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Distributed Adaptive Tracking Control of Pure-feedback Multi-agent Systems with Full State and Control Input Constraints

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Abstract— **The distributed tracking control of multi-agent systems with the general pure-feedback agent dynamics, the full state constraints, and the control input constraints is investigated in this paper. Based on the one-to-one nonlinear mapping, the saturation function transformation, and the degree elevation techniques, the pure-feedback multi-agent system with full state constraints and control input constrains is firstly transformed into a novel one without constraints. Considering the unknown control sign and the unknown dynamic models, a distributed adaptive control law is proposed by leveraging the merits of Nussbaum function and neural networks. The rigorous Lyapunov stability analysis shows that the agent constraints are always satisfied, and the cooperative tracking errors can be made as small as possible by appropriately setting the control parameters. Finally, a numerical simulation is conducted to clarify the effectiveness of the proposed control strategy.**

Index Terms— **Multi-agent systems, full state constraints, unknown control directions, pure-feedback form.**

I. Introduction

Distributed tracking control of multi-agent systems (MAS) has broad applications in multiple unmanned aerial vehicles, attitude alignment, robotic teams and so on [1]. From the aspective of agent dynamics, the first-order linear agent [2], the second-order agent [3],[4], the nonlinear dynamics with external noises or other unpredictable disturbances [5]-[7], the strict-feedback nolinear dynamics [9], the non-affine dynamics [10] or pure-feedback form [11] have been extensively investigated. Based on the backstepping method, the cooperative tracking controller for high-order nonlinear MASs can be desinged and analyzed. Moreover, by applying the dynamic surface control (DSC) technique [8] and the neural network universal approximation property, the explosion of complexity in the conventional backstepping method and the unknown dynamics can be overcome.

In real situation, the constraints often appear owing to physical limitations or the safe operation of systems such as the maximum allowable speed of wheeled mobile robots [12], actuator saturation [13], and so on. To solve the problems of state constraints, the Barrier Lyapunov Function (BLF) and the nonlinear mapping method (NM) have been extensively applied. By employing the BLF, the output-constrained consensus protocol for second-order nonlinear MASs was proposed in [14]. The work in [15] investigated the distributed control of the nonlinear strict-feedback system with state constraints, unmeasured states, and disturbances. For the BLF-based control method, it is inconvenient to make the new designs of controllers to adapt to the changes of Lyapunov functions. As a result, a novel nonlinear mapping methodology was proposed [16]. The NM technique was introduced to transform the strict-feedback system with full state constraints into a novel one without state constraints and then be applied to the non-affine pure-feedback system with full state constraints [17],[18]. Compared with the BLF-based control method, the NM-based design process employed traditional Lyapunov functions rather than redesigning the Lyapunov function. As a result, the proofs of results are more concise and understandable.

For most of the adaptive consensus algorithms, it is a common assumption that the control direction or the sign of control input is known in advance such as [14], [15] and [17]. In [19], the consensus problem with unknown control direction was discussed for single-integrator MASs. The work [20] introduced a neural adaptive consensus control algorithm for high-order nonlinear MASs with unknown control directions. In [21]-[23], the similar problems were addressed for strict-feedback nonlinear MASs with the unknown control direction by applying backstepping adaptive control. To the best of our knowledge, the adaptive consensus problem for pure-feedback nonlinear systems with full state constraints, input constraints, and unknown control direction remains unsolved.

Inspired by the above works, this paper attempts to propose a distributed consensus control protocol for pure-feedback nonlinear MASs with full state constraints, input saturation, and unknown control directions. The contributions of this paper are briefly summarized as follows. (1) The pure-feedback form of MASs discussed in this paper is more general and the more relaxed assumptions are made as compared with the existing works [21]-[23]. (2) The control sign problem appears in the pure-feedback nonlinear multi-agent systems. Unlike the previous results in [14]-[17], the control sign is not required in this work. (3) The pure-feedback nonlinear multi-agent systems with full state constraints and input saturation are considered simultaneously and they are dealt with under the same framework in this paper. (4) We first transform the constrained system into the unconstrained form. By the degree elevation technique, we further transform the non-affine form into the affine-form. The unknown nonlinearities are approximated by the neural networks and the problem of explosion of complexity is avoided by applying the DSC in the backstepping design process.

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II. PRELIMINARIES AND PROBLEM FORMULATION

A. Problem Formulation

Consider a multi-agent system consisting of a leader and N followers. A digraph $\bar{\mathcal{G}}=(\bar{\mathcal{V}},\bar{\mathcal{E}})$ with $\bar{\mathcal{V}}=0,\cdots,N$ is used to describe the communication topology among the $N+1$ agents. The subgraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is applied to represent the topology of N followers with the adjacency matrix $A=[a_{ij}]\in R^{N\times N}$, the in-degree matrix $D=diag\{d_1,\ldots,d_N\}$, and the Laplacian matrix $L=D-A$. The Laplacian matrix $\bar{L}$ corresponding to $\bar{\mathcal{G}}$ is defined as $\bar{L}=\begin{bmatrix}L+B & -b\\ 0 & 0\end{bmatrix}$, where $b=[b_1,...,b_N]^T$, $B=diag(b_1,...,b_N)$ with $b_i=1$ if the leader has access to the ith follower and $b_i=0$ otherwise.

Assumption 1: The directed communication topology $\bar{\mathcal{G}}$ has a directed spanning tree with the leader being the root node.

The ith follower agent with the pure-feedback dynamics is modeled by

$$\begin{cases}\dot{x}_{i,m}=f_{i,m}\left(\bar{x}_{i,m},x_{i,m+1}\right)\\ \dot{x}_{i,n}=f_{i,n}\left(\bar{x}_{i,n},u_i\right)\\ y_i=x_{i,1}\end{cases}\tag{1}$$

where $\bar{x}_{i,m}=[x_{i,1},....x_{i,m}]^T\in R^m$; $x_{i,m}$, y_i, u_i ($m=1,\ldots,n$, $i=1,\ldots,N$) are system state, output, control input of the ith agent, respectively. $f_{i,m}(.), m=1,\ldots,n$ is unknown but smooth function. To circumvent the challenge of controller design for the pure-feedback systems, the degree elevation technique is used here and the following low pass filter is introduced

$$\dot{u}_i=-u_i+\tau_i(v_i)\tag{2}$$

where $u(0)=0$ and $\tau_i(v_i)=sat\,(v_i)$ is the control input with saturation nonlinearity as

$$\tau_i=sat\,(v_i)=\begin{cases}\operatorname{sgn}(v_i)\,U_N, |v_i|\geq U_N\\ v_i, |v_i|<U_N\end{cases}\tag{3}$$

where U_N is a known bound of τ_i.

