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# Robust Online EV Charging Scheduling with Statistical Feasibility

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***Abstract*— With the worldwide adoption of electric vehicles (EVs), charging stations are becoming the bottleneck in delivering high-quality charging service to EVs. Compared to conventional fuel vehicles, EVs require more time to charge at charging stations until their energy requirements are fulfilled. Furthermore, the distribution network capacities frequently limit charging resources at a charging station. As a result, charging station operators must optimize EVs' charging scheduling and allocate the limited charging resources efficiently. Due to the high uncertainty of future EVs' arrival and charging demands, station operators typically schedule the arrived EVs' charging solely based on the charging requirements of these EVs, while disregarding future arrivals. Such a scheduling policy is simple to implement, but it may result in high service drop rate, particularly for charging stations with high occupancy levels. To that end, we develop an EV charging schedule model that includes a reserved charging rate, as well as a robust sample-based approach that incorporates the concept of statistical feasibility to help minimize the service drop rate. Numerical studies further verify the effectiveness of our suggested method.**

***Index Terms*— Smart grid; Robust adaptive control; Data driven control**

## I. INTRODUCTION

Electric vehicles (EVs) are becoming an eco-friendly and cost-effective alternative to fossil-fuel-powered vehicles for reducing carbon emissions. As such, EVs are reshaping the automotive industry. Due to the limited cruising range, EVs must be recharged frequently, making charging stations essential for the increasing adoption of EVs. In contrast to traditional vehicles that can refuel quickly, EVs charging systems require vehicles to stay at the charging station until their charging demands are completed, posing unique challenges for charging station operators (CSO). Furthermore, the distribution network capacities of charging stations often result in limited charging resources, such as maximum charging rate capacity or equivalently the maximum number of charging ports. Consequently, effective EV charging scheduling has received much attention.

CSO often seeks to devise an optimal charging schedule that minimizes charging costs upon the arrival of new EVs. Due to the high uncertainty associated with future EVs' arrival and charging requirements, most existing scheduling strategies only consider the requirements of all EVs that have arrived at the charging station, which are straightforward to implement. However, scheduling plans that disregard future EVs' may increase the probability of rejection of future EV arrivals, compromising the charging station's service quality, which is termed as the high service drop rate. Additionally, stochastic spot market prices coupled with a high service drop rate may further lead to reduced utilization rates and charging profitability, thus negatively affecting charging stations' operation.

The key to controlling the service drop rate is strategically reserving the charging rate in advance, fully utilizing the charging flexibility of the arrived EVs. Nevertheless, excessive reserve rates will result in high charging costs for the arrived EVs, while insufficient reserve capacity cannot effectively prevent rejection of future coming EVs. To this end, we introduce an online scheduling framework for EV charging that considers the reserved charging rate for future arrivals and design a robust sample-based solution to help minimize the service drop rate.

### A. Related Works

As mentioned, our goal is to determine the optimal EV charging schedule while considering the reserved charging rate for future arrivals at the minimum charging cost. The key is appropriately characterizing the future arrivals uncertainty. Accordingly, we identify two major related works for managing EV charging. The first one focuses on characterizing various stochastic factors in EV charging management, while the second introduces classic methods to handling these uncertainties.

While linear or dynamic programming techniques have been applied to implement EV management in deterministic scenarios [1], CSOs inevitably encounter scheduling challenges fraught with uncertainty, e.g., the uncertainty of future EV arrivals and charging requirements, electricity prices, etc. Therefore, the charging scheduling considering various stochastic factors has been investigated. Mouli *et al.* leverage dynamic EV charging profiles to minimize grid dependence and optimize solar power utilization for the direct charging of EVs in [2]. Sánchez-Martín *et al.* propose a model for managing EV charging that takes into account the stochastic nature of vehicle staying patterns in [3]. Sarker *et al.* put forward an optimization framework for the operating model of battery swapping stations that accounts for both battery demand uncertainty and electricity price uncertainty in [4]. In our paper, we seek to obtain the optimal schedule for minimizing the service drop rate by considering the uncertain future EV arrivals and their charging requirements. Specifically, our work focuses on dealing with the stochastic

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impact of future arrivals on the optimal charging scheduling by introducing the reserved charging rate.

