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Observer-based Switched Control of the Three Level Neutral Point Clamped Rectifier

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Abstract— **In this paper, an observer-based switched control law is proposed for the three level neutral point clamped (NPC) converter operating as a rectifier. Modeling the converter as a switched affine system, the proposed control is based on the well known argmin control law to track a varying state reference trajectory. A full-order observer is introduced to compute the control law with only the measure of the input and the output voltages. The control aims at tracking a state reference defined from a power analysis and three objectives are addressed: to stabilize the output at a given DC voltage, to ensure a unit power factor by having the input current and voltage on phase and to have balanced capacitor voltages on the output. Based on a unified modeling methodology, the control and the observer are easily derived from LMI conditions. An outer loop is added to regulate the output when constant perturbations are considered. The results are illustrated by simulations on MATLAB/Simulink.**

I. INTRODUCTION

Neutral Point Clamped (NPC) inverters or rectifiers is one of the most used topology of multilevel converter in high power electrical applications. Indeed, this converter has several advantages like a low total harmonic distortion (THD) of the output and a high efficiency [1], which make it popular for AC/DC applications [2], [3]. The main objectives of the control are to achieve a unit power factor on the AC side while maintaining the DC side at a constant given voltage. Another objective is to ensure that the capacitor voltages are balanced to avoid the switches to undergo excessive voltage stress and preserve the quality of the output voltage [4].

From a control point of view, converter models belong to the class of switched affine systems. This nature comes from the fact that the dynamic of a converter depends on the state of electrical switches in the circuit. In the literature, most widely studied control solutions for power converters are based on averaged linearized models. More recently, an effort has been done to propose control methods that take into account the discontinuous behavior of such systems [5], [6]. However, control oriented papers on converters usually consider simple examples such as the Boost converter.

In the literature, some classical approaches can be found to control the NPC rectifier, like the direct power control methods based on an averaged model and the use of Pulse-Width Modulation (PWM) signal. In this case, linear or nonlinear control techniques are applied, involving several control loops to handle every control objectives [4], [7]. Advanced control strategies have also been considered, like space vector modulation [8] or predictive control methods [8], [9]. Regarding the switching nature of the converter, the number of papers dealing with it in the hybrid control framework is reduced [10].

In the same vein, the observation problem has been even less considered for this specific power converter, except in adaptive control schemes [11]–[13], or based on approximated/averaged model [14]. Indeed, the three level NPC converter have a large number of switches and a dynamic reference which are the sources of difficulties when compared to numerous examples of converters usually considered [15]–[17]. Also, the use of an observer is relevant to reduce the number of sensors and maintenance costs.

This paper addresses the observer-based control problem of the Three Level Neutral Point Clamped Rectifier. Using a unified modeling methodology [18] in Section II, a state space representation of the converter is establish from its electrical equations while considering its switching nature. In Section III-A, a full-order observer is designed through a set of Linear Matrix Inequalities (LMI) deduced from a Lyapunov analysis. Power analysis allows the derivation of a suitable state reference trajectory according to control objectives. The reconstructed state is then used in Section III-B to build an observer based control law. The proposed control is of an argmin state feedback control law type [5], [17], [18], and ensures the global asymptotic stability of the origin of the tracking error. To regulate the output in the presence of constant perturbations, Section III-C proposes to complete the control scheme adding an outer loop that adapts appropriately the state reference. Section IV presents simulation results carried out on MATLAB/Simulink to illustrate the effectiveness of the proposed approach.

II. MODELING

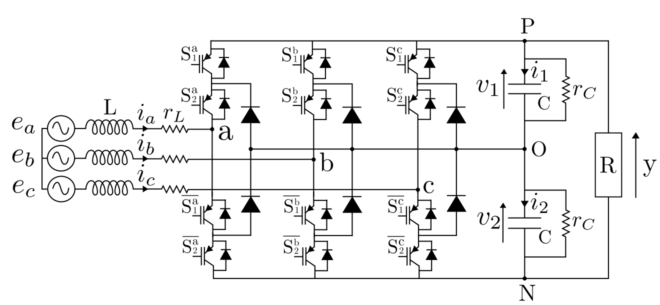

Fig. 1: Three Level Neutral Point Clamped Rectifier Circuit

The considered converter is a three level neutral clamped rectifier presented in Figure 1. The input supplies are three

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phase grid voltages e_a, e_b and e_c, each connected to an inductor L including a parasitic resistance r_L. On the DC side, the load is a resistor R, connected in parallel with two capacitors C with a parasitic resistance r_C. For the AC side, the following Assumption 1 is taken into account.

Assumption 1: The circuit operates in balanced mode. Therefore $\bullet_a + \bullet_b + \bullet_c = 0$ with $\bullet = e, i$, and $e_a = E\cos(\omega t)$, $e_b = E\cos(\omega t - \frac{2\pi}{3})$, and $e_c = E\cos(\omega t - \frac{4\pi}{3})$.

