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Prescribed Performance Output Synchronization for Heterogeneous Uncertain Nonlinear Multi-Agent Systems under Directed Switching Graphs

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Abstract— **In this work, we consider the problem of designing a distributed, output synchronization protocol, for heterogeneous, high-order, uncertain, multi-agent systems in Brunovsky canonical form, operated in a leader-follower scenario. The underlying communication graph is directed and switching. The proposed controller, which is of low-complexity, enforces prescribed performance bounds on the convergence time and the output synchronization accuracy. The leader dynamics are unknown. Each following agent requires relative output information from its neighbors, as well as, measuring its own state. The theoretical findings are highlighted via simulation studies.**

I. INTRODUCTION

Output synchronization of heterogeneous multi-agent systems (MASs), has received significant attention. Its main objective is to reach an agreement between the outputs of the agents, under constrained information exchange and despite the presence of possible uncertain nonlinearities in the dynamics that govern their motion [1].

In the literature, the majority of works consider the case where the underlying communication network that interconnects all agents in the MAS is fixed. However, many operational scenarios exist that violate this unchanged graph hypothesis. The paradigm of sudden (unexpected) communication failures between neighboring agents, falls into this category. A recent survey on this topic can be found in [2]. For linear MASs under time-varying graph topologies, [3] considers the finite-time rendezvous problem, while [4] addresses the event-triggered synchronization, utilizing state estimation. Exponential synchronization in the presence of delays for a randomly switching graph and for a class of nonlinear MASs was discussed in [5]. Despite the switching graphs, all the aforementioned works consider MASs with known dynamics. To alleviate this restriction, [6] incorporated 3rd-order MASs in Brunovsky canonical form connected over an undirected switching graph. It was assumed that all graph nodes that were cut off were replaced by new ones. To estimate the uncertain dynamics, adaptive laws were utilized and combined with a sliding-mode controller. As a consequence, chattering was inevitably introduced around the sliding surface. However, the latter phenomenon is undesirable, as it typically leads to high control activity and may further excite unmodelled high-frequency dynamics. Additionally, the considered nonlinearities were limited, as the control input matrix was taken unitary. Furthermore, in [7] asymptotic synchronization for nonlinear MASs was achieved under the assumption that graph changes were based on average dwell time.

Up to this point, all stated works focus on establishing stability, while performance-related control objectives were typically overlooked. In this direction, the prescribed performance control (PPC) framework was proposed to successfully enforce prescribed and user-defined transient and steady-state performance bounds on the synchronization error, for various classes of high-order, uncertain, nonlinear MASs [8], [9]. However, the underlying communication network was considered fixed. In [10], PPC was utilized in conjunction with an adaptive fuzzy controller, to handle the uncertain nonlinearities of the MAS, while achieving performance attributes on the output synchronization error. The agent dynamics belonged to the strict-feedback class, having though constants to multiply the control input. The communication graph was directed and switching. However, the incorporation of adaptive controllers and of approximation structures (i.e., fuzzy systems), increases the dynamic order of the closed-loop, leading to additional differential equations that have to be solved to produce the control signal. Moreover, hard calculations, analytic and numerical, are also required. The aforementioned questions directly the applicability, as the computational power on-board of typical MAS platforms is limited. Therefore, in this type of research, developing low-complexity control solutions constitutes an inelastic design constraint.

Motivated by the above discussion, we propose in this paper a distributed, output synchronization protocol, for heterogeneous, high-order, uncertain MASs in Brunovsky canonical form, operated in a leader-follower scenario. The underlying communication graph is directed and switching. However, the exact switching time instants and the corresponding graph changes are a priori unknown. The proposed control solution is of low-complexity and achieves preselected and user-defined performance bounds on the convergence time and the synchronization accuracy. The leader dynamics are unknown. Each agent requires measuring its own state, as well as its relative output with its neighbors. Simulation studies clarify and verify the theoretical findings.

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II. PROBLEM FORMULATION

Consider a multi-agent, possibly heterogeneous, system consisting of a leader and $N > 1$ followers. The dynamics of each follower, for $i = 1, \dots, N$, are described by a nonlinear m-th order model of the form:

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1},\ q = 1, \dots, m-1 \\ \dot{x}_{i,m} &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= x_{i,1} \end{aligned} \tag{1}$$

where $x_{i,q} \in \mathbb{R}$, $q = 1, \dots, m$, are the states of each follower, $u_i \in \mathbb{R}$ is the control input and $y_i \in \mathbb{R}$ represents the i-th agent output. Moreover $f_i : \mathbb{R}^m \to \mathbb{R}$ and $g_i : \mathbb{R}^m \to \mathbb{R}$, are defined as nonlinear, locally Lipschitz functions with unknown analytical expressions. We further define $x_i = [x_{i,1} \dots x_{i,m}]^T \in \mathbb{R}^m$ the state vector of the i-th agent and $y = [y_i \dots y_N]^T \in \mathbb{R}^N$ the output vector.

Remark 1: Even though the results can be generalized for different orders m_i, for each agent, for the sake of clarity it has been preferred that the results be presented for the case $m_i = m$.

