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Asynchronous Fuzzy Control for a Class of Constrained Fuzzy Stochastic Switching Systems via Bumpless MPC

Bo Li[1], Rui Weng[2], Bo Cai[1*]

Abstract— **This paper addresses the problem of bumpless stabilization for a class of T-S fuzzy hidden Markovian jump systems with bounded inputs and states. The asynchronous scenario between the real mode and the observed one under consideration is described by hidden Markovian chains. Moreover, the stabilizing controller dependent on both the fuzzy rule and the observed mode is constructed. By virtue of the bumpless model predictive control, the controller not only maintains the system within constraints but also reduces excessive bumps of system states in switching instants. Then, relying on the stochastic stability criteria, a numerically attainable receding horizon optimization problem is established to obtain the recursive feasible and stabilizing controller with the smooth evolution of states by introducing some mathematical techniques. Finally, demonstrating the validity and potential of the developed control strategy is achieved through the presentation of a numerical example.**

I. INTRODUCTION

Markovian jump systems (MJSs), which are a significant category of stochastic switching systems, have been extensively probed and employed in various fields over the past decades [1], [2], [3], [4]. MJSs consist of finite linear or nonlinear subsystems and discrete stochastic switching signals. Owing to the ability to approximate nonlinear systems, Takagi–Sugeno (T–S) fuzzy model is prevalent for modeling the nonlinear subsystems of MJSs. With the real system mode employed directly and integrally, T-S fuzzy Markovian jump systems (F-MJSs) are universally utilized in engineering applicabilities e.g., [5], [6], [7]. In fact, the hypothesis is often unrealistic due to the errors of the measurement appliances and the faults of transducers in practice. Therefore, hidden Markovian jump systems (HMJSs) are introduced to describe the asynchronous scenario [8], [9], where the relation between observed system modes and real switched modes depends on a high-level Markovian chain with emission probabilities (EPs). Based on hidden Markovian chains (HMCs), extensive research has been conducted on asynchronous problems [10], [11], [12].

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[1]Bo Li and Bo Cai are with the School of Astronautics, Harbin Institute of Technology, Harbin, 150080, China. {hitlibo, bcai}@hit.edu.cn.

[2]Rui Weng is with the Faculty of Computing, Harbin Institute of Technology, Harbin, 150080, China. wengrui@hit.edu.cn.

[*]Bo Cai is the corresponding author.

On the other hand, for the switched system, it is noteworthy that the majority of current studies on the stabilization issue have not taken transient characteristics into account. However, the excessive bumps of system states in switching instants may cause performance degradation and actuator damage [13], [14]. It is necessary to balance the steady-state characteristics and the transient characteristics when studying the stabilization of the switched system. In addition, the uncertainty and randomness of the random switching system deepen the difficulty of research. Currently, the bumpless transfer controller for MJSs is designed by imposing constraints on the controller to suppress or eliminate bad bumpless transfer performance during system switching (see [15], [16], [17] and the references therein). However, the controller design only constrains switching bumps to a certain range and does not fully consider system transient performance. Additionally, state and input constraints are often present in practical systems, making the investigations of these problems more challenging. Fortunately, the model predictive control (MPC), a real-time optimized control algorithm, can handle the above issues by virtue of the capability of directly handling control objectives, system constraints, and multiple inputs/outputs [18], [19], [20]. However, as a result of the complexities arising from the randomness and nonlinearity of fuzzy hidden Markovian jump systems (F-HMJSs), the related research has not been reported to the best of the authors' knowledge.

Motivated by the above-mentioned observations, this paper is devoted to addressing the problem of bumpless MPC for a class of constrained F-HMJSs. There are three main contributions. First, it is a new attempt that the asynchronous predictive control is investigated for HMC-based F-MJSs with state bumpless performance. Second, the proposed F-HMJSs are more general and practical, which can cover the MJSs, HMJSs and F-MJSs as three special cases. Third, the concept of bumpless MPC is defined and introduced to deal with the state bump phenomenon. Based on the bumpless MPC framework, the accessible control law is established to ensure both the pivotal feasibility and stability property and the smooth evolution of states for F-HMJSs.

Notation: $\mathbb{R}^n$ represents the n-dimensional Euclidean space. The superscript $\top$ is a vector or matrix transposition. diag$\{...\}$ stands for a block-diagonal matrix. $\mathbf{I}$ and $\mathbf{0}$ are the identity and zero matrices, respectively. $\|\cdot\|$ denotes the Euclidean vector norm. Moreover, $x(n;k)$ and $u(n;k)$ mean the predicted state and input for the time $k{+}n$, with reference to time k, respectively. $*$ stands for the omitted terms in a symmetric matrix. A negative (semi-negative) definite matrix

P is represented by $P \prec \mathbf{0}$ ($P \preceq \mathbf{0}$). $A^{(i)}$ denotes the ith row of the matrix A. $[q]_\alpha$ means the αth element of the vector q. In addition, $\mathbb{E}[x]|_y$ expresses the mathematical expectation of x under the condition y. Given the event Y, the probability of the event X occurring is expressed by notation $\mathbb{P}\{X|Y\}$.