The circuit contains 12 switches and 6 diodes that aim at connecting each phase a,b and c to the point P, O or N. Therefore, we can model the switches with 9 boolean control variables u_{ij}, where $i = a, b, c$ and $j = p, o, n$. A u_{ij} will be equal to one if the phase i is connected to the point j. Since a phase can be connected to only one point at a time, the following constraints exist:

$$u_{\bullet p} + u_{\bullet o} + u_{\bullet n} = 1 \qquad \text{with } \bullet = a, b, c \tag{1}$$

With this modeling, a phase of the circuit can be represented as in Figure 2. Applying Kirchhoff's laws:

$$e_\bullet = r_L i_\bullet + L\dot{i}_\bullet + v_{\bullet o} + v_{ok} \qquad \text{with } \bullet = a, b, c \tag{2}$$

Using Assumption 1 and (2), the sum of the three input voltages is equal to $3v_{ok} + v_{ao} + v_{bo} + v_{co}$. Therefore, we can deduce that

$$\begin{bmatrix} v_{ak} \\ v_{bk} \\ v_{ck} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix} := \mathcal{T}\begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix} \tag{3}$$

Since

$$v_{\bullet o} = \begin{cases} 0 & \text{if } u_{\bullet o} = 1 \\ v_1 & \text{if } u_{\bullet p} = 1 \\ -v_2 & \text{if } u_{\bullet n} = 1 \end{cases} \qquad \text{with } \bullet = a, b, c$$

Then

$$\begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix} = \begin{bmatrix} u_{ap} \\ u_{bp} \\ u_{cp} \end{bmatrix} v_1 - \begin{bmatrix} u_{an} \\ u_{bn} \\ u_{cn} \end{bmatrix} v_2 \tag{4}$$

Substituting (3) and (4) in (2), we have the following model for the AC part:

$$L\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} - r_L\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \mathcal{T}\left(\begin{bmatrix} u_{ap} \\ u_{bp} \\ u_{cp} \end{bmatrix} v_1 - \begin{bmatrix} u_{an} \\ u_{bn} \\ u_{cn} \end{bmatrix} v_2\right)$$

Regarding the DC side, according to Figure 1, i_1 and i_2 are currents flowing through capacitors. Therefore, Kirchhoff's laws give:

$$\begin{aligned} i_1 &= u_{ap}i_a + u_{bp}i_b + u_{cp}i_c - \tfrac{v_1+v_2}{R} - \tfrac{v_1}{r_C} \\ i_2 &= -(u_{an}i_a + u_{bn}i_b + u_{cn}i_c) - \tfrac{v_1+v_2}{R} - \tfrac{v_2}{r_C} \end{aligned}$$

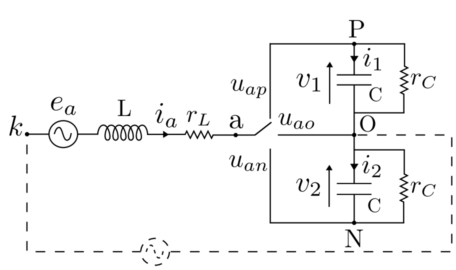

Fig. 2: Schematic of phase a with control variables

Considering the objectives of stabilizing the output and balancing the capacitor voltages, it is interesting to express the model through the variables $v_+ = v_1 + v_2$, $v_- = v_1 - v_2$ and i_a, i_b and i_c. Therefore, the model becomes:

$$\begin{cases} L\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} - r_L\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{1}{2}\mathcal{T}\left(\begin{bmatrix} u_{a-} \\ u_{b-} \\ u_{c-} \end{bmatrix} v_+ + \begin{bmatrix} u_{a+} \\ u_{b+} \\ u_{c+} \end{bmatrix} v_-\right) \\ C\dot{v}_+ = i_1 + i_2 = u_{a-}i_a + u_{b-}i_b + u_{c-}i_c - \frac{v_+}{R_e} \\ C\dot{v}_- = i_1 - i_2 = u_{a+}i_a + u_{b+}i_b + u_{c+}i_c - \frac{v_-}{r_C} \end{cases} \tag{5}$$

where $u_{i\pm} = u_{ip} \pm u_{in}$ for $i = a, b, c$ and $R_e = \frac{Rr_C}{R + 2r_C}$.

In the context of balanced mode operation (see Assumption 1), it is possible to simplify the model using the Clarke-Concordia's transformation, which transforms an abc vector $\begin{bmatrix} g_a & g_b & g_c \end{bmatrix}^T$ into an $\alpha\beta\gamma$ vector $\begin{bmatrix} g_\alpha & g_\beta & g_\gamma \end{bmatrix}^T$ according to:

$$\begin{bmatrix} g_\alpha \\ g_\beta \\ g_\gamma \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix} := \mathcal{C}\begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

Multiplying (5) on the left by $\mathcal{C}$ and after some tedious calculations, the model can be written as:

$$\begin{cases} L\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} - r_L\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \begin{bmatrix} \frac{\sqrt{6}}{6} & \frac{-\sqrt{6}}{12} & \frac{-\sqrt{6}}{12} \\ 0 & \frac{\sqrt{2}}{4} & \frac{-\sqrt{2}}{4} \end{bmatrix}\left(\begin{bmatrix} u_{a-} \\ u_{b-} \\ u_{c-} \end{bmatrix} v_+ + \begin{bmatrix} u_{a+} \\ u_{b+} \\ u_{c+} \end{bmatrix} v_-\right) \\ C\dot{v}_+ = \sqrt{6}\frac{2u_{a-} - u_{b-} - u_{c-}}{6} i_\alpha + \sqrt{2}\frac{u_{b-} - u_{c-}}{2} i_\beta - \frac{v_+}{R_e} \\ C\dot{v}_- = \sqrt{6}\frac{2u_{a+} - u_{b+} - u_{c+}}{6} i_\alpha + \sqrt{2}\frac{u_{b+} - u_{c+}}{2} i_\beta - \frac{v_-}{r_C} \end{cases}$$

where $e_\alpha = V_{\alpha\beta}\cos(\omega t)$, $e_\beta = V_{\alpha\beta}\sin(\omega t)$ and $V_{\alpha\beta} = \sqrt{\frac{3}{2}}E$. If we choose $x = \begin{bmatrix} i_\alpha & i_\beta & v_+ & v_- \end{bmatrix}^T$ as state vector and v_+ as the output, we have the following state space model:

