rho_k) {B_\sigma^j}^\dagger$.

The closed-loop system is then expressed as:

$$\begin{aligned} x_{k+1} = &\left(\mathcal{A}_\sigma(\rho_k) - \mathcal{B}_\sigma(\rho_k)\overline{K}_\sigma - \mathcal{B}_\sigma(\rho_k)\underline{K}_\sigma\right) x_k \\ &- \mathcal{B}_\sigma(\rho_k)\overline{K}_\sigma \overline{e}_{x,k} + \mathcal{B}_\sigma(\rho_k)\underline{K}_\sigma \underline{e}_{x,k} + \Delta\mathcal{A}_\sigma(\rho_k) x_k \\ &- \mathcal{B}_\sigma(\rho_k)\overline{K}_{\sigma,f} \overline{e}_{f,k} + \mathcal{B}_\sigma(\rho_k)\underline{K}_{\sigma,f} \underline{e}_{f,k} + \mathcal{D}_\sigma^w w_k \end{aligned} \tag{18}$$

with $\underline{e}_{x,k} = x_k - \underline{x}_k$, $\overline{e}_{x,k} = \overline{x}_k - x_k$ and $\underline{e}_{f,k} = f - \underline{f}$, $\overline{e}_{f,k} = \overline{f} - f$ are the lower and upper state and fault estimation errors.

The following augmented system is given as:

$$\begin{cases} x_{k+1} = \left(\mathcal{A}_\sigma(\rho_k) - \mathcal{B}_\sigma(\rho_k)\overline{K}_\sigma - \mathcal{B}_\sigma(\rho_k)\underline{K}_\sigma\right)x_k + w_\sigma^1 \\ \overline{e}_{k+1} = \left(\overline{T}_\sigma A_\sigma(\rho_k) - \overline{L}_\sigma(\rho_k)C\right)\overline{e}_k + w_\sigma^2 \\ \underline{e}_{k+1} = \left(\underline{T}_\sigma A_\sigma(\rho_k) - \underline{L}_\sigma(\rho_k)C\right)\underline{e}_k + w_\sigma^3 \end{cases} \tag{19}$$

where $\begin{cases} w_\sigma^1 = -\mathcal{B}_\sigma(\rho_k)\overline{K}_\sigma\overline{e}_{x,k} + \mathcal{B}_\sigma(\rho_k)\underline{K}_\sigma\underline{e}_{x,k} + \Delta\mathcal{A}_\sigma(\rho_k)x_k \\ \quad - \mathcal{B}_\sigma(\rho_k)\overline{K}_{\sigma,f}\overline{e}_{f,k} + \mathcal{B}_\sigma(\rho_k)\underline{K}_{\sigma,f}\underline{e}_{f,k} + \mathcal{D}_\sigma^w w_k \\ w_\sigma^2 = \overline{\nabla}_\sigma - \overline{T}_\sigma\delta_\sigma + \overline{L}_\sigma(\rho_k)D_\sigma^v v_k + \overline{N}_\sigma D_\sigma^v v_{k+1} \\ w_\sigma^3 = \underline{T}_\sigma\delta_\sigma - \underline{\nabla}_\sigma - \underline{L}_\sigma(\rho_k)D_\sigma^v v_k - \underline{N}_\sigma D_\sigma^v v_{k+1} \end{cases}$

The following two vectors are defined for the sake of clarity: $\chi_k = \begin{bmatrix} x_k & \overline{e}_k & \underline{e}_k \end{bmatrix}^T$ and $w_\sigma = \begin{bmatrix} w_\sigma^1 & w_\sigma^2 & w_\sigma^3 \end{bmatrix}^T$.

The following theorem provides sufficient conditions for the existence of the TNL interval observer given in (9), the convergence of the closed-loop system described by (18) and the minimization of the effects of disturbance w_σ. It is the main contribution of this paper.

