2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Decentralized control of intercity electric automated buses via time-varying objective prioritization

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Abstract— **This paper considers electric automated buses traveling in inter-urban roads and following a given route including stops, that must be reached according to a given timetable. Some of these stops are provided with a charging infrastructure allowing to charge the bus batteries. The paper proposes a decentralized control scheme for determining the optimal speed profiles, the dwell and charging times of the buses, by taking into account the traffic conditions along the road through a suitable traffic flow prediction model. Two objectives are considered contemporarily: the minimization of the deviations from the timetable and the minimization of the energy lack at the end of the bus route. To attain both these conflicting objectives, a lexicographic approach is adopted to design the controller which considers that, depending on the system state, the priority of the two objectives can change. Accordingly, the proposed control scheme changes the objective prioritization in real time and switches between two different lexicographic-based optimal control solutions. Some tests are discussed in the paper to show the effectiveness of the proposed control scheme.**

I. Introduction

One of the main challenges of our society is undoubtedly the protection of the environment and the health of our planet. Among the human activities causing damage to the environment, road transport represents one of the main sources. Electric mobility seems to represent a viable alternative to reduce the environmental impact of road transport, especially public transport [1]. The mobility transition for smart cities [2] is not only represented by electrification but also by the development of connected and automated vehicles [3]. In this paper we consider electric automated buses which have to follow a given line in extra-urban roads. Each vehicle has to visit, in given time windows, specific stops which can be provided or not with charging infrastructures. Our main goal is to devise a control strategy in order to regulate the speed of buses along the route, as well as the dwell and charging times at stops. Since each bus is traveling in inter-urban roads, without dedicated lanes, it is important to base the control strategy on the traffic prediction, i.e. considering the traffic state that the bus will encounter along its path. Some research works found in the literature aim at defining eco-driving strategies for electric buses traveling in urban areas where the presence of reserved lanes allows to neglect the influence of traffic [4], [5].

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The main novelty of our work stands, instead, in considering inter-urban roads where the influence of traffic is very relevant for the bus behavior (previous works in this research direction can be found in [6], [7]). In this paper we propose a decentralized control scheme in which the control variables are re-computed in real time when a bus is ready to leave a stop. Such a scheme is composed of two modules: the former allows the prediction of the traffic state and the bus trajectory, while the latter is based on the solution of a multi-objective optimal control problem. Indeed, two main conflicting objectives must be addressed, i.e. the minimization of the deviations from the timetable and the minimization of the energy lack at the end of the bus route. Since the priority between these objectives can vary depending on the system state, the proposed control scheme changes the prioritization between the objectives in real time, requiring the application of two different lexicographic-based algorithms. The multi-objective nature of this problem has been investigated by the authors in [8], where two algorithms based on the ε-constraint approach are applied to find the Pareto frontier in different scenarios. Multi-objective optimization techniques have been studied for some decades [9], with the main focus on theoretical results for the definition of solution methods allowing to provide efficient solutions. Multi-objective optimization has been also applied in feedback and optimal control problems [10] and within model predictive control schemes [11], in which the computational issue becomes much more important for real-time applications.

The paper is organized as follows. In Section II the proposed control scheme is introduced. The two modules of the control scheme are described in Section III and Section IV, respectively dealing with the prediction module and the optimal control module. Section V reports some simulation results, while conclusive remarks are drawn in Section VI.

II. The considered system and the proposed control scheme

We consider electric and automated buses which have to follow specific routes with fixed stops and a given timetable. These routes are in inter-urban roads where the influence of traffic cannot be neglected. Some stops along the route are equipped with a charging infrastructure while others are not. The control variables of the problem are relevant to the bus speed along the route, as well as the dwell times and charging times at stops. The objectives to be considered regard the minimization of the deviations from the timetable at stops, on the one hand, and the respect of a final energy level requirement, on the other hand.