$$\begin{cases} \dot{x} = \big(A_0 + u_{ap}A_{u_{ap}} + u_{an}A_{u_{an}} + u_{bp}A_{u_{bp}} \\ \qquad + u_{bn}A_{u_{bn}} + u_{cp}A_{u_{cp}} + u_{cn}A_{u_{cn}}\big)x + B_0 v_{in} \\ y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x := C_0 x \end{cases} \tag{6}$$

where :

$$A_0 = \begin{bmatrix} \frac{-r_L}{L} & 0 & 0 & 0 \\ 0 & \frac{-r_L}{L} & 0 & 0 \\ 0 & 0 & \frac{-1}{R_eC} & 0 \\ 0 & 0 & 0 & \frac{-1}{Cr_C} \end{bmatrix} \quad B_0 = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad v_{in} = \begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix}$$

$$A_{u_{ap}} = \begin{bmatrix} 0 & 0 & \frac{-\sqrt{6}}{6L} & \frac{-\sqrt{6}}{6L} \\ 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{3C} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{3C} & 0 & 0 & 0 \end{bmatrix} \quad A_{u_{an}} = \begin{bmatrix} 0 & 0 & \frac{\sqrt{6}}{6L} & \frac{-\sqrt{6}}{6L} \\ 0 & 0 & 0 & 0 \\ \frac{-\sqrt{6}}{3C} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{3C} & 0 & 0 & 0 \end{bmatrix}$$

$$A_{u_{bp}} = \begin{bmatrix} 0 & 0 & \frac{\sqrt{6}}{12L} & \frac{\sqrt{6}}{12L} \\ 0 & 0 & \frac{-\sqrt{2}}{4L} & \frac{-\sqrt{2}}{4L} \\ \frac{-\sqrt{6}}{6C} & \frac{\sqrt{2}}{2C} & 0 & 0 \\ \frac{-\sqrt{6}}{6C} & \frac{\sqrt{2}}{2C} & 0 & 0 \end{bmatrix} \quad A_{u_{bn}} = \begin{bmatrix} 0 & 0 & \frac{\sqrt{6}}{12L} & \frac{-\sqrt{6}}{12L} \\ 0 & 0 & \frac{-\sqrt{2}}{4L} & \frac{\sqrt{2}}{4L} \\ \frac{\sqrt{6}}{6C} & \frac{-\sqrt{2}}{2C} & 0 & 0 \\ \frac{-\sqrt{6}}{6C} & \frac{\sqrt{2}}{2C} & 0 & 0 \end{bmatrix}$$

$$A_{u_{cp}} = \begin{bmatrix} 0 & 0 & \frac{\sqrt{6}}{12L} & \frac{\sqrt{6}}{12L} \\ 0 & 0 & \frac{\sqrt{2}}{4L} & \frac{\sqrt{2}}{4L} \\ \frac{-\sqrt{6}}{6C} & \frac{-\sqrt{2}}{2C} & 0 & 0 \\ \frac{-\sqrt{6}}{6C} & \frac{-\sqrt{2}}{2C} & 0 & 0 \end{bmatrix} \quad A_{u_{cn}} = \begin{bmatrix} 0 & 0 & \frac{-\sqrt{6}}{12L} & \frac{\sqrt{6}}{12L} \\ 0 & 0 & \frac{-\sqrt{2}}{4L} & \frac{\sqrt{2}}{4L} \\ \frac{\sqrt{6}}{6C} & \frac{\sqrt{2}}{2C} & 0 & 0 \\ \frac{-\sqrt{6}}{6C} & \frac{-\sqrt{2}}{2C} & 0 & 0 \end{bmatrix}$$

To apply the methodology proposed in [18], modes should be ordered and a change of variable is required to obtain a polytopic model. A mode corresponds to a configuration of the circuit, thus constraints (1) must be considered in their definitions. Modes and new A_{λ_i} matrices are summed up in Table I.
After the change of variables, the model can be expressed as

$$\begin{cases} \dot{x} &= \left(A_0 + \sum_{i=1}^{27} \lambda_i A_{\lambda_i}\right) x + B_0 v_{in} \\ y &= C_0 x \end{cases} \tag{7}$$

This is a polytopic model where the λ_i's are the new control variables. Since only one mode can be active at a time, only one λ_i can be equal to one at a time. Thus, λ_i variables select a circuit configuration, and we have a switched affine system. Let us now gather the λ_i variables into a vector $\lambda \in \Lambda_S$ where $\Lambda_S = \left\{\lambda \in \{0,1\}^{27} : \sum_{i=1}^{27} \lambda_i = 1\right\}$. Finally, the model can be expressed in a compact form expressed as:

$$\begin{cases} \dot{x} = A_0 x + B_0(x)\lambda \\ y = C_0 x \end{cases} \tag{8}$$

where $B_0(x) = \begin{bmatrix} A_{\lambda_1} x + B_0 v_{in} & \dots & A_{\lambda_{27}} x + B_0 v_{in} \end{bmatrix}$.

III. Observation and Control

A. Observer design

The state of a system is not always available to compute a control law. Moreover, the introduction of an observer can be useful to reduce the number of sensors. In this context, we want to design an observer to asymptotically reconstruct the state of system (8). We assume that in addition of v_{in}, v_{C1} and v_{C2} are measured. Thus, let us consider the measured output y_m defined as:

$$y_m = \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} x := C_m x \tag{9}$$

Theorem 1: For a given symmetric positive definite matrix $Q_O \in \mathbb{R}^{4\times 4}$, if it exists a symmetric positive definite matrix $S \in \mathbb{R}^{4\times 4}$ and matrices $W_i \in \mathbb{R}^{4\times 2}$ such that the LMI

$$\left(A_0 + A_{\lambda_i}\right)^T S + S\left(A_0 + A_{\lambda_i}\right) - C_m^T W_i^T - W_i C_m + 2Q_O < 0 \tag{10}$$

holds $\forall i = 1, \dots, 27$, then the observer

$$\begin{cases} \dot{\hat{x}} &= A_0 \hat{x} + B_0(\hat{x})\lambda + \sum_{i=1}^{27} \lambda_i L_{\lambda_i}(y_m - \hat{y}_m) \\ \hat{y}_m &= C_m \hat{x} \end{cases} \tag{11}$$

where $L_{\lambda_i} = S^{-1} W_i$, ensures the convergence of $\hat{x}$ to x.

