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Data-Driven Safety-Critical Control with High-Order Iterative Control Barrier Function

Zi-Yuan Dong, Xin-Yi Yu, Lin-Lin Ou, Yong-Qi Zhang

Abstract— **The stability and safety of uncertain nonlinear systems have always been an essential and hard task in the automation field. For the forward invariance of the safety set of the system under unmodeled dynamics, a new concept of data-driven high-order iterative control barrier function (DHI-CBF) is presented. To overcome the structural complexity and dependence on dynamic models of traditional robot controllers, an iterative learning mechanism is introduced and a model-free iterative predictive controller (MFIPC) is designed. The controller can be progressively optimized by predicting future information so that it can cope with dynamic changes and uncertainties in the environment. On this basis, quadratic programming (QP) that unifies the DHI-CBF with the MFIPC is established, which is able to prioritize the safety of the system in case of conflict between the desired output and the security boundary. Finally, the application for Franka-Panda robot demonstrates the superiority of the presented algorithm.**

I. INTRODUCTION

With the rapid development of control technology, the safety of industrial equipment in complex environments has become one of the mainstream issues in current research [1]. In many control scenarios, the system is hard to model accurately and suffers from model mismatch. Also, many industrial objects, such as robotic arms, are highly nonlinear and coupled [2], [3], which will make the implementation of accurate modeling more challenging. Therefore, it is valuable to conduct safety control studies when model information is not accurate or even completely unknown.

So far, some works on safety control have been developed, see [4], [5], [6]. In [7], the control method based on integral barrier Lyapunov function is proposed that can impose safe state constraints. However, the control input tends to infinity as the robot state approaches the safety bound, which somewhat restricts the practical application. CBFs are an extremely valuable analytical method and have been widely used. In [8], the nominal controller is added to a QP problem to mediate potential conflicts between security and control goals. In [9], a novel CBF with a high-order structural form is constructed for nonlinear systems.

Note that all of the above controllers on CBF are designed based on continuous-time systems. The majority of controllers in practical applications are programmed and realized digitally, so discrete-time CBFs have also received much attention recently. In [10], the discrete-time CBF is combined with the control lyapunov function to give a nonlinear programming problem. Nevertheless, the controllers constructed through the CBFs described above rely on accurate system models, and it may be challenging to secure the system when the model information is unknown. Although the dynamics of the system can be modeled under certain assumptions, it is very time-consuming and relatively complex, making it hard to analyze the controller directly.

Data-driven control (DDC) is increasingly in the spotlight [11], [12], [13], which can effectively overcome the shortcomings of tracking control performance dependent on model information. In [14], the controller with output error rate is derived by the compact format dynamic linearization approach. In [15], the parameter estimation algorithm with a switching mechanism is designed to improve the flexibility of the controller. Iterative learning controllers (ILC) can learn and optimize through feedback information, improve control performance during system operation, and adapt to changes in the system. In [16], a robust tracking ILC is constructed for speed tracking control with iteration-dependent disturbance for the controlled system. Although the above DDCs have achieved excellent results, these methods still do not simultaneously consider the security of the system. Recently, in [17], a novel IL-CBF is proposed for the security of multiple intelligences.

Motivated by above factors, this paper presents a QP-based security control algorithm to solve the trajectory tracking task of systems with unknown dynamics. The main contributions are listed as below:

(1) The DHI-CBF is proposed for safety-critical tasks, which ensures constraint gratification for nonlinear discrete-time systems with high relativity when the model dynamics are completely unknown.

(2) To overcome the structural complexity and the dependence on the dynamics model of conventional controllers, the MFIPC strategy is designed based on a unified framework of data-driven theory and iterative learning. The robustness of controller is enhanced through a stepwise optimization process while an adaptive parameter learning scheme is designed to improve flexibility.

(3) The safe and stable data-driven controller is constructed by unifying designed DHI-CBF and MFIPC in a QP-based formula. The application for the Franka-Panda robot demonstrates the superiority.

The rest of the paper is structured as follows. The specific

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problem formulation is briefly stated in Section II; The proposed algorithm is introduced in Section III. The testing results based on the robot are presented in Section IV; Lastly, the conclusions are given in Section V.