Theorem 1. *If there exist a symmetric positive definite matrix $P_{1,i}$ and positive diagonal matrices $\overline{P}_i$, $\underline{P}_i$ with $P_i = diag(P_{1,i}, \overline{P}_i, \underline{P}_i)$ and multiple ISS Lyapunov function $V_\sigma(\chi_k)$ switching among $V_i(\chi_k) = \chi_k^T P_i \chi_k$, matrices $\underline{W}_i$, $\overline{W}_i$ and $\underline{Y}_i^j$, $\overline{Y}_i^j$ and $\underline{Q}_i$, $\overline{Q}_i$ for all $i \in \mathcal{N}, j \in \mathcal{P}$ and constant scalars $\gamma_i > 0$, $\varepsilon > 0$, $0 < \alpha_1 < \alpha_2$, $\kappa_i > 0$ such that the following LMIs hold:*

$$\min_{P_i, \underline{W}_i, \overline{W}_i, \underline{Q}_i, \overline{Q}_i, \underline{Y}_i^j, \overline{Y}_i^j} \gamma \qquad \gamma_i \leq \gamma \tag{20}$$

$$\alpha_1 I_{3n_x+2n_f} \leq P_i \leq \alpha_2 I_{3n_x+2n_f} \tag{21}$$

$$O_1 = \begin{bmatrix} -P & X_P & P_{23} \\ (*) & (\varepsilon-1)P & 0 \\ (*) & (*) & -\gamma_i \end{bmatrix} \prec 0 \tag{22}$$

$$O_2 = \begin{bmatrix} X_P^{22} + \kappa_i\overline{P}_i & 0 \\ 0 & X_P^{33} + \kappa_i\underline{P}_i \end{bmatrix} \geq 0 \tag{23}$$

then the system (19) robustly estimates the lower and upper bounds of the augmented state and stabilizes the closed-loop system with $\underline{K}_i = \underline{W}_i P_{1,i}^{-1}$, $\overline{K}_i = \overline{W}_i P_{1,i}^{-1}$ and $\underline{L}_i^j = \underline{P}^{-1}\underline{Y}_i^j$, $\overline{L}_i^j = \overline{P}^{-1}\overline{Y}_i^j$ and $\underline{Z}_i = \underline{P}_i^{-1}\underline{Q}_i$, $\overline{Z}_i = \overline{P}_i^{-1}\overline{Q}_i$.

$$\begin{aligned} X_P^{11} &= \mathcal{A}_i(\rho_k)P_{1,i} - \mathcal{B}_i(\rho_k)\overline{W}_i - \mathcal{B}_i(\rho_k)\underline{W}_i \\ X_P^{22} &= \overline{P}_i\Omega_i^\dagger\alpha A_i(\rho_k) + \overline{Q}_i\Theta_i\alpha A_i(\rho_k) - \overline{Y}_i(\rho_k)C \\ X_P^{33} &= \underline{P}_i\Omega_i^\dagger\alpha A_i(\rho_k) + \underline{Q}_i\Theta_i\alpha A_i(\rho_k) - \underline{Y}_i(\rho_k)C \\ X_P &= diag\left(X_P^{11}, X_P^{22}, X_P^{33}\right) \\ P &= \left(P_{1,i}, \overline{P}_i, \underline{P}_i\right), \; P_{23} = \left(I_{n_x}, \overline{P}_i, \underline{P}_i\right) \end{aligned} \tag{24}$$

In addition, the system (19) is ISS with respect to disturbance w_k under any switching signal with ADT (7), then

$$\lim_{k\to\infty} \|\chi_k\|_2 \leq \sqrt{\frac{\gamma}{\alpha_1\varepsilon}} \max \|w_i\|_\infty \tag{25}$$

Proof. Theorem 1. The following multiple quadratic ISS-Lyapunov functions applied to the augmented vector χ_k of the system (19) are defined by

$$V_i(\chi_k) \;= x_k^T P_{1,i}^{-1} x_k + \overline{e}_k^T \overline{P}_i \overline{e}_k + \underline{e}_k^T \underline{P}_i \underline{e}_k \tag{26}$$