II. Problem Formulation and Preliminaries

A. System Description

Consider the class of constrained F-HMJSs in the following form:

Plant rule i: IF $\rho_1(k)$ is $\lambda_{i,1}$, $\rho_2(k)$ is $\lambda_{i,2}$,..., and $\rho_\alpha(k)$ is $\lambda_{i,\alpha}$, THEN

$$x(k+1) = A_{r_k,i}x(k) + B_{r_k,i}u(k), \tag{1}$$

where $x(k) \in \mathbb{R}^{N_x}$ and $u(k) \in \mathbb{R}^{N_u}$ denote the state vector and the control input vectors of the system, respectively; $i \in \mathfrak{I} \triangleq \{1, 2, \cdots, I\}$ is the number of the fuzzy rules, $\rho(k) \triangleq [\rho_1(k), \rho_2(k), ..., \rho_p(k)]^\top$ is the premise variable vector at the time of k and $\{\lambda_{i,1}, \lambda_{i,2}, ..., \lambda_{i,\alpha}\}$ is a fuzzy set for the ith fuzzy rule; the system switched mode r_k takes values in a finite set $\mathfrak{M} \triangleq \{1, 2, \cdots, M\}$, where jump process obeys a Markovian chain; and $A_{r_k,i}$ and $B_{r_k,i}$ represent the system matrices of the real mode r_k. The transition probabilities (TPs) for the Markovian process $\{r_k\}_{k\geq 0}$ are defined by:

$$\mathbb{P}\{r_{k+1} = n | r_k = m\} = \pi_{mn}, \tag{2}$$

where $0 \leq \pi_{mn} \leq 1$, $\sum_{n=1}^{M} \pi_{mn} = 1$ for any $m, n \in \mathfrak{M}$, and the relevant TP matrix is represented by $\Pi \triangleq [\pi_{mn}]_{m,n\in\mathfrak{M}}$.

By the means of the T-S fuzzy approach, (1) with $r_k = m$ can be derived as

$$x(k+1) = \bar{A}_m^k x(k) + \bar{B}_m^k u(k), \tag{3}$$

$$\bar{A}_m^k \triangleq \sum_{i=1}^{I} h_i^k A_{m,i}, \bar{B}_m^k \triangleq \sum_{i=1}^{I} h_i^k B_{m,i},$$

where $h_i^k \triangleq \lambda_i[\rho(k)] / \sum_{i=1}^{I} \lambda_i[\rho(k)]$, $\lambda_i[\rho(k)] \triangleq \prod_{j=1}^{\alpha} \lambda_{i,j}[\rho_j(k)]$, $\lambda_{i,j}[\rho_j(k)]$ is the grade of membership $\rho_j(k)$ in $\lambda_{i,j}$, and h_i^k denotes the normalized membership function of rule i at the time k by using the center-average defuzzification method. Note that for any $i \in \mathfrak{I}$, $0 \leq h_i^k \leq 1$ and $\sum_{i=1}^{I} h_i^k = 1$.

In addition, considering the system performance and actuator ranges in engineering practice, the constraints on states and control inputs are considered as below:

$$|[\Psi x(k)]_p| \leq [\overline{x}]_p, p \in \mathfrak{L}_{N_\psi} \triangleq \{1, 2, \cdots, N_\psi\}, \tag{4}$$

$$|[u(k)]_q| \leq [\overline{u}]_q, q \in \mathfrak{L}_{N_u} \triangleq \{1, 2, \cdots, N_u\}, \tag{5}$$

where $\overline{x} \in \mathbb{R}^{N_\psi}$ and $\overline{u} \in \mathbb{R}^{N_u}$ are the limit vectors on states and control inputs absolute values, respectively; and $\Psi \in \mathbb{R}^{N_\psi \times N_x}$ stands for the state constraint gain matrix.

B. Homogeneous Emission Probability

In order to describe the asynchronous phenomenon between the observed mode and the actual mode caused by the sensor fault and other causes, another Markovian chain $\{\sigma_k\}_{k\geq 0}$ is provided to express the observed mode of Markovian process $\{r_k\}_{k\geq 0}$, which takes values in a finite set $\mathfrak{V} \triangleq \{1, 2, \cdots, V\}$. Moreover, the homogeneous emission probabilities (EPs) are introduced to describe the law between the system mode r_k and the observed mode σ_k as follows:

$$\mathbb{P}\{\sigma_k = v | r_k = m\} = \varphi_{mv}, \tag{6}$$

where $0 \leq \varphi_{mv} \leq 1$, and $\sum_{v=1}^{V} \varphi_{mv} = 1$ for any $m \in \mathfrak{M}$, $v \in \mathfrak{V}$. The EP matrix is defined as $\Phi \triangleq [\varphi_{mv}]_{m\in\mathfrak{M}, v\in\mathfrak{V}}$.

