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Initialization-free distributed constrained optimization algorithms with a pre-specified time

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Abstract— **The distributed constrained optimization problem over an undirected communication topology is investigated in this study. It focuses on addressing a global coupled equality constraint that applies to all agents. To tackle this problem, a distributed approach with arbitrary initialization is developed by virtue of the aperiodic sampling control idea and the consensus-based multi-agent system(MAS) technology. This approach is developed to address constrained optimization problems within a pre-specified time. In addition, this predefined time is freely defined by users and irrelevant to the initial states, control coefficients, and network structure of systems. The Lyapunov stability theory completes the convergence proof of the developed method. Then, the developed method is extended to handle distributed nonlinear constrained optimization problems. Finally, The availability of two developed methods is demonstrated through two simulation examples.**

Index Terms— **distributed constrained optimization, pre-specified time, nonlinear, non-periodical sampling**

I. INTRODUCTION

Recently, the field of distributed optimization problems over MASs has gained significant attention due to its applicability in various practical examples. These examples include process operational optimization in chemical industries, spread control of epidemic diseases, source localization in UAV systems, and more. The primary objective of distributed optimization problems is to develop efficient methods that enable agents to collaborate and minimize the global objective function by leveraging local communication among themselves [1].

Therefore, to handle optimization problems in a distributed way, various discrete-time algorithms have been developed by using consensus-based MAS technology and gradient dependent strategy. For instance, in the case of distributed convex optimal coordination problem over time-varying jointly connected undirected networks, a well-known subgradient-based distributed method with diminishing step size is created in [2]. For time-varying cost function-based distributed optimization problem over an undirected communication topology, a quantized distributed method utilizing random quantization operation is introduced [3]. In the realm of the distributed constrained optimization, a proximal primal-dual method with diminishing and nonsummable step size is proposed [4]. However, the above diminishing step size leads to a lower convergence rate, i.e., a sublinear convergence rate. To address this limitation, an exact first-order method with constant control step size is designed in [5] to reach a linear convergence rate. Additionally, a Nesterov accelerated optimization method is suggested in [6] to further enhance the convergence performance for problems over the undirected communication topology. For distributed optimization over an unbalanced directed network, a gradient tracking-based distributed method is proposed in [7] to achieve linear convergence. In addition, a surplus-based accelerated method with an uncoordinated step size is provided in [8] for achieving a linear convergence rate. A general unified framework for distributed optimization is proposed in [9].

Compared with discrete-time algorithms, continuous-time distributed methods have attracted significant attention because of their real-time solutions and the realization for hardware implementation, which guarantees the more straightforward performance of the proposed methods in physical systems. Furthermore, more robust convergence properties can be ensured through Lyapunov stability, such as the fixed time convergence theory [10]. Similar to discrete-time algorithms, the linear(or exponential) convergence of the distributed algorithm is designed first. For example, a distributed primal-dual distributed method combined by fixed control parameters is designed for solving distributed optimization problems over weight-balanced digraphs [11]. For reducing the communication consumption, a distributed method with on demanded communication scheme is introduced in [12]. However, these linear protocol-based approaches handle distributed optimization problems with an exponential or asymptotical convergence rate, meaning that the methods achieve the optimal result as time goes to infinity. Nevertheless, there is a need to address the finite-time requirement optimization problems in various applications.

To address this requirement, an adaptive distributed gradient optimization algorithm with scale sign consensus scheme is proposed [13], enabling the establishment of optimal solutions within a finite time. Considering the unknown disturbance of agent, the finite-time result is guaranteed using a sliding mode control scheme [14]. For the distributed optimization over an unbalanced directed communication topology, by using the nonsmooth theory, a distributed

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finite-time optimization method is created [15]. However, the setting time of these finite-time distributed methods is usually influenced by the primary state value, which may be unavailable and hard to off-line pre-assign the setting time. Therefore, to overcome this restriction, for the time-varying cost function-based distributed optimization problem, the optimal result is obtained within fixed time using three scale sign terms [16]. In view of the zero-gradient-sum framework and the scale sign function, the fixed-time result is obtained for the quadratic optimization problems [17]. For distributed optimization problems coupled with an equality constraint, the fixed-time convergence is obtained via communicating local gradient information among agents [18]. In more complex cases involving global equality and local inequality constraints, a projection-gradient-based method is developed in [19].