Taking the derivative of (26) along the trajectory of (19) and with any $0 < \varepsilon < 1$, $\gamma_i > 0$ one can have:

$$\Delta V_i = \begin{bmatrix} \chi_k^T & w_k^T \end{bmatrix} \Xi \begin{bmatrix} \chi_k \\ w_k \end{bmatrix} - \varepsilon\chi_k^T P \chi_k + \gamma_i w_i^T w_i \tag{27}$$

where $\begin{cases} \Xi = \begin{bmatrix} X^T P' X + (\varepsilon-1)P' & X^T P' \\ (*) & P' - \gamma_i \end{bmatrix} \\ X = diag(X_{11}, X_{22}, X_{33}), \; P' = diag(P_1^{-1}, \overline{P}_i, \underline{P}_i) \\ X_{11} = \mathcal{A}_i(\rho_k) - \mathcal{B}_i(\rho_k)\overline{K}_i - \mathcal{B}_i(\rho_k)\underline{K}_i \\ X_{22} = \overline{T}_i A_i(\rho_k) - \overline{L}_i(\rho_k)C, \; X_{33} = \underline{T}_i A_i(\rho_k) - \underline{L}_i(\rho_k)C \end{cases}$

The following condition is a sufficient condition for system (19) to be input-to-state-stable:

$$\Xi \prec 0 \tag{28}$$

The term Ξ can be rewritten as follows:

$$\Xi = \begin{bmatrix} (\varepsilon-1)P' & 0 \\ 0 & -\gamma_i \end{bmatrix} + \begin{bmatrix} (P'X)^T \\ P' \end{bmatrix} P'^{-1} \begin{bmatrix} P'X & P' \end{bmatrix} \tag{29}$$

Applying Schur complement to (29), one can get:

$$\begin{bmatrix} -P' & P'X & P' \\ (*) & (\varepsilon-1)P' & 0 \\ (*) & (*) & -\gamma_i \end{bmatrix} \prec 0 \tag{30}$$

Pre-and post multiply (30) by $P_p = diag\left(P_I, P_I, I_{n_x+n_f}\right)$ with $P_I = diag\left(P_1, I_{n_x+n_f}, I_{n_x+n_f}\right)$, one obtains LMI (22). If the sufficient condition (28) holds, it follows that

$$\Delta V_i \prec -\varepsilon\chi_k^T P_i \chi_k + \gamma_i w_i^T w_i \tag{31}$$

it indicates that

$$V_i(\chi_{k+1}) \prec (1-\varepsilon)V_i(\chi_k) + \gamma_i\|w_i\|^2 \tag{32}$$

Then integrating (32) on the interval $[k_0, k)$ one can get the inequality

$$V(\chi_k) \prec (1-\varepsilon)^{k-k_0}V(\chi_{k_0}) + \gamma_i \textstyle\sum_{m=0}^{k-k_0-1}(1-\varepsilon)^m\|w_i\|^2 \tag{33}$$

with LMI condition (21), it allows that

$$\|\chi_k\|_2 \leq \sqrt{\frac{(1-\varepsilon)^{k-k_0}\|\chi_{k_0}\|_2^2 + \gamma_i\sum_{m=0}^{k-k_0-1}(1-\varepsilon)^m\|w_i\|_2^2}{\alpha_1}} \tag{34}$$

Once $k \to \infty$ and $\gamma_i \leq \gamma$, LMI condition (25) is obtained.