Remark 1: EP can be used to represent the degree of detection device failure and the accuracy of the detection of system modes. When $\mathfrak{M} = \mathfrak{V}$ and $\sigma_k = \mathbf{I}$, the system detection device is trouble-free and the system modes can be accurately observed. When $\varphi_{mv} = 0$ for $m = v$, the system detection device is completely faulty and cannot observe accurate modes.

C. Asynchronous Fuzzy Controller

In view of the system nonlinearity and the system mode asynchrony, a fuzzy-rule and observed-mode-dependent state feedback controller will be applied as follows:

$$u(k) = \bar{K}_v^k x(k) = \sum_{j=1}^{I} h_j^k K_{v,j} x(k), \tag{7}$$

where $K_{v,j}$ is the controller gain to be determined. Combining (3) and (7), the overall closed-loop F-HMJSs with the practical system mode $r_k = m$ and the observed mode $\sigma_k = v$ can be rewritten as

$$x(k+1) = \bar{\mathcal{A}}_{mv}^k x(k), \tag{8}$$

where

$$\bar{\mathcal{A}}_{mv}^k \triangleq \sum_{i=1}^{I}\sum_{j=1}^{I} h_i^k h_j^k \mathcal{A}_{mvij}, \mathcal{A}_{mvij} \triangleq A_{m,i} + B_{m,i}K_{v,j},$$

with constraints

$$|[\Psi x(k)]_p| \leq [\bar{x}]_p, p \in \mathfrak{L}_{N_\psi}, \tag{9}$$

$$|[\bar{K}_v^k x(k)]_q| \leq [\bar{u}]_q, q \in \mathfrak{L}_{N_u}. \tag{10}$$

D. Bumpless MPC Framework

Thanks to the exceptional ability to handle hard constraints and favorable control performance, an MPC framework is introduced to design the asynchronous fuzzy controller to stabilize the system (3) while satisfying the states and inputs constraints (4) (5) in this study. By this means, at each time k, a receding horizon optimization problem is solved to get real-time feedback gains. In order to reduce the bump phenomenon of the system state caused by mode switching, a major innovation is that the improved infinite horizon cost function in this optimization problem is constructed in the following form:

$$J^{\infty}(k) \triangleq \mathbb{E}\Bigg[\sum_{n=0}^{\infty}||x(n;k)||^2_{\mathcal{Q}}+||u(n;k)||^2_{\mathcal{R}} + ||\Delta x(n;k)||^2_{\mathcal{S}}\Bigg]\Bigg|_{x(k),r_k}, \quad (11)$$

where $||x(n;k)||^2_{\mathcal{Q}}$ and $||u(n;k)||^2_{\mathcal{R}}$ are for steering the states and inputs to the origin, respectively; $||\Delta x(n;k)||^2_{\mathcal{S}}$ is adopted to smooth the evolution of system states; $\mathcal{Q},\mathcal{R},\mathcal{S}\succ \mathbf{0}$ denote the known positive definite weight matrices; and $\Delta x(n;k)\triangleq x(n+1;k)-x(n;k)$. Then, $l(x,u)\triangleq ||x||^2_{\mathcal{Q}}+||u||^2_{\mathcal{R}}+||\Delta x||^2_{\mathcal{S}}$ is defined as the stage cost. Compared to the traditional MPC, this MPC framework contains the cost of state changes, so it is called bumpless MPC for brevity.

To sum up, at each time k, the controller gains are calculated via the following receding horizon optimization problem:

$$\min_{K_{v,j},v\in\mathfrak{V},j\in\mathfrak{J}} J^{\infty}(k) \quad (12)$$

$$\text{s.t. } x(n+1;k)=\bar{A}^{k+n}_{r_{k+n}}x(n;k)+\bar{B}^{k+n}_{r_{k+n}}u(n;k) \quad (12a)$$

$$u(n;k)=\bar{K}^{k+n}_{\sigma_{k+n}}x(n;k) \quad (12b)$$

$$|[\Psi x(n;k)]_p|\le[\overline{x}]_p, p\in\mathfrak{L}_{N_\psi} \quad (12c)$$

$$|[\bar{K}^{k+n}_{\sigma_{k+n}}x(n;k)]_q|\le[\overline{u}]_q, q\in\mathfrak{L}_{N_u} \quad (12d)$$

Prior to continuing, several definitions and lemma are provided to facilitate the derivation of subsequent results.

Definition 1 (see [10]): The F-HMJS (3) is said to be stochastically stable, if $u(k)\equiv 0$, and the following equation holds:

$$\lim_{k\to\infty}\mathbb{E}\left[\|x(k)\|^2\right]\Big|_{x_0,r_0}=0, \quad (13)$$

for $\forall x(0)=x_0,\ r_0\in\mathfrak{M}$.

Definition 2 (see [21]): For F-HMJS (3), the set Ω_{r_k} is said to be a positive invariant set in the stochastic sense, if the current states belong to the set Ω_{r_k}, and the future system states $x(k+n)$ for $n\in\mathbb{N}^+$ belong to such a set $\Omega_{r_{k+n}}$ in the stochastic sense.