Existing finite- and fixed-time distributed algorithms are known to exhibit setting times that are significantly correlated with algorithm parameters and communication topologies, such as the agent number and its Laplacian matrix eigenvalues. Therefore, studying predefine-time approaches for distributed optimization problems is meaningful. Thus, some excellent distributed methods have been designed recently. One approach that has been explored is the the time-based generator scheme, which has been used to derive many predefined-time distributed methods for distributed convex optimization problems. However, it is notable that the time-based generator scheme can merely get its underutilized solution. In order to obtain the exact most favorable solution, a new class of predefined-time distributed methods has been designed based on the sign function. These methods have been applied to solve optimal coordination problems [20] and resource allocation problems [21]. Nevertheless, sign function leads to frequent trembling behavior. Currently, the specified-time theory has gained considerable attention. For instance, a new out-degree-based distributed method with pre-specified time is designed in [22], where the convergence time has nothing to do with other parameters. a novel predefined time coordination algorithm is described in [23] utilizing a multi-step planning, which requires the initialization of the initial state of all agents. However, an efficient method is still lacking to achieve exact most favorable solutions within a pre-specified time without the need for initializing all agent states.

Motivated by the aforementioned discussions, this study further researches the distributed optimization problem, specifically focusing on cases where a global coupled equality constraint exists. To address this problem, a novel initialization-free distributed method with pre-specified time is designed, leveraging a non-periodic sampling control strategy. The contributions of this study can be summarized in the following two folds: 1) Different from the existing results [22], [23], our developed methods eliminate the requirement for initialization, enhancing its practicality and efficiency; 2) Moreover, this study takes into account the presence of a global nonlinear constraint, expanding the scope of the problem addressed.

This study is organized as below. Section II provides several preliminaries and presents the problem formulation. Section III gives the main results achieved in this study. Section IV illustrates the validity of our created approaches through illustrative examples. Section V concludes the whole work, highlighting potential avenues for future research.

II. Preliminaries and problem formulation

Notations: Let the set $\mathbb{R}$ indicate all real numbers. For a differentiable function $f(z)$, its gradient function and Hessian matrix are denoted by $\nabla f(z)$ and $\nabla^2 f(z)$ in turn. Here, z represents the input variable. For a positive integer n, the symbols 0_n and 1_n respectively represent the all-zero and all-one vector. Additionally, the notation span$\{1_n\}$ refers to the vector space spanned by the all-one vector 1_n.

A. Graph theory

Given a MAS consisting of n n agents. Let $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ denote its communication topology. Particularly, $\mathcal{V} = \{1, 2, ..., n\}$ represents the agent set, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes its communication edge set, and $\mathcal{A} := [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weight adjacency matrix. If the information from agent j can be acquired by agent i, i.e., $(i, j) \in \mathcal{E}$, then agent j is considered a neighbor of agent i and thus $a_{ij} > 0$, and $a_{ij} = 0$ otherwise. Additionally, we define $a_{ii} = 0$ even if the information of i can be used by itself. This implies that the graph $\mathcal{G}$ is assumed to have no self-loop. If at least one neighbor agent exists for all agents in graph $\mathcal{G}$, then $\mathcal{G}$ is a connected graph. For the graph $\mathcal{G}$, the corresponding Laplacian matrix $L := [l_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $l_{ij} = -a_{ij}, \forall i \neq j \in \mathcal{V}$ and $l_{ii} = \sum_{j=1}^{n} a_{ij}, \forall i \in \mathcal{V}$. It is evident that $L1_n = 0$. Furthermore, we have $1_n^T L = 0$ and the ordered eigenvalues $0 = \rho_1(L) < \rho_2(L) \leq, ..., \leq \rho_n(L)$ when $\mathcal{G}$ is undirected.

B. Aperiodic sampling

Different from the periodic sampling and the event-triggered sampling control, the aperiodic sampling is defined as $\{t_k\}_{k=1,2,...,\infty}$ with $t_k = \sum_{p=1}^{k} \frac{6T_f}{(\pi p)^2}$, where T_f is the pre-specified time. Based on the fact $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$, then we have $\lim_{k\to\infty} t_k = T_f$.

C. Problem formulation

The dynamic of agent $i, \forall i \in \mathcal{V}$, is given by

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

in which $x_i(t) \in \mathbb{R}^N$ and $u_i(t) \in \mathbb{R}^N$ respectively denote their state variable and control input. For simplicity, let $N = 1$ in the following context. The case $N > 1$ can be similarly deduced by the Kroncker product.