The optimal uncertainty management for EVs has been well investigated. Just name a few, Wang *et al.* propose a real-time EV charging scheduling model that considers uncertainties in EV availability using a scenario-based method in [5]. Sun *et al.* develop a robust cost-minimizing day-ahead scheduling approach for EV charging in low voltage networks given uncertain individual EV-users' behavior, variable renewable generation, and spot market prices in [6]. Zhang *et al.* design an optimal pricing scheme to minimize the service drop rate of charging stations with dual charging modes based on the queue theory in [7]. Wu *et al.* address the challenges of energy scheduling in office buildings integrated with photovoltaic systems and uncertain workplace EV charging by stochastic programming in [8]. In contrast, our work adopts the notion of statistical feasibility and aims to achieving an effective robust sample-based solution.

### B. Our Contributions

By adopting the notion of statistical feasibility, we propose an effective sample-based solution to the online EV charging scheduling problem that considers the uncertain reserved charging rate for future EV arrivals. Our major contributions can be summarized as follows:

- *Online EV Charging Scheduling with Reserved Charging Rates*: We develop an EV charging scheduling model that accounts for the reserved charging rate to minimize the risk of charging unavailability for future EV arrivals.
- *EV Charging Scheduling with Statistical Feasibility*: We incorporate the notion of statistical feasibility and customize an EV charging scheduling problem with statistical feasibility.
- *Robust Sample-based Solution*: We propose a robust sample-based approach, in seek of jointly mitigating the reliance on the distribution information and solutions' conservation.

## II. Deterministic EV Charging Scheduling and Chance-constrained Formulation

This section first revisits the classic EV charging scheduling problem and recasts a light-weight formulation to focus on the charging plan for the newly arrived EVs. To address the uncertainty associated with future EV arrivals and their charging requirements, we introduce the reserved charging rate and provide a chance-constrained formulation.

### A. Deterministic EV Charging Scheduling Model

We focus on an online EV charging management for single EV charging station. Specifically, a series of EVs arrive at the station for charging, and a CSO manages the charging station with a fixed overall charging rate capacity $C$ and faces a time-varying electricity price. CSO aims to effectively manage the station when new EVs arrive by solving the following optimization problem to fully exploit the flexibility from all the arrived EVs in the station:

$$\min_{v_{n,t}} \sum\nolimits_{n=1}^{N} \sum\nolimits_{t=1}^{T} v_{n,t} p_t \Delta_t \tag{1}$$

$$s.t. \quad \sum\nolimits_{t=T_n^a}^{T_n^d} v_{n,t} \geq E_n, \ \forall n, \tag{1a}$$

$$0 \leq v_{n,t} \leq V_n, \ \forall t, \ \ \forall n, \tag{1b}$$

$$v_{n,t} = 0, \ \forall t \notin [T_n^a, T_n^d], \ \forall n, \tag{1c}$$

$$\sum\nolimits_{n=1}^{N} v_{n,t} \leq C, \ \forall t, \tag{1d}$$

where $v_{n,t}$ is the decision variable, representing the charging rate of the EV $n$ at time $t$; $N$ is the number of EVs in the charging station; $T$ is the length of scheduling interval; $p_t$ is the electricity price at time $t$[1]; $\Delta_t$ is the simulation step[2]; $T_n^a$ and $T_n^d$ represent the arrival and departure time of the EV $n$, respectively, which are deterministic for each EV in our model[3]; $E_n$ is the EV $n$'s charging demand; $V_n$ is the EV $n$'s maximum charging rate.

Constraints (1a) and (1c) require the charging station to fully charge all EVs up to their demands in the required charging time windows. Constraint (1b) ensures that EVs' charging rates cannot exceed their maximal. Constraint (1d) guarantees that the total charging rate at each time $t$ is within the charging station capacity.

**Remark**: Note that Model (1) may result in frequent rescheduling of the arrived EVs when new EVs arrive, which may lead to fluctuations in the charging completion times and costs for historically arrived EVs. However, EV owners typically expect relatively accurate estimates of the charging cost and time. For example, Uber provides estimated prices and arrival times based on the current distance to the destination before passengers place their orders. Therefore, we propose a light-weight version of Model (1) that only focuses on the charging schedule for newly arrived EVs given the charging schedules of all historically arrived EVs.