Lemma 1 (see [11]): Consider a discrete-time stochastic switching system $x(k+1)=f(x(k),r_k)$, where r_k denotes the system switch mode, and suppose $X\subset\mathbb{R}^{N_x}$ is a positive invariant in the stochastic sense for this system. The system is stochastically stable if there exists a set of functions $V(x(k),r_k):\mathbb{R}^{N_x}\to\mathbb{R}$, three class $\mathcal{K}_\infty$ functions α_1, α_2, and α_3, such that for $\forall x(k)\in X$, $r_k\in\mathfrak{M}$,

$$\alpha_1(||x(k)||)\le V(x(k),r_k)\le\alpha_2(||x(k)||), \quad (14)$$

$$\mathbb{E}\left[V(x(k+1),r_{k+1})\right]\big|_{x(k),r_k}-V(x(k),r_k)\le-\alpha_3(||x(k)||). \quad (15)$$

III. Main Results

In this section, the feasibility and stability analysis for the system (8) based on Lemma 1 will be determined and an asynchronous fuzzy control strategy under the framework of bumpless MPC will be further presented.

A. Feasibility and Stability Analysis

First of all, a Lyapunov function depends not only on system mode but also on the fuzzy basis and optimization is constructed as below:

$$V(x(k),r_k=m)\triangleq x^\top(k)\bar{P}^k_m(k)x(k), \quad (16)$$

where $\bar{P}^k_m(k)\triangleq\sum_{i=1}^{I}h^k_iP_{m,i}(k)$, and $P_{m,i}(k)$ is obtained at each time k via the MPC approach.

Then, the following definitions of F-HMJS (3) are presented based on the constructed Lyapunov function (16).

Definition 3: (Invariance) The set represented as

$$\Omega_{r_k}\triangleq\left\{x(k)|x^\top(k)\bar{P}^k_{r_k}(k)x(k)\le\gamma\right\}$$

is a positive invariant set for F-HMJS (8) in the stochastic sense, i.e., for any $n\in\mathbb{N}^+$

$$\mathbb{E}\left[x^\top(k+n)\bar{P}^{k+n}_{r_{k+n}}(k)x(k+n)\right]\Big|_{x(k),r_k}\le\gamma. \quad (17)$$

Definition 4: (Stability) Considering the closed-loop system (8) for any initial condition $x(k)\in\Omega_{r_k}$, $r_k\in\mathfrak{M}$, it holds that

$$\begin{aligned}&\mathbb{E}\left[V(x(n+1;k),r_{k+n+1})-V(x(n;k),r_{k+n})\right]\big|_{x(n;k),r_{k+n}}\\ &\le-\mathbb{E}\left[l(x(n;k),u(n;k))\right]\big|_{x(n;k),r_{k+n}},\end{aligned} \quad (18)$$

where $V(x(n;k),r_{k+n})\triangleq x^\top(n;k)\bar{P}^{k+n}_{r_{k+n}}(k)x(n;k)$.

In the following, based on the above definitions, it is possible to initiate the sufficient conditions to ensure that the closed-loop system (8) is both feasible and stable.

Theorem 1: For F-HMJSs (3) with constraints (4) and (5), if there exists a solution at initial time $k=0$ of the following optimization problem for any $x(0)\in\Omega_{r_0}$

$$\min_{K_{v,i},P_{m,i},i\in\mathfrak{J},v\in\mathfrak{V},m\in\mathfrak{M}}\gamma, \quad (19)$$
$$\text{s.t. (12a)-(12d), (17), (18)}$$

then feasible solutions at each time k, $k\in\mathbb{N}^+$ will still exist, and the closed-loop F-HMJS (8) can be stochastically stable.

Proof: (Upper bound) Taking the summation on both sides of (18) from 0 to ∞, it implies that $J^\infty(k)\le V(x(k),r_k)\le\gamma$, so that, γ is the upper bound of the cost function $J^\infty(k)$. The upper bound can be minimized in place of the cost function to obtain the controller.

(Feasibility) By denoting the solution at the time k as $\mathbf{u}^*\triangleq[u^*(k),u^*(k+1),\cdots]$, it holds that $\bar{A}^k_mx(k)+\bar{B}^k_mu^*(k)=x(1;k)$. Then, the feasibility of the control law $\mathbf{u}^*$ for the successor state $x(k+1)$ at time $k+1$ can be demonstrated.