Assumption 1: The sign of the nonlinear functions g_i is either strictly positive or strictly negative. Without loss of generality, we assume it is positive and it is considered known with $g_i(x_i) \geq \underline{g}$.

Assumption 2: Let $x_0 = [x_{0,1} \dots x_{0,m}]^T \in \mathbb{R}^m$ denote the state of the leader and $y_0 = x_{0,1} \in \mathbb{R}$ is its output. The leader's dynamics are such that x_0 and its first derivative $\dot{x}_0$, are bounded, continuous, and unknown in advance. Moreover, only the leader's output is available for control design.

The communication between the agents can be modeled by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the set of nodes $\mathcal{V} = \{v_1, \dots, v_N\}$ represents the agents and the set of edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, the exchange of information between them. By virtue of the definition of the adjacency matrix $A = [\alpha_{i,j}] \in \mathbb{R}^{N \times N}$, if agent i receives information from agent j, (i.e $(v_i, v_j) \in \mathcal{E}$), $\alpha_{i,j} = 1$, else $\alpha_{i,j} = 0$. It is presumed that no self loops exist, thus $\alpha_{i,i} = 0$, $(v_i, v_i) \notin \mathcal{E}$. Let us also denote the neighborhood of node i as $N_i = \{v_j : (v_i, v_j) \in \mathcal{E}\}$ and the indegree matrix as $D = \text{diag}(d_1, \dots, d_N)$, where $d_i = \sum_{j=1}^{N} \alpha_{i,j}$, $i = 1, \dots, N$. The graph Laplacian matrix can be defined as $L = D - A$. Symbolizing the augmented graph containing the leader as $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{E}})$, with $\overline{\mathcal{V}} = \{v_0, v_1, \dots, v_N\}$ and $\overline{\mathcal{E}} \in \overline{\mathcal{V}} \times \overline{\mathcal{V}}$, we can represent the leaders state information, provided to a subgroup of N, as a diagonal matrix $B = \text{diag}(b_1, \dots, b_N)$, where $b_i = 1$, when the follower i is directly connected to the leader node and $b_i = 0$ otherwise. Additionally, a digraph has a directed spanning tree, if there exists a node, that has no parent, called the root, such that there is a directed path from it to every other node in the graph.

For every follower, the output disagreement error is defined as: $\delta_{i,1} = x_{i,1} - y_0 \in \mathbb{R}$, $i = 1, \dots, N$ and thus

$$\delta_1 = [\delta_{1,1} \dots \delta_{N,1}]^T = y - \underline{1} \bigotimes y_0 \in \mathbb{R}^N, \tag{2}$$

where $\bigotimes$ is the Kronecker product and $\underline{1} = [1, \dots, 1]^T \in \mathbb{R}^N$.

In this work, we consider that the graph is not constant; instead, it is discontinuously varied. Let $\mathbb{J} = \{t_k : k \in \mathbb{N}_+\}$ constitutes the set of time instants, where agent i loses or regain communication with its neighboring agent j. This results to a change in the graph topology, from $\overline{\mathcal{G}}_{k-1}$ to $\overline{\mathcal{G}}_k$

Assumption 3: Every augmented graph $\overline{\mathcal{G}}_k$ contains a directed spanning tree for all $k \in N$, with the leader as its root 0.

Remark 2: The aforementioned assumption can be boiled down to i) $\mathcal{G}_k$ contains a directed spanning tree and at least one node is connected to the root or ii) $\overline{\mathcal{G}}_k$ has a hierarchical structure, meaning that every node except for the root has indegree d_i equal to 1 [11].

Consequently, for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}_+$, the neighborhood synchronization error is defined as:

$$e_{i,1}(t) = \sum_{j \in N_i} \alpha_{i,j}^k (x_{i,1}(t) - x_{j,1}(t)) + b_i^k (x_{i,1}(t) - y_0(t)),$$

where $\alpha_{i,j}^k$ and b_i^k are elements of the adjacency matrix A and the B-matrix that correspond to the augmented graph $\overline{\mathcal{G}}_k$. Denoting $e_1 = [e_{1,1} \dots e_{N,1}]^T \in \mathbb{R}^N$, and owing to (2), the neighborhood synchronization error can be rewritten in the compact form for all $t \in [t_k, t_{k+1})$:

$$e_1(t) = (L_k + B_k)(x_1(t) - \underline{1} \bigotimes y_0(t)) = H_k \delta_1(t), \tag{3}$$

where $H_k = L_k + B_k$.

Owing to Assumption 3, H_k is a nonsingular M-Matrix. The main results of the work will utilize the subsequent technical lemma concerning M-matrices.

Lemma 1: [12, p. 168] Consider a nonsingular M-Matrix $S \in \mathbb{R}^{N \times N}$. There exists a diagonal positive matrix $P = \text{diag}(1/v_1, \dots, 1/v_N)$, where $v = [v_1 ... v_N]^T = (S)^{-1}\underline{1}$, such that $PS + S^T P$ is also positive definite.

Utilizing the nonsingularity of H_k, owing to Assumption 3, we deduce $|\delta_1| \leq \frac{|e_1|}{\underline{\sigma}(H_k)}$, where $\underline{\sigma}(H_k)$ represents the minimum singular value of H_k.