### B. Light-Weight Reformulation

Mathematically, upon the arrival of new EVs, CSO schedules their charging by solving the following problem:

$$\min_{v_{n,t}} \sum\nolimits_{n=1}^{\tilde{N}} \sum\nolimits_{t=1}^{T} p_t v_{n,t} \tag{2}$$

$$s.t. \quad h_t + \sum\nolimits_{n=1}^{\tilde{N}} v_{n,t} \leq C, \ \forall t, \tag{2a}$$

$$\text{Constraints} \quad (1a)-(1c),$$

where $\tilde{N}$ is the number of newly arrived EVs; $h_t$ is the aggregated historical charging rate (calculated by the historically arrived EVs) at time $t$, which is known and fixed.

[1]We assume the electricity price $p_t$ is known to CSO in advance, e.g., based on the prediction from the historical data.

[2]Note that $\Delta_t$ is constant and hence will not affect the solution to Model (1). In the subsequent analysis, we will ignore $\Delta_t$ in all the modeling and decision-making processes.

[3]Clearly, for each EV, the arrival time $T_n^a$ is deterministic. It is reasonable to assume that the available window of EV charging is bounded, i.e., $T_n^d - T_n^a \in [T_n^{\min}, T_n^{\max}]$. In our paper, we only focus on the deterministic departure case, i.e., $T_n^d = T_n^a + T_n^{\min}$ or $T_n^d = T_n^a + T^{\max}$ to provide reference for the maximum or minimum charging cost.

**Remark**: Fixing $h_t$ makes the charging schedule for historical arrived EVs deterministic. Although we give up fully utilizing the historically arrived EVs' flexibility, the schedule performance may not be significantly affected, which will be further verified in numerical study. Nevertheless, Model (2) only focuses on the schedule for newly arrived EVs, but completely ignores the future arrivals, making it likely for the future arrivals to wait or even to be rejected due to the limit charging rate capacity $C$. This observation motivates us to reserve some charging rate in advance. Next, we introduce the reserved charging rate $s_t$ to help reduce the charging rejection in the future.

### C. Involving Stochastic Consideration

After involving a consideration for future EV arrivals and their charging requirements, we consider the EV charging scheduling problem with a reserved charging rate as follows,

$$\min_{v_{n,t}} \sum_{n=1}^{\tilde{N}} \sum_{t=1}^{T} p_t v_{n,t} \tag{3}$$

$$s.t. \quad h_t + \sum_{n=1}^{\tilde{N}} v_{n,t} \leq C - s_t, \ \forall t, \tag{3a}$$

$$\text{Constraints} \quad (1a)-(1c).$$

**Remark**: Note that we consider $s_t$ in constraint (3a) to characterize the future EV arrivals' uncertainty aggregately, including their arrival times, departure times, and charging demands. By introducing $s_t$, we want to fully utilize the newly arrived EVs' flexibility and reduce charging rejections for future arrivals.

In Model (3), the predicted $s_t$ can be directly utilized to help obtain the optimal charging scheduling for newly arrived EVs[4]. However, in practice, accurate prediction for $s_t$ is challenging, as these parameters are often highly volatile and time-dependent (which is related to future EV arrivals and their charging requirements). Hence, in most cases, the uncertainty in $s_t$ is often characterized by an error term $\omega_t$ on top of the prediction $\hat{s}_t$, i.e.,

$$s_t = \hat{s}_t + \omega_t. \tag{4}$$

Considering the future EV arrivals' uncertainty, our scheduling problem turns into

$$\min_{v_{n,t}} \sum_{n=1}^{\tilde{N}} \sum_{t=1}^{T} p_t v_{n,t} \tag{5}$$

$$s.t. \quad h_1 + \sum_{n=1}^{\tilde{N}} v_{n,1} \leq C, \tag{5a}$$

$$h_t + \sum_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \omega_t), \ \forall t \in [2, T], \tag{5b}$$

$$\text{Constraints} \quad (1a)-(1c),$$

where there is no uncertainty at $t=1$ as shown in constraint (5a) (the arrived EVs at $t=1$ is deterministic).