The considered road stretch is divided into N sections, each one having length L_i [km], $i = 1, \ldots, N$, in which some external flows can enter and from which traffic flows can exit. Let us denote the set of road sections as $\mathcal{I} = \{i : i = 1, \ldots, N\}$. According to the length of each section, it is possible to compute the position p_i of the beginning of section i as $p_1 = 0$ and $p_i = \sum_{j=1}^{i-1} L_j$, $i \in \mathcal{I} \setminus 1$. In the considered road stretch a set $\mathcal{B}$ of electric and automated buses is considered and a set $\mathcal{S}$ of bus stops is present. The position of each bus stop s in the road stretch is denoted as π_s [km].

In order to control all the buses, a decentralized control scheme is proposed in which each bus is controlled independently. A sketch of the control logic is provided in Fig. 1. Such a logic is applied every time a bus is ready to leave a stop, and the considered control horizon covers all the time necessary for the bus to reach the last stop. The proposed control scheme consists of two main modules:

1) the *prediction module*, which allows to make a prediction of the traffic state in the road and of the bus trajectory; the model is initialized with the state measured in real time;
2) the *optimal control module*, which allows to find the optimal values of the control variables considering as inputs both the average traffic speed computed by the prediction model and the traffic state of buses measured in real time. This module can be applied with two different lexicographic approaches, giving priority to the timetable or to the final energy level, on the basis of a condition which verifies if the bus is expected to be late at the next stop or not.

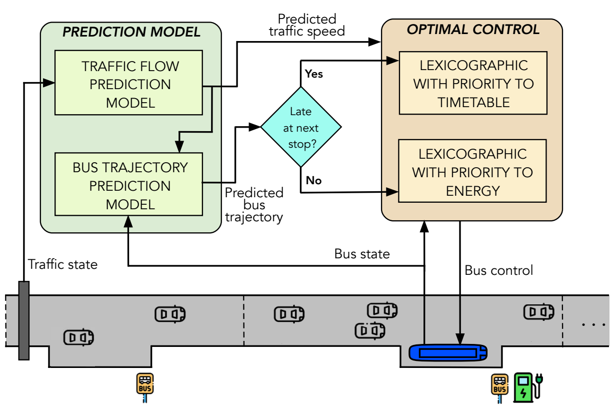

Fig. 1: The proposed control scheme

Both modules are based on a time discretization that is, in general, different. Let us denote with $T^{\rm pred}$ and T the sampling times of the prediction model and the optimal control problem, respectively, and let us introduce $\Gamma = T/T^{\rm pred}$. Moreover, for the prediction model and the optimal control problem different time step indexes are used, i.e. k and h respectively.

As aforementioned, the proposed control strategy is run at each time instant in which bus b is ready to leave a stop, denoted as t_s^b for each stop s. Let us indicate with $k(t_s^b)$ and $h(t_s^b)$ the time steps corresponding to t_s^b in the two discretization mechanisms. Then, at each time instant t_s^b, the prediction model is run for $k \in \mathcal{K}(t_s^b) = \{k(t_s^b), \ldots, K^b - 1\}$, in which K^b is the last time step of the considered horizon corresponding to the scheduled arrival time of the bus at the last stop plus the waiting time in the last stop. The optimal control problem, instead, refers to time steps $h \in \mathcal{H}(t_s^b) = \{h(t_s^b), \ldots, H^b - 1\}$, where H^b is the last time step of the considered horizon, such that $H^b = 1/\Gamma K^b$. When applying the control strategy at time t_s^b, the sets $\mathcal{S}(t_s^b) \subseteq \mathcal{S}$ and $\mathcal{I}(t_s^b) \subseteq \mathcal{I}$ must be updated, including the bus stops and the road sections after the position of the bus at time t_s^b.

III. The Prediction Module

The prediction module run at time t_s^b over the prediction horizon $k \in \mathcal{K}(t_s^b)$ is composed of a traffic flow model for predicting the traffic state along the road and a prediction model for the bus.