Let $x = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$ denotes the stack vector of variables $x_1, x_2, ..., x_n$. The goal of all agents is to minimize the total cost function $f(x)$ while satisfying a global coupled equality constraint. Mathematically, the following distributed constrained optimization problem is provided:

$$\begin{aligned} \min_x f(x) &= \sum_{i=1}^n f_i(x_i), \\ \sum_{i=1}^n x_i &= \sum_{i=1}^n d_i, \end{aligned} \tag{2}$$

where $f_i(x_i)$ and d_i represent its local cost function and demand resource, respectively. To ensure the following convergence analysis, the convex assumption is provided as below.

Assumption 1: For agent $i, \forall i \in \mathcal{V}$, let the differentiable function $f_i(x_i)$ satisfy $0 < \nabla^2 f_i(x_i) \leq m_i$ $(m_i > 0)$.

Remark 1: The aforementioned Assumption 1 ensures the uniqueness of the optimal solution for problem (2), which has been employed in various existing methods [22], [23]. Unlike the strong convexity requirements in most existing literature [5]–[7], [9], the above smooth function requirement is more practical. For instance, the exponential function is convex but not strongly convex.

Based on Assumption 1 and the Karush-Kuhn-Tucker conditions [22], the optimal result $x^* \in \mathbb{R}^n$ needs to satisfy that

$$\begin{aligned} \nabla f(x^*) &\in \text{span}\{1_n\}, \\ \sum_{i=1}^n x_i^* &= \sum_{i=1}^n d_i. \end{aligned} \tag{3}$$

III. Main results

A. Algorithm design

From (3), to acquire the optimal result x^*, it is necessary to achieve the consensus on gradient function and the global coupled constraint should be also satisfied. Thus, in view of the aperiodic sampling control approach and the consensus scheme in multi-agent systems, we design the following distributed optimization method:

$$\begin{aligned} \dot{x}_i(t) =& -\frac{1}{t_{l+1} - t_l} e_i(t_l) \\ & - \alpha \sum_{j=1}^n a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l))), \end{aligned} \tag{4a}$$

$$\begin{aligned} e_i(t) =& x_i(t) - d_i \\ & + \alpha \int_0^t \sum_{j=1}^n a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l)))d\tau \end{aligned} \tag{4b}$$

where $t \in [t_l, t_{l+1})$ and α is the control parameter. We define $\alpha = \frac{h}{t_{l+1} - t_l}$ with $h > 0$. The sampling time t_l is defined as $t_l = \begin{cases} \sum_{p=1}^l \frac{6T_f}{(\pi l)^2}, & 0 \leq t \leq T_f \\ T_f + \sum_{p=1}^l \frac{6T_f}{(\pi l)^2}, & T_f \leq t \leq 2T_f \end{cases}$. Without loss of generality, let $t_0 = 0$. From the aforementioned definition, the proposed method (4) can be divided into two stages. In stage 1 ($0 \leq t \leq T_f$), the global equality constraint is solved. In stage 2 ($T_f \leq t \leq 2T_f$), the optimality is guaranteed. The integral term $\int_0^t \sum_{j=1}^n a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l)))d\tau$ is designed to balance the real state value $x_i(t)$ and the demand state d_i. The initial value of x_i can be randomly selected. Moreover, let $e_i(0) = x_i(0) - d_i$.

Remark 2: Different from the time-based generator and sign function method [20], [21], our proposed algorithm (4) utilize the aperiodic sampling control approach to address the problem (2) with pre-specified time. In addition, the designed algorithm not merely obtain the exact most favorable result but avoids chattering behavior as well. A distinctive feature of our approach is that it allows for the random selection of the initial state, which distinguishes it from the existing work [22], [23].

From the definition of aperiodic sampling in subsection II-B, the pre-specified time is ensured when k goes to infinity. However, Zeno's behavior will occur. To avoid Zeno's behavior, the sampling interval sequence is modified as below:

$$t_l = t_{l-1} + \begin{cases} \frac{6T_f}{(\pi l)^2}, & \frac{6T_f}{(\pi l)^2} > \epsilon \\ \epsilon, & \frac{6T_f}{(\pi l)^2} \leq \epsilon \end{cases}, \tag{5}$$

where ϵ is a small positive constant. It implies that the periodic sampling will stand in for the aperiodic sampling when the sampling interval is relatively small. As proved in [22], the convergence precision can be adjusted by choosing proper ϵ.