It is obvious that the relationship between τ_i and v_i has a sharp corner when $|v_i|=U_N$. In order to employ the backstepping method directly in the following design, the saturation is approximated by a smooth function defined as

$$P_i\left(v_i\right)=U_N\tanh\left(\frac{v_i}{U_N}\right)=U_N\frac{e^{v_i/U_N}-e^{-v_i/U_N}}{e^{v_i/U_N}+e^{-v_i/U_N}}\tag{4}$$

Then τ_i can be written as $\tau_i\left(v_i\right)=sat\left(v_i\right)=P_i\left(v_i\right)+q_i\left(v_i\right)$, where $|q_i\left(v_i\right)|=|sat\left(v_i\right)-P_i\left(v_i\right)|\leq U_N\left(1-\tanh\left(1\right)\right)=\bar{q}_i$. By using the mean-value theorem, there exists a constant $\mu_i(0<\mu_i<1)$, such that

$$P_i\left(v_i\right)=P_i\left(v_i^0\right)+\frac{\partial P_i\left(v_i\right)}{\partial v_i}\,|_{v_i=v_i{}^{\mu_i}}\left(v_i-v_i^0\right)\tag{5}$$

where $v_i^{\mu_i}=\mu_i v_i+(1-\mu_i)\,v_i^0$. Let $v_i^0=0$, and $\left(\partial P_i\left(v_i\right)/\partial v_i\right)|_{v_i=v_i{}^{\mu_i}}=P_{i,0}\left(v_i{}^{\mu_i}\right)$, the equation (5) can be rewritten as

$$P_i\left(v_i\right)=\frac{\partial P_i\left(v_i\right)}{\partial v_i}\,|_{v_i=v_i{}^{\mu_i}}\cdot v_i=P_{i,0}\left(v_i{}^{\mu_i}\right)v_i\tag{6}$$

In this paper, the function $P_{i,0}$ and its sign are not known, which is challenging in the controller design.

The objective of this paper is to design a distributed adaptive consensus protocol for the MAS (1) with input saturation such that the outputs of the followers y_i track the reference trajectory y_r and all the signals in the closed loop system are bounded. Meanwhile, all states are required to remain in an open set $\Omega_{x_{i,m}}=\{x_{i,m}:-b_{m1}<x_{i,m}<b_{m2}\}$, where b_{m1} and b_{m2} are known positive design constants.

Assumption 2: The reference trajectory y_r is continuous with $|y_r|<B_1<\min\{b_{m1},b_{m2}\}$, where B_1 is a known positive constant. In addition, the derivative $|\dot{y}_r|$ satisfies $|\dot{y}_r|\leq r<\infty$ with r being an unknown positive constant.

B. Radial Basis Function Neural Network Approximation

In this paper, the radial basis function neural network (RBFNN) is used to approximate the unknown continuous functions. According to the universal approximation property, for any given known continuous functions $\psi(\xi)$, there exists $\phi(\xi)$ such that

$$\psi(\xi)=W^{*T}\phi(\xi)+\varepsilon(\xi)\tag{7}$$

where $\xi\in R^q$ is the input vector, $\varepsilon(\xi)$ is the approximation error satisfying $|\varepsilon(\xi)|\leqslant\bar{\varepsilon}$. $W^*=[W_1^*,...,W_l^*]^T\in R^l$ is an idealized constant weight vector.

C. Nussbaum-type Function

A function $N(\vartheta)$ is called Nussbaum type function if it has the properties $\limsup_{h\to\infty}\frac{1}{h}\int_0^h N(\vartheta)d\vartheta=+\infty, \liminf_{h\to\infty}\frac{1}{h}\int_0^h N(\vartheta)d\vartheta=-\infty$. There are many functions that satisfy these conditions such as $exp(\vartheta^2)\cos(\vartheta)$, $\vartheta^2\cos(\vartheta)$, $\ln(\vartheta+1)\cos(\sqrt{\ln(\vartheta+1)})$ and so on. In this paper, the Nussbaum function $\vartheta^2\cos(\vartheta)$ is utilized. The following result is important in the subsequent analysis [24].

Lemma 1: Let $V(t)$ and $\vartheta(t)$ be smooth functions with $V(t)\geq 0$ for $\forall t\in[0,t_f)$. If $V(t)$ satisfies the inequality $V(t)\leq\int_0^{t_f}(G(t)N(\vartheta(t))+1)\dot{\vartheta}(t)dt+const$ with $G(t)$ being a bounded function, $N(\vartheta(t))$ being a smooth Nussbaum function, and $const$ representing the suitable constant, then $V(t)$, $\vartheta(t)$ and $\int_0^{t_f}(G(t)N(\vartheta(t))+1)\dot{\vartheta}(t)dt$ must be bounded on $[0,t_f)$.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

A. System Transformation

By utilizing (2), (3), and (6), the system (1) can be rewritten as

$$\begin{cases}\dot{x}_{i,m}=f_{i,m}\left(\bar{x}_{i,m},x_{i,m+1}\right)\\ \dot{x}_{i,n}=f_{i,n}\left(\bar{x}_{i,n},u_i\right)\\ \dot{u}_i=-u_i+P_{i,0}v_i+q_i\left(v_i\right)\\ y_i=x_{i,1}\end{cases}\tag{8}$$