[4]For example, we can utilize the historical data in a charging station to make a prediction $\hat{s}_t$ for $s_t$, which may be dependent on time $t$.

### D. Chance-Constrained Solution

Given the distribution information of $\omega_t$ or the estimated distributions captured from the historical prediction errors, the chance-constrained (CC) method can be utilized to solve Model (5). Mathematically, the chance-constrained EV charging scheduling model is formulated as follows,

$$\min_{v_{n,t}} \sum_{n=1}^{\tilde{N}} \sum_{t=1}^{T} p_t v_{n,t} \tag{6}$$

$$s.t. \quad \mathbf{P}\left(h_t + \sum_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \omega_t)\right) \geq 1 - \delta, \quad \forall t \in [2, T], \tag{6a}$$

$$\text{Constraints} \quad (1a)-(1c), (5a).$$

**Remark**: Although CC methods have demonstrated their effectiveness for dealing with optimizations with uncertain parameters, standard CC algorithms frequently rely on transforming the problem into an equivalent deterministic formulation. Such approaches require the distribution information of the random variables ($\omega_t$ in our case), which can be difficult to obtain precisely. To alleviate the reliant on the distribution information, robust optimization (RO) is often adopted.

## III. Robust Sample-based Scheduling for Online EV Charging

RO is a well-established approach to deal with uncertainty in optimization problems. The key of such a method involves constructing an uncertainty set from historical samples. However, RO methods ensure that the optimal solution is feasible under worst-case scenarios (in terms of random variables). In our problem, this means that CSO must ensure that the maximum possible future EVs' charging demands are satisfied while scheduling the newly arrived EVs, which may be unnecessary since the probability of such extreme demand cases is often low and the obtained solutions may be too conservative. Consequently, in this section, we design a robust sample-based solution that addresses the dependence on distribution capture in CC methods and the conservative issue of classic RO methods.

### A. Statistical Feasibility and Robust Sample-based Model

Before diving into the details, we first introduce the notion of statistical feasibility.

**Definition 1 (Statistical Feasibility) [9]:** For a CC program,

$$\min f(x), \quad s.t. \ \mathbf{P}(g(x;\vartheta) \in \mathcal{V}) \geq \alpha, \tag{7}$$

where $f(x)$ is the objective function, $x$ is the decision variable, $\vartheta$ is the random vector characterizing uncertainties over a probability measure $\mathbf{P}$, $\mathcal{V}$ is the feasible region, and $\alpha$ is the probability requirement for the chance constraints. Assume a dataset $\boldsymbol{\vartheta} = \{\vartheta_1, \vartheta_2, \cdots, \vartheta_n\}$ is sampled from $\mathbf{P}$ and $\chi(\boldsymbol{\vartheta})$ is constructed based on $\boldsymbol{\vartheta}$. An algorithm is statistically feasible if the resulting solution $x^*$ is feasible for the chance constraint with confidence $\beta$, i.e.,

$$\mathbf{P}_{\boldsymbol{\vartheta}}(\mathbf{P}(g(x^*; \vartheta \in \chi(\boldsymbol{\vartheta})) \in \mathcal{V}) \geq \alpha) \geq \beta. \tag{8}$$

We directly adopt statistical feasibility in our setting. That is, for the CC model (6), an algorithm is statistically feasible means that the resulting solution $\boldsymbol{v} = \{v_{n,t},\ \forall n \in [1, \tilde{N}],\ \forall t \in [1, T]\}$ is feasible for the chance constraint with confidence $1-\eta$:

$$\mathbf{P}_{\boldsymbol{v}_t}\left(\mathbf{P}_{\omega_t}\left(h_t + \sum\nolimits_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \omega_t)\right) \geq 1-\delta\right) \geq 1-\eta,\ \forall t \in [2, T]. \tag{9}$$

**Remark**: Given any sample set, statistical feasibility implies that the optimal solutions guarantee at least $1-\eta$ confidence level for the chance constraints in the original CC problem. Obviously, $\boldsymbol{v}_t = \sum_{n=1}^{\tilde{N}} v_{n,t}$ is dependent on the historical samples $\omega_t$. Thus, $\boldsymbol{v}_t$ can also be treated as random variables, and the randomness comes from $\omega_t$. Here, for simplicity, we consider the same $\delta$ and $\eta$ for all $t$.

