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Input-Output Data-Driven Sensor Selection

Filippos Fotiadis[1], Kyriakos G. Vamvoudakis[1]

***Abstract*— In this paper, we design a learning-based sensor selection procedure for an unknown cyber-physical system. In particular, a set of sensors that maximize a metric of observability of the system is chosen, but without using knowledge of the system's dynamics. The metric of observability is related to the notion of the $\mathcal{H}_2$ norm, which quantifies the strength of the sensor signals generated under a given control input excitation. It is shown that the evaluation of this metric boils down to solving a set of model-based Lyapunov equations which, however, is a task that cannot be carried out directly since the system is unknown. Nevertheless, we tackle this by expressing the metric solely with respect to input-output data, and we use the new expression to choose the best sensors for the system in a model-free manner, and in polynomial time. Simulations are performed to demonstrate the efficiency of the proposed approach.**

***Index Terms*— Sensor selection, input-output data, unknown systems.**

I. Introduction

Cyber-physical systems (CPS) are, by definition, complex systems that combine multiple software and hardware units, along with the communication channels through which these units exchange data [1], [2]. One of the major focuses of the systems and controls community is the design of decision-making/control policies for such systems, with objectives including regulation around some nominal operation point, safety in terms of avoiding hazardous areas of operation [3], or security against adversarial attacks [4]. Nevertheless, the design of the very components of the CPS is also of equal significance because these have a direct impact on the efficiency of any underlying control design. For example, the actuators of the CPS should be selected to render it easy to control, whereas its sensors should be chosen to provide as much information about the CPS state as possible.

This paper focuses on the problem of sensor selection, that is, the problem of choosing the best sensors to utilize in a CPS out of a larger set of available sensors [5]. This is a task that has been extensively studied in the literature, for example, [6]–[9] designed algorithms to select the sensors that maximize $\mathcal{H}_2$ norm-related metrics of the system. The intuition is that more potent sensors usually output stronger signals, and this potency is captured by the $\mathcal{H}_2$ norm. Similarly, the problem sensor selection has been studied from the perspective of optimizing Gramian-related metrics [10]–[13], linear-quadratic regulation [14]–[16], and security [17], [18]. Nevertheless, most existing works assume that the model of the system is completely known during the sensor selection procedure. This can often be a restrictive requirement for CPS, which are complex by definition and thus often contain unmodeled dynamics.

Learning and adaptive control are often used to deal with unknown dynamics, with some notable examples including model-free optimal control [19]–[21] and adaptive backstepping [22]. Several learning techniques were also recently developed to deal with unknown or uncertain models in the context of sensor and actuator selection. For example, to deal with a system model that is constantly changing, [23] developed a real-time learning-based algorithm that decides, at each time instant, which sensors the system should use, and which should be discarded. Moreover, in the context of actuator selection, [24]–[26] developed algorithms to choose the best-performing actuators for the system without knowledge its state/input matrices. Nevertheless, one of the main limitations is the requirement for full-state feedback. On the other hand, when all system matrices are unknown, and more importantly, when only output data are available for use, the problem of sensor selection becomes more difficult.

Motivated by the discussion above, in this paper, we consider the problem of sensor selection for unknown CPS using input-output data. We choose the cost function of the problem to be related to the $\mathcal{H}_2$ norm of the system, which is known to be optimizable in polynomial time [9]. However, since this function inherently depends on the system's matrices, we leverage techniques from the reinforcement learning literature [27] to express it solely with respect to input-output data. The data-driven expression of the cost function subsequently enables solving the sensor selection problem, despite the restrictiveness of the considered setup.

Notation: For any signal $x : \mathbb{N} \to \mathbb{R}^n$, we use the notation $x(t_1 : t_2) = [x^{\mathrm{T}}(t_2)\ x^{\mathrm{T}}(t_2-1)\ \dots\ x^{\mathrm{T}}(t_1+1)\ x^{\mathrm{T}}(t_1)]^{\mathrm{T}}$ for $t_1, t_2 \in \mathbb{N}$ such that $t_2 \geqslant t_1$. We use $\|\cdot\|$ to denote the standard Euclidean vector norm, and $\|\cdot\|_\infty$ to denote the infinity vector norm. For a matrix A, $A^\dagger$ denotes its Moore–Penrose inverse, and $\mathrm{vec}(A) = [A_{1,1}\ A_{2,1}\ \dots\ A_{n-1,n}\ A_{n,n}]^{\mathrm{T}}$ its vectorized form, with the inverse operation denoted with the operator $\mathrm{vec}^{-1}(\cdot)$. If A is symmetric, then $\mathrm{vech}(A) = [A_{1,1}\ A_{2,1}\ \dots\ A_{n,1}\ A_{2,2}\ A_{3,2}\ \dots\ A_{n,n}]^{\mathrm{T}}$ denotes its half-vectorized form, while $\mathrm{vecs}(A) = [A_{1,1}\ 2A_{2,1}\ \dots\ 2A_{n,1}\ A_{2,2}\ 2A_{3,2}\ \dots\ A_{n,n}]^{\mathrm{T}}$ denotes its scaled half-vectorized form. For a square matrix A, $\mathrm{tr}(A)$ denotes its trace, and $\rho(A)$ its spectral radius. For any two matrices A, B, $A \otimes B$ denotes their Kronecker product. The identity matrix of order n is denoted as I_n.