II. PROBLEM FORMULATION

A. System description

Consider a class of nonlinear systems with discrete-time forms [18], described as follows:

$$y_i(\iota+1) = f[\bar{Y}_\iota^{\iota-n_a}, u_i(\iota), \bar{U}_{\iota-1}^{\iota-n_b}] \tag{1}$$

where $\bar{Y}_\iota^{\iota-n_a} = \{y_i(\iota), y_i(\iota-1), \cdots y_i(\iota-n_a)\}$, $\bar{U}_{\iota-1}^{\iota-n_b} = \{u_i(\iota-1), u_i(\iota-2), \cdots u_i(\iota-n_b)\}$. $y_i(\iota)$, $u_i(\iota)$ denote the system output and the control input respectively. $f(.)$ is the nonlinear function with unknown relationships. $\iota \in \{0, 1, \cdots T\}$ represents sampling instant. i represents the number of iterations. n_a and n_b are non-negative factors.

Similar to [16], the following assumptions are introduced:

Assumption 2.1: The partial derivatives of $f(.)$ with respect to $u_i(\iota)$ exist and are continuous.

Assumption 2.2: The nonlinear system satisfies the generalized Lipschitz condition.

$$|\Delta y_i(\iota+1)| \leq b|\Delta u_i(\iota)| \tag{2}$$

where Δ is the iterative difference operator, $\Delta\Upsilon_i(\iota) = \Upsilon_i(\iota) - \Upsilon_{i-1}(\iota)$, b is a normal number.

Subsequently, if $|\Delta u(k)| \neq 0$, there exist time-varying pseudo-partial derivative (PPD) $\Psi_i(\iota)$, such that system (1) can be transformed into the dynamic data model.

$$y_i(\iota) = y_{i-1}(\iota) + \Psi_i^T(\iota-1)\Delta u_i(\iota-1) \tag{3}$$

Remark 1: The obtained data model (2) conforms to the time-varying linear incremental form of the input-output relationship. The structural form shows that an unknown relationship between the control inputs and outputs is established in the interval between two adjacent iterations.

B. Problem Formulation

To make sure the system converges to the desired position $y_d(\iota)$, the tracking error $e_i(\iota)$ is defined based on the desired output trajectory as

$$e_i(\iota) = y_d(\iota) - y_i(\iota) \tag{4}$$

In previous designed frameworks, the dynamics model is usually determined. However, it is hard to obtain in practice and very time-consuming.

Another issue concerns safety constraints, which are common but essential for robotic tasks, such as robotic assembly, robotic surgery, etc. To evaluate the safety of the system output, the output $y_i(\iota)$ in (3) is defined as a safe output if it satisfies the following criterion.

$$w_d(\iota) - y_i(\iota) \geq 0 \tag{5}$$

where $w_d(\iota)$ represents the security boundary.

Therefore, it is desired that the system output $y_i(\iota)$ is kept within a safety region denoted by the set C.

$$\begin{aligned} \mathcal{C} &:= \{y_i(\iota) \in \mathcal{D} \mid \Theta(y_i(\iota)) \geq 0\} \\ Int(\mathcal{C}) &:= \{y_i(\iota) \in \mathcal{D} \mid \Theta(y_i(\iota)) > 0\} \\ \partial\mathcal{C} &:= \{y_i(\iota) \in \mathcal{D} \mid \Theta(y_i(\iota)) = 0\} \end{aligned} \tag{6}$$

where $\Theta(.)$ is a function changing along the iteration axis.

Before presenting the concept of DHI-CBF, some necessary definitions are explained below.

Definition 1: In nonlinear system (3), the set C satisfies forward invariance along the iterative dimension, if $y_i(\iota) \in C, \forall \iota \in N$ for every $y_i(0) \in C$.

Definition 2: Define the function $\alpha : [0, \alpha) \to [0, \infty)$ is belong to class κ function. It is increasing and $\alpha(0) = 0$.

To satisfy safety, that is, to ensure that the system maintains the forward invariance of the safety set C during operation, the concept of CBF can be used to establish corresponding conditions and constraints.

Definition 3: For the nonlinear system (3) with iterative form, let $C \subset R^n$ define a function $\Theta : R^n \to R$ with an iterative structure. Θ is a IL-CBF if the κ function α satisfying $\alpha(r) < r$ for all $r > 0$ such as

$$\Delta\Theta(y_i(\iota), u_i(\iota)) \geq -\alpha(\Theta(y_i(\iota), u_i(\iota))). \forall y_i(\iota) \in D \tag{7}$$

where $\Delta\Theta(y_i(\iota), u_i(\iota)) = \Theta(y_{i+1}(\iota)) - \Theta(y_i(\iota))$.