To provide a statistically feasible solution to Model (6), we can solve the following robust sample-based optimization problem,

$$\min_{v_{n,t}} \sum\nolimits_{n=1}^{\tilde{N}} \sum\nolimits_{t=T^a}^{T^d} p_t v_{n,t} \tag{10}$$

$$s.t. \quad \text{Constraints} \quad (1a)-(1c), (5a)-(5b),\ \boldsymbol{\omega} \in \mathcal{U}.$$

where $\mathcal{U}$ is constructed to satisfy the statistical feasibility of the chance constraint, i.e., constraint (9). Note that for any $\boldsymbol{v}$ feasible to Model (10), $\boldsymbol{\omega} \in \mathcal{U}$ ($\boldsymbol{\omega} = [\omega_2, \cdots, \omega_T]$) implies that all the constraints in Model (10) are satisfied. Intuitively, by choosing $\mathcal{U}$ that covers $1-\delta$ portion of historical sample set $\boldsymbol{\Omega} = [\Omega_2, \cdots, \Omega_T]$ (i.e., $\mathcal{U}$ satisfies $\mathbf{P}(\boldsymbol{\omega} \in \boldsymbol{\Omega}) \geq 1-\delta$), any $\boldsymbol{v}$ feasible to Model (10) is also feasible to Model (6) because

$$\mathbf{P}\left(h_t + \sum\nolimits_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \omega_t)\right) \geq \mathbf{P}(\omega_t \in \Omega_t) \geq 1-\delta. \tag{11}$$

### *B. Uncertainty Set Construction and Solution*

The remaining hurdle is to construct a suitable $\mathcal{U}$ from $\boldsymbol{\Omega}$ that ensures statistically feasible solutions to Model (10). If $\mathcal{U}$ encompasses the extreme cases and covers $1-\delta$ portion of $\boldsymbol{\Omega}$, conservative solutions are likely to arise. To overcome this issue, we design a two-stage strategy to construct $\mathcal{U}$. Specifically, we divide the dataset $\boldsymbol{\Omega}$ into two parts $\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2$ with size of $m_1$ and $m_2$. We leverage $\boldsymbol{\Omega}_1$ for shape approximation and uncertainty set localization, while $\boldsymbol{\Omega}_2$ is used for shape calibration.

**Shape Localization**: We utilize ellipsoids as an example to help approximate the uncertainty set[5] and fix the position for the possibly uncertainty set. Specifically, we utilize the ellipsoidal uncertainty set to approximate $\boldsymbol{\Omega}_1$, i.e., $\{\omega_t | \frac{(\omega_t-\mu)^2}{\sigma_t} \leq q_t\}$, for some $q_t > 0$. Parameters $\mu_t$ and $\sigma_t$ can be directly chosen as the sample mean and variance of $\boldsymbol{\Omega}_1$, i.e., $\mu_t = \frac{1}{m_1}\sum_{\omega_t \in \Omega_1^{(t)}} \omega_t$, $\sigma_t = \frac{1}{m_1-1}\sum_{\omega_t \in \Omega_1^{(t)}} (\omega_t - \mu_t)^2$ with $\Omega_1^{(t)}$ representing the historical samples at time $t$.

[5]Classical RO approach also uses polytypes to approximate the uncertainty set. Such polyhedron uncertainty set can be constructed following the similar routine as the construction procedure for the ellipsoid uncertainty set.