Therefore, transient and state-state bounds on e_1 can be directly translated into performance bounds on δ_1; thus, rendering e_1 an effective synchronization metric. Nevertheless, knowledge of $\underline{\sigma}(H_k)$, requires knowledge of the global graph topology, and as a result it is not available for the design of a distributed control protocol. This problem can be alleviated by employing the estimate $\left(\frac{N-1}{N^3+N^2-N}\right)^{\frac{N-1}{2}} \leq \underline{\sigma}(H_k)$ [9], that depends only on the number of agents, which remains invariant despite the change in graph topology and as a result we obtain:

$$|\delta_1(t)| \leq \left(\frac{N^3 + N^2 - N}{N-1}\right)^{\left(\frac{N-1}{2}\right)} |e_1(t)|, \forall t \geq t_0 \geq 0. \tag{4}$$

To continue, notice that for any $t_k \in \mathbb{J}$, a graph switch has no effect on δ_1 (i.e., $\delta_1(t_k^-) = \delta_1(t_k)$). However, it results in an instantaneous variation of the neighborhood synchronization error from $e_1(t_k^-) = H_{k-1}\delta_1(t_k^-)$ to $e_1(t_k) = H_k \delta_1(t_k)$.

Hence, $e_1(t_k) = H_{k-1}\delta_1(t_k) + \Delta H_k \delta_1(t_k)$, where $\Delta H_k \coloneqq H_k - H_{k-1}$; or in compact form,

$$e_1(t) = e_1(t^-) + \Delta e_1(t),\ \forall t \in \mathbb{J}, \tag{5}$$

with $\Delta e_1(t_k) \coloneqq \Delta H_k \delta_1(t_k)$.

Assumption 4: For the time instants of any two consecutive graph changes, it holds $t_{k+1} - t_k \geq \overline{\tau} > 0$, where $\overline{\tau}$ is a known constant. However, the actual time instants t_k, $k \in \mathbb{N}_+$ are unknown in advance.

Remark 3: When modeling the removal of some edges in the graph, $||\Delta H_k||$ is bounded by the maximal eigenvalue of H_k, as long as the directed spanning tree property is preserved. Correspondingly, when modeling the addition of edges in the graph, $||\Delta H_k||$ is bounded by the maximal eigenvalue of the complete graph consisting of N nodes. Therefore, in any case, $||\Delta H_k||$ is bounded.

Remark 4: It should be noted that while no impulsive behavior appears in the output disagreement error, δ_1 is not available for use by the controller, as in the opposite it would require knowledge of the global graph topology. The neighborhood synchronization error e_1 is available for use. However, it is affected by impulsive behavior. The proposed controller should therefore compensate for the appearance of impulsive phenomena.

Control Objective: Design a distributed control protocol, of low computational complexity, for the heterogeneous multi-agent system (1) satisfying Assumption 1, having a leader that satisfies Assumption 2, under a directed switching graph, obeying Assumptions 3 and 4, utilizing state and relative output-feedback, such that the output disagreement error $\delta_1(t)$ is driven to an arbitrarily small neighborhood of zero, whose size is predetermined by the user-defined positive constants $w_1 = [w_{1,1}, \ldots, w_{N,1}]^T$, for all $t \in [t_k + \tau, t_{k+1}]$, with $\tau > 0$ being a prespecified fixed-time satisfying $\overline{\tau} > \tau$. Moreover, all signals in the closed-loop should remain bounded.

Remark 5: The term low complexity is attributed to i) not incorporating prior knowledge regarding the agent nonlinearities; ii) not employing approximating structures (i.e. fuzzy systems or neural networks) to acquire such knowledge; iii) not requiring hard calculations analytic or numerical to produce the control protocol; iv) the controller being static, thus avoiding the expansion of the dynamic order of the closed-loop.

III. MAIN RESULTS

The main results of this work are summarized in the following theorem:

Theorem 1: Consider the heterogeneous multi-agent system (1) satisfying Assumption 1, having a leader that satisfies Assumption 2. Consider, also, a directed switching graph obeying Assumptions 3 and 4. Given the constants $w_{i,1} > 0$ and $\tau > 0$ with $\overline{\tau} > \tau$ and any initial conditions $x_i(0) \in \mathbb{R}^m$, $i = 1, \ldots, N$, the impulsive distributed control protocol:

$$\xi_{i,1} = \frac{e_{i,1}}{p_{i,1}}, \tag{6a}$$

$$a_{i,1} = -2K_{i,1}\frac{\ln\left(\frac{1+\xi_{i,1}}{1-\xi_{i,1}}\right)}{p_{i,1}(1-(\xi_{i,1})^2)} - 4K'_{i,1}\frac{\ln\left(\frac{1+\xi_{i,1}}{1-\xi_{i,1}}\right)^3}{p_{i,1}(1-(\xi_{i,1})^2)}, \tag{6b}$$