Lemma 2.1: Given the set $C \subseteq D \subset R^n$ defined by (6) for the continuous function $\Theta : R^n \to R$, any discrete-time control input $u_i(\iota)$ that meets (7) will make the set C forward invariant if Θ is an IL-CBF on D.

Proof: The proof is similar to the analysis in [17]. ■

As a conclusion, the design objectives are listed as:

1) The tracking control error satisfies $\lim_{i\to\infty} |e_i(\iota)| = \lim_{i\to\infty} |y_d(\iota) - y_i(\iota)| = 0$.
2) In a finite time, the control output $y_i(\iota)$ of the nonlinear system is kept within a safe area.
3) When control objectives (1) and (2) are in conflict, priority will be given to guaranteeing the safety.

It needs to be noted that most of the existing CBFs are designed based on continuous-time systems. Nevertheless, most of the controllers in real applications are implemented through digital programming, so it is challenging to design discrete-time CBFs when the system model is completely unknown.

III. MAIN RESULTS

This section is divided into three parts. The design of MFIPC and the DHI-CBF will be given. The QP problem is then constructed on this basis.

A. The Design of MFIPC Strategy

When the data model (2) with an iterative structure is considered as a prediction equation, a new prediction model can be obtained:

$$
\begin{aligned}
y_{i+1}(\iota) &= y_i(\iota) + \Psi_{i+1}^T(\iota-1)\Delta u_{i+1}(\iota-1) \\
y_{i+2}(\iota) &= y_{i+1}(\iota) + \Psi_{i+2}^T(\iota-1)\Delta u_{i+2}(\iota-1) \\
&= y_i(\iota) + \Psi_{i+1}^T(\iota-1)\Delta u_{i+1}(\iota-1) \\
&+ \Psi_{i+2}^T(\iota-1)\Delta u_{i+2}(\iota-1) \\
&\vdots \\
y_{i+N}(\iota) &= y_{i+N-1}(\iota) + \Psi_{i+N}^T(\iota-1)\Delta u_{i+N}(\iota-1) \\
&= y_i(\iota) + \Psi_{i+1}^T(\iota-1)\Delta u_{i+1}(\iota-1) \\
&+ \cdots + \Psi_{i+N}^T(\iota-1)\Delta u_{i+N}(\iota-1)
\end{aligned} \tag{8}
$$

Define

$$
\begin{cases}
Y_N(\iota) = [y_{i+1}(\iota), \cdots, y_{i+N}(\iota)]^T \\
\Delta U_N(\iota) = [\Delta u_{i+1}(\iota), \Delta u_{i+2}(\iota) \ldots \Delta u_{i+N}(\iota)] \\
A(\iota) = \\
\begin{bmatrix}
\Psi_{i+1}^T(\iota) & & & \\
\Psi_{i+1}^T(\iota) & \Psi_{i+2}^T(\iota) & & \\
\cdots & & & \\
\Psi_{i+1}^T(\iota) & \Psi_{i+2}^T(\iota) & \cdots & \Psi_{i+N}^T(\iota)
\end{bmatrix}_{N*N} \\
E = [\ 1 \quad 1 \quad \cdots \quad 1\ ]^\tau
\end{cases} \tag{9}
$$

where $Y_N(\iota)$ represents the forward prediction vector, $\Delta U_N(\iota)$ is the increment vector of control torque. The dynamic model (8) can be rewritten as the matrix form

$$
Y_N(\iota) = A(\iota-1) * \Delta U_N^T(\iota-1) + E * y_i(\iota) \tag{10}
$$

Remark 2: The option of N determines the size of prediction step. The choice of N ought to be adjusted to the actual requirements and characteristics in order to obtain better performance. If the system has a fast dynamic response, the size of prediction step N may need to be smaller in order to more accurately capture the rapid changes in the system. If the system has large inertia or a slower dynamic response, the prediction step N may need to be larger to better predict the long-term behavior of the system.

Based on the obtained prediction model (10), the following cost function is considered

$$
J(\Delta U_N(\iota)) = \sum_{j=1}^{N} \left[(y_{d,i+j}(\iota) - y_{i+j}(\iota))^2 + \lambda \Delta U_{i+j}^2(\iota) \right] \tag{11}
$$

where $\lambda > 0$ can prevent the drastic changes of inputs.