Proof: Let $\varepsilon = x - \hat{x}$ be the reconstruction error signal. Its dynamic is:

$$\dot{\varepsilon} = \left(A_0 + \sum_{i=1}^{27} \lambda_i \left(A_{\lambda_i} - L_{\lambda_i} C_m\right)\right) \varepsilon$$

Consider the Lyapunov function $V(\varepsilon) = \varepsilon^T S \varepsilon$ where $S = S^T > 0$. $V(0) = 0$ and $V(\varepsilon) > 0$,$\forall \varepsilon \neq 0$. Regarding its time derivative, we have

$$\begin{aligned} \dot{V}(\varepsilon) = \varepsilon^T \Big(&\left(A_0 + \textstyle\sum_{i=1}^{27} \lambda_i A_{\lambda_i}\right)^T S + S\left(A_0 + \textstyle\sum_{i=1}^{27} \lambda_i A_{\lambda_i}\right) \\ &- \textstyle\sum_{i=1}^{27} \lambda_i \left(S L_{\lambda_i} C_m - C_m^T L_{\lambda_i}^T S\right)\Big) \varepsilon < -2\varepsilon^T Q_O \varepsilon < 0 \end{aligned}$$

The last inequality is obtained from LMI condition (10) by defining $W_i = S L_{\lambda_i}$. Since $\dot{V}(\varepsilon) < 0$,$\forall \varepsilon \neq 0$ and the dynamic of ε doesn't jump when the control change, we can conclude that the origin of ε is globally asymptotically stable (GAS). ■

B. Observer based control

The control aims at tracking a state reference computed according to three objectives: to stabilize the output y at a given DC voltage setpoint y_e, to ensure a unit power factor by having the input currents and input voltages on phase, and to have balanced capacitor voltages on the output.

The first and third objectives directly set a part of the state reference since $y = v_+$ and the difference between the capacitor voltages is v_-. The second objective suggests that the currents references should be of the form $I_{\alpha\beta}\cos(\omega t)$ for i_α and $I_{\alpha\beta}\sin(\omega t)$ for i_β. Therefore, the state reference can be expressed as:

$$x_e(t) = \begin{bmatrix} i_{\alpha e}(t) \\ i_{\beta e}(t) \\ v_{+e} \\ v_{-e} \end{bmatrix} = \begin{bmatrix} I_{\alpha\beta}\cos(\omega t) \\ I_{\alpha\beta}\sin(\omega t) \\ y_e \\ 0 \end{bmatrix} \tag{12}$$

where $I_{\alpha\beta}$ has to be determined. To achieve this, the power balance of the system can be studied. Using (6), we can calculate instantaneous powers in capacitors and inductors, and recover the rest of the power balance expression:

$$\begin{aligned} &L i_\alpha \dot{i}_\alpha + L i_\beta \dot{i}_\beta + \frac{C}{2} v_+ \dot{v}_+ + \frac{C}{2} v_- \dot{v}_- \\ &= -r_L(i_\alpha^2 + i_\beta^2) + e_\alpha i_\alpha + e_\beta i_\beta - \frac{v_+^2}{2R_e} - \frac{v_-^2}{2r_C} \end{aligned} \tag{13}$$

For $x = x_e$ given by (12), the power balance expression reduces to:

$$0 = -r_l I_{\alpha\beta}^2 + V_{\alpha\beta} I_{\alpha\beta} - \frac{y_e^2}{2R_e} \tag{14}$$

To simplify the problem, the objectives can be expressed in terms of instantaneous active and reactive power p and q instead of currents and voltages. They are expressed as [19]:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{15}$$

With those definitions, the control objectives can now be summarized as follows: $p = p^*$, $q = 0$, $v_+ = y_e$ and $v_- = 0$, where p^* has to be determined. The power balance is then expressed as follows:

$$0 = -r_L \frac{p^{*2}}{V_{\alpha\beta}^2} + p^* - \frac{y_e^2}{2R_e}$$

Hence, for a fixed y_e, p^* is expressed as:

$$p^* = \frac{V_{\alpha\beta}^2}{2r_L}\left(1 \pm \sqrt{1 - \frac{2r_L}{V_{\alpha\beta}^2 R_e} y_e^2}\right) \text{ with } y_e < V_{\alpha\beta}\sqrt{\frac{R_e}{2r_L}} \tag{16}$$

To reduce the power in the system, the smallest solution should be selected. Finally, by definition (15), $I_{\alpha\beta} = \frac{p^*}{V_{\alpha\beta}}$.

Remark 1: Since the system is a switched system, the state reference is considered in the sense of Filipov [20].