$$\xi_{i,q} = \frac{x_{i,q} - a_{i,q-1}}{p_{i,q}},\ q = 2, \ldots, m, \tag{6c}$$

$$a_{i,q} = -2K_{i,q}\frac{\ln\left(\frac{1+\xi_{i,q}}{1-\xi_{i,q}}\right)}{p_{i,q}(1-(\xi_{i,q})^2)} - 4K'_{i,q}\frac{\ln\left(\frac{1+\xi_{i,q}}{1-\xi_{i,q}}\right)^3}{p_{i,q}(1-(\xi_{i,q})^2)}, \tag{6d}$$

$$u_i = a_{i,m} \tag{6e}$$

where $K_{i,q} > 0$, $K'_{i,q} > 0$, $q = 1, \ldots, m$ and

$$p_{i,q}(t) = (p^k_{i,q} - p^\infty_{i,q})e^{-l^k_{i,q}(t-t_k)} + p^\infty_{i,q},\ t \notin \mathbb{J}, \tag{6f}$$
$$p_{i,q}(t) = p_{i,q}(t^-) + \Delta p_{i,q}(t),\ t \in \mathbb{J}, \tag{6g}$$
$$\Delta a_{i,q}(t) = a_{i,q}(t) - a_{i,q}(t^-),\ t \in \mathbb{J}, \tag{6h}$$
$$\Delta p_{i,1}(t) = \gamma_{i,1}|\Delta e_{i,1}(t)|,\ t \in \mathbb{J}, \tag{6i}$$
$$\Delta p_{i,q}(t) = \gamma_{i,q}|\Delta a_{i,q-1}(t)|,\ q = 2, \ldots, m,\ t \in \mathbb{J}, \tag{6j}$$

where $\gamma_{i,1}, \gamma_{i,q} > 1$ and for $k \in \mathbb{N}$

$$p_{i,1}(0) \equiv p^0_{i,1} > |e_{i,1}(0)|,\ w_{i,q} > p^\infty_{i,q} > 0, \tag{6k}$$
$$p_{i,q}(0) \equiv p^0_{i,q} > |x_{i,q}(0) - a_{i,q-1}(0)|, \tag{6l}$$
$$p^k_{i,q} \geq p^\infty_{i,q} > 0,\ l^k_{i,q} > \ln\left(\frac{p^k_{i,q} - p^\infty_{i,q}}{w_{i,q} - p^\infty_{i,q}}\right)/\tau. \tag{6m}$$

guarantees that:

i) all the closed-loop signals remain bounded,
ii) $|\delta_{i,1}(t)| \leq (\frac{N^3+N^2-N}{N-1})^{\frac{N-1}{2}}|w_{i,1}|$, $\forall t \in [t_k + \tau, t_{k+1}]$.

Remark 6: The impulsive dynamics (6g)-(6h) define an event-like triggering mechanism. The time instants t_k are unknown in advance, however, they become available for use as soon as an impulse appears. Thus, the discontinuities indicated in (6g)-(6h) and the calculation of the lower bound of $l^k_{i,q}$ in (6m) take place simultaneously at the time instant t_k, $k \in \mathbb{N}_+$.

Remark 7: The proposed controller guarantees that prior to any graph switching, the output disagreement error will enter a neighborhood of the origin of predetermined size $w_{i,1}$ within a prescribed fixed-time τ. Notice that both $w_{i,1}$ and τ are not only prespecified but they are also user-defined.

Remark 8: Regarding the choice of design parameters, the following conclusions have been reached through experimenting with simulation and in accordance with similar studies. While $p_{i,1}$ is the only function responsible for imposing the prescribed characteristics on $\delta_{i,1}$, the choice of $p_{i,q}$, $k_{i,q}$ and $k'_{i,q}$ directly influence the actual state evolution inside the performance envelopes. Selecting larger values for $l_{i,q}$ and smaller $w_{i,q}$ brings satisfactory steady-state performance, but leads to larger control effort during the transient. Alternatively, the transient response is improved, but heavier oscillations are observed during steady-state. In the same manner, larger $k_{i,q}$, $k'_{i,q}$ introduce desirable behavior at steady state, at the cost of larger control signals during the transient. When it comes to choosing $\gamma_{i,q}$, values closer to 1 encourage stricter bounds at the moment of switching, whereas a more relaxed choice results in a significantly larger control effort after the appearance of the impulse.

IV. PROOF OF THEOREM 1

For $q = 1, \ldots, m$, we define: $x_q = [x_{1,q} \ldots x_{N,q}]^T \in \mathbb{R}^N$, $u = [u_1 \ldots u_N]^T \in \mathbb{R}^N, F(x) = [f_1(x_1) \ldots f_N(x_N)]^T \in \mathbb{R}^N, G(x) = \text{diag}(g_1(x_1) \ldots g_N(x_N)) \in \mathbb{R}^{N\times N}$, $x = [x_1^T \ldots x_m^T]^T \in \mathbb{R}^{Nm}, R_q = \text{diag}(p_{1,q}, \ldots, p_{N,q}) \in \mathbb{R}^{N\times N}$, $K_q = \text{diag}(K_{1,q}, \ldots, K_{N,q}) \in \mathbb{R}^{N\times N}, K'_q = \text{diag}(K'_{1,q}, \ldots, K'_{N,q}) \in \mathbb{R}^{N\times N}$, $a_q = [a_{1,q} \ldots a_{N,q}]^T \in \mathbb{R}^N$. Then

$$\begin{aligned}&\xi_1 = (R_1)^{-1} e_1,\\ &\xi_q = (R_q)^{-1}(x_q - a_{q-1}),\ q = 2, \ldots, m\end{aligned} \tag{7}$$

and (1) becomes:

$$\begin{aligned}\dot{x}_q &= x_{q+1},\\ \dot{x}_m &= F(x) + G(x)u\end{aligned} \tag{8}$$