To proceed, the non-affine constrained MAS (8) is transformed into a strict feedback one by introducing the one-to-one NM technique [17]. Define

$$z_{i,m}=M(b_{m1},b_{m2},x_{i,m})=\log\frac{b_{m1}+x_{i,m}}{b_{m2}-x_{i,m}}\tag{9}$$

where b_{m1} and b_{m2} are known positive constants. From (9), it is easy to obtain that $x_{i,m} = \frac{b_{m2}e^{z_{i,m}} - b_{m1}}{e^{z_{i,m}}+1}$. Then, we further get that $\dot{z}_{i,m} = h_{i,m}(z_{i,m})\dot{x}_{i,m}$, where $h_{i,m} = \frac{e^{z_{i,m}}+e^{-z_{i,m}}+2}{b_{m1}+b_{m2}}$. Let $z_{i,n+1} = u_i$. Then, the system (8) is rewritten as:

$$\begin{cases} \dot{z}_{i,m} = F_{i,m} + z_{i,m+1} \\ \dot{z}_{i,n} = F_{i,n} + z_{i,n+1} \\ \dot{z}_{i,n+1} = -z_{i,n+1} + P_{i,0}v_i + q_i(v_i) \\ \hat{y}_i = z_{i,1} \end{cases} \tag{10}$$

where $F_{i,m}(\bar{z}_{i,m}, z_{i,m+1}) = h_{i,m}(z_{i,m})f_{i,m}(\bar{x}_{i,m}, x_{i,m+1}) - z_{i,m+1}, F_{i,n}(\bar{z}_{i,n}) = h_{i,n}(z_{i,n})f_{i,n}(\bar{x}_{i,n}, z_{i,n+1}) - z_{i,n+1}$.

Remark 1: After the system transformation, the objective of this paper is converted to design a distributed adaptive consensus protocol for the transformed multi-agent systems (10) such that the output $\hat{y}_i$ of the followers will track the reference trajectory $\hat{y}_r = [\log(b_{01} + y_r)/(b_{02} - y_r)]$, while the system states remain bounded.

B. Controller Design

For the strict feedback system (10), based on the backstepping method, the DSC technique, and RBFNNs, the true control law is constructed through n steps.

Step 1: This step is to design the virtual control $\alpha_{i,2d}$. The local error surface $e_{i,m}$ and the boundary layer error $S_{i,m}$ for the ith agent can be defined as

$$e_{i,1} = \sum_{j=1}^{N} a_{ij}\left(z_{i,1} - z_{j,1}\right) + b_i\left(z_{i,1} - \hat{y}_r\right) \tag{11}$$

$$e_{i,m} = z_{i,m} - \alpha_{i,m} \tag{12}$$

$$S_{i,m} = \alpha_{i,m} - \alpha_{i,md}, m = 2, \ldots, n, \tag{13}$$

where $\alpha_{i,m}$ refers to the output of the first-order filter with a time constant $\tau_{i,m}$ which is of the form:

$$\tau_{i,m}\dot{\alpha}_{i,m} + \alpha_{i,m} = \alpha_{i,md}, \ \ \alpha_{i,m}(0) = \alpha_{i,md}(0) \tag{14}$$

The first part of the Lyapunov function candidate for the ith agent can be constructed as

$$V_{i,1} = \frac{1}{2\left(b_i + d_i\right)}e_{i,1}^2 + \frac{1}{2\gamma_{1,1}}\tilde{\chi}_{i,1}^2 + \frac{1}{2}S_{i,2}^2 \tag{15}$$

where $\gamma_{i,1}$ is the design parameter and $\tilde{\chi}_{i,1}$ is the estimation error which is expressed as $\tilde{\chi}_{i,1} = \chi_{i,1} - \hat{\chi}_{i,1}$. $\hat{\chi}_{i,1}$ is the estimation of $\chi_{i,1}$ defined as $\chi_{i,1} = \|W_{i,1}^*\|_2^2$. $z_{i,2} = e_{i,2} + S_{i,2} + \alpha_{i,2d}$. The derivative of $e_{i,1}$ is computed as $\dot{e}_{i,1} = (b_i + d_i)\left(F_{i,1} + z_{i,2}\right) - \sum\limits_{j=1}^{N} a_{ij}\left(F_{j,1} + z_{j,2}\right) - b_i\dot{\hat{y}}_r$. Taking the time derivative of $V_{i,1}$, we have

$$\begin{aligned} \dot{V}_{i,1} &= \frac{1}{b_i + d_i}e_{i,1}\dot{e}_{i,1} + \frac{\tilde{\chi}_{i,1}\left(-\dot{\hat{\chi}}_{i,1}\right)}{\gamma_{i,1}} + S_{i,2}\dot{S}_{i,2} \\ &= -\frac{e_{i,1}}{b_i + d_i}\sum_{j=1}^{N} a_{ij}\left(F_{j,1} + z_{j,2}\right) + e_{i,1}\left(F_{i,1} + z_{i,2}\right) \\ &\quad - \frac{e_{i,1}b_i\dot{\hat{y}}_r}{b_i + d_i} + \frac{\tilde{\chi}_{i,1}}{\gamma_{i,1}}\left(-\dot{\hat{\chi}}_{i,1}\right) + S_{i,2}\dot{S}_{i,2} \end{aligned} \tag{16}$$

Let $\Phi_{i,1}(\xi_{i,1}) = W_{i,1}^{*T}\phi_{i,1} + \delta_{i,1} = F_{i,1} - \frac{1}{b_i+d_i}\left[\sum\limits_{j=1}^{N} a_{ij}\left(F_{j,1} + z_{j,2}\right) + b_i\dot{\hat{y}}_r\right]$. Then, we have

$$\begin{aligned} \dot{V}_{i,1} =& e_{i,1}\left(W_{i,1}^{*T}\phi_{i,1} + \delta_{i,1} + e_{i,2} + \alpha_{i,2d} + S_{i,2}\right) \\ &+ \frac{\tilde{\chi}_{i,1}}{\gamma_{1,1}}\left(-\dot{\hat{\chi}}_{i,1}\right) + S_{i,2}\dot{S}_{i,2} \end{aligned} \tag{17}$$