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**Algorithm 1** Robust Sample-based Solution

**Input** $\{\mathbf{s}, \hat{\mathbf{s}}\}$, $\delta$, $\eta$, $m_1$, $m_2$;
**Output** solution $\boldsymbol{v}$ with statistical feasible guarantee;

**Uncertainty Set Construction:**
1: Initialize $t = 2$;
2: Compute predicted error dataset $\boldsymbol{\Omega}^t = \mathbf{s} - \hat{\mathbf{s}}$;
3: Divide $\boldsymbol{\Omega}^t$ into two groups, denoted by $\boldsymbol{\Omega}_1$ and $\boldsymbol{\Omega}_2$ with sizes $m_1$ and $m_2$;
4: **while** $t \leq T$ **do**
5: Approximate the high-probability region of $\boldsymbol{\omega}_t$ using $\Omega_1^{(t)}$ and denote it by $\{\omega_t | \frac{(\omega_t-\mu_t)^2}{\sigma_t} \leq q_t\}$;
6: Reformulate the obtained geometric shape in step 5 in the form of $\{\omega_t : r(\omega_t) \leq q_t\}$;
7: Sort observations of $\{r(\omega_t)\}$ for $\omega_t \in \Omega_2^{(t)}$, and obtain $r(\omega_t)_{(1)} < r(\omega_t)_{(2)} < \cdots < r(\omega_t)_{(m2)}$;
8: Calculate the index of calibration value of $q_t$ according to Eq. (12);
9: Denote $\tilde{q}_t = r(\omega_t)_{(i^*)}$, and calibrate the uncertainty set as $r(\omega_t) \leq \tilde{q}_t$;
10: $t = t + 1$;
11: **end while**
12: **return** $\{\mu_t, \sigma_t, \tilde{q}_t\}$.

**Optimization:**
1: For $t \in [2, T]$, transform Eq. (13) into Eq. (14) using $\{\mu_t, \sigma_t, \tilde{q}_t\}$;
2: Solve the equivalent problem to get the solution $\boldsymbol{v}$ of Model (10).

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**Shape Calibration**: After obtaining the approximated ellipsoidal set, we need to guarantee the statistical feasibility using $\boldsymbol{\Omega}_2$. We set $r(\omega_t) = \frac{(\omega_t-\mu_t)^2}{\sigma_t}$. Then the ellipsoidal set is $r(\omega_t) \leq q_t$. Sort the values of $r(\omega_t)$ for $\omega_t \in \Omega_2^{(t)}$: $[r(\omega_t)_{(1)}, r(\omega_t)_{(2)}, \cdots, r(\omega_t)_{(m_2)}]$. We can find the critical sample index $i^*$, which satisfies that,

$$i^* = \min\left\{j : \sum_{k=0}^{j-1} \begin{pmatrix} m_2 \\ k \end{pmatrix} (1-\delta)^k \delta^{m_2-k} \geq 1-\eta\right\} \tag{12}$$

for $1 \leq j \leq m_2$ if $1-(1-\delta)^{m_2} \geq 1-\eta$. This allows us to calibrate the ellipsoidal set with $r(\omega_t) \leq \tilde{q}_t$, where $\tilde{q}_t = r(\omega_t)_{(i^*)}$.

We can reformulate Model (10) into the equivalent deterministic tractable problem by using the shape parameters of the constructed uncertainty set. Mathematically, the constraint

$$h_t + \sum\nolimits_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \omega_t),\ \omega_t \in \mathcal{U}_t,\ \forall t \in [2, T], \tag{13}$$

with $\mathcal{U}_t$ characterized by $\{\omega_t | \frac{(\omega_t-\mu_t)^2}{\sigma_t} \leq \tilde{q}_t\}$, can be transformed into the equivalent linear constraint as follows,

$$h_t + \sum\nolimits_{n=1}^{\tilde{N}} v_{n,t} \leq C - (\hat{s}_t + \mu_t + \sqrt{\tilde{q}_t \sigma_t}),\ \forall t \in [2, T]. \tag{14}$$

By such a transformation, many existing optimization packages, e.g., CVX [10], can be utilized to effectively handle Model (10). The whole procedure is detailed in Algorithm 1.

## IV. Numerical Studies

This section conducts extensive numerical studies aimed at validating the efficiency of our proposed robust sample-based approach. All the experiments are performed on a laptop with an Intel ® i5 4.5GHz CPU and 16GB RAM. The optimization model is implemented using MATLAB ® 2022a and solved by Gurobi 9.5.2 [11].