The traffic flow prediction model considered in this work is METANET [12] referred to a single road stretch in an extra-urban environment (more details on traffic models can be found in [13]). For each section $i \in \mathcal{I}(t_s^b)$, and for each time step $k \in \mathcal{K}(t_s^b)$, the following variables are defined:

- $\rho_i^{\rm pred}(k)$ is the predicted traffic density in section i at time $kT^{\rm pred}$ [veh/km];
- $v_i^{\rm pred}(k)$ is the predicted mean traffic speed in section i at time $kT^{\rm pred}$ [km/h];
- $q_i^{\rm pred}(k)$ is the predicted traffic flow from section i to section $i+1$ during time interval $[kT^{\rm pred}, (k+1)T^{\rm pred})$ [veh/h];
- $r_i^{\rm pred}(k)$ is the predicted traffic flow entering section i from external roads during time interval $[kT^{\rm pred}, (k+1)T^{\rm pred})$ [veh/h];
- $f_i^{\rm pred}(k)$ is the predicted traffic flow exiting section i to external roads during time interval $[kT^{\rm pred}, (k+1)T^{\rm pred})$ [veh/h].

The METANET model is given by the following finite difference equations

$$\rho_i^{\rm pred}(k+1) = \rho_i^{\rm pred}(k) + \frac{T^{\rm pred}}{L_i}\Big[q_{i-1}^{\rm pred}(k) - q_i^{\rm pred}(k) + r_i^{\rm pred}(k) - f_i^{\rm pred}(k)\Big] \quad (1)$$

$$\begin{aligned} v_i^{\rm pred}(k+1) = {} & v_i^{\rm pred}(k) + \frac{T^{\rm pred}}{\tau}\Big[V\Big(\rho_i^{\rm pred}(k)\Big) - v_i^{\rm pred}(k)\Big] \\ & + \frac{T^{\rm pred}}{L_i} v_i^{\rm pred}(k)\Big[v_{i-1}^{\rm pred}(k) - v_i^{\rm pred}(k)\Big] \\ & - \frac{\nu T^{\rm pred}\Big[\rho_{i+1}^{\rm pred}(k) - \rho_i^{\rm pred}(k)\Big]}{\tau L_i\Big[\rho_i^{\rm pred}(k) + \chi\Big]} \\ & - \delta_{\rm on} T^{\rm pred}\frac{v_i^{\rm pred}(k) r_i^{\rm pred}(k)}{L_i\Big[\rho_i^{\rm pred}(k) + \chi\Big]} \end{aligned} \quad (2)$$

where $i \in \mathcal{I}(t_s^b)$, $k \in \mathcal{K}(t_s^b)$, with τ, ν, χ, δ_{on} being model parameters. The traffic flow to be used in (1) is given by $q_i^{\text{pred}}(k) = \rho_i^{\text{pred}}(k) v_i^{\text{pred}}(k)$ and the steady-state speed-density relation adopted in (2) can be expressed as

$$V(\rho_i^{\text{pred}}(k)) = v_i^{\text{f}} \exp\left[-\frac{1}{a}\left(\frac{\rho_i^{\text{pred}}(k)}{\rho_i^{\text{cr}}}\right)^a\right] \tag{3}$$

where v_i^{f} is the free-flow speed [km/h] of section i, ρ_i^{cr} is the critical density [veh/km] of section i, a is another parameter.

This prediction model is initialized with the traffic state measured in real time at time instant t_s^b, and enables to compute all the traffic variables along the prediction horizon. Among them, the mean traffic speed $v_i^{\text{pred}}(k)$, $k \in \mathcal{K}(t_s^b)$, is an input both for the bus trajectory prediction model and for the optimal control module.