Owing to Assumption 1 and $G(x)$ being a diagonal matrix, there exists a constant $\underline{g}$ such that:

$$G(x) \geq \underline{g} I_n > 0. \tag{9}$$

Further defining the generalized state vector as $\phi = [\xi_1^T \ldots \xi_m^T]^T$ and utilizing the property $\dot{x}_{i,q} = \dot{\xi}_{i,q} p_{i,q} + \xi_{i,q}\dot{p}_{i,q}$ we obtain for all $t \in [t_k, t_{k+1})$:

$$\dot{\phi}(t) = \begin{bmatrix} (R_1)^{-1}(H_k(x_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1) \\ (R_2)^{-1}((x_3 + -\dot{a}_1) - \dot{R}_2\xi_2) \\ \vdots \\ (R_m)^{-1}((F(x) + G(x)u - \dot{a}_{m-1}) - \dot{R}_m\xi_m) \end{bmatrix} \tag{10}$$

The proof of Theorem 1 is decomposed into two lemmas, where Lemma 2 is responsible for enforcing the performance characteristics, while Lemma 3 ascertains the boundedness of all closed-loop signals.

Lemma 2: Consider the multi-agent system (1), the proposed control protocol, (6), guarantees that $|\delta_{i,1}| \leq (\frac{N^3+N^2-N}{N-1})^{\frac{N-1}{2}} |w_{i,1}|$, for all $t \in [t_k + \tau, t_{k+1})$.

Proof: The proof consists of three phases. In Phase A the existence and uniqueness of a maximal solution for $\phi(t)$ in $[t_0, \tau_{max}), \tau_{max} \in [t_0, t_1)$ is addressed. In Phase B τ_{max} is extended up to t_1 and in Phase C the methodology presented in Phase B is generalized for all intervals $[t_k, t_{k+1}), k \geq 2$.

Phase A: Initially let us define an open non-empty set $\Omega_\phi = (-1, 1)^{mN} \subset \mathbb{R}^{mN}$. Owing to (6k), (6l), (6m) it holds that $|\xi_{i,q}(t_0)| < 1$, which guarantees $\phi(t_0) \in \Omega_\phi$. Moreover the right-hand side of (10) is continuously differentiable with respect to t and locally Lipschitz with respect to ϕ in Ω_ϕ. As a result following standard arguments [13], there exists a unique maximal solution $\phi : [t_0, \tau_{max}) \to \Omega_\phi$, in $[t_0, \tau_{max})$, with $\tau_{max} \in [t_0, t_1)$, thus $\phi(t) \in \Omega_\phi$, for all $t \in [t_0, \tau_{max})$.

Phase B: To proceed we define the continuous, monotonically increasing, auxiliary function $T : (-1, 1) \to \mathbb{R}$ of the form $T(z) = \frac{1+z}{1-z}$. Obviously, it holds that $\lim_{x\to 1} T(x) \to \infty$ and $\lim_{x\to -1} T(x) \to -\infty$. Hence, setting $\xi_{i,j}$ as its argument and exploiting this property, the initial control problem is transformed into a bounding problem concerning $e_{i,j}$. To implement the aforementioned methodology let us define:

$$\epsilon_{i,q} = T(\xi_{i,q}),\ \epsilon_q = [\epsilon_{1,q} \ldots \epsilon_{N,q}]^T \tag{11}$$

Noticing that $\frac{\partial \epsilon_{i,q}}{\partial \xi_{i,q}} = \frac{2}{1-(\xi_{,q})^2}$, we set $r_{i,q} = \frac{2}{p_{i,q}(1-(\xi_{i,q})^2)}$, $r_q = \text{diag}(r_{1,q}, \ldots, r_{N,q})$. Further defining $s_{i,q} = T^3(\xi_{i,q})$, $s_q = [s_{1,q} \ldots s_{N,q}]^T$, the control signals can be rewritten in the compact form:

$$\begin{aligned}a_q &= -K_q r_q \epsilon_q - K''_q r_q s_q,\ q = 1, \ldots, m-1\\ u = a_m &= -K_m r_m \epsilon_m - K''_m r_m s_m,\end{aligned} \tag{12}$$

where $K''_{i,q} = 2K'_{i,q}$ and thus $K''_q = 2K'_q$.
Note that, owing to Phase A, $\epsilon_{i,q}(t)$ are well defined in $[t_0, t_{max})$. Therefore, differentiating we obtain the error dynamics:

$$\begin{aligned}\dot{\epsilon}_1 &= r_1(H_k(x_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1)\\ \dot{\epsilon}_q &= r_q((\dot{x}_q - \dot{a}_{q-1}) - \dot{R}_q\xi_q),\ q = 2, \ldots, m.\end{aligned} \tag{13}$$

To show that τ_{max} can be extended to t_1 we follow a recursive design procedure.

