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Synchronous of Multiagent Systems over Finite Fields via Event-triggered Control

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Abstract— **This paper studies the leader-follower synchronous of multiagent systems (MASs) over finite fields by the event-triggered control (ETC). Firstly, the MASs over finite fields with ETC are modeled as algebraic form by the semi-tensor product (STP) method, based on which, the event-triggered synchronous problem is transformed into the set stabilization problem about the system with algebraic form. Secondly, the maximum invariant set is selected as the target set to reduce the number of controller execution times. Thirdly, the necessary and sufficient conditions of the event-triggered synchronous of the MAS as well as the design method of the optimal state feedback event-triggered controller are given. Finally, an example is presented to illustrate the conclusion of this paper.**

I. INTRODUCTION

Multiagent systems (MASs) aim to achieve some goals by designing control strategies for a group of agents or nodes that communicate with each other locally. Synchronization is a fundamental problem in MASs as it allows agents to coordinate their actions and achieve a collective objective. In MASs, synchronization has a wide range of applications, including robotic swarm coordination [1], [2], power grid control [3], [4], traffic flow control [5], [6], and social network analysis [7], [8].

In MASs, the finite field synchronization problem refers to how to achieve synchronization among a group of agents with discrete states and finite dynamic range under mutual communication and interaction. This synchronization problem often involves fields such as discrete event systems, computer networks, and distributed control. The synchronization problem of finite fields has been gradually studied by scholars [9], [10].

The event-triggered control (ETC) becomes an effective method to deal with the synchronization problem in MASs [11], [12], as it reduces the number of communication exchanges between agents and can be more efficient than traditional time-triggered control methods.

Recently, semi-tensor product (STP), proposed by Professor Cheng, has become a powerful tool for dealing with finite-valued systems. The classical linear system theory can be used to deal with the finite-valued systems, such as logical networks [13], [14], networked evolutionary games [15], [16], finite automata [17], [18] and so on. Since the finite field network is also a special kind of finite-valued systems, it is very convenient to deal with problems of the finite field network by STP [19], [20].

In this study, the event-triggered synchronous of MASs over finite fields is investigated by STP. There are three main contributions. Firstly, the MASs over finite fields are transformed into an equivalent algebraic form by STP. Based on this, the event-triggered synchronous problem is transformed into the set stabilization problem of the MASs. Secondly, according to the event-triggered mechanism, the global reachability of the maximum invariant set is considered. If the maximum invariant set is not globally reachable, then the maximum control invariant set (MCIS) is selected. Thirdly, the states of the event-triggered set are processed to reduce the number of controlled states and the set of attractors is considered to optimize the controller.

The rest of the article is structured as follows. Some notations used in this paper are presented in Section 2. Section 3 presents the definition of leader-follower synchronous, and converts the MASs into the equivalent algebraic form. In section 4, the main results of the paper are shown, including some criteria of event-triggered synchronous and the design method of the time-optimal state feedback event-triggered controller. In section 5, an example is presented to demonstrate the result. Section 6 is a brief conclusion.

II. PRELIMINARIES

Lemma 1: ([21]) Let $f : \mathscr{D}_k^n \to \mathscr{D}_k$ be a k-valued logical function. Then there exists a unique matrix $M_f \in \mathscr{L}_{k\times k^n}$, called the structural matrix of f, such that in the vector form we have

$$f(x_1,x_2,\ldots,x_n) = M_f \ltimes_{i=1}^n x_i,$$

where $x_i \in \Delta_k, i \in \{1,2,\ldots,n\}$.

III. PROBLEM FORMULATION

The dynamics of the leader have the following form as

$$X_0(t+1) = A_0X_0(t). \tag{1}$$

The dynamics of the i-th follower have the following form as

$$X_i(t+1) = \sum_{j\in\mathscr{N}_i\bigcup\{i\}} a_{ij}X_j(t). \tag{2}$$

Lemma 2: ([22]) Let $\vec{X},\vec{Y} \in \Delta_p$ be the vector form of $X,Y \in \mathbb{F}_p$.