In the bus trajectory prediction model, for each bus $b \in \mathcal{B}$ the following variables are defined:

- $p^{b,\text{pred}}(k)$ is the predicted position of bus b at time kT^{pred} [km];
- $v^{b,\text{pred}}(k)$ is the predicted speed of bus b at time kT^{pred} [km/h];
- $\delta_i^{b,\text{pred}}(k)$ is equal to 1 if bus b is predicted to travel in section i at time kT^{pred}, 0 otherwise;
- $t_{s+1}^{b,\text{pred}}$ is the predicted departure time at the next stop.

The bus trajectory prediction model computes the predicted position of the bus along the road as

$$p^{b,\text{pred}}(k+1) = p^{b,\text{pred}}(k) + v^{b,\text{pred}}(k) T^{\text{pred}} \tag{4}$$

where the predicted speed is computed taking into account the mean traffic speed associated with the section in which the bus travels, i.e.

$$v^{b,\text{pred}}(k) = \min\left\{\sum_{i \in \mathcal{I}(t_s^b)} \delta_i^{b,\text{pred}}(k) v_i^{\text{pred}}(k), v^{b,\max}\right\} \tag{5}$$

with $v^{b,\max}$ indicating the maximum speed of bus b.

According to the predicted position of each bus and considering the minimum dwell time indicated in the timetable, it is possible to compute $t_{s+1}^{b,\text{pred}}$, i.e. the predicted departure time at the next stop, in order to compare it with $\bar{t}_{s+1}^b$, i.e. the scheduled departure time at the next stop.

IV. THE OPTIMAL CONTROL MODULE

In order to optimally control each bus in the road stretch, an optimal control problem must be solved at each time t_s^b, considering a planning horizon with $h \in \mathcal{H}(t_s^b)$. In the next sub-sections, the multi-objective optimization problem to be solved is firstly presented, then the two lexicographic approaches are reported, along with the algorithm summarizing the whole control strategy.

A. The multi-objective optimal control problem

In this paper we consider a decentralized control scheme in which an optimal control problem is stated and solved for each bus $b \in \mathcal{B}$, for which we report here the main variables.

The state variables are defined for each time step $h \in \mathcal{H}(t_s^b) \cup \{H^b\}$:

- $p^b(h)$ is the position of bus b at time step h [km];
- $e^b(h)$ is the energy stored in the battery of bus b at time step h [kWh].

The control variables are defined for time steps $h \in \mathcal{H}^b(t_s^b)$:

- $v^b(h)$ is the speed of bus b at time step h [km/h];
- $w_s^b(h) \in \{0,1\}$ is equal to 1 if bus b is in stop s at time step h, 0 otherwise;
- $c_s^b(h) \in \{0,1\}$ is equal to 1 if bus b is charging its battery in stop s at time step h, 0 otherwise.

Some auxiliary variables are needed in the optimal control problem, for $i \in \mathcal{I}(t_s^b)$, $h \in \mathcal{H}^b(t_s^b)$, in order to define the position of the bus in the road sections:

- $z_i^b(h) \in \{0,1\}$ is equal to 1 if $p^b(h) \geq p_i$, i.e. if bus b is after the beginning of section i at time h, 0 otherwise;
- $y_i^b(h) \in \{0,1\}$ is equal to 1 if $p^b(h) \leq p_{i+1}$, i.e. if bus b is before the beginning of section $i+1$ at time h, 0 otherwise;
- $\lambda_i^b(h) \in \{0,1\}$ is equal to 1 if bus b is in section i at time step h, 0 otherwise.

Two other sets of auxiliary variables are defined for $s \in \mathcal{S}(t_s^b)$, $h \in \mathcal{H}^b(t_s^b)$, and are related to the arrivals and departures of the bus at bus stops:

- $\gamma_s^{b,\text{arr}}(h) \in \{0,1\}$ is equal to 1 if bus b arrives at stop s between time step h and $h+1$, 0 otherwise;
- $\gamma_s^{b,\text{dep}}(h) \in \{0,1\}$ is equal to 1 if bus b departs from stop s between time step h and $h+1$, 0 otherwise.