Define $Y_d(\iota) = [y_{d,i+1}(\iota), y_{d,i+2}(\iota) \ldots y_{d,i+N}(\iota)]$ and the new cost function is designed as

$$
\begin{aligned}
J = &|Y_d(\iota) - Y_N(\iota)|^T |Y_d(\iota) - Y_N(\iota)| \\
&+ \lambda \Delta U_N(\iota)^T \Delta U_N(\iota)
\end{aligned} \tag{12}
$$

Conduct $\partial J / \partial U_N(\iota) = 0$, and the optimal control law can be derived as

$$
\begin{aligned}
\Delta U_N(\iota) = &\left[A^T(\iota-1)A(\iota-1) + \lambda I\right]^{-1} A^T(\iota-1) \\
&[Y_d(\iota) - E * y_{i-1}(\iota)]
\end{aligned} \tag{13}
$$

The calculation can be simplified by introducing the parameter $0 < \rho \leq 1$.

$$
\Delta U_N(\iota) = \frac{\rho A^T(\iota-1)\left[Y_d(\iota) - E * y_{i-1}(\iota)\right]}{\|A(\iota-1)\|^2 + \lambda I} \tag{14}
$$

According to the principle of recursive optimal control, the optimal control input for applying the current time step can be calculated

$$
u_i(\iota) = u_{i-1}(\iota) + g^T * \Delta U_N(\iota) \tag{15}
$$

where $g = [\ 1 \quad 0 \quad \cdots \quad 0\ ]^T$. For $\Psi_i(\iota)$, we design the following index function of PPD estimation

$$
\begin{aligned}
J(\Psi_i(\iota-1)) = &\left|\Delta y_{i-1}(\iota) - \Psi_i^T(\iota-1)\Delta u_{i-1}(\iota-1)\right|^2 \\
&+ \mu \left|\Psi_i(\iota-1) - \hat{\Psi}_{i-1}(\iota-1)\right|^2
\end{aligned} \tag{16}
$$

where μ is a weighting factor that serves to prevent the PPD from varying too much effectively. By minimizing the above index function, one can obtain

$$
\begin{aligned}
\hat{\Psi}_i(\iota-1) = \hat{\Psi}_{i-1}(\iota-1) + &\frac{\eta \Delta u_{i-1}(\iota-1)}{\mu + \|\Delta u_{i-1}(\iota-1)\|^2} \\
&\times [\Delta y_{i-1}(\iota) - \hat{\Psi}_{i-1}^T(\iota-1)\Delta u_{i-1}(\iota-1)]
\end{aligned} \tag{17}
$$

Then, the autoregressive model by the sequence $\hat{\Psi}_1(\iota-1), \hat{\Psi}_2(\iota-1), \cdots \hat{\Psi}_i(\iota-1)$ is developed below

$$
\begin{aligned}
\hat{\Psi}_{i+1}(\iota-1) = &\theta_1(\iota-1)\hat{\Psi}_i(\iota-1) + \theta_2(\iota-1)\hat{\Psi}_{i-1}(\iota-1) \\
&+ \cdots + \theta_{n_p}(\iota-1)\hat{\Psi}_{i-n_p+1}(\iota-1)
\end{aligned} \tag{18}
$$

where $\theta_j(\iota-1)(j = 1, 2 \cdots n_p)$ represent the appropriate coefficients. n_p denotes the order of the model.

Define $\theta(\iota-1) = [\theta_1(\iota-1), \cdots \theta_{n_p}(\iota-1)]$, and the estimation algorithm of PPD becomes

$$
\begin{aligned}
\hat{\Psi}_{i+j}(\iota-1) = &\theta_1(\iota-1)\hat{\Psi}_{i+j-1}(\iota-1) \\
&+ \theta_2(\iota-1)\hat{\Psi}_{i+j-n_p}(\iota-1) \\
&+ \cdots + \theta_{n_p}(\iota-1)\hat{\Psi}_{i-n_p+1}(\iota-1)
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\theta_i(\iota-1) = &\theta_{i-1}(\iota-1) + \frac{\hat{\Psi}_i(\iota-1)}{\delta + \left\|\hat{\Psi}_i(\iota-1)\right\|^2}[\hat{\Phi}_i(\iota-1) \\
&+ \hat{\Psi}_i(\iota-1)\theta_{\iota-1}(S-1)]
\end{aligned} \tag{20}
$$

where $j = 1, 2 \cdots N$ and $0 < \delta \leq 1$.