[1]F. Fotiadis and K. G. Vamvoudakis are with the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA, USA. Email: {ffotiadis, kyriakos}@gatech.edu.

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II. Problem Statement

Consider, for $t \in \mathbb{N}$, a discrete-time system of the form:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t),\ x(0) = x_0, \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector with initial value $x_0 \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are the system matrices. Each row of the matrix C represents a sensor of the system, whereas each column of the matrix B represents an actuator of the system. We have:

$$\begin{aligned} C &= \begin{bmatrix} c_1^{\mathrm{T}} & c_2^{\mathrm{T}} & \dots & c_p^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ B &= \begin{bmatrix} b_1 & b_2 & \dots & b_m \end{bmatrix}, \end{aligned}$$

where $c_j \in \mathbb{R}^{1\times n}$, $j = 1,\dots,p$ represents a sensor, whereas $b_i \in \mathbb{R}^n$, $i = 1,\dots,m$, represents an actuator.

Within the context of the sensor selection problem, we assume that, due to computational, communicational, or energy constraints, we want to select only $p' < p$ out of the p sensors available for use at each time instant. That is, we seek a selection matrix

$$\mathbb{S}_\gamma = \begin{bmatrix} e_{\gamma_1} & e_{\gamma_2} & \dots & e_{\gamma_{p'}} \end{bmatrix}^{\mathrm{T}},$$

with $\gamma = \{\gamma_1,\dots,\gamma_{p'}\} \subset \{1,\dots p\}$, that will choose which rows of the sensing matrix C will be used, so that the new sensing matrix is $C' = \mathbb{S}_\gamma C$. This problem has been studied extensively in the literature, and various algorithms to tackle it efficiently have been proposed [9]–[12], [16]. However, in this study we consider the sensor selection problem under a restrictive setup in which

1) All system matrices A, B, C are unknown.
2) Only the output provided by the selected sensors at the instant $t \in \mathbb{N}$ is available for measurement.

While data-driven approaches to perform sensor/actuator selection under the assumption of unknown A or B matrices have also been proposed in the literature [24], [25], [28], those rely on the availability of full-state feedback. On the other hand, here we restrict ourselves not just to output feedback, but also to the assumption that the C matrix in (1) is unknown. As a minimum requirement, we still require the following observability Assumption, which implies we know a set of sensors that renders the system observable.

Assumption 1. There exists a known selection index $\hat{\gamma} = \{\hat{\gamma}_1,\dots,\hat{\gamma}_r\} \subset \{1,\dots,p\}$, such that $(A,\ \mathbb{S}_{\hat{\gamma}}C)$ is observable.

A. Observability Metric and $\mathcal{H}_2$ Norm

The next step is to define the cost function that will distinguish which set of sensors is the "best" to select, with metrics of observability becoming relevant here. Nevertheless, since all system matrices A, B, C are unknown, there is no unique state-space representation for the input-output behavior of (1). This limitation restricts us to metrics of observability that apply only to the transfer function G of the tuple (A, B, C).

One metric that jointly quantifies controllability and observability is the $\mathcal{H}_2$ norm of the transfer function G [6]. This is defined as the infinite-horizon expected output energy

$$\|G\|_2^2 = \mathbb{E}\left[\sum_{t=0}^{\infty} \|y(t)\|^2\right] \tag{2}$$

under the condition that $x_0 = 0$ and $u \sim \mathcal{N}(0,1)$. The motivation behind choosing this norm as the cost function for the sensor selection problem is that "more observable" sensors will generally yield larger output signals [6]. However, a limitation of (2) is that it applies only to asymptotically stable systems, owing to the infinite summation in (2) being divergent if A is unstable. To circumvent this issue, following [29], [30] one can discount the output measurements in (2) with a parameter $a \in (0,1)$, and instead define a "discounted $\mathcal{H}_2$ norm" as:

$$\|G\|_{2,a}^2 = \mathbb{E}\left[\sum_{t=0}^{\infty} a^{2t} \|y(t)\|^2\right]. \tag{3}$$