TABLE I: Modes and change of variables from the physical model (6) to the polytopic model (7)

Mode(i)	u_{ap}	u_{an}	u_{ao}	u_{bp}	u_{bn}	u_{bo}	u_{cp}	u_{cn}	u_{co}	A_{λ_i}
1	0	0	1	0	0	1	0	0	1	0
2	0	0	1	0	0	1	0	1	0	$A_{u_{cn}}$
3	0	0	1	0	0	1	1	0	0	$A_{u_{cp}}$
4	0	0	1	0	1	0	0	0	1	$A_{u_{bn}}$
5	0	0	1	0	1	0	0	1	0	$A_{u_{bn}} + A_{u_{cn}}$
6	0	0	1	0	1	0	1	0	0	$A_{u_{bn}} + A_{u_{cp}}$
7	0	0	1	1	0	0	0	0	1	$A_{u_{bp}}$
8	0	0	1	1	0	0	0	1	0	$A_{u_{bp}} + A_{u_{cn}}$
9	0	0	1	1	0	0	1	0	0	$A_{u_{bp}} + A_{u_{cp}}$
10	0	1	0	0	0	1	0	0	1	$A_{u_{an}}$
11	0	1	0	0	0	1	0	1	0	$A_{u_{an}} + A_{u_{cn}}$
12	0	1	0	0	0	1	1	0	0	$A_{u_{an}} + A_{u_{cp}}$
13	0	1	0	0	1	0	0	0	1	$A_{u_{an}} + A_{u_{bn}}$
14	0	1	0	0	1	0	0	1	0	$A_{u_{an}} + A_{u_{bn}} + A_{u_{cn}}$
15	0	1	0	0	1	0	1	0	0	$A_{u_{an}} + A_{u_{bn}} + A_{u_{cp}}$
16	0	1	0	1	0	0	0	0	1	$A_{u_{an}} + A_{u_{bp}}$
17	0	1	0	1	0	0	0	1	0	$A_{u_{an}} + A_{u_{bp}} + A_{u_{cn}}$
18	0	1	0	1	0	0	1	0	0	$A_{u_{an}} + A_{u_{bp}} + A_{u_{cp}}$
19	1	0	0	0	0	1	0	0	1	$A_{u_{ap}}$
20	1	0	0	0	0	1	0	1	0	$A_{u_{ap}} + A_{u_{cn}}$
21	1	0	0	0	0	1	1	0	0	$A_{u_{ap}} + A_{u_{cp}}$
22	1	0	0	0	1	0	0	0	1	$A_{u_{ap}} + A_{u_{bn}}$
23	1	0	0	0	1	0	0	1	0	$A_{u_{ap}} + A_{u_{bn}} + A_{u_{cn}}$
24	1	0	0	0	1	0	1	0	0	$A_{u_{ap}} + A_{u_{bn}} + A_{u_{cp}}$
25	1	0	0	1	0	0	0	0	1	$A_{u_{ap}} + A_{u_{bp}}$
26	1	0	0	1	0	0	0	1	0	$A_{u_{ap}} + A_{u_{bp}} + A_{u_{cn}}$
27	1	0	0	1	0	0	1	0	0	$A_{u_{ap}} + A_{u_{bp}} + A_{u_{cp}}$

The addressed problem is to design a control law $\lambda : \mathbb{R} \to \Lambda_S$ which asymptotically stabilizes the state of the system (7) at the state reference x_e defined by (12), only measuring the output y_m. The control is expressed based on the reconstructed state provided by the observer (11). The observer and the control law can be designed separately, leading to a separation principle.

Theorem 2: Let $\tilde{e} := \hat{x} - x_e$ be the estimated tracking error signal, where $\hat{x}$ is the state estimated by the observer (11) and x_e is defined by (12). For a given symmetric positive definite matrix $Q_C \in \mathbb{R}^{4\times 4}$, if it exists a symmetric positive definite matrix $P \in \mathbb{R}^{4\times 4}$ solution of

$$A_{\lambda_i}^T P + P A_{\lambda_i} + 2Q_C < 0 \tag{17}$$

$\forall i = 1 \dots 27$, then the control law

$$\begin{aligned}\lambda^* &= arg \min_{d\in\Lambda_S} \Big(\tilde{e}^T P\Big(A_0\hat{x} + B_0(\hat{x})d\Big)\Big) \\ &= arg \min_{i=1\dots 27} \big(\, \tilde{e}^T P A_{\lambda_i}\hat{x}\,\big)\end{aligned} \tag{18}$$

makes the origin of the tracking error $e := x - x_e$ globally asymptotically stable (GAS).

Proof: Consider the Lyapunov function $V(\tilde{e}, \varepsilon) = \frac{1}{2}\big(\tilde{e}^T P\tilde{e} + \alpha\varepsilon^T S\varepsilon\big)$ where $P = P^T > 0$ is solution of (17), $S = S^T > 0$ is solution of (10) and $\alpha > 0$ is a scalar. $V(0,0) = 0$ and $V(\tilde{e},\varepsilon) > 0$, $\forall \tilde{e}, \varepsilon \neq 0$. The time derivative is expressed as

$$\begin{aligned}\dot{V}(\tilde{e},\varepsilon) = \tilde{e}^T P &\left(A_0\hat{x} + B_0(\hat{x})\lambda + \textstyle\sum_{i=1}^{27}\lambda_i L_{\lambda_i} C_m\varepsilon - \dot{x}_e\right) \\ &+ \alpha\varepsilon^T S\left(\textstyle\sum_{i=1}^{27}\lambda_i\big(A_0 + A_{\lambda_i} - L_{\lambda_i}C_m\big)\right)\varepsilon\end{aligned}$$

For the proposed switching control λ^* (18), by construction and convexity

$$\tilde{e}^T P\Big(A_0\hat{x} + B_0(\hat{x})\lambda^*\Big) \leq \tilde{e}^T P\Big(A_0\hat{x} + B_0(\hat{x})\lambda_e\Big)$$