B. Convergence analysis

Theorem 1: Consider that Assumption 1 is satisfied, and the graph $\mathcal{G}$ is undirected and connected, the developed distributed method (4) achieves the optimal result of (2) within pre-specified time $2T_f$. In addition, the control parameter h satisfies $0 < h < \frac{2}{m\rho_2(L)}$.

Proof: the above result is proved by the following two steps. In step I, the global coupled equality constraint is guaranteed with pre-specified time; In step II, $x(t)$ achieve the most favorable result within pre-specified time.

Step I. if $t \in [0, T_f]$, one has

$$\dot{e}_i(t) = -\frac{1}{t_{l+1} - t_l} e_i(t_l), t \in [t_l, t_{l+1}). \tag{6}$$

Integrating (6) over time $[t_l, t_{l+1})$, it can be obtained that

$$\begin{aligned} e_i(t_{l+1}) &= e_i(t_l) - \int_{t_l}^{t_{l+1}} \frac{1}{t_{l+1} - t_l} e_i(t_l) d\tau \\ &= e_i(t_l) - e_i(t_l) \\ &= 0. \end{aligned} \tag{7}$$

It is notable that $e_i(t_l) = 0, \forall l \geq 1$. it followings this result and (4b) that

$$\sum_{i=1}^n e_i(t_l) = \sum_{i=1}^n (x_i(t_l) - d_i) = 0, \tag{8}$$

in which the result $1_n^T L = 0$ is used. Namely, the coupled equality limitation $\sum_{i=1}^n x_i = \sum_{i=1}^n d_i$ is guaranteed within pre-specified time T_f.

Step II. When $t \in [T_f, 2T_f]$, we have

$$\dot{x}_i(t) = \frac{h}{t_{l+1} - t_l} L\nabla f(x(t_l)), t \in [t_l, t_{l+1}). \tag{9}$$

Integrating (9) over time $[t_l, t_{l+1})$ yields

$$x(t_{l+1}) = x(t_l) - hL\nabla f(x(t_l)). \tag{10}$$

Let $V(l) = f(x(t_l)) - f(x^*)$, in which x^* is the theoretical optimal result of problem (2). Since the global coupled equality limitation holds the whole time, $V(l)$ is radially unbounded and semi-positive. The difference of $V(l)$ along system (10) is provided by

$$\Delta V(l) = V(l+1) - V(l). \tag{11}$$

As $f(x)$ is convex, then one has

$$\begin{aligned} f(y) =& f(z) + \nabla^T f(z)(y - z) \\ &+ \frac{1}{2}(y - z)^T \nabla^2 f(\bar{y})(y - z), \end{aligned} \tag{12}$$

in which $\bar{y} \in (y, z)$. Let $y = x(t_{l+1})$ and $z = x(t_l)$, substituting the result (12) into (11) yields

$$\begin{aligned} \Delta V(l) =& \frac{1}{2}(x(t_{l+1}) - x(t_l))^T \nabla^2 f(\bar{y}) \\ &\cdot (x(t_{l+1}) - x(t_l)) \\ &+ \nabla^T f(x(t_l))(x(t_{l+1}) - x(t_l)) \\ =& \frac{h^2}{2}\nabla^T f(x(t_l)) L^T \nabla^2 f(\bar{y}) L\nabla f(x(t_l)) \\ &- h\nabla^T f(x(t_l)) L\nabla f(x(t_l)). \end{aligned} \tag{13}$$

We have used the fact (10) in the second equality of (13). It follows Assumption 1 that

$$\begin{aligned} \Delta V(l) \leq& \frac{h^2 m}{2}\nabla^T f(x(t_l)) L^T L\nabla f(x(t_l)) \\ &- h\nabla^T f(x(t_l)) L\nabla f(x(t_l)), \end{aligned} \tag{14}$$

in which $m = \max_i m_i$. It follows the undirected and connected assumption for graph $\mathcal{G}$ that $L^T L \leq \rho_n(L)L$ with the largest eigenvalue $\rho_n(L)$. It follows (14) that

$$\Delta V(l) \leq -h(1 - \frac{hm}{2})\nabla^T f(x(t_l)) L\nabla f(x(t_l)). \tag{15}$$

If $0 < h < \frac{2}{m\rho_n(L)}$, we have $\Delta(l) \leq 0$. Additionally, in view of the definition of Laplacian matrix L, $\nabla f(x(t_l))$ achieves consensus iff $L\nabla f(x(t_l)) = 0$. Thus, in view of (3), the most favorable result is achieved within pre-specified time due to the fact $\lim_{l\to\infty} t_l = 2T_f$ when $t \in [T_f, 2T_f]$. ■