If $a < \frac{1}{\rho(A)}$, similar to [29], it is straightforward to verify that (3) is well-defined. In particular, it follows that $\|G\|_{2,a}^2 = \left\|\tilde{G}\right\|_2^2$ where $\tilde{G}$ is the transfer function of the tuple (aA, B, C). Defining $\tilde{A} = aA$, it then follows from standard linear systems theory that

$$\|G\|_{2,a}^2 = \mathrm{tr}(CW_cC^{\mathrm{T}}) = \mathrm{tr}(B^{\mathrm{T}}W_oB) \tag{4}$$

where W_o, W_c are observability and controllability Gramians for the pairs $(\tilde{A}, C)$ and $(\tilde{A}, B)$, which satisfy the Lyapunov equations (LEs):

$$\begin{aligned} \tilde{A}W_c\tilde{A}^{\mathrm{T}} - W_c + BB^{\mathrm{T}} &= 0, \\ \tilde{A}^{\mathrm{T}}W_o\tilde{A} - W_o + C^{\mathrm{T}}C &= 0. \end{aligned}$$

In the rest of this paper, we focus on finding the sensors that maximize the cost function (3)-(4) using input-output data, and without using the system matrices A, B, C. In particular, for the sensor selection problem, we want to find the optimal selection matrix $\mathbb{S}_{\gamma^\star}$ that attains:

$$\mathbb{S}_{\gamma^\star} = \arg\max_{\mathbb{S}_\gamma} \mathrm{tr}(\mathbb{S}_\gamma CW_cC^{\mathrm{T}}\mathbb{S}_\gamma^{\mathrm{T}}). \tag{5}$$

III. Data-Driven Estimation of the Cost for Fixed Sensor Sets

The first step towards solving the sensor selection problem with unknown system matrices A, B, C, is being able to express the cost function in (5) using input-output data only. In this section, we perform this task for a fixed choice of the selection matrix $\mathbb{S}_\gamma$, and we will subsequently use this in the next section to perform data-driven sensor selection.

Before proceeding, note that this task requires i) being able to take measurements from the sensors involved in the matrix $\tilde{C} = \mathbb{S}_\gamma C$, whose $\mathcal{H}_2$ norm we want to evaluate[1]; and ii) concurrently being able to take measurements from the sensors involved in $\hat{C} = \mathbb{S}_{\hat{\gamma}}C$, as a minimum observability

[1] Since C is unknown, such a requirement is natural.

requirement. In short, this reduces the output measurements we can access to:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t),\ x(0) = x_0, \\ \hat{y}(t) &= \hat{C}x(t),\ \tilde{y}(t) = \tilde{C}x(t). \end{aligned} \tag{6}$$

A. Expressing the State with Input-Output Trajectories

Apart from the matrices A, B, C being unknown, a major restriction of the present setup is not having access to full-state feedback, rather, only to the measured outputs available at each time. Nevertheless, directly following [27] the state $x(t)$ can be reconstructed using a history stack of past outputs and control inputs.

Lemma 1. *[27] Let Assumption 1 hold, so that $(A, \hat{C})$ is observable. Define the matrices*

$$\begin{aligned} U_N &= \begin{bmatrix} B & AB & A^2B & \dots & A^{N-1}B \end{bmatrix}, \\ V_N &= \begin{bmatrix} (\hat{C}A^{N-1})^{\mathrm{T}} & \dots & (\hat{C}A)^{\mathrm{T}} & \hat{C}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ T_N &= \begin{bmatrix} 0 & \hat{C}B & \hat{C}AB & \dots & \hat{C}A^{N-2}B \\ 0 & 0 & \hat{C}B & \dots & \hat{C}A^{N-3}B \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & & 0 & \hat{C}B \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

If $N \geqslant K$, where K is the observability index[2], then at any $t \geqslant N$:

$$x(t) = Mz(t)$$

where

$$z(t) = \begin{bmatrix} u(t-1:t-N) \\ \hat{y}(t-1:t-N) \end{bmatrix},$$

and $M = \begin{bmatrix} M_u & M_y \end{bmatrix}$, $M_y = A^N V_N^{\dagger}$, $M_u = U_N - M_y T_N$.

A clear limitation of Lemma 1 is that it relies on prior knowledge of the system matrices A, B, C. However, this requirement is relaxed in the following subsection.

B. Expressing the Cost Function with Input-Output Data

Denote $J(\mathbb{S}_\gamma) = \mathrm{tr}(\mathbb{S}_\gamma C W_c C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}})$ as the cost function (5), which we want to estimate for a fixed sensor set $\mathbb{S}_\gamma$ using input-output data. The following Lemma re-expresses this cost with respect to a dual LE, which will subsequently facilitate the task of its data-driven estimation.