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1) The algebraic form of $\vec{X}+_p\vec{Y}$ can be expressed as

$$\vec{X}+_p\vec{Y}=K_{+|p}\ltimes\vec{X}\ltimes\vec{Y},$$

where

$$K_{+|p}=[V_1\ V_2\ \cdots\ V_p]$$

is the structural matrix of addition "$+_p$", $V_1=\delta_p[1\ \cdots\ p]$ and $V_s=\delta_p[s\ \cdots\ p\ 1\ \cdots\ s-1]$, $s=2,\ldots,p$.

2) The algebraic form of $\vec{X}\times_p\vec{Y}$ can be expressed as

$$\vec{X}\times_p\vec{Y}=K_{\times|p}\ltimes\vec{X}\ltimes\vec{Y},$$

where

$$K_{\times|p}=[U_1\ U_2\ \cdots\ U_p]$$

is the structural matrix of multiplication "$\times_p$" and $U_s=\delta_p[(0\times(s-1))mod(p)+1\quad(1\times(s-1))mod(p)+1\ \cdots\ ((p-1)\times(s-1))mod(p)+1]$, $s=1,\ldots,p$.

Denote the vector forms of $X_i(t)$ and $\vec{a}_{ij}\in\Delta_p$ by $x_i(t)$ and $a_{ij}\in\mathbb{F}_p$. Let $x(t)=\ltimes_{i=0}^{N}x_i(t)\in\Delta_{p^{n(N+1)}}$, $x_i(t)=\ltimes_{r=1}^{n}x_i^r(t)\in\Delta_{p^n}$, then the algebraic form of (1) can be described as

$$\begin{aligned}&x_0^r(t+1)\\=&\vec{a}_{r1}^0\times_p x_0^1(t)+_p\cdots+_p\vec{a}_{rn}^0\times_p x_0^n(t)\\=&(K_{+|p})^{n-1}K_{\times|p}\vec{a}_{r1}^0x_0^1(t)\cdots K_{\times|p}\vec{a}_{rn}^0x_0^n(t)\\=&(K_{+|p})^{n-1}K_{\times|p}\vec{a}_{r1}^0D_f^{[p,p^{nN+n-1}]}x(t)\cdots\\&K_{\times|p}\vec{a}_{rn}^0D^{[p^{n-1},p,p^{nN}]}x(t)\\=&\overline{L}_0^rx(t),\end{aligned}\tag{3}$$

where $\overline{L}_0^r=(K_{+|p})^{n-1}(K_{\times|p}\vec{a}_{r1}^0D_f^{[p,p^{nN+n-1}]}*\cdots*K_{\times|p}\vec{a}_{rn}^0D^{[p^{n-1},p,p^{nN}]})\in\mathscr{L}_{p\times p^{n(N+1)}}$.

Then we have

$$x_0(t+1)=\overline{L}_0x(t),\tag{4}$$

where $\overline{L}_0=\overline{L}_0^1*\overline{L}_0^2*\cdots*\overline{L}_0^n\in\mathscr{L}_{p^n\times p^{n(N+1)}}$.

According to (2), the algebraic form of the i-th follower's dynamics is shown as

$$\begin{aligned}&x_i^r(t+1)\\=&(K_{+|p})^NK_{\times|p}\vec{a}_{i0}x_0^r(t)\cdots K_{\times|p}\vec{a}_{iN}x_N^r(t)\\=&(K_{+|p})^NK_{\times|p}\vec{a}_{i0}D^{[p^{r-1},p,p^{nN+n-r}]}x(t)\cdots\\&K_{\times|p}\vec{a}_{iN}D^{[p^{nN+r-1},p,p^{n-r}]}x(t)\\:=&\overline{L}_i^rx(t).\end{aligned}$$

Then the algebraic form of the i-th follower is shown as

$$x_i(t+1)=\overline{L}_ix(t),\ i=1,2,\ldots,N,\tag{5}$$

where $\overline{L}_i=\overline{L}_i^1*\overline{L}_i^2*\cdots*\overline{L}_i^n\in\mathscr{L}_{p^n\times p^{n(N+1)}}$.