For improving the estimation performance of the time-varying parameters, the following iterative axis-based reset algorithm is designed. The MFILPC scheme is as follows

$$
\begin{aligned}
\hat{\Psi}_{i+j}(\iota-1) = &\theta_1(\iota-1)\hat{\Psi}_{i+j-1}(\iota-1) \\
&+ \theta_2(\iota-1)\hat{\Psi}_{i+j-n_p}(\iota-1) \\
&+ \cdots + \theta_{n_p}(\iota-1)\hat{\Psi}_{i-n_p+1}(\iota-1)
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
&\hat{\Psi}_i(\iota-1) = \hat{\Psi}_1(\iota-1), \\
&if \left|\hat{\Psi}_i(\iota-1)\right| \leq \varphi, or\, |\Delta u_i(\iota-1)| \leq \varphi
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\theta_i(\iota-1) = &\theta_{i-1}(\iota-1) + \frac{\hat{\Psi}_i(\iota-1)}{\delta + \left\|\hat{\Psi}_i(\iota-1)\right\|^2}[\hat{\Phi}_i(\iota-1) \\
&+ \hat{\Psi}_i(\iota-1)\theta_{\iota-1}(S-1)]
\end{aligned} \tag{23}
$$

$$\theta_i(\iota-1)=\theta_1(\iota-1), if\ \|\theta_i(\iota-1)\|\geq M \tag{24}$$

$$u_i(\iota)=u_{i-1}(\iota)+g^T*\Delta U_N(\iota) \tag{25}$$

where $M>0, \varphi>0$.

Remark 3: Inspired by previous work related to iterative learning [17] and model predictive control [19], parameter adaptive mechanisms can be designed to improve controller flexibility based on a unified framework of DDC and ILC theory. To overcome the shortcomings of existing DDC schemes [12], [15] which are mainly limited to the time domain, discrete control of the robot is addressed by extending MFAC to the iterative domain for the first time to fully utilize more system information. In addition, this scheme utilizes only terminal I/O information throughout the design process, thus allowing for additional degrees of freedom, such as faster processing speeds.

B. DHI-CBF for Discrete-Time Systems

The conventional CBFs [5] cannot be directly employed to secure the system since dynamic information is completely unknown. To handle this issue, we proposed the concept of DHI-CBF based on the design of the dynamic data model with iterative structure.

For the time-varying system (3), in order to obtain the ability to satisfy more strict output constraint guarantees, we define a list of functions by primitive function $\Theta: R^n \rightarrow R$ with a relative degree of m.

$$\begin{aligned}
&\Gamma_0(y_\iota):=\Theta(y_\iota)\\
&\Gamma_1(y_\iota):=\Delta\Gamma_0(y_\iota,u_\iota)+\alpha_1\Gamma_0(y_\iota)\\
&\vdots\\
&\Gamma_m(y_\iota):=\Delta\Gamma_{m-1}(y_\iota,u_\iota)+\alpha_m\Gamma_{m-1}(y_\iota)
\end{aligned} \tag{26}$$

where $y_i(\iota)$ and $u_i(\iota)$ are replaced by the simplified symbols y_ι and u_ι. $\Delta\Gamma_\delta(y_\iota,u_\iota)=\Gamma_\delta(y_{\iota+1})-\Gamma_\delta(y_\iota)$, $\delta=0,1\cdots m-1$, $\alpha_j(j=1,2,\cdots m)$ are class κ functions.

A list of sets C_j, $j=0,1,\cdots m-1$ are generated as follows:

$$\begin{aligned}
\mathcal{C}_0&:=\{y_\iota\in\mathcal{D}\mid\Gamma_0(y_\iota)\geq 0\\
\mathcal{C}_1&:=\{y_\iota\in\mathcal{D}\mid\Gamma_1(y_\iota)\geq 0\\
&\vdots\\
\mathcal{C}_{m-1}&:=\{y_\iota\in\mathcal{D}\mid\Gamma_{m-1}(y_\iota)\geq 0
\end{aligned} \tag{27}$$

Definition 4: For the discrete-time system (2), the continuous function $\Theta: R^n\rightarrow R$ is a DHI-CBF of relative degree m if there exist $\Gamma_\delta(y_\iota), \delta\in\{0,1,\cdots,m-1\}$ defined by (27) such that

$$\Gamma_m(y_\iota)\geq 0 \tag{28}$$

for all $y_\iota\in\cap_{i=0}^{m-1}C_i$.