Lemma 2. *It holds that*

$$J(\mathbb{S}_\gamma) = \mathrm{tr}(B^{\mathrm{T}} W_o^\gamma B), \tag{7}$$

where W_o^γ uniquely solves the LE

$$\tilde{A}^{\mathrm{T}} W_o^\gamma \tilde{A} - W_o^\gamma + C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}} \mathbb{S}_\gamma C = 0. \tag{8}$$

Proof. Following standard linear systems theory, we have $W_c = \sum_{t=0}^{\infty} \tilde{A}^t B B^{\mathrm{T}} (\tilde{A}^t)^{\mathrm{T}}$. Therefore:

$$\begin{aligned} J(\mathbb{S}_\gamma) &= \mathrm{tr}(\mathbb{S}_\gamma C W_c C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}}) \\ &= \mathrm{tr}\left(\mathbb{S}_\gamma C \sum_{t=0}^{\infty} \tilde{A}^t B B^{\mathrm{T}} (\tilde{A}^t)^{\mathrm{T}} C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}}\right) \\ &= \mathrm{tr}\left(B^{\mathrm{T}} \sum_{t=0}^{\infty} (\tilde{A}^t)^{\mathrm{T}} C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}} \mathbb{S}_\gamma C \tilde{A}^t B\right) \\ &= \mathrm{tr}(B^{\mathrm{T}} W_o^\gamma B) \end{aligned}$$

where $W_o^\gamma = \sum_{t=0}^{\infty} (\tilde{A}^t)^{\mathrm{T}} C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}} \mathbb{S}_\gamma C \tilde{A}^t$. Since $\tilde{A}$ is Schur stable, W_o^γ uniquely solves (8). ■

Using Lemma 1, and inspired from reinforcement learning methods [27], the following theorem provides a data-driven procedure for estimating the cost function (7) of the sensor selection problem.

Theorem 1. *Consider system* (6) *and let Assumption 1 hold. Then*

$$J(\mathbb{S}_\gamma) = \mathrm{tr}\left(E_1^{\mathrm{T}} \bar{W}_o^\gamma E_1\right), \tag{9}$$

where $E_1 = \begin{bmatrix} I_m & 0_{m \times (Nr + (N-1)m)} \end{bmatrix}^{\mathrm{T}}$, and $\bar{W}_o^\gamma$ is a symmetric matrix that satisfies, for all $t \geqslant N$, the data-driven equation

$$\Phi^{\mathrm{T}}(t) \mathrm{vech}(\bar{W}_o^\gamma) + \|\tilde{y}(t)\|^2 = 0, \tag{10}$$

with

$$\begin{aligned} \Phi(t) =& \mathrm{vecs}\Big(\mathrm{vec}^{-1}\Big(a^2((z(t+1) - E_1 u(t)) \\ & \otimes (z(t+1) - E_1 u(t))) - z(t) \otimes z(t)\Big)\Big). \end{aligned} \tag{11}$$

Proof. Multiplying both sides of equation (8) of Lemma 2 with $x(t)$, $t \geqslant N$, and using (6) as well as the definition $\tilde{A} = aA$, we obtain

$$a^2 x^{\mathrm{T}}(t) A^{\mathrm{T}} W_o^\gamma A x(t) - x^{\mathrm{T}}(t) W_o^\gamma x(t) + \|\tilde{y}(t)\|^2 = 0. \tag{12}$$

Moreover, from the system definition (6) one has $Ax(t) = x(t+1) - Bu(t)$. Substituting this relation in (12) yields

$$\begin{aligned} a^2 (x(t+1) - Bu(t))^{\mathrm{T}} W_o^\gamma (x(t+1) - Bu(t)) \\ - x^{\mathrm{T}}(t) W_o^\gamma x(t) + \|\tilde{y}(t)\|^2 = 0. \end{aligned} \tag{13}$$

Employing Lemma 1, we have $x(t+1) = Mz(t+1)$ and $x(t) = Mz(t)$. Moreover, notice that $B = ME_1$. Hence, (13) becomes

$$\begin{aligned} a^2 (Mz(t+1) - ME_1 u(t))^{\mathrm{T}} W_o^\gamma (Mz(t+1) - ME_1 u(t)) \\ - z^{\mathrm{T}}(t) M^{\mathrm{T}} W_o^\gamma M z(t) + \|\tilde{y}(t)\|^2 = 0. \end{aligned}$$

Defining the symmetric matrix $\bar{W}_o^\gamma = M^{\mathrm{T}} W_o^\gamma M$, we obtain,

$$\begin{aligned} a^2 (z(t+1) - E_1 u(t))^{\mathrm{T}} \bar{W}_o^\gamma (z(t+1) - E_1 u(t)) \\ - z^{\mathrm{T}}(t) \bar{W}_o^\gamma z(t) + \|\tilde{y}(t)\|^2 = 0. \end{aligned} \tag{14}$$