Therefore

$$\begin{aligned}\dot{V}(\tilde{e},\varepsilon) &\leq \tilde{e}^T P\Big(A_0\tilde{e} + \big(B_0(\hat{x}) - B_0(x_e)\big)\lambda_e + \textstyle\sum_{i=1}^{27}\lambda_i L_{\lambda_i}C_m\varepsilon\Big) \\ &\quad + \alpha\varepsilon^T S\left(\textstyle\sum_{i=1}^{27}\lambda_i\big(A_0 + A_{\lambda_i} - L_{\lambda_i}C_m\big)\right)\varepsilon \\ &= \tilde{e}^T P\Big(\big(A_0 + \textstyle\sum_{i=1}^{27}\lambda_{ei}A_{\lambda_i}\big)\tilde{e} + \textstyle\sum_{i=1}^{27}\lambda_i L_{\lambda_i}C_m\varepsilon\Big) \\ &\quad + \alpha\varepsilon^T S\left(\textstyle\sum_{i=1}^{27}\lambda_i\big(A_0 + A_{\lambda_i} - L_{\lambda_i}C_m\big)\right)\varepsilon\end{aligned}$$

Inequalities (17) and (10) imply

$$\begin{aligned}\dot{V}(\tilde{e},\varepsilon) &\leq -\tilde{e}^T Q_C\tilde{e} + \tilde{e}^T P\textstyle\sum_{i=1}^{27}\lambda_i L_{\lambda_i}C_m\varepsilon - \alpha\varepsilon^T Q_O\varepsilon \\ &= -\begin{bmatrix}\tilde{e}\\ \varepsilon\end{bmatrix}^T \begin{bmatrix}Q_C & \gamma \\ \gamma^T & \alpha Q_O\end{bmatrix}\begin{bmatrix}\tilde{e}\\ \varepsilon\end{bmatrix} := -\begin{bmatrix}\tilde{e}\\ \varepsilon\end{bmatrix}^T \mathbf{Q}\begin{bmatrix}\tilde{e}\\ \varepsilon\end{bmatrix}\end{aligned}$$

where $\gamma = -\frac{1}{2}P\sum_{i=1}^{27}\lambda_i L_{\lambda_i}C_m$. Consequently, $\dot{V}(\tilde{e},\varepsilon) < 0$ if $\mathbf{Q} > 0$. By Schur complement, this condition holds for

$$\alpha Q_O - \tfrac{1}{4}\textstyle\sum_{i=1}^{27}\lambda_i C_m^T L_{\lambda_i}^T P Q_C^{-1} P L_{\lambda_i} C_m > 0$$

Hence, a sufficient condition is

$$\alpha > \max_{i=1\dots 27}\left\{\lambda_{max}\left(\tfrac{1}{4}Q_O^{-\frac{1}{2}}C_m^T L_{\lambda_i}^T P Q_C^{-1} P L_{\lambda_i} C_m Q_O^{-\frac{1}{2}}\right)\right\}$$

Finally, for a sufficiently large α and since the state doesn't jump when the control change, we can conclude that the origins of ε and $\tilde{e}$ are GAS. Noticing that $e = \tilde{e} + \varepsilon$, it proves that the origin of e is GAS too. ■

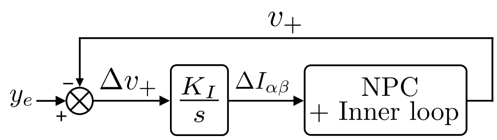

Fig. 3: Block diagram of the output regulation loop

C. Regulation of the output

In this section, the objective is to regulate the output v_+ when constant perturbations are considered. Such perturbations may stem from, for example, some variations of the resistive load R or some variations of the amplitude of the input source voltages $V_{\alpha\beta}$. The idea consists in modifying the current reference amplitude $I_{\alpha\beta}$ adding an extra term $\Delta I_{\alpha\beta}$. This extra term is obtained from an outer loop on v_+ with an integral action whose output $\Delta I_{\alpha\beta}$ is expressed as

$$\Delta I_{\alpha\beta}(t) = K_I \int_0^t (y_e - v_+(u))du$$

where K_I is a constant gain. The control scheme is depicted in Figure 3 where the inner loop corresponds to the closed-loop system designed in the previous paragraph: the converter controlled by the observer based control (18). To design gain K_I, it is assumed that the dynamic of the inner loop is much faster than the one of the introduced outer loop. In that context, the transfer function between $\Delta I_{\alpha\beta}$ and $\Delta v_+ = y_e - v_+$ can be approximated from the power balance equation. Introducing $i_\alpha = (I_{\alpha\beta} + \Delta I_{\alpha\beta})\cos(\omega t)$, $i_\beta = (I_{\alpha\beta} + \Delta I_{\alpha\beta})\sin(\omega t)$, $v_+ = y_e - \Delta v_+$ and $v_- = 0 - \Delta v_-$, the power balance equation (13) becomes

$$\frac{C}{2}(y_e - \Delta v_+)\Delta \dot{v}_+ = -r_L(I_{\alpha\beta} + \Delta I_{\alpha\beta})^2$$
$$+V_{\alpha\beta}(I_{\alpha\beta} + \Delta I_{\alpha\beta}) - \frac{(y_e - \Delta v_+)^2}{2R_e} - \frac{(0 - \Delta v_-)^2}{2r_C}$$

Taking into account (14) and retaining the first order terms, we obtain

$$\frac{Cy_e}{2}\Delta \dot{v}_+ = -2r_L I_{\alpha\beta}\Delta I_{\alpha\beta} + V_{\alpha\beta}\Delta I_{\alpha\beta} - \frac{y_e}{R_e}\Delta v_+$$

The transfer function is given by

$$G_{v_+}(s) = \frac{\Delta v_+}{\Delta I_{\alpha\beta}} = \frac{\frac{(V_{\alpha\beta} - 2r_L I_{\alpha\beta})R_e}{y_e}}{1 + \frac{R_e C}{2}s} := \frac{K}{1 + Ts}$$

Considering the control scheme in Figure 3, the outer open-loop transfer function is given by

$$\frac{KK_I}{s(1 + Ts)}$$

If the desired phase margin is $\pi/3$ radians, a simple calculation gives $K_I = \frac{2}{3KT} = \frac{4y_e}{3R_e^2 C(V_{\alpha\beta} - 2r_L I_{\alpha\beta})}$.