Theorem 1: Given sets $C_j, j\in\{0,1,\cdots,m-1\}$ defined by (27) and a function $\Theta: R^n\rightarrow R$. If Θ is a DHI-CBF of relative degree m defined on $\cap_{i=0}^{m-1}C_i$, any discrete-time controller $u_i(\iota)$ ensuring (28) will render the set $\cap_{i=0}^{m-1}C_i$ forward invariant.

Proof: When Θ is DHI-CBF, condition (27) holds, which yields

$$\begin{aligned}
\Gamma_m(y_\iota)&=\Delta\Gamma_{m-1}(y_\iota,u_\iota)+\alpha_m\Gamma_{m-1}(y_\iota)\\
&\geq 0,\forall\iota\in N.
\end{aligned}$$

By Lemma 2.1, it is obvious that the corresponding set C_{m-1} satisfies forward invariance, that is, $y_\iota\in C_{m-1},\forall\iota\in N$ if $y_0\in C_{m-1}$. Then,

$$\begin{aligned}
\Gamma_{m-1}(y_\iota)&=\Delta\Gamma_{m-2}(y_\iota,u_\iota)+\alpha_{m-1}\Gamma_{m-2}(y_\iota)\\
&\geq 0,\forall\iota\in N.
\end{aligned} \tag{29}$$

which verifies the forward invariance of set C_{m-2} through Lemma 2.1 again.

By iterative induction, it can be derived that if $y_0\in C_j$, then $y_\iota\in C_j,\forall\iota\in N,\forall j\in\{0,1\cdots m-1\}$. Therefore, the set $\cap_{j=0}^{m-1}C_j$ is forward invariant. ■

Remark 4: The above design concepts can be viewed as a generalization of the high-order CBFs [20] from continuous-time systems to discrete-time systems. Another noteworthy point is that previous design of higher-order CBFs relied on dynamical models, otherwise the mathematical relationship between the input u_ι and the output y_ι could not be inscribed in $\Theta(\cdot)$. Therefore the safety of the system will be poorly ensured if the model information is inaccurate.

C. Optimization Problem Unifying MFIPC and DHI-CBF

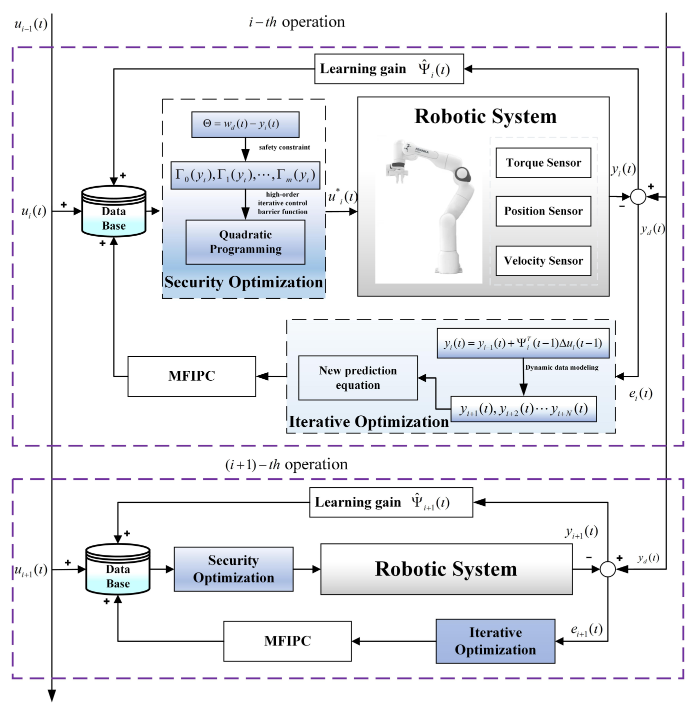


Fig. 1: Control algorithm framework.

To design control inputs that can simultaneously meet multiple objectives such as system safety and stability, the MFIPC and DHI-CBF are unified into a QP-based solution formula.