Moreover, $\underline{T}_i A_i(\rho_k) - \underline{L}_i(\rho_k)C$ and $\overline{T}_i A_i(\rho_k) - \overline{L}_i(\rho_k)C$ are Metzler matrices, if $\underline{T}_i A_i(\rho_k) - \underline{L}_i(\rho_k)C + \kappa_i I_{n_x+n_f} \geq 0$ and $\overline{T}_i A_i(\rho_k) - \overline{L}_i(\rho_k)C + \kappa_i I_{n_x+n_f} \geq 0$ with $\kappa_i > 0, \forall i \in \mathcal{N}$. Multiplying by $\underline{P}_i$ and $\overline{P}_i$ on the left side and performing the change of variables $\underline{Y}_i(\rho_k) = \underline{P}_i\underline{L}_i(\rho_k)$, $\overline{Y}_i(\rho_k) = \overline{P}_i\overline{L}_i(\rho_k)$, one can obtain the LMI in (23). This completes the proof. □

IV. Application to Lateral Vehicle Dynamics

This section applies the developed procedure to the lateral vehicle dynamics model. A single-track 2-DOF model is obtained from the nonlinear vehicle dynamics model with the second law of Newton and some simplifying assumptions [17]. The following structure of lateral vehicle dynamics is considered:

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{c_f+c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} - v_x \\ \frac{c_r l_r - c_f l_f}{I_z v_x} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix} u + \begin{bmatrix} \frac{c_f}{m} \\ \frac{c_f l_f}{I_z} \end{bmatrix} \delta_f \tag{35}$$

where v_y is lateral velocity, r is yaw rate, δ_f is front steering angle commanded by the driver. The controller outputs is $u = \Delta M_z$ which is a yaw moment obtained from differential wheels braking. The distances from the front and rear axles to the center of gravity are represented by l_f and l_r. Moreover, vehicle mass m, longitudinal vehicle velocity v_x, moment of inertia I_z are other important parameters that determine how the vehicle responds to the lateral dynamics.

The cornering stiffness parameters c_f and c_r depend on the friction of the road surface and cannot be directly observable. To improve the accuracy of the model, the correction variables Δc_f and Δc_r are introduced to account for the errors in the cornering stiffness as follows: $c_f = c_{f_0} + \Delta c_f$ and $c_r = c_{r_0} + \Delta c_r$ where c_{f_0} and c_{r_0} are the nominal values of the front and rear tire cornering stiffness while Δc_f and Δc_r are the unknown but bounded uncertainties in the front and rear one. For the simulation scenario, it is assumed that the nominal values of the cornering stiffness parameters have an uncertainty of 20%.

By using the Euler's discretization approach along with sampling time $T_s = 0.01\,\text{sec}$, a discrete-time switched LPV system of lateral vehicle dynamics model (35) with actuator fault, process disturbances and measurement noise can be obtained by as follows:

$$\begin{cases} x_{k+1} = \big(\mathcal{A}_\sigma(\rho_k) + \Delta\mathcal{A}_\sigma(\rho_k)\big)x_k + \mathcal{B}u_k \\ \qquad\quad + (\mathcal{D} + \Delta\mathcal{D})\delta_f + F f_k + \mathcal{D}^w w_k \\ y_k = \mathcal{C}x_k + D^v v_k \end{cases} \tag{36}$$

where the distribution matrices $\mathcal{A}_\sigma(\rho_k) = I_{n_x} + \begin{bmatrix} -\frac{c_{f_0}+c_{r_0}}{mv_x} & \frac{c_{r_0}l_r - c_{f_0}l_f}{mv_x} - v_x \\ \frac{c_{r_0}l_r - c_{f_0}l_f}{I_z v_x} & -\frac{c_{r_0}l_r^2 + c_{f_0}l_f^2}{I_z v_x} \end{bmatrix} \times T_s$, $\mathcal{B} = F = \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix} \times T_s$, $\mathcal{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $\mathcal{D} = \begin{bmatrix} \frac{c_{f_0}}{m} \\ \frac{c_{f_0}l_f}{I_z} \end{bmatrix} \times T_s$, $\Delta\mathcal{A}_\sigma(\rho_k) = \begin{bmatrix} -\frac{\Delta c_f+\Delta c_r}{mv_x} & \frac{\Delta c_r l_r - \Delta c_f l_f}{mv_x} - v_x \\ \frac{\Delta c_r l_r - \Delta c_f l_f}{I_z v_x} & -\frac{\Delta c_r l_r^2 + \Delta c_f l_f^2}{I_z v_x} \end{bmatrix} \times T_s$, $\Delta\mathcal{D} = \begin{bmatrix} \frac{\Delta c_f}{m} \\ \frac{\Delta c_f l_f}{I_z} \end{bmatrix} \times T_s$, $\mathcal{D}^w = \begin{bmatrix} \frac{1}{m} \\ \frac{l_w}{I_z} \end{bmatrix} \times T_s$, $D^v = 1$ and w_k is disturbance wind gust, measurement noise v_k, distance wind fore action l_w.