By employing Young's inequality, we have $e_{i,1}W_{i,1}^{*T}\phi_{i,1} \le \frac{\chi_{i,1}}{2\eta_{i,1}^2}\phi_{i,1}^T\phi_{i,1}e_{i,1}^2 + \frac{\eta_{i,1}^2}{2}$, $\quad e_{i,1}\delta_{i,1} \le \frac{e_{i,1}^2}{2} + \frac{\bar{\varepsilon}_{i,1}^2}{2}$, $\quad e_{i,1}e_{i,2} \le \frac{e_{i,2}^2}{2} + \frac{e_{i,1}^2}{2}$, $\quad e_{i,1}S_{i,2} \le \frac{S_{i,2}^2}{2} + \frac{e_{i,1}^2}{2}$, where $\eta_{i,1}$ is the design parameter. Now, we design the virtual controller $\alpha_{i,2d}$ and the update law $\hat{\chi}_{i,1}$ as

$$\alpha_{i,2d} = - k_{i,1}e_{i,1} - \frac{3}{2}e_{i,1} - \frac{\hat{\chi}_{i,1}}{2\eta_{i,1}^2}\phi_{i,1}^T\phi_{i,1}e_{i,1} \tag{18}$$

$$\dot{\hat{\chi}}_{i,1} = \gamma_{i,1}\left(\frac{e_{i,1}^2}{2\eta_{i,1}^2}\phi_{i,1}^T\phi_{i,1} - \sigma_{i,1}\hat{\chi}_{i,1}\right) \tag{19}$$

where $k_{i,1}, \eta_{i,1}, \sigma_{i,1}, \gamma_{i,1}$ are design parameters. Substituting the virtual control law (18) and the update law (19) into (17) yields

$$\begin{aligned} \dot{V}_{i,1} \le& - k_{i,1}e_{i,1}^2 + \frac{e_{i,2}^2}{2} + \sigma_{i,1}\tilde{\chi}_{i,1}\hat{\chi}_{i,1} \\ &+ \frac{S_{i,2}^2}{2} + S_{i,2}\dot{S}_{i,2} + \frac{\eta_{i,1}^2}{2} + \frac{\bar{\varepsilon}_{i,1}^2}{2} \end{aligned} \tag{20}$$

Using the fact that $\hat{\chi}_{i,1}\tilde{\chi}_{i,1} = \tilde{\chi}_{i,1}\left(\chi_{i,1} - \tilde{\chi}_{i,1}\right) \le -\frac{1}{2}\tilde{\chi}_{i,1}^2 + \frac{1}{2}\chi_{i,1}^2$, one can obtain that

$$\begin{aligned} \dot{V}_{i,1} \le& -k_{i,1}e_{i,1}^2 + \frac{e_{i,2}^2}{2} - \frac{\sigma_{i,1}\tilde{\chi}_{i,1}^2}{2} \\ &+ \frac{S_{i,2}^2}{2} + S_{i,2}\dot{S}_{i,2} + c_{i,1} \end{aligned} \tag{21}$$

where $c_{i,1} = \sigma_{i,1}\chi_{i,1}^2/2 + \eta_{i,1}^2/2 + \bar{\varepsilon}_{i,1}^2/2$ is a constant.

Step m ($2 \leqslant m \leqslant n$): The Lyapunov function for $m = 2, \ldots, n$ can be constructed as

$$V_{i,m} = \frac{1}{2}e_{i,m}^2 + \frac{1}{2\gamma_{i,m}}\tilde{\chi}_{i,m}^2 + \frac{1}{2}s_{i,m+1}^2 \tag{22}$$

where $\gamma_{i,m}$ is the design parameter and $\tilde{\chi}_{i,m}$ is the estimation error which is expressed as $\tilde{\chi}_{i,m} = \chi_{i,m} - \hat{\chi}_{i,m}$. $\hat{\chi}_{i,m}$ is the estimation of $\chi_{i,m}$. Let $\Phi_{i,m}(\xi_{i,m}) = W_{i,m}^{*T}\phi_{i,m}\left(\xi_{i,m}\right) + \delta_{i,m}\left(\xi_{i,m}\right) = F_{i,m} - \dot{\alpha}_{i,m}$. Taking the time derivative of $V_{i,m}$ yields that

$$\begin{aligned} \dot{V}_{i,m} =& e_{i,m}\dot{e}_{i,m} + \frac{\tilde{\chi}_{i,m}\left(-\dot{\hat{\chi}}_{i,m}\right)}{\gamma_{i,m}} + S_{i,m+1}\dot{S}_{i,m+1} \\ =& e_{i,m}\left(W_{i,m}^{*T}\phi_{i,m} + \delta_{i,m} + e_{i,m+1} + S_{i,m+1} + \alpha_{i,(m+1)d}\right) \\ &+ \frac{\tilde{\chi}_{i,m}\left(-\dot{\hat{\chi}}_{i,m}\right)}{\gamma_{i,m}} + S_{i,m+1}\dot{S}_{i,m+1} \end{aligned} \tag{23}$$

Since $z_{i,m+1} = e_{i,m+1} + S_{i,m+1} + \alpha_{i,(m+1)d}$, the time derivative of $e_{i,m}$ can be represented as

$$\dot{e}_{i,m} = \dot{z}_{i,m} - \dot{\alpha}_{i,m}$$

$$=F_{i,m}+e_{i,m+1}+S_{i,m+1}+\alpha_{i,(m+1)d}-\dot{\alpha}_{i,m} \quad (24)$$

By applying Young's inequality, we have $e_{i,m}W^{*T}_{i,m}\phi_{i,m} \leq \frac{\chi_{i,m}}{2\eta^2_{i,m}}\phi^T_{i,m}\phi_{i,m}e^2_{i,m} + \frac{\eta^2_{i,m}}{2}$, $e_{i,m}\delta_{i,m} \leq \frac{e^2_{i,m}}{2} + \frac{\bar{\varepsilon}^2_{i,m}}{2}$, $e_{i,m}e_{i,m+1} \leq \frac{e^2_{i,m+1}}{2} + \frac{e^2_{i,m}}{2}$, $e_{i,m}S_{i,m+1} \leq \frac{S^2_{i,m+1}}{2} + \frac{e^2_{i,m}}{2}$, where $\eta_{i,m}$ is the design parameter. The virtual controller $\alpha_{i,(m+1)d}$ and the update law $\dot{\hat{\chi}}_{i,m}$ are proposed as

$$\alpha_{i,(m+1)d}=-k_{i,m}e_{i,m}-2e_{i,m}-\frac{\hat{\chi}_{i,m}}{2\eta^2_{i,m}}\phi^T_{i,m}\phi_{i,m}e_{i,m} \quad (25)$$

$$\dot{\hat{\chi}}_{i,m}=\gamma_{i,m}\left(\frac{\phi^T_{i,m}\phi_{i,m}}{2\eta^2_{i,m}}e^2_{i,m}-\sigma_{i,m}\hat{\chi}_{i,m}\right) \quad (26)$$

where $k_{i,m},\eta_{i,m},\sigma_{i,m},\gamma_{i,m}$ are design parameters.