(Stability) First, we define the optimal values at the time k and $k+1$ as $P_{m,i}(k),P_{m,i}(k+1)$, for any $m\in\mathfrak{M}$ and $i\in\mathfrak{J}$, respectively, and it follows from (18) that

$$\begin{aligned}&\mathbb{E}\left[x^\top(1;k)\bar{P}^{k+1}_{r_{k+1}}(k)x(1;k)\right]\Big|_{x(k),r_k}\\ &\quad-x^\top(k)\bar{P}^k_{r_k}(k)x(k)\le-l(x(k),u(k)),\end{aligned} \quad (20)$$

Next, by the optimum principle, it can obtain that

$$\begin{aligned}&x^\top(k+1)\bar{P}^{k+1}_{r_{k+1}}(k+1)x(k+1)\\ &\quad\le\mathbb{E}\left[x^\top(k+1)\bar{P}^{k+1}_{r_{k+1}}(k)x(k+1)\right]\Big|_{x(k),r_k}.\end{aligned} \quad (21)$$

By taking mathematical expectation $\mathbb{E}\left[\cdot\right]|_{x(k),r_k}$ on both sides of (21) and together with (20), the following holds:

$$\mathbb{E}\left[x^\top(k+1)\bar{P}^{k+1}_{r_{k+1}}(k+1)x(k+1)\right]\Big|_{x(k),r_k} - x^\top(k)\bar{P}^k_{r_k}(k)x(k) \leq -l(x(k),u(k)). \quad (22)$$

Apparently, the following relation can be ensured

$$l(x(k),u(k)) \leq V(x(k),r_k) \leq x^\top(k)\bar{P}^k_{r_k}(0)x(k). \quad (23)$$

Then, as a result of combining (22) and (23) using Lemma 1, it can be inferred that the closed-loop system (8) is stochastically stable. ∎

B. Asynchronous Fuzzy Control Strategy

Building upon the aforementioned theorem, the tractable controller design method are further explored.

Theorem 2: For the discrete closed-loop F-HMJS (8) in the framework of bumpless MPC (12), if there exists a finite constant γ, sets of matrices $\mathcal{P}_{m,i}, \mathcal{Z}^{m,v}_{s,t,i} \succ \mathbf{0}$, invertible matries Y_v, and matries $\mathcal{K}_{v,i}$, for any s, t, $i \in \mathfrak{I}$, $m \in \mathfrak{M}$, and $v \in \mathfrak{V}$, such that it holds that

$$\begin{bmatrix} -Y_v - Y_v^\top + \mathcal{Z}^{m,v}_{s,t,i} & * & * \\ \Theta_m\Gamma^{m,v}_{s,t} & -\boldsymbol{P}_i & * \\ \Upsilon^{m,v}_{s,t} & \mathbf{0} & -\gamma\mathbf{I} \end{bmatrix} \preceq \mathbf{0}, \quad (24)$$

$$\begin{bmatrix} -4\mathcal{P}_{m,s} & * & * \\ \Xi_m\mathcal{P}_{m,s} & -\boldsymbol{Z}^m_{s,t,i} & * \\ 2\mathcal{Q}^{\frac{1}{2}}\mathcal{P}_{m,s} & \mathbf{0} & -\gamma\mathbf{I} \end{bmatrix} \preceq \mathbf{0}, \quad (25)$$

$$\begin{bmatrix} -1 & * \\ x(k) & -\mathcal{P}_{m,i} \end{bmatrix} \preceq \mathbf{0}, \quad (26)$$

where

$$\begin{aligned} \Theta_m &\triangleq [\sqrt{\pi_{m1}}, \sqrt{\pi_{m2}}, \cdots, \sqrt{\pi_{mM}}]^\top, \\ \Xi_m &\triangleq [\sqrt{\varphi_{m1}}, \sqrt{\varphi_{m2}}, \cdots, \sqrt{\varphi_{mV}}]^\top, \\ \boldsymbol{P}_i &\triangleq \text{diag}\{\mathcal{P}_{1,i}, \mathcal{P}_{2,i}, \cdots, \mathcal{P}_{M,i}\}, \\ \boldsymbol{Z}^m_{s,t,i} &\triangleq \text{diag}\{\mathcal{Z}^{m,1}_{s,t,i}, \mathcal{Z}^{m,2}_{s,t,i}, \cdots, \mathcal{Z}^{m,V}_{s,t,i}\}, \\ \Upsilon^{m,v}_{s,t} &\triangleq [\mathcal{S}^{\frac{1}{2}}(\Gamma^{m,v}_{s,t} - 2Y_v)^\top, \mathcal{R}^{\frac{1}{2}}\mathcal{K}^\top_{v,s}]^\top, \\ \boldsymbol{A}^{m,v}_{s,t} &\triangleq A_{m,s}Y_v + B_{m,s}\mathcal{K}_{v,t}, \\ \Gamma^{m,v}_{s,t} &\triangleq \boldsymbol{A}^{m,v}_{s,t} + \boldsymbol{A}^{m,v}_{t,s}, \end{aligned}$$

then the optimization problem (19) has an optimal solution for the initial state $x(k) \in \Omega_{r_k}$ and recursion is feasible. Moreover, the control laws are as follows

$$K_{\sigma_{k+n},i} \triangleq \mathcal{K}_{\sigma_{k+n},i}Y^{-1}_{\sigma_{k+n}}, \forall\sigma_{k+n} \in \mathfrak{V}, \forall i \in \mathfrak{I}, \quad (27)$$

which can guarantee the stochastic stability of the closed-loop system (8).