Note now that

$$\begin{aligned} &(z(t+1) - E_1 u(t))^{\mathrm{T}} \bar{W}_o^\gamma (z(t+1) - E_1 u(t)) \\ &= \Big((z(t+1) - E_1 u(t)) \otimes (z(t+1) - E_1 u(t))\Big)^{\mathrm{T}} \mathrm{vec}(\bar{W}_o^\gamma), \\ &z^{\mathrm{T}}(t) \bar{W}_o^\gamma z(t) = (z(t) \otimes z(t))^{\mathrm{T}} \mathrm{vec}(\bar{W}_o^\gamma). \end{aligned}$$

Using these relations, one can linearly parameterize (14) as:

$$\bar{\Phi}^{\mathrm{T}}(t) \mathrm{vec}(\bar{W}_o^\gamma) + \|\tilde{y}(t)\|^2 = 0,$$

[2]The observability index is upper-bounded as $K \leqslant n$.

where $\bar{\Phi}(t) = a^2\Big((z(t+1) - E_1u(t)) \otimes (z(t+1) - E_1u(t))\Big) - z(t)\otimes z(t)$. Applying half-vectorization thanks to the symmetricity of $\bar{W}_o^\gamma$ gives equations (10)-(11). Finally, recall that $\bar{W}_o^\gamma = M^{\mathrm{T}} W_o^\gamma M$, and $ME_1 = B$. Therefore, it follows that

$$\begin{aligned}\mathrm{tr}(E_1^{\mathrm{T}}\bar{W}_o^\gamma E_1) &= \mathrm{tr}(E_1^{\mathrm{T}} M^{\mathrm{T}} W_o^\gamma M E_1) \\ &= \mathrm{tr}(B^{\mathrm{T}} W_o^\gamma B) = J(\mathbb{S}_\gamma),\end{aligned}$$

with the last equality following from Lemma 2. This yields equation (9), and concludes the proof. ■

Note now that equation (10) is linearly parameterized with respect to $\bar{W}_o^\gamma$. As a result, it is straightforward to solve it for $\bar{W}_o^\gamma$ with a least-squares procedure, provided a number of sufficiently rich input-output measurements is available. This claim is summarized in the following Corollary.

Corollary 1. *Let $t_0, t_1, \ldots, t_k$, $k \geqslant N$, be measurement time instants, and let Assumption 1 hold. Denote*

$$\begin{aligned}\Psi &:= \begin{bmatrix}\Phi(t_0) & \Phi(t_1) & \ldots & \Phi(t_k)\end{bmatrix}, \\ Y_\gamma &:= \begin{bmatrix}\|\tilde{y}(t_0)\|^2 & \|\tilde{y}(t_1)\|^2 & \ldots & \|\tilde{y}(t_k)\|^2\end{bmatrix}^{\mathrm{T}}.\end{aligned}$$

If Ψ has full row rank, then

$$\mathrm{vech}(\bar{W}_o^\gamma) = -(\Psi^{\mathrm{T}})^\dagger Y_\gamma. \tag{15}$$

Proof. Stacking equations of the form (10) for $t = t_1, \ldots, t_k$ in a vertical matrix, we obtain

$$\Psi^{\mathrm{T}}\mathrm{vech}(\bar{W}_o^\gamma) = -Y_\gamma. \tag{16}$$

Multiplying both sides by Ψ, one has $\Psi\Psi^{\mathrm{T}}\mathrm{vech}(\bar{W}_o^\gamma) = -\Psi Y_\gamma$. Since Ψ has full row rank, $\Psi\Psi^{\mathrm{T}}$ is invertible. Hence, the solution to this equation is given by $\mathrm{vech}(\bar{W}_o^\gamma) = -(\Psi\Psi^{\mathrm{T}})^{-1}\Psi Y_\gamma$, and $(\Psi^{\mathrm{T}})^\dagger = (\Psi\Psi^{\mathrm{T}})^{-1}\Psi$ since Ψ has full row rank, which yields equation (15). ■

Combining the assumptions and conditions of Theorem 1 and Corollary 1, we finally obtain a complete data-driven expression for the cost function $J(\mathbb{S}_\gamma)$.