The vehicle motion is managed by two parameters known as longitudinal velocity and steering angle as depicted at the top of Figure 1. The steering angle $\delta_f(t)$ is the known control input which is controlled by the driver through the steering wheel of the vehicle. As shown in Figure 1, the longitudinal velocity is considered as a time-varying parameter varying in a range from 12.5 to $28.5(m/s)$. The polytopic representation is thus obtained by considering the two-dimensional vector $\rho_k = [\rho_{1,k}, \rho_{2,k}] = \left[v_x, \frac{1}{v_x}\right]$.

Moreover, three subsystems are considered with switching signal $\sigma(t)$ given as:

$$\sigma(t) = \begin{cases} 1, & 12.5(m.s^{-1}) \le v_x < 17.5(m.s^{-1}) \\ 2, & 17.5(m.s^{-1}) \le v_x < 22.5(m.s^{-1}) \\ 3, & 22.5(m.s^{-1}) \le v_x \le 28.5(m.s^{-1}) \end{cases} \tag{37}$$

The disturbance wind gust and measurement noise are shown at the bottom of Figure 1. The wind gust affects the vehicle dynamics from 50 sec to 200 sec with the variations between $250N$ and $450N$. The measurement noise is normally distributed random signal bounded by $\begin{bmatrix} -1, & 1 \end{bmatrix}$. As shown at the top of Figure 2, the actuator fault profile consists of a constant and sinusoidal form of frequency $0.35Hz$ between 20 sec and 145 sec and $0.5Hz$ from 160 sec 225 sec. This fault profile is applied as a differential torque input in lateral vehicle dynamics.

Using the Matlab SEDUMI optimization tools [18], Theorem 1 provides a set of LMIs that is solved by minimizing γ. The observer and controller gain matrices are obtained by solving LMIs in Theorem 1. The simulation results obtained by the proposed method are depicted in Figure 2 to 3.

The bottom of Figure 2 shows a good estimation of lateral velocity v_y and yaw rate r of the vehicle in which the solid red lines represent the actual system states and the dashed green and blue lines represent the lower and upper bounds. At any instant time, the actual state is always bounded by the lower and upper estimation.

The top of Figure 2 shows the estimation of actuator fault signal in which $\underline{f}_k$ and $\overline{f}_k$ are denoted by the dashed green and blue lines, respectively. Additionally, the figure illustrates the control input signal. The designed controller ensures the desired stability and performance of the closed-loop system under actuator fault conditions. The proposed FTC system can also effectively compensate for the actuator fault effects and maintain the vehicle operation as demonstrated in Figure 3. The faulty system without FTC presented by the orange line is unable to deal with the fault and exhibits significant deviation from the fault-free system due to the fault effects. This result is achieved by the design of FTC that adapts the gain according to increasing actuator fault thanks to the reconfiguration mechanism as demonstrated at the top right of Figure 2.