Proof: First, for the sake of simplicity, let's define some abbreviations $h_i \triangleq h^{n;k}_i$, $r^+_{n;k} \triangleq r_{n+1;k}$, $h^+_i \triangleq h^{n+1;k}_i$, and $P_{r_{n;k},i} \triangleq P_{r_{n;k},i}(k)$. For any $r_{n;k} = m \in \mathfrak{M}$ and $\sigma_{n;k} = v \in \mathfrak{V}$, from (18), one can obtain

$$\begin{aligned} &\mathbb{E}[V(x(n+1;k),m^+) - V(x(n;k),m) + ||x(n;k)||^2_{\mathcal{Q}} \\ &\quad + ||u(n;k)||^2_{\mathcal{R}} + ||\Delta x(n;k)||^2_{\mathcal{S}}]|_{x(n;k),m} \\ =&\mathbb{E}\Bigg\{x^\top(n;k)\Bigg[(\bar{\mathcal{A}}^k_{mv})^\top\sum_{i=1}^{I}h^+_iP_{m^+,i}\bar{\mathcal{A}}^k_{mv} - \bar{P}_m + \mathcal{Q} \\ &\quad + (\bar{K}^k_v)^\top\mathcal{R}\bar{K}^k_v + (\bar{\mathcal{A}}^k_{mv} - \mathbf{I})^\top\mathcal{S}(\bar{\mathcal{A}}^k_{mv} - \mathbf{I})\Bigg]x(n;k)\Bigg\} \\ =&\mathbb{E}\Bigg\{x^\top(n;k)\sum_{i=1}^{I}\sum_{p=1}^{I}\sum_{q=1}^{I}\sum_{s=1}^{I}\sum_{t=1}^{I}h^+_ih_ph_qh_sh_t\Big[\mathcal{A}^\top_{mvpq} \\ &\quad \times P_{m^+,i}\mathcal{A}_{mvst} - P_{m,p} + \mathcal{Q} + K^\top_{v,p}\mathcal{R}K_{v,q} \\ &\quad + (\mathcal{A}_{mvpq} - \mathbf{I})^\top\mathcal{S}(\mathcal{A}_{mvst} - \mathbf{I})\Big]x(n;k)\Bigg\} \\ \leq&\mathbb{E}\Bigg\{x^\top(n;k)\sum_{i=1}^{I}\sum_{p=1}^{I}\sum_{q=1}^{I}\sum_{s=1}^{I}\sum_{t=1}^{I}h^+_ih_ph_qh_sh_t\frac{1}{4}\Big[(\mathcal{A}^\top_{mvst} \\ &\quad + \mathcal{A}^\top_{mvts})P_{m^+,i}(\mathcal{A}_{mvst} + \mathcal{A}_{mvts}) - 4P_{m,s} + 4\mathcal{Q} \\ &\quad + 4K^\top_{v,s}\mathcal{R}K_{v,s} + (\mathcal{A}_{mvst} + \mathcal{A}_{mvts} - 2\mathbf{I})^\top\mathcal{S}(\mathcal{A}_{mvst} \\ &\quad + \mathcal{A}_{mvts} - 2\mathbf{I})\Big]x(n;k)\Bigg\} \leq 0. \end{aligned} \quad (28)$$

Then, according to the Markovian property of the system (8), following (28), it can be deduced that

$$\begin{aligned} &\sum_{v=1}^{V}\varphi_{mv}\Bigg[(\mathcal{A}^\top_{mvst} + \mathcal{A}^\top_{mvts})\sum_{m^+=1}^{M}\pi_{m,m^+}P_{m^+,i} \\ &\times(\mathcal{A}_{mvst} + \mathcal{A}_{mvts}) + (\mathcal{A}_{mvst} + \mathcal{A}_{mvts} - 2\mathbf{I})^\top\mathcal{S}(\mathcal{A}_{mvst} \\ &+ \mathcal{A}_{mvts} - 2\mathbf{I}) + 4K^\top_{v,s}\mathcal{R}K_{v,s}\Bigg] - 4P_{m,s} + 4\mathcal{Q} \preceq \mathbf{0}. \end{aligned} \quad (29)$$

Further, condition (18) become that for any s, t, $i \in \mathfrak{I}$, $m \in \mathfrak{M}$, and $v \in \mathfrak{V}$,

$$\begin{aligned} &(\mathcal{A}^\top_{mvst} + \mathcal{A}^\top_{mvts})\sum_{m^+=1}^{M}\pi_{m,m^+}P_{m^+,i}(\mathcal{A}_{mvst} + \mathcal{A}_{mvts}) \\ &+(\mathcal{A}_{mvst} + \mathcal{A}_{mvts} - 2\mathbf{I})^\top\mathcal{S}(\mathcal{A}_{mvst} + \mathcal{A}_{mvts} - 2\mathbf{I}) \\ &+4K^\top_{v,s}\mathcal{R}K_{v,s} - Z^{m,v}_{s,t,i} \preceq \mathbf{0}, \end{aligned} \quad (30)$$

$$\sum_{v=1}^{V}\varphi_{mv}Z^{m,v}_{s,t,i} - 4P_{m,s} + 4\mathcal{Q} \preceq \mathbf{0}. \quad (31)$$

Then, by defining the matrix variables $\mathcal{P}_{m,i} \triangleq \gamma P^{-1}_{m,i}$, $\mathcal{Z}^{m,v}_{s,t,i} \triangleq \gamma(Z^{m,v}_{s,t,i})^{-1}$ and using the Schur complement and the congruence transformation, (30) and (31) can be guaranteed by (24) and (25), respectively.