Step 1 ($q = 1$, $t \in [t_0, \tau_{max})$): H_0 is a nonsingular M-matrix., and employing Lemma 1, there exist a diagonal positive matrix P. It is now possible to define the Lyapunov candidate function:

$$V_1 = \frac{1}{2}\epsilon_1^T P \epsilon_1, \tag{14}$$

and differentiating with respect to time for all $t \in [t_0, \tau_{max})$ it yields : $\dot{V}_1 = 2\epsilon_1^T P r_1 \dot{\xi}_1$ or equivalently

$$\dot{V}_1 = \epsilon_1^T P r_1 (H_0(x_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1). \tag{15}$$

Solving (7) for x_2, invoking (12) and because r_1, P, K_1, and K''_1 are diagonal, expressing PH_0 as a sum of its symmetric and skew-symmetric parts, it can be shown that:

$$\begin{aligned}\dot{V}_1 = &-\frac{1}{2}\epsilon_1^T r_1 K_1 (PH_0 + H_0^T P) K_1 r_1 \epsilon_1\\ &-\frac{1}{2}\epsilon_1^T r_1 K''_1 (PH_0 + H_0^T P) K''_1 r_1 s_1\\ &+\epsilon_1^T r_1 P (H_0(R_2\xi_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1).\end{aligned} \tag{16}$$

Furthermore, defining $Q_1 = K_1(PH_0 + H_0^T P)K_1$, as well as $Q''_1 = K''_1(PH_0 + H_0^T P)K''_1$, both being positive definite, due to lemma 1, (16) becomes:

$$\begin{aligned}\dot{V}_1 = &-\frac{1}{2}\epsilon_1^T r_1 Q_1 r_1 \epsilon_1 - \frac{1}{2}\epsilon_1^T r_1 Q''_1 r_1 s_1\\ &+\epsilon_1^T r_1 P (H_0(R_2\xi_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1)\end{aligned} \tag{17}$$

Owing to $\phi \in \Omega_\phi$, ξ_1, ξ_2 remain bounded for $t \in [t_0, \tau_{max})$. Moreover $\dot{R}_1$,R_2, are bounded by construction and $\dot{x}_{0,1}$ is bounded owing to Assumpiton 2. Hence, utilizing the extreme value theorem, we conclude, that for some unknown constant C_1, it holds $|P(H_0(R_2\xi_2 - \underline{1} \bigotimes \dot{x}_{0,1}) - \dot{R}_1\xi_1)| \leq C_1$. Moreover, the definition of r_1 implies $\lambda_{min}(r_1) \geq \lambda_{r_1} = 2/\overline{p}_1$, where $\overline{p}_1 = \max_i p_{i,1}(0)$. Employing the inequality $\epsilon_1^T s_1 < ||\epsilon_1^4||/N$ and after some straightforward algebraic manipulations, yields:

$$\dot{V}_1 \leq \frac{C_1^2}{2\lambda_{min}(Q)} - \frac{\lambda_{min}(Q'')\lambda_{r_1}^2}{2N}||\epsilon_1||^4, \tag{18}$$

with $\lambda_{min}(*)$ denoting the minimum eigenvalue of the matrix. Therefore, $\dot{V}_1 < 0$, when $|\epsilon_1| > \sqrt[4]{\frac{NC_1^2}{\lambda_{min}(Q)\lambda_{min}(Q'')\lambda_{r_1}^2}}$, resulting in:

$$||\epsilon_1|| \leq \overline{\epsilon}_1 = \max\left\{||\epsilon_1(0)||, \sqrt[4]{\frac{NC_1^2}{\lambda_{min}(Q)\lambda_{min}(Q'')\lambda_{r_1}^2}}\right\}, \tag{19}$$

for all $t \in [t_0, \tau_{max})$ and taking the inverse logarithmic function from the definition of ϵ_1 we get for $i = 1, \ldots, N$:

$$-1 < T^{-1}(-\overline{\epsilon}_1) \leq \xi_{i,1} \leq T^{-1}(\overline{\epsilon}_1) < 1. \tag{20}$$

Additionally (12) guarantees that a_1 is bounded, and owing to $x_2 = R_2\xi_2 + a_1$, we conclude the boundedness of x_2 as well. Taking advantage of $\dot{e}_1 = H_0(x_2 - \underline{1} \bigotimes \dot{x}_{0,1})$ we also deduce that $\dot{e}_1$ remains bounded. Finally, owing to (6b), $\dot{a}_{i,1}$ is bounded.

Step q ($2 < q \leq m-1$, $t \in [t_0, \tau_{max})$): By choosing the Lyapunov candidate function $V_q = \frac{1}{2}\epsilon_q^T\epsilon_q$ and following the same line of argument as Step 1, it can easily be verified, that $-1 < T^{-1}(-\overline{\epsilon}_q) \leq \xi_{i,q} \leq T^{-1}(\overline{\epsilon}_q) < 1$ confirming the boundedness of a_q and $\dot{a}_{i,q}$ for all $i = 1, \ldots, N$.