Corollary 2. *Let Assumption 1 hold and Ψ have full row rank. Then:*

$$J(\mathbb{S}_\gamma) = -\mathrm{tr}\left(E_1^{\mathrm{T}}\mathrm{vech}^{-1}\left(((\Psi^{\mathrm{T}})^\dagger Y_\gamma\right) E_1\right). \tag{17}$$

Remark 1. The operator $\mathrm{vecs}(\mathrm{vec}^{-1}(\cdot))$ in (11), which transforms the full-vectorized form to a half-vectorized form, can be implemented using an elimination matrix. Similarly, a duplication matrix can help speed up the computation of $\mathrm{vech}^{-1}(\cdot)$ in (17). These two properties are crucial when implementing the proposed data-driven scheme in large-scale systems, in which $n, m \gg 1$. □

Remark 2. Although Corollary 2 requires Ψ to have full row rank, it can be proved that formula (17) is valid even if this is not the case, but where the weaker condition $\mathrm{rank}(\Psi) = (Nm+n)(Nm+n+1)/2$ holds. The proof of this will be presented in a future extended version of this paper. □

Algorithm 1 Data-Driven Sensor Selection

```
procedure
    for j = 1, ..., p do
        C̃ ← S_j C.
        Gather input-output data from (6).
        Evaluate J(S_j) from (17).
    end for
    Sort J(S_j) for j = 1, ..., p in decreasing order,
    let γ* = {γ*_1, ..., γ*_p'} contain the p' indices with
    the highest scores.
    Select C' ← S_γ* C.
end procedure
```

IV. Data-Driven Sensor Selection

A naive way to use the results of the previous section and perform data-driven sensor selection would be to evaluate the cost $J(\mathbb{S}_\gamma)$ for all possible selection matrices $\mathbb{S}_\gamma$ using data, and then choose the selection matrix that maximizes $J(\mathbb{S}_\gamma)$. However, not only is such a brute-force procedure of extraordinarily high computational complexity, but it also requires a lot of input-output data for the data-driven evaluation of the cost of each distinct sensor set.

Towards obtaining a less naive data-driven selection algorithm, the following theorem provides a rather unsurprising result, given the modularity of the $\mathcal{H}_2$ norm proved in [9]. That is, to evaluate $J(\mathbb{S}_\gamma)$, the theorem shows that one only needs to evaluate $J(\mathbb{S}_{\gamma_i})$ for $i = 1, \ldots, p'$.

Theorem 2. *It holds that $J(\mathbb{S}_\gamma) = \sum_{i=1}^{p'} J(\mathbb{S}_{\gamma_i})$.*

Proof. It suffices to see that

$$\mathbb{S}_\gamma C W_c C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}} = \begin{bmatrix} c_{\gamma_1} W_c c_{\gamma_1}^{\mathrm{T}} & c_{\gamma_1} W_c c_{\gamma_2}^{\mathrm{T}} & \ldots & c_{\gamma_1} W_c c_{\gamma_{p'}}^{\mathrm{T}} \\ c_{\gamma_2} W_c c_{\gamma_1}^{\mathrm{T}} & c_{\gamma_2} W_c c_{\gamma_2}^{\mathrm{T}} & \ldots & c_{\gamma_2} W_c c_{\gamma_{p'}}^{\mathrm{T}} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\gamma_{p'}} W_c c_{\gamma_1}^{\mathrm{T}} & c_{\gamma_{p'}} W_c c_{\gamma_2}^{\mathrm{T}} & \ldots & c_{\gamma_{p'}} W_c c_{\gamma_{p'}}^{\mathrm{T}} \end{bmatrix}.$$

Therefore,

$$J(\mathbb{S}_\gamma) = \mathrm{tr}(\mathbb{S}_\gamma C W_c C^{\mathrm{T}} \mathbb{S}_\gamma^{\mathrm{T}}) = \sum_{i=1}^{p'} \mathrm{tr}(c_{\gamma_i} W_c c_{\gamma_i}^{\mathrm{T}}) = \sum_{i=1}^{p'} J(\mathbb{S}_{\gamma_i}),$$

which is the required result. ■

The implication of Theorem 2 is that, to solve the sensor selection problem, one does not need to perform data-driven estimation of $J(\mathbb{S}_\gamma)$ for all possible selection matrices $\mathbb{S}_\gamma$; rather, one only needs to evaluate the cost $J(\mathbb{S}_j)$, $j = 1, \ldots, p$, of each distinct sensor row in C. Subsequently, sorting the scores $J(\mathbb{S}_j)$ for all $j = 1, \ldots, p$, and choosing $\mathbb{S}_{\gamma^\star}$ to contain the p' sensors with the highest scores indeed solves the sensor selection problem (5). This procedure is summarized in the following Algorithm.

As discussed above, the convergence of Algorithm 1 follows directly from Corollary 2 and Theorem 2.

Corollary 3. *Under the Assumptions of Corollary 2, Algorithm* 1 *converges to the optimal sensor selection matrix* (5).