Combining (4) and (5), we get the equivalent form of system (1) and (2) as

$$x(t+1)=\overline{L}x(t),\tag{6}$$

where $\overline{L}=\overline{L}_0*\overline{L}_1*\cdots*\overline{L}_N\in\mathscr{L}_{p^{n(N+1)}\times p^{n(N+1)}}$.

IV. MAIN RESULTS

The dynamics of the i-th follower with controls have the following form as

$$X_i(t+1)=\sum_{j\in\mathscr{N}_i\bigcup\{i\}}a_{ij}X_j(t)+b_iU_i(t),\quad i\in\{1,2,\ldots,N\}.\tag{7}$$

Similarly, using STP, we can obtain the algebraic form of (7) as shown in (6),

$$x(t+1)=\tilde{L}u(t)x(t),\tag{8}$$

where $\tilde{L}\in\mathscr{L}_{p^{n(N+1)}\times p^{n(N+1)+N}}$.

Denote the event-triggered set

$$\Pi:=\{\delta_{p^{n(N+1)}}^{j_1},\ldots,\delta_{p^{n(N+1)}}^{j_r}\},$$

Proposition 1: System is event-triggered synchronous over $\mathbb{F}_p$, if and only if there exists an ETC and an integer $\mu\in\mathbb{Z}_+$ such that

$$x(t)\in\Lambda$$

holds for any $x_0\in\Delta_{p^{n(N+1)}}$ and any $t\geq\mu$.

Proposition 1 transforms the event-triggered synchronous problem into the set stabilization problem of system.

V. EXAMPLE

Consider an MAS with three agents over $\mathbb{F}_3$.

The dynamics of the MAS are shown as follows

$$\begin{cases}x_0(t+1)=2x_0(t),\\x_1(t+1)=x_0(t)+2x_1(t),\\x_2(t+1)=2x_0(t)+x_1(t)+x_2(t),\end{cases}\tag{9}$$

and the dynamics of the MAS with controls are given as

$$\begin{cases}x_0(t+1)=2x_0(t),\\x_1(t+1)=x_0(t)+2x_1(t)+u_1(t),\\x_2(t+1)=2x_0(t)+x_1(t)+x_2(t)+u_2(t),\end{cases}\tag{10}$$

where $x_i(t)\in\mathbb{F}_3$ and $u_i(t)\in\mathbb{F}_3$, $i=1,2$.

We have the following algebraic form of (9) and (10) as

$$x(t+1)=\overline{L}x(t),$$

and

$$x(t+1)=\tilde{L}u(t)x(t),$$

where $\overline{L}=\delta_{27}[1\ 2\ 3\ 8\ 9\ 7\ 6\ 4\ 5\ 24\ 22\ 23\ 19\ 20\ 21\ 26\ 27\ 25\ 17\ 18\ 16\ 15\ 13\ 14\ 10\ 11\ 12]\in\mathscr{L}_{3^3\times3^3}$ and $\tilde{L}=\delta_{27}[1\ 2\ 3\ 8\ 9\ 7\ 6\ \cdots\ 12\ 10\ 18\ 16\ 17]\in\mathscr{L}_{3^3\times3^5}$.

Then we can get the algebraic form of the MAS with ETC as

$$x(t+1)=L\overline{u}(t)x(t),\tag{11}$$

where $L=\delta_{27}[1\ 2\ 3\ 8\ 9\ 7\cdots12\ 10\ 18\ 16\ 17]\in\mathscr{L}_{3^3\times(3^3+3^5)}$, and $\overline{u}(t)\in\Delta_{10}$.