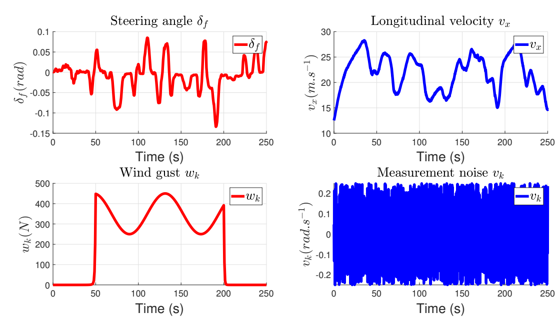

Fig. 1. Steering angle δ_f, longitudinal velocity v_x, disturbance wind gust w_k and measurement noise v_k profiles.

To demonstrate the efficacy of co-designing the observers and controllers in this paper, the proposed approach is compared to the conventional separate-design strategy discussed in [11]. The comparative analysis of both methodologies is illustrated in Figure 3. The results depicted at the top of Figure 3 show that the augmented state estimation accuracy achieved by the proposed approach surpasses that of the approach investigated in [11].

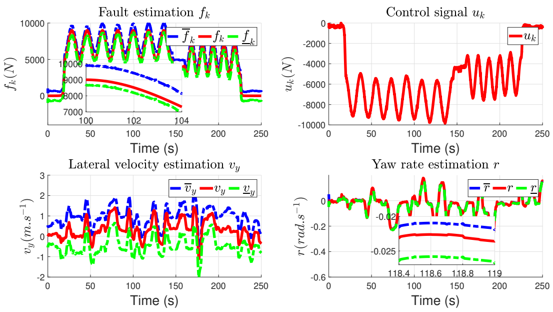

Fig. 2. Actuator fault estimation and control input signal u_k, lateral velocity v_y and yaw rate r estimations.

In contrast to the Separate Principle-based design methodology, the co-design of observer and controller shows better stability and performance characteristics as evidenced at the bottom of Figure 3. The outcomes obtained from both methods are compared to the state of the ideal fault-free system without the uncertainties, external disturbances and measurement noise. Notably, the system states obtained through the co-design framework remain closer to the reference states, highlighting its effectiveness over the conventional separate design approach. This improvement is reasonable since the bi-directional interaction between observers and controllers is taken into consideration during the control system design process.

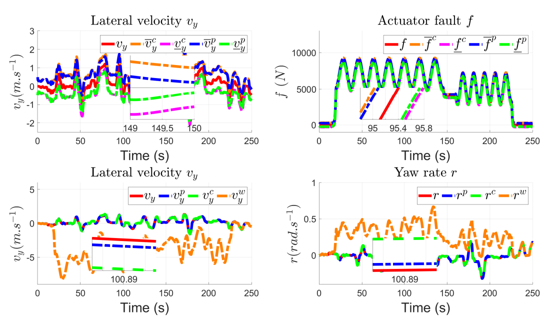

Fig. 3. The estimation between the proposed and compared method, the performance of the system with and without FTC.

V. Conclusions and Future work

This paper presented a co-design method of an integrated observer and FTC for uncertain switched LPV systems subjected to faults and unknown but bounded uncertainties, disturbances and measurement noise. The co-design approach allows for tighter integration between the observer and controller. A TNL interval observer is designed to jointly estimate the lower and upper bounds of the system states and the actuator faults. Afterward, a FTC is synthesized to stabilize the closed-loop system and compensate for fault effects. The existence conditions are formulated in terms of LMIs with multiple ISS-switched Lyapunov functions under the ADT. These conditions ensure stability, positivity and error attenuation. Introducing weighting matrices T and N provides more design degrees of freedom in the determination of the observer gain matrices. The proposed approach proves its efficiency when applied to lateral vehicle dynamics estimation and control as shown by the simulation results conducted on varying longitudinal velocity and steering angle profiles. Future works will consider the validation on experimental data obtained with a prototype vehicle. The conservativeness of the LMI conditions will be further reduced by exploring the use of parameter-dependent Lyapunov functions.

References

[1] H. L. Trentelman and P. Antsaklis, "Observer-based control," *Encyclopedia of Systems and Control*, pp. 1–6, 2013.