IV. Simulation

To illustrate the performance of the proposed control, simulations were performed on MATLAB/Simulink. Components parameters are summed up in Table II. With those values, the equilibrium power is $p^* = 782.02$ W.

TABLE II: Parameters of simulations

Parameters	Values	Parameters	Values
L	15 mH	r_L	0.4 Ω
C	1500 μ F	r_C	20 kΩ
E	72 V	R	30 Ω
y_e	150 V	ω	$2\pi\times 50$ rad/s

Using MATLAB and a SDP solver to minimize the trace of P and S while satisfying (17) for $Q_C = \mathbb{I}^4$, (10) and $S > 10^{-4}\mathbb{I}^4$ for $Q_O = 10^{-2}\mathbb{I}^4$, we obtain:

$$P = diag(600, 600, 23, 30) \text{ and } S = diag(4, 4, 1, 1) \times 10^{-4}$$

and 27 L_{λ_i} gains.

During simulation, the outer loop is turned on after 0.2s to prevent overshoots at start-up and to observe the system behavior without it. For a simulation step of $T = 5 \times 10^{-5}$s and with the initial condition $x_0 = \begin{bmatrix} 0 & 0 & 15 & 5 \end{bmatrix}^T$, Figure 4 presents the evolution of the currents at start-up. The observer states reaches system states as expected, and currents follows their references.

Perturbations that affect the output and the amplitude of the input voltage are reported in Figure 5, along with the evolution of the amplitude of the reference currents $I_{\alpha\beta} + \Delta I_{\alpha\beta}$ due to the outer loop. As expected, the reference is adapted when a perturbation affects the system, which ensures that the control objectives are still met.

The evolution of instantaneous powers p and q, currents stabilization and reconstruction errors, as well as system and observer voltages with references are presented in Figure 6. It can be noticed that the output $y = v_+$ and p present a steady state error before the outer loop is activated, of 2% and 3.2% respectively. Once the outer loop is turned on, steady state errors are cancelled and the output is regulated to its reference. p, q and system states are affected by perturbations, but are regulated too. The reactive power q converge to 0 after each perturbation, the unit power factor is therefore maintained. As expected, all reconstruction errors converge to 0 despite the perturbations, which ensures that the observer-based control law works properly.

V. Conclusion

In this paper, an observer-based switched control law for the three-level neutral point clamped rectifier has been designed. The modeling process was based on the modeling methodology proposed in [18] and adapted to take into account constraints between control variables specific to this converter. Despite the complexity and the large number of configurations, a full order observer is easily designed based on LMI conditions that can be solved with numerical tools. It allows to reconstruct the state of the system with the measurement of the input and both capacitors voltages. The stability of the system with the proposed control law is assessed with a Lyapunov analysis, and illustrated by simulations on MATLAB/Simulink. To complete the control scheme, an outer loop is proposed to regulate the output for constant perturbations. The future work could consist in validating those results on an experimental setup.

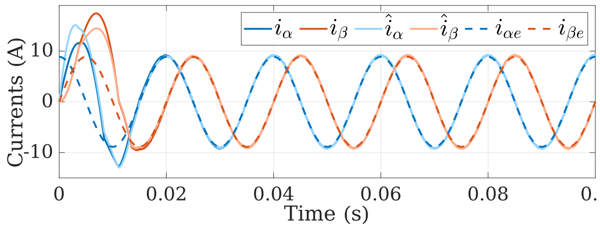

Fig. 4: System and observer currents with references

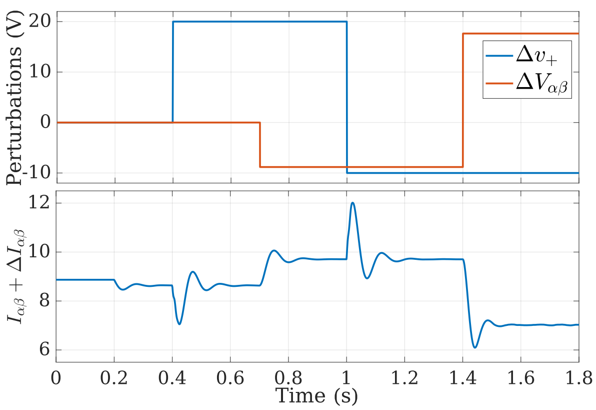

Fig. 5: Perturbations and adapted amplitude of the reference currents

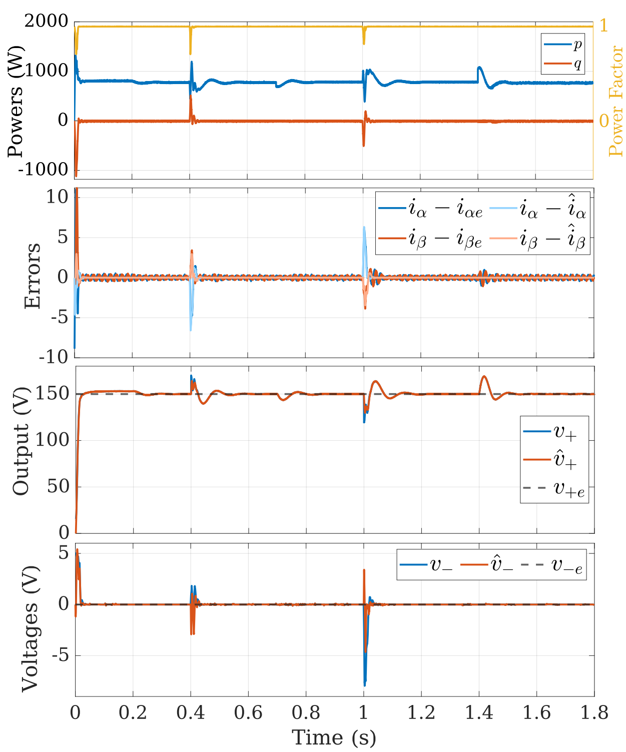

Fig. 6: Active and reactive powers, currents stabilization and reconstruction errors, system and observer voltages

References

[1] A. Nabae, I. Takahashi, and H. Akagi, "A New Neutral-Point-Clamped PWM Inverter," *IEEE Trans. on Industry Applications*, vol. IA-17, no. 5, pp. 518–523, 1981.