Taking the virtual control law (25) and the update law (26) into (23), we have

$$\begin{aligned}\dot{V}_{i,m} \leq &-k_{i,m}e^2_{i,m}-\frac{1}{2}e^2_{i,m}+\sigma_{i,m}\tilde{\chi}_{i,m}\hat{\chi}_{i,m}+\frac{e^2_{i,m+1}}{2}\\ &+\frac{S^2_{i,m+1}}{2}+\frac{\eta^2_{i,m}}{2}+\frac{\bar{\varepsilon}^2_{i,m}}{2}+S_{i,m+1}\dot{S}_{i,m+1} \quad (27)\end{aligned}$$

Similarly, we have

$$\hat{\chi}_{i,m}\tilde{\chi}_{i,m}=\tilde{\chi}_{i,m}\left(\chi_{i,m}-\tilde{\chi}_{i,m}\right) \leq -\frac{1}{2}\tilde{\chi}^2_{i,m}+\frac{1}{2}\chi^2_{i,m} \quad (28)$$

From (27) and (28), we get

$$\begin{aligned}\dot{V}_{i,m} \leq &-k_{i,m}e^2_{i,m}-\frac{1}{2}e^2_{i,m}+\frac{1}{2}e^2_{i,m+1}-\frac{1}{2}\sigma_{i,m}\tilde{\chi}^2_{i,m}\\ &+\frac{1}{2}S^2_{i,m+1}+S_{i,m+1}\dot{S}_{i,m+1}+c_{i,m} \quad (29)\end{aligned}$$

where $c_{i,m}=\sigma_{i,m}\chi^2_{i,m}/2+\eta^2_{i,m}/2+\bar{\varepsilon}^2_{i,m}/2$ is a constant.

Step n+1: In this step, the actual control law will be designed. Considering a Lyapunov function as

$$V_{i,n+1}=\frac{1}{2}e^2_{i,n+1}+\frac{1}{2\gamma_{i,n+1}}\tilde{\chi}^2_{i,n+1} \quad (30)$$

The derivative of the first term of the right side of (30) is

$$\begin{aligned}\dot{e}_{i,n+1}=&\dot{z}_{i,n+1}-\dot{\alpha}_{i,n+1}\\ =&-z_{i,n+1}+P_{i,0}v_i+q_i(v_i)-\dot{\alpha}_{i,n+1} \quad (31)\end{aligned}$$

Let $\Phi_{i,n+1}=W^{*T}_{i,n+1}\phi_{i,n+1}+\delta_{i,n+1}=-z_{i,n+1}-\dot{\alpha}_{i,n+1}$. By applying Young's inequality, we have $e_{i,n+1}W^{*T}_{i,n+1}\phi_{i,n+1} \leq \frac{\chi_{i,n+1}}{2\eta^2_{i,n+1}}\phi^T_{i,n+1}\phi_{i,n+1}e^2_{i,n+1}+\frac{1}{2}\eta^2_{i,n+1}$, $e_{i,n+1}\delta_{i,n+1} \leq \frac{e^2_{i,n+1}}{2}+\frac{\bar{\varepsilon}^2_{i,n+1}}{2}$, where $\eta_{i,n+1}$ is the design parameter. Taking the time derivative of $V_{i,n+1}$ yields that

$$\begin{aligned}\dot{V}_{i,n+1}=&e_{i,n+1}\dot{e}_{i,n+1}-\frac{\tilde{\chi}_{i,n+1}\dot{\hat{\chi}}_{i,n+1}}{\gamma_{i,n+1}}\\ \leq&\frac{\chi_{i,n+1}}{2\eta^2_{i,n+1}}e^2_{i,n+1}\phi^T_{i,n+1}\phi_{i,n+1}+\frac{1}{2}\eta^2_{i,n+1}+\frac{1}{2}\bar{\varepsilon}^2_{i,n+1}\\ &+e^2_{i,n+1}+\frac{1}{2}\bar{q}^2_i+P_{i,0}e_{i,n+1}v_i-\frac{\tilde{\chi}_{i,n+1}\dot{\hat{\chi}}_{i,n+1}}{\gamma_{i,n+1}} \quad (32)\end{aligned}$$

The control law v_i and the update law $\dot{\hat{\chi}}_{i,n+1}$ are designed as

$$\begin{aligned}v_i=&N_i\left(\zeta_i\right)\\ &\left[k_{i,n+1}e_{i,n+1}+\frac{3}{2}e_{i,n+1}+\frac{\hat{\chi}_{i,n+1}}{2\eta^2_{i,n+1}}\phi^T_{i,n+1}\phi_{i,n+1}e_{i,n+1}\right]\end{aligned} \quad (33)$$

$$\dot{\zeta}_i=k_{i,n+1}e^2_{i,n+1}+\frac{3}{2}e^2_{i,n+1}+\frac{\hat{\chi}_{i,n+1}}{2\eta^2_{i,n+1}}\phi^T_{i,n+1}\phi_{i,n+1}e^2_{i,n+1} \quad (34)$$

$$\dot{\hat{\chi}}_{i,n+1}=\gamma_{i,n+1}\left(\frac{e^2_{i,n+1}}{2\eta^2_{i,n+1}}\phi^T_{i,n+1}\phi_{i,n+1}-\sigma_{i,n+1}\hat{\chi}_{i,n+1}\right) \quad (35)$$

where $k_{i,n+1},\eta_{i,n+1},\sigma_{i,n+1}$ and $\gamma_{i,n+1}$ are positive design parameters. Based on (33), (34) and (35), the time derivative of $V_{i,n+1}$ is obtained as follows

$$\begin{aligned}\dot{V}_{i,n+1} \leq&\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i-k_{i,n+1}e^2_{i,n+1}\\ &-\frac{1}{2}e^2_{i,n+1}-\frac{1}{2}\sigma_{i,n+1}\tilde{\chi}^2_{i,n+1}+c_{i,n+1} \quad (36)\end{aligned}$$

where $c_{i,n+1}=\sigma_{i,n+1}\chi^2_{i,n+1}/2+\eta^2_{i,n+1}/2+\bar{q}^2_i/2+\bar{\varepsilon}^2_{i,n+1}/2$ is a constant.