### A. Dataset Description and Simulation Design

Due to the lack of real-world EV charging scheduling data, synthetic datasets are generated by drawing upon Chargepoint's EV charging session data from San Francisco, CA [12]. The dataset furnishes pertinent information on EVs' $T_n^a$, $T_n^d$, and $E_n$. In particular, the value of $h_t$ is derived by consecutively solving Model (2) contingent upon the random arrival of EVs (assuming the EVs' arrival follows the Poisson process), with pricing data obtained from [13].

We assume $s_t$ is inversely proportional to electricity price and is generated from a Gaussian distribution, i.e., $s_t = \mathcal{N}(10/p_t, 1)$. $\omega_t$ is generated from a Gaussian distribution or Weibull distribution, and $\hat{s}_t$ is calculated by Eq. (4). For simplicity, we utilize polytypes to approximate the uncertainty set. We repeat the experiments to schedule EVs' charging for 400 times and divide the generated dataset into two parts with equal size: one is acted as a historical sample set, and the other is utilized as a testing set. We focus on the performance analysis on the testing set.

### B. Benchmarks and Evaluation Metrics

We compare our proposed robust sample-based method, denoted as RSO, with several benchmarks:

**Chance-constrained Method (CC)**: We compare our proposed method with the conventional stochastic programming approach, CC method. We assume $\omega_t$ is Gaussian distributed and the parameters of the Gaussian distribution is estimated from the historical sample set.

**Classic Robust Optimization (CRO)**: The classic RO method is also used for comparison and we obtain the feasible solutions for the maximum reserved charging rate in terms of $\omega_t$.

**Deterministic methods (OPT and DM)**: The optimal baseline solutions are obtained from the deterministic optimization with perfect predictions for $s_t$ (OPT). To provide a more thorough experimental comparison, we additionally include a deterministic case with predicted information $\hat{s}_t$, abbreviated as DM.

To evaluate the performance of different methods, over 200 testing cases, we compare the total charging cost (TCC), the average charging unit price (ACP), and the average shortening period (ASP) for all solvable cases, as well as the service drop rate (SDR). We define ASP as the mean value of the shortening time comparing the EVs' departure time across different methods. SDR is calculated as follows

$$\mathrm{SDR} = 1 - \frac{\text{the number of feasible solutions}}{200}, \qquad (15)$$

where the feasible solutions are required to satisfy Eq. (3a) for $\forall t \in [2, T]$. For example, if CC obtains 180 solvable cases and only 104 feasible cases which succeed in handling the future EVs' demands, then TCC, ACP, and ASP is calculated by 180 solvable cases while $\mathrm{SDR} = 1 - 104/200$.

### C. Performance Evaluation

*1) Effect of Using Light-Weight Reformulation:* We start by evaluating the effect of using the light-weight Model (2) as opposed to the classic Model (1) on the solutions. Table I presents the results. The values in brackets under the column labeled "No Solutions" indicate the number of cases that yield infeasible solutions. As depicted in Table I, fixing $h_t$ in the light-weight model increases the possibility of the failure case and decreases the charging profit (from the CSO's perspective), which is reasonable because the arrived EVs' charging flexibility is not fully utilized in Model (2). Nevertheless, from the perspective of EVs' owner, ACP is lower and ASP is higher, corresponding to a lower unit charging cost and enhanced departure flexibility.

*2) Effect of Introducing the Reserved Charging Rate:* Table II presents a comparison of the performance between Model (2) and Model (3) under the conditions of complete information $s_t$ (OPT) or predicted information $\hat{s}_t$ (DM). It is evident that by incorporating the reserved charging rate in advance and utilizing perfect prediction for $s_t$, SDR experiences a decrease, which means reserving charging rate in advance has a positive effect on SDR. Additionally, due to the imprecise characterization of the future EV arrivals, there exists a high possibility (32.92%) that solvable cases may fail to offer charging services in the future because of the limited $C$, implying that the performance of DM is quite unstable. Hence, in the subsequent analysis, we will concentrate on the performance comparison between CC, CRO, and RSO.