TABLE I
VALUES OF ESTIMATED COST AND RELATIVE ERRORS

Sensor j	1	2	3	4	5
$\hat{J}(\mathbb{S}_j)$	0.320	0.0024	0.5882	0.3673	0.0326
$\frac{J(\mathbb{S}_j)-\hat{J}(\mathbb{S}_j)}{J(\mathbb{S}_j)}$	$4.5{\cdot}10^{-12}$	$1.1{\cdot}10^{-7}$	$1.2{\cdot}10^{-8}$	$3.0{\cdot}10^{-12}$	$1.2{\cdot}10^{-10}$

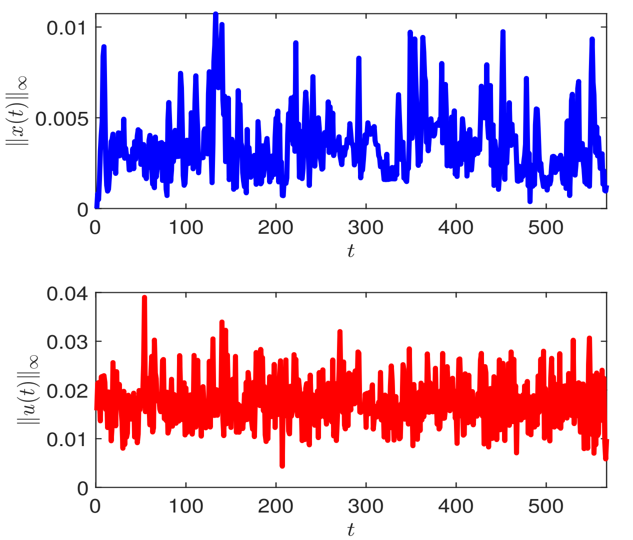

Fig. 1. Evolution of the state and control input norm during the data-gathering phase of Algorithm 1.

Remark 3. While Algorithm 1 sequentially evaluates $J(\mathbb{S}_j)$ from (17) using different input-output data at each iteration j, this evaluation can also take place using the same data matrix Ψ for all j. However, this requires having the resources to employ all to-be-evaluated sensors at once. □

V. SIMULATIONS

A. Benchmark Aircraft

We consider the Aero-Data Model in Research Environment (ADMIRE) benchmark aircraft, with its continuous-time plant and input matrices A_c, B_c given as in [31]. The discrete-time equivalents of these matrices are subsequently obtained using Euler integration with a sample rate of $h = 0.01$ seconds, i.e., $A = I_5 + A_c h$ and $B = B_c h$.

We assume there are $p = 5$ sensors available for selection, each of which measures a distinct state of the system, i.e., $C = I_5$. Out of these sensors, we want to select $p' = 4$ of them to maximize the cost (5) with $a = 0.98$. To this end, we apply Algorithm 1, gathering input-output data from the system under the control input proposed in [27] augmented with random exploration noise, and using these data to estimate $J(\mathbb{S}_j)$ for all $j = 1, \ldots, p$ without knowledge of the system. The known observable sensor set is selected as $\hat{\gamma} = \{1, 3\}$, and $N = 3$.

The trajectories of the infinity norm of the control input and state vector during the data-gathering phase are shown in Figure 1, illustrating that the system had to be perturbed only slightly with exploration noise in order to implement the data-driven Algorithm 1. In addition, the estimated values of $J(\mathbb{S}_j)$, denoted as $\hat{J}(\mathbb{S}_j)$, as well as the relative estimation

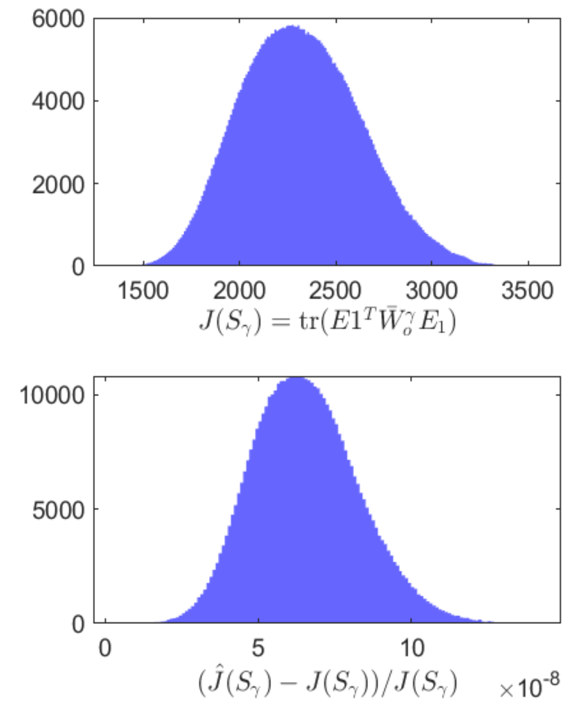

Fig. 2. Histogram of a) the cost value of $J(\mathbb{S}_\gamma)$ for all values of γ with cardinality 7; and b) the relative cost error of $\frac{J(\mathbb{S}_\gamma)-\hat{J}(\mathbb{S}_\gamma)}{J(\mathbb{S}_\gamma)}$ for all values of γ with cardinality 7.