C. Stability Analysis

From the boundary layer errors $S_{i,m+1}$ (13) and the filter for $\alpha_{i,(m+1)}$ (14), we have $\dot{\alpha}_{i,m+1}=\frac{\alpha_{i,(m+1)d}-\alpha_{i,m+1}}{\tau_{i,m+1}}=-\frac{S_{i,m+1}}{\tau_{i,m+1}}$. Furthermore, $\dot{S}_{i,m+1}=\dot{\alpha}_{i,m+1}-\dot{\alpha}_{i,(m+1)d}=-\frac{S_{i,m+1}}{\tau_{i,m+1}}-\dot{\alpha}_{i,(m+1)d}$. It is obvious that $\dot{\alpha}_{i,(m+1)d}$ is the function of variables $e_{i,1},\ldots,e_{i,m},\hat{\chi}_{i,1},\ldots,\hat{\chi}_{i,m}$. Thus,

$$\left|\dot{S}_{i,m+1}+\frac{S_{i,m+1}}{\tau_{i,m+1}}\right| \leq J_{i,m+1}(e_{i,1},\ldots,e_{i,m},\hat{\chi}_{i,1},\ldots,\hat{\chi}_{i,m}) \quad (37)$$

where $J_{i,m+1}(.)$ is a continuous function, $m=1,\ldots,n$. Now, construct the Lyapunov function candidate for the ith agent as $V_i=\sum_{m=1}^{n+1}V_{i,m}$. By applying (21), (29), (36) and taking the time derivative of V_i, we have

$$\begin{aligned}\dot{V}_i \leq&-\sum_{m=1}^{n+1}k_{i,m}e^2_{i,m}-\sum_{m=1}^{n+1}\frac{1}{2}\sigma_{i,m}\tilde{\chi}^2_{i,m}\\ &+\sum_{m=1}^{n}\left(\frac{1}{2}S^2_{i,m+1}+S_{i,m+1}\dot{S}_{i,m+1}\right)\\ &+\sum_{m=1}^{n+1}c_{i,m}+\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i \quad (38)\end{aligned}$$

In view of (37), all the variables in the function $J_{i,m+1}(.)$ are bounded, and thus we assume that there exists an upper bound on $J_{i,m+1}(.)$ such that $|J_{i,m+1}(\cdot)| \leq N_{i,m+1}$ [8]. Using Young's inequality, we have that

$$S_{i,m+1}\dot{S}_{i,m+1} \leq -\frac{S^2_{i,m+1}}{\tau_{i,m+1}}+\frac{1}{2}S^2_{i,m+1}+\frac{1}{2}N^2_{i,m+1} \quad (39)$$

Substituting (39) into (38), it follows that

$$\begin{aligned}\dot{V}_i \leq & -\sum_{m=1}^{n+1} k_{i,m} e_{i,m}^2 - \sum_{m=1}^{n+1} \frac{1}{2}\sigma_{i,m}\tilde{\chi}_{i,m}^2 + C_i \\ & -\sum_{m=1}^{n}\left(\frac{1}{\tau_{i,m+1}}-1\right) S_{i,m+1}^2 + \left(N_i\left(\zeta_i\right) P_{i,0}+1\right)\dot{\zeta}_i\end{aligned} \tag{40}$$

where $C_i = \sum_{m=1}^{n+1} c_{i,m} + \sum_{m=1}^{n} \frac{1}{2} N_{i,m+1}^2$ is a constant. Let $K_1 = \min\left\{2k_{i,1}\left(b_i+d_i\right), 2k_{i,m}, \frac{2}{\tau_{i,m}}-2\right\}, m = 2,\cdots,n+1, K_2 = \min\left\{\sigma_{i,m}\gamma_{i,m}\right\}, m = 1,\cdots,n+1$. Then, we have

$$\begin{aligned}\dot{V}_i \leq & -K\sum_{m=1}^{n+1}\left(\frac{1}{2}e_{i,m}^2 + \frac{1}{2\gamma_{i,m}}\tilde{\chi}_{i,m}^2\right) + C_i \\ & -K\sum_{m=1}^{n} S_{i,m+1}^2 + \left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i \\ \leq & -KV_i + C_i + \left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i\end{aligned} \tag{41}$$

where $K = \min\{K_1, K_2\}$. Integrating the differentiation inequality (41) with respect to $[0,t)$ yields that $0 \leq V_i(t) \leq e^{-Kt}\int_0^t e^{K\tau}\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i d\tau + \left(V_i(0) - \frac{C_i}{K}\right)e^{-Kt} + \frac{C_i}{K}$. According to Lemma 1, we have that V_i, $\int_0^t \left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i d\tau$ and ζ_i are bounded on $[0,t)$. Therefore, it can be concluded that for the ith subsystem all the signals in the closed-loop remain bounded.