Moreover, together with (17) and (18), it is easily shown that $x(n;k) \in \Omega_{r_{k+n}}$ in the stochastic sense if for any $m \in \mathfrak{M}$ the following holds

$$x^\top(k)\sum_{i=1}^{I}h^k_iP_{m,i}x(k) \leq \gamma. \quad (32)$$

Then, by utilizing the Schur complement, (26) can be ensured from the achieved inequality. Now, both (18) and (17) are achieved. The remaining part of the proof can be derived directly from Theorem 1. ■

Furthermore, Theorem 3 presented below can address the constraints (9) and (10).

Theorem 3: Consider a F-HMJS (8) with constraints (9) and (10). If there exists sets of matrices $\mathcal{P}_{m,i} \succ \mathbf{0}$, invertible matrices Y_v, and matrices $\mathcal{K}_{v,s}$, for any $s, i \in \mathfrak{I}$, $m \in \mathfrak{M}$, and $v \in \mathfrak{V}$, such that

$$\begin{bmatrix} -Y_v - Y_v^\top + \mathcal{P}_{m,i} & * \\ \mathcal{K}_{v,s}^{(q)} & -[\overline{u}]_q^2 \end{bmatrix} \preceq \mathbf{0}, \tag{33}$$

$$\begin{bmatrix} -\mathcal{P}_{m,i} & * \\ \Psi^{(p)}\mathcal{P}_{m,i} & -[\overline{x}]_p^2 \end{bmatrix} \preceq \mathbf{0}, \tag{34}$$

for any $q \in \mathfrak{L}_{N_u}$, $p \in \mathfrak{L}_{N_\psi}$, then both the predicted state and control input satisfy constraints (12c) and (12d).

Proof: First, due to $0 \le h_s^k \le 1$, and $\sum_{s=1}^{I} h_s^k = 1$ for any $s \in \mathfrak{I}$, (12d) can be guaranteed by

$$\left| [K_{v,s}x(n;k)]_q \right| \le [\overline{u}]_q. \tag{35}$$

Further, squaring the left and right sides of (35) and together with (26), one can arrive at

$$x^\top(n;k)K_{v,s}^{(q)\top}K_{v,s}^{(q)}x(n;k) \le [\overline{u}]_q^2 x^\top(n;k)\gamma^{-1}P_{m,i}x(n;k).$$

Then, it can be deduced that

$$K_{v,s}^{(q)\top}K_{v,s}^{(q)} - [\overline{u}]_q^2\gamma^{-1}P_{m,i} \preceq \mathbf{0}. \tag{36}$$

Thus, by the Schur complement and performing the congruence transformation by diag$\{Y_v, \mathbf{I}\}$, (33) can be derived from (36). Likewise, (12c) can be achieved by (34), which completes the proof. ■

Drawing on the theorems stated above, the solvable optimization problem presented below can be used to derive a state feedback control law that is dependent on both fuzzy rules and observed modes, at every time step k.

$$\min_{\mathcal{P}_{m,i},\mathcal{Z}_{s,t,i}^{m,v},Y_v,\mathcal{K}_{v,i},s,t,i\in\mathfrak{I},m\in\mathfrak{M},v\in\mathfrak{V}} \gamma. \tag{37}$$
$$\text{s.t. (12a), (12b), (24)-(26), (33), (34)}$$

IV. Illustrative Example

Consider the following F-HMJS of three modes and two fuzzy rules:

$$\begin{cases} A_{1,1} = \begin{bmatrix} 1.16 & 0.31 \\ 0.42 & 0.81 \end{bmatrix}, B_{1,1} = \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix} \\ A_{2,1} = \begin{bmatrix} 0.15 & 0.82 \\ 0.32 & 0.18 \end{bmatrix}, B_{2,1} = \begin{bmatrix} 1.1 \\ 0.8 \end{bmatrix} \\ A_{3,1} = \begin{bmatrix} 0.32 & 0.21 \\ 0.15 & 1.09 \end{bmatrix}, B_{3,1} = \begin{bmatrix} 0.8 \\ 1.1 \end{bmatrix} \\ A_{1,2} = \begin{bmatrix} 1.21 & 0.41 \\ 0.74 & 0.43 \end{bmatrix}, B_{1,2} = \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix} \\ A_{2,2} = \begin{bmatrix} 0.13 & 1.21 \\ 0.34 & 0.11 \end{bmatrix}, B_{2,2} = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix} \\ A_{3,2} = \begin{bmatrix} 0.25 & 0.13 \\ 0.29 & 0.92 \end{bmatrix}, B_{3,2} = \begin{bmatrix} 0.8 \\ 1.1 \end{bmatrix} \end{cases}$$