A final set of auxiliary variables is required to represent the final energy state of the bus:

- $\zeta^{b,\text{fin}} = \max\{\bar{e}^{b,\text{fin}} - e^b(h^{b,\text{fin}}), 0\}$ is the final lack of energy in bus b compared with the desired level $\bar{e}^{b,\text{fin}}$.

Since the prediction model described in Section III has a different time discretization compared with the optimal control problem, the predicted traffic speed transmitted by the prediction model at time instant t_s^b must be processed in order to be used in the optimal control problem, where the traffic speed must be associated with the time steps $h \in \mathcal{H}(t_s^b)$. This speed is denoted as $v_i(h)$ and is computed as follows:

$$v_i(h) = \sum_{\ell=(h-1)\Gamma+1}^{h\Gamma} \frac{v_i^{\text{pred}}(\ell)}{\Gamma} \tag{6}$$

The multi-objective optimal control problem to be solved by bus b at time t_s^b can be stated with the following mixed-integer linear quadratic formulation.

Problem 1: Multi-objective optimal control problem

$$\min\left(F_1, F_2\right) \tag{7}$$

where

$$F_1 = \sum_{s \in \mathcal{S}(t_s^b)} \sum_{h \in \mathcal{H}^b} \alpha_s^b(h)\left(\bar{w}_s^b(h) - w_s^b(h)\right)^2 \tag{8}$$

$$F_2 = \zeta^{b,\text{fin}} \tag{9}$$

subject to:

$$p^b(h+1) = p^b(h) + v^b(h) T \qquad h \in \mathcal{H}(t_s^b) \tag{10}$$

$$e^b(h+1) = e^b(h) + T \sum_{s\in\mathcal{S}(t_s^b)} \varsigma_s c_s^b(h) - \delta^b \left[p^b(h+1) - p^b(h)\right] - T\beta^b \qquad h \in \mathcal{H}(t_s^b) \quad (11)$$

$$p^b(h) - \big(\pi_s - \Delta\big) \geq \sigma - \big(L+\sigma\big)\big(1 - w_s^b(h)\big) \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (12)$$

$$\big(\pi_s + \Delta\big) - p^b(h) \geq \sigma - \big(L+\sigma\big)\big(1 - w_s^b(h)\big) \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (13)$$

$$\sum_{s\in\mathcal{S}(t_s^b)} w_s^b(h) \leq 1 \qquad h \in \mathcal{H}(t_s^b) \quad (14)$$

$$\sum_{h\in\mathcal{H}(t_s^b)} w_s^b(h) \geq \psi_s \qquad s \in \mathcal{S}(t_s^b) \quad (15)$$

$$c_s^b(h) \leq w_s^b(h) \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (16)$$

$$c_s^b(h) \leq \eta_s \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (17)$$

$$\gamma_s^{b,\mathrm{arr}}(h) - \gamma_s^{b,\mathrm{dep}}(h) = w_s^b(h+1) - w_s^b(h) \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \setminus H^b - 1 \quad (18)$$

$$\gamma_s^{b,\mathrm{arr}}(h) + \gamma_s^{b,\mathrm{dep}}(h) \leq 1 \qquad s \in \mathcal{S}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (19)$$

$$\sum_{h\in\mathcal{H}(t_s^b)} \gamma_s^{b,\mathrm{arr}}(h) + \gamma_s^{b,\mathrm{dep}}(h) \leq 2 \qquad s \in \mathcal{S}(t_s^b) \quad (20)$$

$$p^b(h) - p_i + M\big(1 - z_i^b(h)\big) \geq \sigma \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (21)$$

$$p_i - p^b(h) + M z_i^b(h) \geq 0 \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (22)$$

$$p_{i+1} - p^b(h) + M\big(1 - y_i^b(h)\big) \geq 0 \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (23)$$