[2] R. Lamouchi, T. Raissi, M. Amairi, and M. Aoun, "Interval observer framework for fault tolerant control of linear parameter-varying systems," *International Journal of Control*, vol. 91, pp. 1–16, 01 2017.

[3] K. H. Degue, D. Efimov, J. Le Ny, and E. Feron, "Interval observers for secure estimation in cyber-physical systems," *2018 IEEE Conference on Decision and Control (CDC)*, pp. 4559–4564, 2018.

[4] D. Efimov, S. Li, Y. Hu, S. Muldoon, H. Javaherian, and V. O. Nikiforov, "Application of interval observers to estimation and control of air-fuel ratio in a direct injection engine," *2015 American Control Conference (ACC)*, pp. 25–30, 2015.

[5] H. Ethabet, D. Rabehi, D. Efimov, and T. Raïssi, "Interval estimation for continuous-time switched linear systems," *Automatica*, vol. 90, pp. 230–238, 2018.

[6] E. Chambon, L. Burlion, and P. Apkarian, "Overview of lti interval observer design: towards a non-smooth optimisation-based approach," *IET Control Theory & Applications*, vol. 10, pp. 1258–1268, 07 2016.

[7] T. Chevet, A. Rauh, T. N. Dinh, J. Marzat, and T. Raïssi, "Robust interval observer for systems described by the fornasini–marchesini second model," *IEEE Control Systems Letters*, vol. 6, pp. 1940–1945, 2022.

[8] T. Chevet, T. N. Dinh, J. Marzat, and T. Raissi, "Robust sensor fault detection for linear parameter-varying systems using interval observer," *Proceedings of the 31st European Safety and Reliability Conference (ESREL 2021)*, pp. 1486–1493, 01 2021.

[9] T. Chevet, T. N. Dinh, J. Marzat, and T. Raïssi, "Interval estimation for discrete-time linear parameter-varying system with unknown inputs," *2021 60th IEEE Conference on Decision and Control (CDC)*, pp. 4002–4007, 2021.

[10] C. Zammali, J. Van Gorp, X. Ping, and T. Raïssi, "Simultaneous interval state and fault estimation for continuous-time switched systems," *IFAC-PapersOnLine*, vol. 55, no. 25, pp. 73–78, 2022, 10th IFAC Symposium on Robust Control Design ROCOND 2022.

[11] R. Lamouchi, T. Raissi, M. Amairi, and M. Aoun, "On interval observer design for active fault tolerant control of linear parameter-varying systems," *Systems & Control Letters*, vol. 164, p. 105218, 2022.

[12] J. Lan and R. J. Patton, "A new strategy for integration of fault estimation within fault-tolerant control," *Automatica*, vol. 69, pp. 48–59, 2016.

[13] T. N. Dinh, G. Marouani, T. Raïssi, Z. Wang, and H. Messaoud, "Optimal interval observers for discrete-time linear switched systems," *International Journal of Control*, pp. 1–9, 2020.

[14] Q. Yu and H. Lv, "Stability analysis for discrete-time switched systems with stable and unstable modes based on a weighted average dwell time approach," *Nonlinear Analysis: Hybrid Systems*, vol. 38, p. 100949, 2020.

[15] J. Li, Z. Wang, Y. Shen, and Y. Wang, "Interval observer design for discrete-time uncertain takagi–sugeno fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 4, pp. 816–823, 2019.

[16] C. Zammali, J. V. Gorp, X. Ping, and T. Raïssi, "Interval estimation for discrete-time lpv switched systems," *2019 IEEE 58th Conference on Decision and Control (CDC)*, pp. 2479–2484, 2019.

[17] B. Lacroix, Z. Liu, and P. Seers, "A comparison of two control methods for vehicle stability control by direct yaw moment," *Applied Mechanics and Materials*, vol. 120, pp. 203–217, 01 2012.

[18] J. F. Sturm, "Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones," *Optimization Methods and Software*, vol. 11, no. 1-4, pp. 625–653, 1999.