The Lyapunov function candidate for the whole MAS is selected as $V = \sum_{i=1}^N V_i$. Applying (41) yields $\dot{V} = \sum_{i=1}^{N}\dot{V}_i \leq \sum_{i=1}^{N}\left(-KV_i + C_i\right) + \sum_{i=1}^{N}\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i \leq -KV + C + \sum_{i=1}^{N}\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i$, where $C = \sum_{i=1}^{N} C_i$. Then, we have $V \leq \sum_{i=1}^{N} e^{-Kt}\int_0^t e^{K\tau}\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i d\tau + (V(0) - \frac{C}{K})e^{-Kt} + \frac{C}{K}$. Note that $\int_0^t\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i d\tau$ is bounded on $[0,t)$. Let $\mathcal{M}_i$ be upper bound of $\left|e^{-Kt}\int_0^t e^{K\tau}\left(N_i\left(\zeta_i\right)P_{i,0}+1\right)\dot{\zeta}_i d\tau\right|$. Finally, we get $V \leq \mathcal{M} + (V(0) - \frac{C}{K})e^{-Kt} + \frac{C}{K}$ where $\mathcal{M} = \sum_{i=1}^N \mathcal{M}_i$. It shows that $V(t)$, $e_{i,m}$, $\tilde{\chi}_m(m = 1,\ldots,n+1)$, $S_{i,m}(m = 2,\ldots,n+1)$ are bounded. Furthermore, $\alpha_{i,md}$, $\alpha_{i,m}(m = 2,\ldots,n+1)$ are also uniformly ultimately bounded. Let $E_1 = (e_{1,1},\cdots,e_{N,1})^T$ and $z_1 = (z_{1,1},\cdots,z_{N,1})^T$. From (11), we have $E_1 = (L+B)(z_1 - \hat{y}_r)$. Then, it is obvious that $\|z_1 - \hat{y}_r\| \leq \|E_1\|/(\lambda_{\min}(L+B))$ is bounded. Based on (12) and (13), $z_{i,m}(m = 2,\ldots,n+1)$ are also bounded. By appropriately setting the parameters $k_{i,m}, \tau_{i,m}, \sigma_{i,m}, \gamma_{i,m}$, the value E_1 can be made as small as possible. By setting $b_{01} = b_{11}, b_{02} = b_{12}$, the tracking error $|x_{i,1} - y_r|$ is also made as small as possible. All system states are remained in the constrains $x_{i,m} \in \Omega_{x_{i,m}}, m = 1,\ldots,n$.

From the above analysis, we derive the main result in the following theorem.

Theorem 1: Consider the non-affine pure-feedback MAS (1) under Assumptions 1-2, with the virtual control laws (18), (25), the adaptive laws (19), (26), (35) and the actual controller (33). For any initial conditions satisfying $\hat{\chi}_{i,m}(0) > 0$ and $x_{i,m}(0) \in \Omega_{x_{i,m}}$, by appropriately setting the parameters $k_{i,m}, \tau_{i,m}, \sigma_{i,m}, \gamma_{i,m}$, the cooperaitve tracking errors can be made as small as possible while the full states are bounded in $\Omega_{x_{i,m}}$ for any $t \geq 0$ and the control inputs are also bounded.

IV. Simulation Results

To confirm the effectiveness of the proposed control, the simulation is conducted in this section.

Consider a MAS including four agents modeled by

$$\begin{cases}\dot{x}_{i,1} = x_{i,2} + 0.05\sin\left(x_{i,2}\right) \\ \dot{x}_{i,2} = 0.1\sin\left(u_i\right) + \left(0.9 + 0.05e^{-x_{i,1}^2}\right)u_i \\ y_i = x_{i,1}\end{cases} \tag{42}$$

The communication topology is set as $1 \leftarrow 0 \rightarrow 2 \rightarrow 4, 2 \rightarrow 3$. The trajectory of the leader node labeled by 0 is $y_r = \sin(0.5t)$. The input saturation for $\tau_i(v_i)$ are considered with $U_N = 3$. The initial conditions of the followers are set as $x_{i,1}(0) = [0.5, -0.4, 0.8, -0.1]^T$, $x_{i,2}(0) = [-0.7, 1, 0.9, 0]^T$. The design parameters are selected as $k_{i,1} = [5, 6, 6, 6]^T$, $k_{i,2} = [2, 1, 2, 1]^T$, $\gamma_{i,1} = \gamma_{i,2} = 2$, $\sigma_{i,1} = \sigma_{i,2} = 0.5$, $\eta_{i,1} = \eta_{i,2} = 2$, $\tau_{i,2} = 0.001$, $b_{11} = 1.3$, $b_{12} = 1.6$, $b_{21} = 1$, $b_{22} = 1.1$. The tracking performance is illustrated in Fig.1(a). By applying the proposed control method, it shows that outputs of the followers can track the desired trajectory and always remain in $\Omega_{x_{i,1}} = \{x_{i,1} : -1.3 < x_{i,1} < 1.6\}$. Fig.1(b) shows that the states $x_{i,2}$ of the followers also remain in $\Omega_{x_{i,2}} = \{x_{i,2} : -1 < x_{i,2} < 1.1\}$. Fig.2 represents the control inputs of the four agents. Fig.3 is the estimations of the unknown parameters $\chi_{i,1}$ and $\chi_{i,2}$. As Fig.4 shows, there is a significant difference between system states $x_{i,2}$ with or without constraints. As we see, it is apparent that the proposed control based on NM method guarantees that the system states always stay in the constraints $\Omega_{x_{i,2}} = \{x_{i,2} : -1 < x_{i,2} < 1.1\}$.

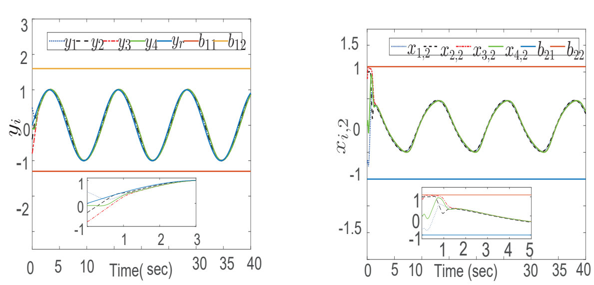

Fig. 1. (a)The tracking performance; (b) $x_{i,2}$ with constraints.

V. Conclusion

This paper addresses the tracking problem of a class of pure-feedback nonlinear uncertain MASs with full state

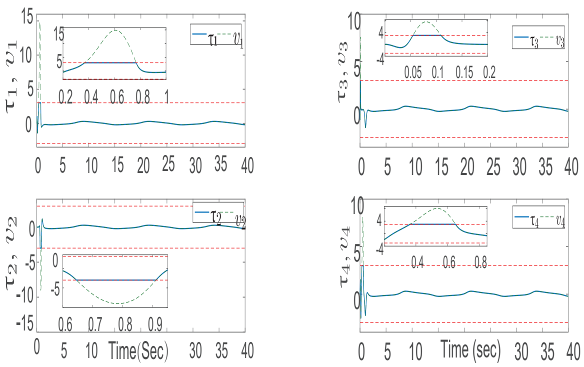

Fig. 2. Control signals τ_i and v_i

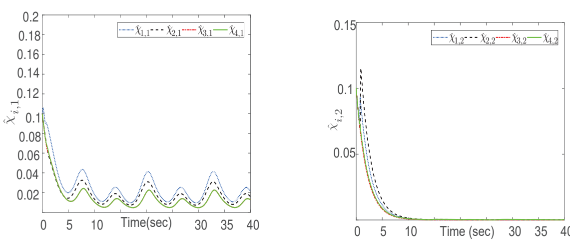

Fig. 3. The adaptive parameters $\hat{\chi}_{i,1}$ and $\hat{\chi}_{i,2}$.