Step m ($q = m$, $t \in [t_0, \tau_{max})$): Consider the Lyapunov candidate function $V_m = \frac{1}{2}\epsilon_m^T\epsilon_m$. Differentiating with respect to time and utilizing (12), we derive:

$$\begin{aligned} V_m = &- \epsilon_m^T r_m G(x) K_m r_m \epsilon_m - \epsilon_m^T r_m G(x) K_m'' r_m s_m + \\ &\epsilon_m^T r_m (F(x) - \dot{a}_{m-1} - \dot{R}_m\xi_m). \end{aligned} \tag{21}$$

where $\dot{R}_m$ is bounded by construction and ξ_m is bounded owing to $\phi \in \Omega_\phi$. Moreover $\dot{a}_{m-1}$ has been proven bounded for all $t \in [t_0, \tau_{max})$ in Step $m-1$. Utilizing the extreme value theorem, for F, leads to $|F(x) - \dot{a}_{m-1} - \dot{R}_m\xi_m| \leq C_m$, where C_m is an unknown constant. Utilizing the user-selected diagonal matrices K_m, K_m'', and (9), we can derive the existence of positive constants $g_k = \lambda_{min}(K_m)\underline{g}$ and $g_{k''} = \lambda_{min}(K_m'')\underline{g}$ such that $G(x)K_m \geq g_k I_n$, and $G(x)K_m'' \geq g_{k''} I_n$. Furthermore, by definition of r_m, it holds that $\lambda_{min}(r_m) \geq \lambda_{r_m} = 2/\overline{p}_m$, where $\overline{p}_m = \max_i p_{i,m}(0)$. Subsequently, utilizing $\epsilon_m^T s_m < ||\epsilon_m^4||/N$, (21) yields:

$$\dot{V}_m \leq -g_{k''}\lambda_{r_m}^2||\epsilon_m||^4/N + C_m^2/4g_k. \tag{22}$$

Therefore $\dot{V}_m < 0$, when $||\epsilon_m|| \leq \sqrt[4]{\frac{NC_m^2}{4g_k g_{k''}\lambda_{r_m}^2}}$ and so:

$$||\epsilon_m|| \leq \overline{\epsilon}_m = \max\left\{||\epsilon_m(0)||, \sqrt[4]{\frac{NC_m^2}{4g_k g_{k''}\lambda_{r_m}^2}}\right\}. \tag{23}$$

Employing the inverse logarithmic function, it can be straightforwardly shown that:

$$-1 < T^{-1}(-\overline{\epsilon}_m) \leq \xi_{i,m} \leq T^{-1}(\overline{\epsilon}_m) < 1, i = 1, \ldots, N. \tag{24}$$

Thus, owing to (12), (20), and (24), it is now clear that all control signals remain bounded in $[t_0, \tau_{max})$.

Finally, let us define $\Omega'_{\phi_k} = \prod_{q=1}^{m}(-\overline{\epsilon}_q, \overline{\epsilon}_q)$. Then, $\phi(t)$ evolves strictly within this compact subset of Ω_ϕ, meaning that $\phi(t) \in \Omega'_{\phi_k} \subset \Omega_\phi$ and using standard arguments [13], τ_{max} can be extended up to t_1.

Phase C: To conclude to proof, it remains to be shown that $\phi(t_1^-) \in \Omega_\phi$, implies $\phi(t_1) \in \Omega_\phi$. The latter can be easily proven following a recursive procedure and therefore the proof is omitted owing to space limitations.

The line of analysis, that was presented, consisting of phases A, B, and C can be recursively repeated, for all intervals $[t_k, t_{k+1}), k \in N_+$, concluding the proof of Lemma 2. Lastly, combing (4), (7) and (6m) achieves $||\delta_{i,1}|| \leq \left(\frac{N^3+N^2-N}{N-1}\right)^{\left(\frac{N-1}{2}\right)}||w_{i,1}||$, for all $t \in [t_k + \tau, t_{k+1}]$.

Up to this point, we have proven that $\phi(t)$ evolves strictly within Ω'_{ϕ_k}, $\forall t \in [t_k, t_{k+1})$, $k \in N_+$. Nevertheless, this compact subset can vary between each time interval, due to its dependency on C_q, which in turn depends on the bounds of ξ_q and R_q at t_k. To ensure the boundedness of all closed loop signals, it remains to be shown that, these bounds are non-increasing with respect to k. In this direction, exploiting Assumption 4 and Remark 3, the energy injected in the system at t_k, as a result of the impulsive behavior stays bounded. Thus it suffices to prove that $V_q(t_k)$, are upper bounded by constants independent of $V_q(t_{k-1})$. To achieve this, the following lemma is introduced:

Lemma 3: Given a system subject to (1), satisfying Assumption 1, having a leader that satisfies Assumption 2, with graph satisfying Assumptions 3, and 4, the formerly presented control protocol, (6), assures that, $x_{i,j}, a_{i,j}$, $i = 1, \ldots, N$, $j = 1, \ldots, m$, are bounded, while simultaneously guaranteeing $\xi_{i,j} \in (-1, 1)$.

Proof: The proof follows similar argumentation to [14] and therefore it is omitted, bringing Theorem 1 to a close.