Next, we design the state feedback event-triggered controller. Denote $S_0=\Delta_{27}\setminus\overline{\Gamma}$ and construct $T_{\overline{\Gamma}|S_0}$.

We further reduce the control execution times. The set of attractors of system (9) is

$$\begin{aligned}
&\Omega_1=\{\delta_{27}^1\},\Omega_2=\{\delta_{27}^2\},\Omega_3=\{\delta_{27}^3\},\\
&\Omega_4=\{\delta_{27}^4,\delta_{27}^8\},\Omega_5=\{\delta_{27}^5,\delta_{27}^9\},\Omega_6=\{\delta_{27}^6,\delta_{27}^7\},\\
&\Omega_7=\{\delta_{27}^{10},\delta_{27}^{24},\delta_{27}^{14},\delta_{27}^{20},\delta_{27}^{18},\delta_{27}^{25}\},\\
&\Omega_8=\{\delta_{27}^{11},\delta_{27}^{22},\delta_{27}^{15},\delta_{27}^{21},\delta_{27}^{16},\delta_{27}^{26}\},\\
&\Omega_9=\{\delta_{27}^{12},\delta_{27}^{23},\delta_{27}^{13},\delta_{27}^{19},\delta_{27}^{17},\delta_{27}^{27}\}.
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
&\mathrm{Col}_2(\hat{H})=\mathrm{Col}_2(H),\mathrm{Col}_3(\hat{H})=\mathrm{Col}_3(H),\\
&\mathrm{Col}_4(\hat{H})=\mathrm{Col}_4(H),\mathrm{Col}_5(\hat{H})=\mathrm{Col}_5(H),\\
&\mathrm{Col}_6(\hat{H})=\mathrm{Col}_6(H),\mathrm{Col}_7(\hat{H})=\delta_{10}^{10},\\
&\mathrm{Col}_8(\hat{H})=\delta_{10}^{10},\mathrm{Col}_9(\hat{H})=\delta_{10}^{10},\\
&\mathrm{Col}_{10}(\hat{H})=\mathrm{Col}_{20}(\hat{H})=\mathrm{Col}_{18}(\hat{H})=\mathrm{Col}_{25}(\hat{H})=\delta_{10}^{10},\\
&\mathrm{Col}_{11}(\hat{H})=\mathrm{Col}_{11}(H),\\
&\mathrm{Col}_{22}(\hat{H})=\mathrm{Col}_{15}(\hat{H})=\mathrm{Col}_{21}(\hat{H})\\
&=\mathrm{Col}_{16}(\hat{H})=\mathrm{Col}_{26}(\hat{H})=\delta_{10}^{10},\\
&\mathrm{Col}_{12}(\hat{H})=\mathrm{Col}_{23}(\hat{H})=\mathrm{Col}_{13}(\hat{H})=\mathrm{Col}_{19}(\hat{H})=\delta_{10}^{10},
\end{aligned}$$

then $\hat{\Pi}=\{\delta_{27}^2,\delta_{27}^3,\delta_{27}^4,\delta_{27}^5,\delta_{27}^6,\delta_{27}^{11},\delta_{27}^{14},\delta_{27}^{27}\}$ and the optimal state feedback event-triggered gain matrix $\overline{G}$ is

$$\overline{G}=\delta_{27}[10\ 3\ 2\ 6\cdots 10\ 10\ 10\ 10\ 10\ 10\ 6].$$

VI. CONCLUSIONS

In this paper, we have studied the event-triggered synchronous of the MAS over finite fields. First, we have given the definition of leader-follower synchronous, and converted the system into the equivalent algebraic form. Based on the algebraic form, the optimal state feedback ETC is design by the truth matrix method. Selecting the maximum invariant set as the target set can reduce the number of controller execution times. When the system can not be stabilized to the maximum invariant set, the MCIS is considered. The future work is to consider stabilizing the system to a control invariant set rather than the MCIS, thereby further reducing the number of controller execution times.

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