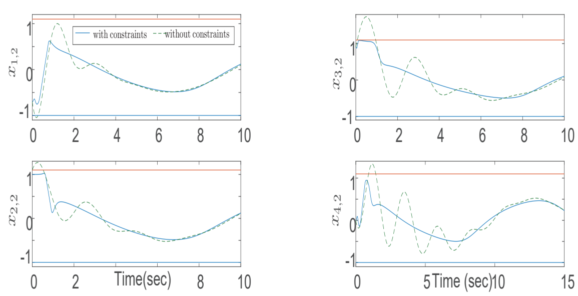

Fig. 4. Comparisons of $x_{i,2}$ with or without constraints.

constraints and input saturation under the unknown control direction. Based on the NM, Nussbaum functions,and degree elevation technique, a distributed neural adaptive control law is proposed recursively and the proposed scheme is very concise. It is proven that the tracking errors can be made as small as possible while all the state and control input constraints are always guaranteed.

References

[1] W. Ren, R.W. Beard, E.M. Atkins, "Information consensus in multi-vehicle cooperative control," IEEE Control Systems Magazine 2007, 27(2):71-82.

[2] H. Zhang, F. L. Lewis, Z. Qu, "Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs," IEEE Transactions on Industrial Electronics, 2012, 59(7): 3026-3041.

[3] X. Liu, B. Xu, L. Xie, "Distributed tracking control of second-order multi-agent systems under measurement noises," Journal of Systems Science and Complexity, 2014, 27(5): 853-865.

[4] Z. Meng, Z. Lin, W. Ren. "Robust cooperative tracking for multiple non-identical second-order nonlinear systems," Automatica, 2013, 49(8):2363-2372.

[5] C. L. P. Chen, G. X. Wen, Y. J. Liu, et al. "Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks," IEEE Transactions on Neural Networks and Learning Systems, 2014, 25(6): 1217-1226.

[6] S.El-Ferik, A. Qureshi, F. L. Lewis, "Neuro-adaptive cooperative tracking control of unknown higher-order affine nonlinear systems," Automatica, 2014, 50(3): 798-808.

[7] A. Das, F. L. Lewis, "Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities," International Journal of Robust and Nonlinear Control, 2011, 21(13):1509-1524.

[8] D. Swaroop, J. K. Hedrick, P. P. Yip, J.C. Gerdes, "Dynamic surface control for a class of nonlinear systems,"IEEE Transactions on Automatic Control, 2000, 45(10): 1893-1899.

[9] J. Qiu, K. Sun, I. J. Rudas, H. Gao,"Command filter-based adaptive nn control for mimo nonlinear systems with full-state constraints and actuator hysteresis," IEEE Trans. on Cybernetics, 2020, 50(7):2905-2915.

[10] Y. Yang, D. Yue, C. Dou, "Distributed adaptive output consensus control of a class of heterogeneous multi-agent systems under switching directed topologies," Information Sciences, 2016, 345: 294-312.

[11] Q. Zhu, B. Niu, D. Wang, S. Li, et.al., "Cooperative etm-based adaptive neural network tracking control for nonlinear pure-feedback mass: a special-shaped laplacian matrix method," IEEE Transactions on Neural Networks and Learning Systems, 2022, 1-11.

[12] M. Meza-Sanchez, E. Clemente, M.C. Rodriguez-Linan, G. Olague, "Synthetic-analytic behavior-based control framework: Constraining velocity in tracking for nonholonomic wheeled mobile robots," Information Sciences, 2019, 501(2019): 436-459.

[13] B. Zhou, Z. Li, Z. Lin, "Discrete-time l_∞ and l_2 norm vanishment and low gain feedback with their applications in constrained control," Automatica, 2013, 49(1):111-123.

[14] Y. Yu, W. Wang, Kang-Hyun Jo, "Adaptive consensus control of output-constrained second-order nonlinear systems via neurodynamic optimization," Neurocomputing, 2018, 295: 1-7.

[15] Y. H. Zhang, H. J. Liang, H. Ma, Q. Zhou, Z. D. Yu, "Distributed adaptive consensus tracking control for nonlinear multi-agent systems with state constraints," Applied Mathematics and Computation, 2018, 326: 16-32.

[16] T. Guo, X. Wu, "Backstepping control for output-constrained nonlinear systems based on nonlinear mapping," Neural Computing and Applications, 2014, 25(7): 1665-1674.

[17] T. Zhang, M. Xia, Y. Yi, "Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics," Automatica, 2017, 81:232-239.

[18] T. Zhang, M. Xia, Y. Yi and Q. Shen, "Adaptive neural dynamic surface control of pure-feedback nonlinear systems with full state constraints and dynamic uncertainties," IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2017, 47(8): 2378-2387.

[19] J. Peng, X. Ye, "Cooperative control of multiple heterogeneous agents with unknown high-frequency-gain signs," Systems & Control Letters, 2014, 68: 51-56.

[20] H. Rezaee, F. Abdollahi, "Adaptive consensus control of nonlinear multiagent systems with unknown control directions under stochastic topologies," IEEE Transactions on Neural Networks and Learning Systems, 2018, 29(99): 3538-3547.

[21] W. Wang, D. Wang, Z. Peng and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2016, 46(9): 1279-1286.

[22] M. Shahvali, J. Askari, "Distributed containment output-feedback control for a general class of stochastic nonlinear multi-agent systems,"Neurocomputing, 2016, 179: 202-210.

[23] Y. H. Choi, S. J. Yoo, "Minimal-approximation-based distributed consensus tracking of a class of uncertain nonlinear multiagent systems with unknown control directions," IEEE Transactions on Cybernetics, 2017, 47(8): 1994-2007.

[24] X. Ye, J. Jiang, "Adaptive nonlinear design without a priori knowledge of control directions," IEEE Transactions on Automatic Control, 1998, 43(11): 1617-1621.