V. SIMULATIONS

To demonstrate the performance of our proposed controller, we assume a multi-agent system consisting of a leader and $N = 6$ followers. For simulation purposes, we assume that the leader dynamics satisfy $\dot{x_0} = \sin(5t)$. While the followers are assumed to be inverted pendulums with different masses and lengths:

$$\left.\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= g\sin(x_{i,1})/h_i + u_i/(M_i h_i^2) \end{aligned}\right\} i = 1, \ldots, 6 \tag{25}$$

where for $i = 1, \ldots, 6$, $x_{i,1}[rad]$ and $x_{i,2}[rad/s]$ denotes the pendulums position and velocity respectively. The control input is presented by u_i. The system parameters are assumed to be: $g = 9.81$ $[m/s^2], M = [M_1, \ldots, M_6] = [0.5, 0.2, 1.0, 0.7, 0.4, 0.6]$ $[Kg]$ and $h = [h_1, \ldots, h_6] =$

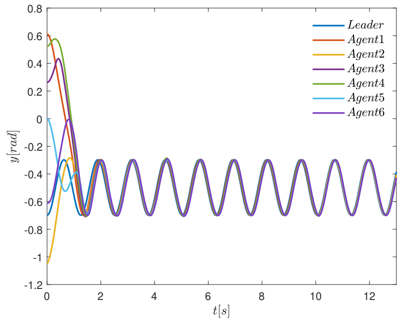

Fig. 1. Evolution of the outputs y_i, $i = 0, \ldots, 6$.

$[1.0, 0.8, 1.6, 1.5, 1.2, 1.8]$ $[m]$. The initial communication topology is described by the following augmented neighboring sets $N_1 = \{0\}$, $N_2 = \{0, 1\}$, $N_3 = \{1\}$, $N_4 = \{1, 3\}$, $N_5 = \{2\}$ and $N_6 = \{2, 4\}$, where the leader is indicated with 0. The graph is subject to switching satisfying $t_{k+1} - t_k \geq 4$. We assume the first change occurs at $t_1 = 4$s affecting agents 2 and 4 with their new augmented neighboring sets be $N_2' = \{1\}$ and $N_4' = \{1\}$. The second change occurs at $t_2 = 10$s affecting agents 2 and 6 with their new augmented neighboring sets be $N_2'' = \{0\}$ and $N_6'' = \{2\}$. The requirement for the neighborhood synchronization errors $e_{i,1}$, $i = 1, \ldots, 6$ is to converge to the set of $\{e_{i,1} \in \mathbb{R} : |e_{i,1}(t)| \leq \pi/45\}$ no later that the fixed time $\tau = 2.5$s. Therefore, we set $w_{i,1} = \pi/45$, $i = 1, \ldots, 6$ and we implement the proposed controller as presented in Theorem 1.

In addition, to achieve reasonable control effort with smooth state evolution we chose for all $i = 1, \ldots, 6$ the following parameters $K_{i,1} = K_{i,2} = 1$, $K'_{i,1} = K'_{i,2} = 0.05$, $\gamma_{i,1} = \gamma_{i,2} = 1.1$, and $w_{i,2} = \pi/18$. The prescribed performance functions are selected to satisfy (6k), (6l) and (6m) with $p^{\infty}_{i,1} = \pi/60$ and $p^{\infty}_{i,2} = \pi/36$.

The simulation results are depicted in Figs. 1-2. In Fig. 1, we illustrate the output of all agents and the leader, verifying the achievement of output consensus. The intermediate errors $x_{i,2} - a_{i,1}$ are presented alongside their performance envelopes in Fig. 2. Clearly, all disagreement errors converge to a neighborhood of zero after every change in the communication topology.

VI. CONCLUSIONS

A low-complexity, distributed, output synchronization protocol was designed in this paper to confront the heterogeneous, high-order, uncertain, multi-agent systems in Brunovsky canonical form, operated in a leader-follower scenario. The underlying communication graph was considered directed and switching. Receiving only relative output information from its neighbors and measuring its own state, the proposed controller achieved prescribed performance bounds on the convergence time and the output synchronization accuracy. The leader dynamics were considered unknown. Simulation studies clarified and verified the theoretical findings

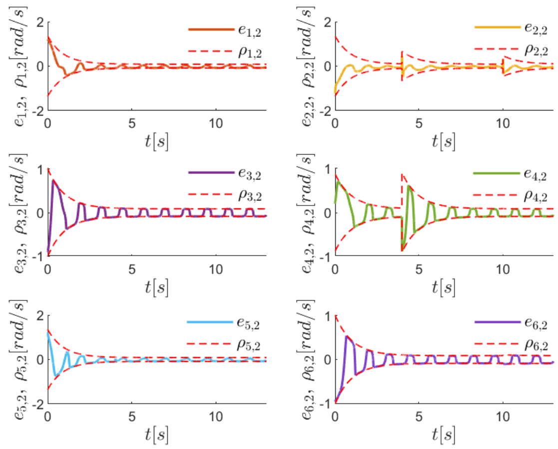

Fig. 2. Evolution of the intermediate error $x_{i,2} - a_{i,1}$, $i = 1, \ldots, 6$.

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