C. Applications to economic dispatch problems

The global coupled constraint is considered linear in the problem (2). However, the nonlinear term is more general in applications, such as the transmission line loss in the economic dispatch problem of energy internet. Next, we extend the proposed algorithm (4) to the global nonlinear constrained distributed optimization problem shown below:

$$\begin{aligned} \min_x f(x) = \sum_{i=1}^n f_i(x_i), \\ \sum_{i=1}^n g_i(x_i) = 0, \end{aligned} \tag{16}$$

where $g_i(x_i)$ is a nonlinear function. For instance, the local function $g_i(x_i)$ is defined as $g_i(x_i) = x_i - d_i - B_i x_i^2$ with loss coefficient B_i. Generally, B_i is very small. The following distributed optimization approach is created to obtain the optimal solution of the problem (16) within a pre-specified time.

$$\begin{aligned} \dot{x}_i(t) =& -\frac{1}{(t_{l+1} - t_l)\nabla g_i(x_i(t))} e_i(t_l) \\ &- \frac{\alpha}{\nabla g_i(x_i(t))} \sum_{j=1}^n a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l))), \end{aligned} \tag{17a}$$

$$\begin{aligned} e_i(t) =& g_i(x_i) \\ &+ \alpha \int_0^t \sum_{j=1}^n a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l))) d\tau \end{aligned} \tag{17b}$$

where $t \in [t_l, t_{l+1})$ and α is the control parameter, which is defined the same with algorithm (4). To obtain the optimal solution of (16), we provide following assumption.

Assumption 2: For each agent i, the condition $\nabla g_i(x_i) > 0$ holds.

By similar convergence proof in Theorem 1, we establish the following conclusion.

Corollary 1: Consider that Assumptions 1 and 2 hold, and the graph $\mathcal{G}$ is undirected and connected, the proposed distributed method (17) achieves the optimal result of (16) within pre-specified time $2T_f$. In addition, the control parameter h satisfies $0 < h < \frac{2\max_i \nabla g_i(x_i)}{m\rho_2(L)}$.

IV. Simulation

In the following, we will provide some cases to indicate the accuracy for our designed algorithm.

Example 1. The economic dispatch problem without transmission line loss over smart grids in [23] is devoted to illustrating the algorithm (2). For each generator i, let $f_i(x_i) = a_i x_i^2 + b_i x_i + c_i$ with coefficients a_i, b_i, c_i, where x_i indicates its generation power output. The coefficients are shown in Table. I. The economic dispatch problem aims to design a generation plan for some distributed generators while minimizing the total generation cost function. Let the initial state be $x(0) = [60, 0, 0, 0, 0, 0]^T \in \mathbb{R}^6$. It can be seen that the global coupled equality constraint does not hold. Thus, existing results [22], [23] fail to solve the economic dispatch problem under the same initial states. From Theorem 1, it can be calculated that $0 < h <= \frac{2}{m\rho_2} = \frac{2}{2\times 0.105 * 5.7} = 1.77$. Let the control parameter h be $h = 1$, the detailed trajectories of variable x_i and $e_i, \forall i \in \mathcal{V}$, are shown in Figs. 1 and 2. From Fig. 1, the most favorable result $x^* = [23.47, 16.09, 15.22, 10.40, 18.12, 16.70]^T \in \mathbb{R}^6$ is obtained within setting time $2T_f$. Additionally, the setting time $2T_f$ is independent on the constrained optimization problem and its network knowledge. From 2, the error variable $e_i(t)$ reaches zero within a given time $T_f = 5$s, meaning that the coupled equality limitation is ensured by a pre-specified time. Additionally, the error is zero after one iteration from the analysis of the theorem. After one iteration, the coupling constraint in constraint optimization problem (2) holds and remains unchanged.

TABLE I

THE COEFFICIENTS OF EACH GENERATOR.

generator i \ coefficients	a_i	b_i	c_i	d_i
1	0.096	1.22	51	20
2	0.072	3.41	31	20
3	0.105	2.53	78	20
4	0.078	4.02	42	20
5	0.078	2.90	67	10
6	0.090	2.72	49	10

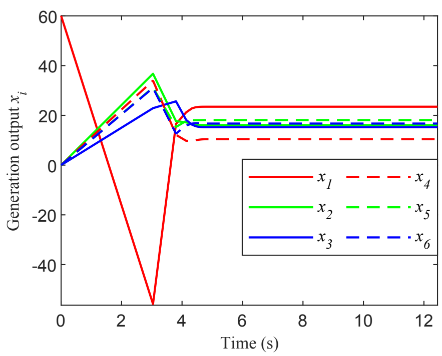

Fig. 1. Transient behavior of $x_i, \forall i \in \mathcal{V}$, in Example 1.