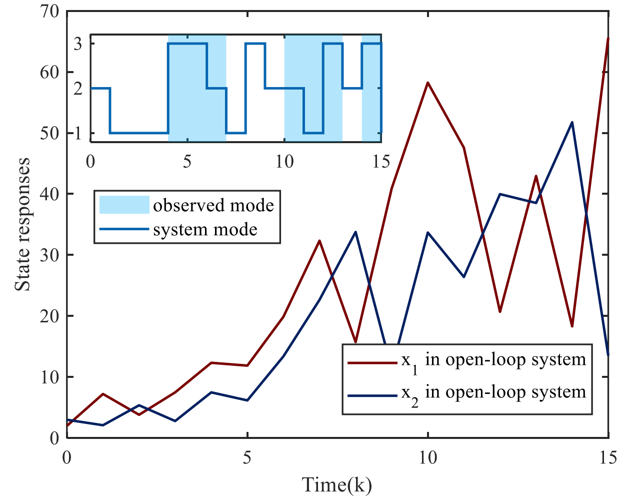

Fig. 1. One group of open-loop state responses with the system mode and observation attached.

and the constraints are considered to be $|x_1| \le 5$, $|x_2| \le 5$, and $|u| \le 5$. The fuzzy basis functions are selected as

$$h_1(x_1(k)) = 0.5\left|\sin(x_1(k)) + \cos(x_1(k))\right|,$$
$$h_2(x_1(k)) = 1 - h_1(x_1(k)).$$

Then, the TP matrix of the Markovian process $\{r_k\}_{k\ge0}$ is as follows

$$\Pi = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.4 & 0.2 & 0.4 \\ 0.2 & 0.4 & 0.2 \end{bmatrix}.$$

Moreover, without loss of generality, the homogeneous EP matrix of the Markovian chain $\{\sigma_k\}_{k\ge0}$ is given as below:

$$\Phi = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}.$$

In this example, setting $x(0) = [2\ 3]^\top$, the state responses of the open-loop system with random generating jumping sequences are displayed in Fig. 1 and the light blue block represents the asynchronous moments. It can be readily found that the stability of the open-loop system cannot be guaranteed. Further, the cost function is set as $\mathcal{Q} = 0.5\mathbf{I}$,$\mathcal{R} = 0.2$, and $\mathcal{S} = 8\mathbf{I}$ for the proposed MPC, while $\mathcal{Q} = 0.5\mathbf{I}$, $\mathcal{R} = 0.2$, and $\mathcal{S} = \mathbf{0}$ for the standard MPC. Besides, for the purpose of contrast, the evolution of the closed-loop system state and control input via two different MPC strategies are illustrated in Fig. 2. It is evident that, in due course, every state response tends towards zero and that both the state variables and control inputs conform to the constraints. The efficacy of the developed strategy is further demonstrated by the smooth evolution of the state via the proposed bumpless MPC method.

V. Conclusion

The asynchronous predictive control problem for the constrained F-HMJSs with homogeneous EPs has been investigated. Both the state bump phenomenon and asynchronous

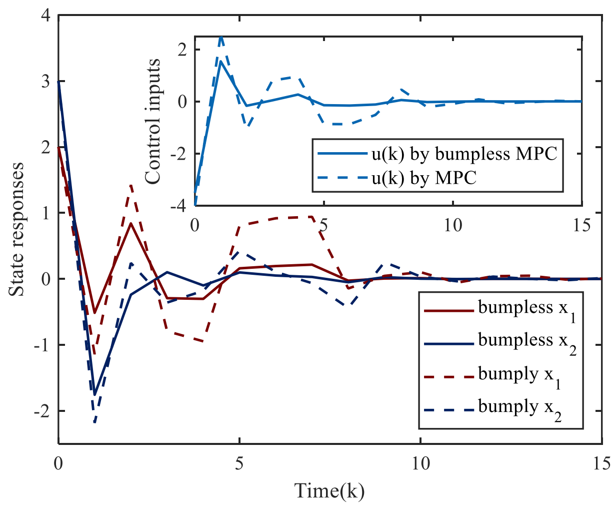

Fig. 2. Corresponding closed-loop state responses and control inputs under different control approach.

phenomenon are taken into account. The family of F-HMJSs in question has the capability to encompass both F-MJSs and F-HMJSs with homogeneous EPs. Then, the asynchronous fuzzy-rule and observed-mode-dependent state feedback control law has been employed. Furthermore, a so-called bumpless MPC framework has been proposed to handle the state bump phenomenon. By solving a convex optimization problem, a tractable controller based on MPC has been designed suring feasibility and stability. Eventually, the efficiency and supremacy of the methodology developed in this study have been shown and contrasted by a numerical simulation.

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