[2] J. Carrasco, L. Franquelo, J. Bialasiewicz, E. Galvan, R. PortilloGuisado, M. Prats, J. Leon, and N. Moreno-Alfonso, "Power-electronic systems for the grid integration of renewable energy sources: A survey," *IEEE Trans. on Industrial Electronics*, vol. 53, no. 4, pp. 1002–1016, 2006.

[3] L. G. Franquelo, J. Rodriguez, J. I. Leon, S. Kouro, R. Portillo, and M. A. Prats, "The age of multilevel converters arrives," *IEEE Industrial Electronics Magazine*, vol. 2, no. 2, pp. 28–39, 2008.

[4] F. Umbría, F. Gordillo, F. Salas, and S. Vázquez, "Voltages balance control in three phase three-level NPC rectifiers," in *IEEE International Symposium on Industrial Electronics*, 2010, pp. 3018–3023.

[5] G. Deaecto, J. Geromel, F. Saldanha Garcia, and J. Pomilio, "Switched affine systems control design with appliction to DC-DC converters," *Control Theory & Applications, IET*, vol. 4, pp. 1201–1210, 2010.

[6] A. H. R. Rosa, L. M. F. Morais, G. O. Fortes, and S. I. Seleme Júnior, "Practical considerations of nonlinear control techniques applied to static power converters: A survey and comparative study," *International Journal of Electrical Power & Energy Systems*, vol. 127, p. 106545, 2021.

[7] L. Yacoubi, K. Al-Haddad, L.-A. Dessaint, and F. Fnaiech, "Linear and nonlinear control techniques for a three-phase three-level npc boost rectifier," *IEEE Trans. on Industrial Electronics*, vol. 53, no. 6, pp. 1908–1918, 2006.

[8] D. Zhou, C. Jiang, Z. Quan, and Y. R. Li, "Vector shifted model predictive power control of three-level neutral-point-clamped rectifiers," *IEEE Trans. on Industrial Electronics*, vol. 67, no. 9, pp. 7157–7166, 2020.

[9] J. D. Barros and J. F. Silva, "Optimal Predictive Control of Three-Phase NPC Multilevel Converter for Power Quality Applications," *IEEE Trans. on Industrial Electronics*, vol. 55, no. 10, pp. 3670–3681, 2008.

[10] P. Montero-Robina, C. Albea, F. Gómez-Estern, and F. Gordillo, "Hybrid modeling and control of three-level npc rectifiers," *Control Engineering Practice*, vol. 130, p. 105374, 2023.

[11] F. Umbría, S. Vázquez, F. Gordillo, and F. Gómez-Estern, "Observer-based Direct Power Control for three-level NPC rectifiers," in *35th Annual Conference of IEEE Industrial Electronics*, 2009, pp. 1663–1668.

[12] R. Portillo, S. Vazquez, J. I. Leon, M. M. Prats, and L. G. Franquelo, "Model based adaptive direct power control for three-level npc converters," *IEEE Trans. on Industrial Informatics*, vol. 9, no. 2, pp. 1148–1157, 2013.

[13] Y. Yin, J. Liu, J. A. Sánchez, L. Wu, S. Vazquez, J. I. Leon, and L. G. Franquelo, "Observer-based adaptive sliding mode control of npc converters: An rbf neural network approach," *IEEE Trans. on Power Electronics*, vol. 34, no. 4, pp. 3831–3841, 2019.

[14] F. Umbría, F. Gordillo, F. Gómez-Estern, F. Salas, R. C. Portillo, and S. Vázquez, "Voltage balancing in three-level neutral-point-clamped converters via Luenberger observer," *Control Engineering Practice*, vol. 25, pp. 36–44, Apr. 2014.

[15] N. Gonzalez-Fonseca, J. de Leon-Morales, and J. Leyva-Ramos, "Observer-Based Controller for Switch-Mode DC-DC Converters," in *Proceedings of the 44th IEEE Conference on Decision and Control*, 2005, pp. 4773–4778.

[16] S. Hadjeras, C. A. Sanchez, and G. Garcia, "Hybrid adaptive control of the boost converter," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, 2017, pp. 5720–5725.

[17] G. Beneux, P. Riedinger, J. Daafouz, and L. Grimaud, "Adaptive stabilization of switched affine systems with unknown equilibrium points: Application to power converters," *Automatica*, vol. 99, pp. 82–91, 2019.

[18] G. Garcia, O. Lopez-Santos, and L. Martinez-Salamero, "A Unified Approach for the Control of Power Electronics Converters. Part II: Tracking," *Applied Sciences*, vol. 11, no. 16, p. 7618, 2021.

[19] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous Reactive Power Compensators Comprising Switching Devices without Energy Storage Components," *IEEE Trans. on Industry Applications*, vol. IA-20, no. 3, pp. 625–630, 1984.

[20] B. Ingalls, E. D. Sontag, and Y. Wang, "An Infinite-Time Relaxation Theorem for Differential Inclusions," *Proceedings of the American Mathematical Society*, vol. 131, no. 2, pp. 487–499, 2003.