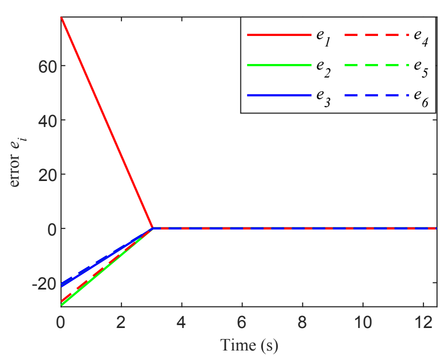

Fig. 2. Transient behavior of $e_i, \forall i \in \mathcal{V}$, in Example 1.

Example 2. For the economic dispatch problem shown in [24], whose coefficients are given by $a = [0.094, 0.078, 0.105, 0.082, 0.074]^T \in \mathbb{R}^5, b = [1.22, 3.41, 2.53, 4.02, 3.17]^T \in \mathbb{R}^5, c = [51, 31, 78, 42, 62]^T \in \mathbb{R}^5, d = [20, 20, 20, 20, 40]^T \in \mathbb{R}^5$ and $B = [0.0021, 0.0031, 0.0011, 0.0022, 0.0041]^T \in \mathbb{R}^5$. The considered network is an undirected ring topology. Let the control parameter be $h = \begin{cases} 1, & t \in [0, T_f] \\ 0.1, & t \in [T_f, 2T_f] \end{cases}$ and the initial state be $x(0) = [60, 0, 0, 0, 0]^T \in \mathbb{R}^5$, the detailed trajectories of variables x_i and $e_i, \forall i \in \mathcal{V}$, are depicted in Figs. 3 and 4. From (16), the algorithm is executed in two steps. The consensus term $\frac{\alpha}{\nabla g_i(x_i(t))} \sum_{j=1}^{n} a_{ij}(\nabla f_i(x_i(t_l)) - \nabla f_j(x_j(t_l)))$ with big control parameter h always working. Therefore, the optimal solution can be achieved far earlier than the pre-specified time $2T_f$. It has been verified in Fig. 3. Thus, the developed algorithms are valid for distributed constrained optimization problems with pre-specified time.

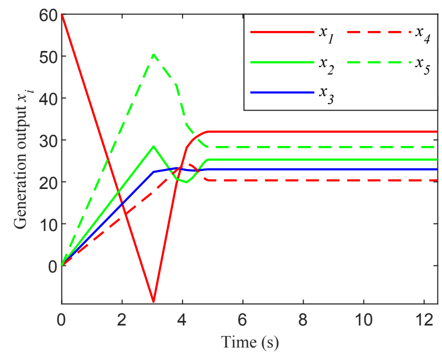

Fig. 3. Transient behavior of $x_i, \forall i \in \mathcal{V}$, in Example 2.

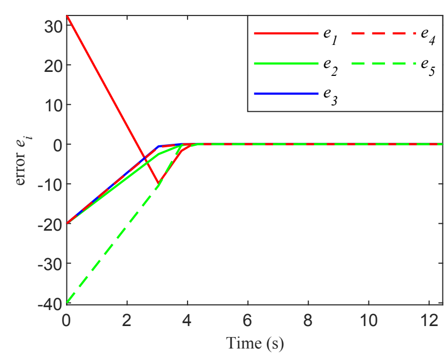

Fig. 4. Transient behavior of $e_i, \forall i \in \mathcal{V}$, in Example 2.

V. CONCLUSIONS AND FUTURE WORKS

This paper introduces two distributed algorithms for solving distributed constrained optimization problems over undirected MASs. In view of the aperiodic sampling control approach, our developed methods converge to the optimal result within a pre-specified time. The developed optimization approaches have been proved to be convergent using the Lyapunov stability theory. Furthermore, the efficiency illustration is completed by economic dispatch problems. In future work, we plan to incorporate the consideration of local constraints for each agent in the design of pre-specified-time distributed optimization problems. Besides, the exact value of gradient function is required to develop the optimization approaches, which heavily increased the system bandwidth consumption. Consequently, we will also study the quantized optimization approaches within pre-specified time.

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