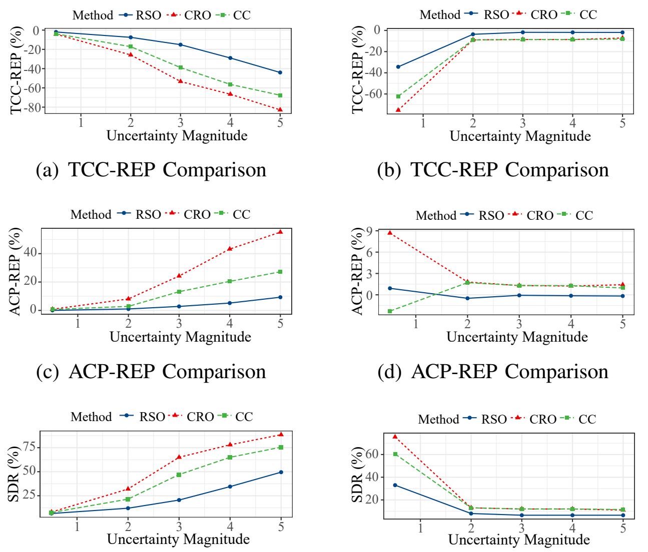
*3) Performance Comparison between Different Methods:* We adopt Gaussian and Weibull noises for $\omega_t$ to model the uncertainty and employ the standard deviation value (shape parameter for the Weibull distribution) to quantify the uncertainty magnitude. To enable more effective comparison, we consider the savings in TCC and ACP (in percentage) compared to the optimal benchmark OPT. We term this gap the regret percentage, denoted by REP (the lower absolute REP means the better performance). We set $m_1 + m_2 = 200$, $\delta = \eta = 0.9$ and the comparison results for different methods are presented in Fig. 1.

TABLE I
Comparison between Light-Weight Reformulation Model (2) and Classic Model (1).

| Model | TCC | ACP | No Solutions | ASP |
|---|---|---|---|---|
| Model (1) | **15130.71** | 6.703 | **4.5%** (191/200) | 0.6755 |
| Model (2) | 14742.76 | **6.695** | 6.0% (188/200) | **0.7128** |

TABLE II
PERFORMANCE COMPARISON BETWEEN MODELS (3) AND (2).

| Model | TCC | ACP | SDR | ASP |
|---|---|---|---|---|
| Model (2) | **14742.76** | 6.695 | 48.0% | **0.7128** |
| Model (3) - OPT | 12553.17 | 6.801 | **19.5%** | 0.6584 |
| Model (3) - DM | 12360.56 | **6.748** | 46.0% | 0.7019 |




(a) TCC-REP Comparison (b) TCC-REP Comparison

(c) ACP-REP Comparison (d) ACP-REP Comparison

(e) SDR Comparison (f) SDR Comparison

Fig. 1. Performance Comparison using Different Methods (left: Gaussian case, right: Weibull case). SDR of OPT is 4.5%.

From Fig.1, we conclude that our proposed RSO achieves lower SDR (see Fig. 1(e)(f)), higher charging profit (see Fig. 1(a)(b)) for CSO, and lower charging unit price (see Fig. 1(c)(d)) for EV owners than the other two methods. Besides, RSO changes slower with increasing uncertainty magnitudes, which means RSO provides more stable control performance than CC and CRO. For ASP, all the three methods do not show apparent improvement.

## V. CONCLUDING REMARKS

This paper formulates an online EV charging scheduling framework from the perspective of CSO. The proposed framework incorporates a reserved charging rate to ensure sufficient power supply for future EV arrivals and maintain service quality for the arrived EVs. We adopt the notion of statistical feasibility to design an effective sample-based solution that both reduces reliance on the distribution knowledge and relaxes the solutions' conservative issue of classic RO. Numerical experiments demonstrate the superiority of our proposed method.

We want to emphasize that our work can be extended by considering more practical factors, e.g., renewable energy [14], [15], energy storage [16], [17], stochastic electricity prices [18], [19]. Moreover, for the uncertainty set construction procedure of our proposed robust solution, more improvements can be explored, e.g., integrating the domain knowledge related to the EV scheduling problem into uncertainty set reconstruction [20], [21].

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