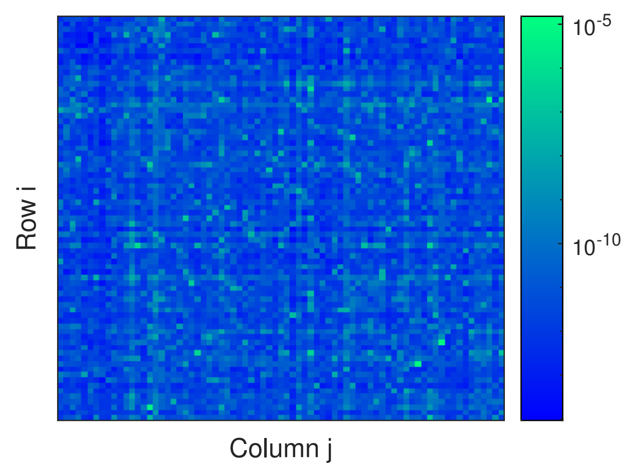

Fig. 3. Heatmap of the mean squared value of the relative error $\frac{[\hat{W}_o^\gamma - \bar{W}_o^\gamma]_{ij}}{[\bar{W}_o^\gamma]_{ij}}$ for each entry i, j.

errors $(J(\mathbb{S}_j) - \hat{J}(\mathbb{S}_j))/J(\mathbb{S}_j)$, are shown in Table I. From this table, we notice that the relative estimation error is practically zero, implying an accurate enough estimation of $J(\mathbb{S}_j)$. Moreover, from this table we conclude that the optimal sensor choice is $\gamma^\star = \{1, 3, 4, 5\}$.

B. Random System

To further test the proposed data-driven sensor selection scheme, we implement it on a larger system with $n = 25$ states, $m = 10$ actuators, $p = 25$ available sensor choices, and where the system matrices A, B, C are randomly generated. Similarly to the previous example, the purpose is, out of the 25 available sensors, to find the $p' = 7$ of them that maximize the cost (5) with $a = 0.1$. To this end, we apply Algorithm 1, gathering input-output data from the system and using them to estimate $J(\mathbb{S}_j)$ for all $j = 1, \ldots, p$. The known observable sensor set is chosen as $\hat{\gamma} = \{1, 2, 3, 4, 5\}$, and $N = 5$.

The estimated optimal set of sensors is found to be $\gamma^\star = \{5, 7, 10, 12, 15, 17, 24\}$, with a value of $J(\mathbb{S}_{\gamma^\star}) = 3553.7$. To verify that this estimated optimal value is true, Figure 2 shows the histogram of the value of the cost $J(\mathbb{S}_\gamma)$ for all possible choices of γ, demonstrating that the estimated $\gamma^\star$ indeed maximizes $J(\mathbb{S}_\gamma)$. Figure 2 also depicts the histogram of the relative estimation errors $\frac{J(\mathbb{S}_\gamma) - \hat{J}(\mathbb{S}_\gamma)}{J(\mathbb{S}_\gamma)}$ for all possible choices of γ, showing that those errors remain strictly below 10^{-6}. Finally, to ascertain that the estimated value $\hat{W}_o^\gamma$ of $\bar{W}_o^\gamma = M^{\mathrm{T}} W_o^\gamma M$ used in (9) is close to its real one, Figure 3 presents a heatmap of the mean squared value of the relative error $\frac{[\hat{W}_o^\gamma - \bar{W}_o^\gamma]_{ij}}{[\bar{W}_o^\gamma]_{ij}}$ for each entry i, j of the matrices $\bar{W}_o^\gamma, \hat{W}_o^\gamma$, averaged over all of $\gamma = \{1\}, \{2\}, \ldots, \{p\}$. From this figure, we notice that all entries have a mean squared relative error of less than 10^{-5}, hence $\hat{W}_o^\gamma$ indeed provided an accurate enough estimation of $\bar{W}_o^\gamma$.

VI. Conclusion

We studied the problem of sensor selection for systems whose dynamics are unknown. The cost function of the sensor selection problem was related to the notion of the $\mathcal{H}_2$ norm, the computation of which requires knowledge of the system's dynamics. Nevertheless, we relaxed this requirement of system knowledge by expressing the cost with respect to input-output data and used the resulting expression to perform model-free sensor selection in polynomial time.

Future work includes extending the proposed scheme to joint sensor-actuator selection and control design.

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