Since the designed MFIPC has a display expression, the safety constraints described by DHI-CBF can be embedded in the closed-loop system through the QP problem. The control framework is given as Fig. 1, and the optimization problem can be rewritten as follows:

$$\begin{cases} u_i^*(\iota) = \underset{U_i(\iota)}{\operatorname{argmin}} \frac{1}{2} \parallel u_i(\iota) - \Xi_i(\iota) \|^2 \\ U_i(\iota) = \left[\begin{array}{c} u_i(\iota) \end{array} \right] \\ s.t. \\ \quad \Gamma_m(y_\iota) \geq 0 \\ \quad u_{\min} \leq u_i(\iota) \leq u_{\max} \end{cases} \tag{30}$$

where $u_{\min}$ and $u_{\max}$ are the lower limit and upper limit of the control input respectively, $\Xi_i(\iota)$ represents the nominal control law of Equation (25).

In this sense, the designed DHI-CBF can be viewed as a "filter" capable of filtering out control inputs that undermine security. In other words, when the control objectives of the system do not conflict with the safety constraints described by the DHI-CBF, then the robotic system will achieve the control objectives without violating the safety constraints; otherwise, the priority will be given to ensuring system safety.

IV. EXPERIMENTS

In this section, a nonlinear robotic system with time-varying constraints is used to demonstrate the effectiveness of the proposed optimal control scheme.

The overview of the hardware configuration is shown in Fig. 2. The Franka-Panda robot is composed of highly flexible flex joints. In addition, the Panda robot is equipped with a variety of sensors. The host computer is an Intel Core i7-8700K computer with Ubuntu 16.04 LTS operating system installed on it and the Ubuntu system uses the real-time kernel Linux 4.14.12-rt10. The software platform is based on the Robot Operating System (ROS), version Kinetic Kame. Furthermore, a distributed communication architecture with master and slave nodes is designed. The master node is mainly responsible for sending the control torque to the robot as well as sending the information such as the position of the robot itself to the slave node. On the basis of the state information, the control torque is calculated by the slave node through the control algorithm.

Run the proposed algorithm with $\eta = 0.65, \rho = 0.7, \mu = 0.1$ for the application. See Table I for other parameter settings. The A1 joint of the robot with brushless dc motor is initialized from $q(0) = 0$ at rest. The control frequency is set to 1000Hz to realize high-speed real-time communication based on User Datagram Protocol.

Based on the concept of DHI-CBF (26), the following function is set up:

$$\begin{aligned} \Gamma_0(y_\iota) &:= \Theta(y_\iota) \\ \Gamma_1(y_\iota) &:= \Delta\Gamma_0(y_\iota, u_\iota) + \alpha_1\Gamma_0(y_\iota) \\ \Gamma_2(y_\iota) &:= \Delta\Gamma_1(y_\iota, u_\iota) + \alpha_2\Gamma_1(y_\iota) \end{aligned} \tag{31}$$

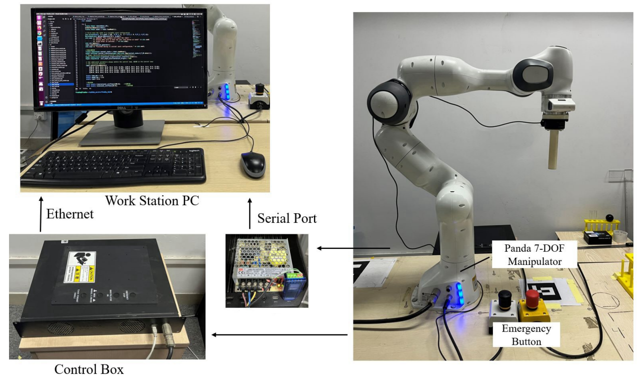


Fig. 2: Franka-Panda robot experiment platform.

TABLE I: Control Parameter

Parameter	Value	Parameter	Value
μ	0.1	N	3
θ	$[0.6, 0.4, 0.4]$	ρ	0.7
η	0.65	$\Psi_1(1)$	0.1
α_1	0.4	u_{max}	0.7
α_2	0.35	u_{min}	-0.7
m	2	n_p	3

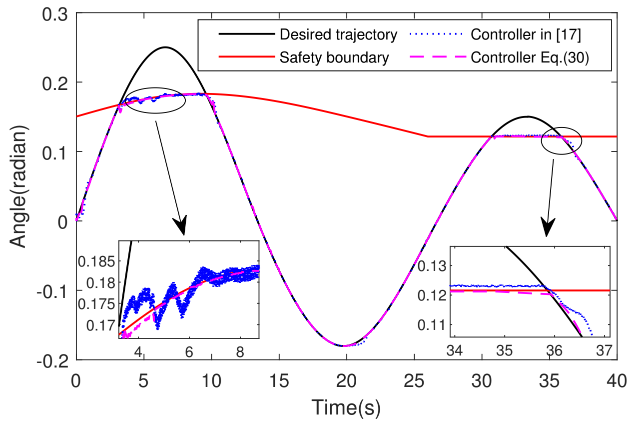
To verify the superiority of the proposed QP-based control algorithm, it is compared with the safety controller [17] under dynamic constraints, and the results in 50-th iterations are shown in Fig. 3 (a) - Fig. 3 (b). Note that time-varying constraints of joint position were added to the experiments to fully demonstrate the guarantees for safety of the proposed method.

When there is no conflict between the control target and the security boundary (between about time 0s and 3.1s), the system runs along the desired trajectory. From the point of steady state performance, the proposed method has a smaller tracking error and higher tracking accuracy. Meanwhile, it can be clearly seen that when the reference trajectory conflicts with the dynamic constraints (between about time 3.1s and 9.5s), the position outputs under the present scheme are strictly limited within the safety boundaries and the trajectory runs smoothly. In terms of transition performance, when the conflict disappears (around 36s), the proposed method has a faster response time without large overshooting.

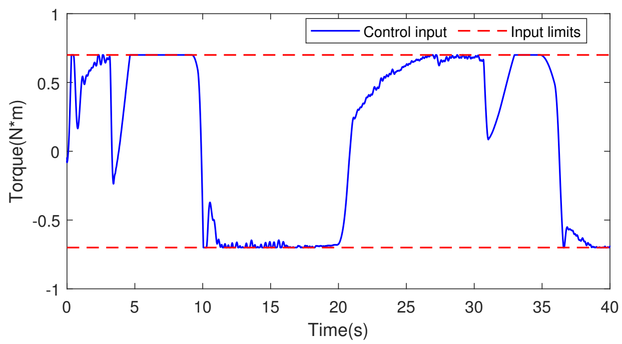
Besides, it can be seen from Fig. 3 (b) - Fig. 3 (c) that the control torque is strictly limited to the preset safety range throughout the control process with satisfactory real-time performance. The average calculation time for each control cycle does not exceed 0.5 ms. Therefore, the proposed control scheme has great reliability in practical applications.

V. CONCLUSION

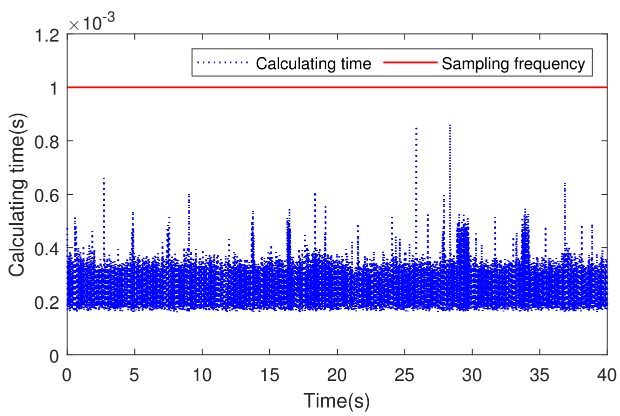
A data-driven safety-critical control scheme was proposed in this paper, where the concept of DHI-CBF was formulated to enable set invariance. To overcome the structural complexity of conventional controllers and the dependence



(a) The trajectory of robot system with time-varying constraints



(b) Control torque for robotic systems with constraints



(c) Execution time of proposed strategy

Fig. 3: Trajectory tracking control under safety constraints.

on the dynamics model, an MFIPC control method was developed in the framework of ILC. Meanwhile, the control sequence was optimized by iteratively solving for the optimal control inputs. The safe and stable data-driven controller was constructed by unifying designed DHI-CBF and MFIPC in a QP-based formula. The application of the Franka-Panda robot demonstrated the superiority.

Since sensor information is susceptible to high-frequency noise due to external environmental influences, uncertainties in output signals of DDC will be investigated and dealt with in the future.

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