$$p^b(h) - p_{i+1} + M y_i^b(h) \geq \sigma \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (24)$$

$$\lambda_i^b(h) \leq z_i^b(h) \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (25)$$

$$\lambda_i^b(h) \leq y_i^b(h) \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (26)$$

$$\lambda_i^b(h) \geq z_i^b(h) + y_i^b(h) - 1 \qquad i \in \mathcal{I}(t_s^b),\ h \in \mathcal{H}(t_s^b) \quad (27)$$

$$\zeta^{b,\mathrm{fin}} \geq \bar{e}^{b,\mathrm{fin}} - e^b(H^b) \quad (28)$$

$$\zeta^{b,\mathrm{fin}} \geq 0 \quad (29)$$

$$0 \leq v^b(h) \leq \sum_{i\in\mathcal{I}(t_s^b)} \lambda_i^b(h) v_i(h) \qquad h \in \mathcal{H}(t_s^b) \quad (30)$$

$$v^b(h) \leq \left(1 - \sum_{s\in\mathcal{S}(t_s^b)} w_s^b(h)\right) \cdot v^{b,\max} \qquad h \in \mathcal{H}(t_s^b) \quad (31)$$

$$-\overline{\Delta V} \leq v^b(h+1) - v^b(h) \leq \overline{\Delta V} \qquad b \in \mathcal{B},\ h \in \mathcal{H}(t_s^b) \setminus H^b - 1 \quad (32)$$

$$e^{b,\min} \leq e^b(h) \leq e^{b,\max} \qquad b \in \mathcal{B},\ h \in \mathcal{H}(t_s^b) \quad (33)$$

where σ is a small quantity arbitrarily chosen, M is a large quantity arbitrarily chosen, Δ is a tolerance, L is computed as $L = \sum_{i\in\mathcal{I}(t_s^b)} L_i$.

Problem 1 is a multi-objective optimal control problem, including two cost terms, F_1 and F_2, respectively defined in (8) and (9). In particular, F_1 is related to the deviation from the timetable, i.e. it is the quadratic difference between the control variable $w_s^b(h)$ and its expected value $\bar{w}_s^b(h)$ (related to the predefined path and timetable of the bus), multiplied by $\alpha_s^b(h)$, which allows to properly weigh early arrivals and late departures from stops. The second objective F_2 penalizes the final lack of energy for the bus with respect to the prescribed final energy level.

Constraints (10) and (11) are the bus state equations. (10) computes the distance covered by the bus on the basis of its speed and is initialized with the measured bus state $p^b(h(t_s^b))$. Constraints (11) update the energy stored in the bus battery by increasing it on the basis of the charging power ς_s associated with stop s [kW], and decreasing it by considering the electric consumption δ^b when traveling [kWh/km] and the electric consumption β^b for the auxiliary equipment [kW]. (11) is initialized with the state $e^b(h(t_s^b))$ measured in real time.

Constraints (12)-(13) impose that $w_s^b(h)$ is equal to 0 if $p^b(h) \leq \pi_s - \Delta$ or $p^b(h) \geq \pi_s + \Delta$, i.e. when the bus is not in the proximity of stops. Constraints (14) ensure that the bus can be at most in one stop in each time step, while (15) guarantee that the bus stays in a stop for at least ψ_s time steps. Constraints (16) and (17) impose, respectively, that the bus can be charged in a bus stop only if it is waiting there and if the stop is provided with a charging infrastructure, considering that $\eta_s \in \{0, 1\}$ is equal to 1 if bus stop s is provided with a charging infrastructure and 0 if not.

Constraints (18) and (19) ensure that, when $w_s^b(h) = 0$ and $w_s^b(h+1) = 1$, it yields $\gamma_s^{b,\mathrm{arr}}(h) = 1$ and $\gamma_s^{b,\mathrm{dep}}(h) = 0$. When instead $w_s^b(h) = 1$ and $w_s^b(h+1) = 0$, constraints (18) and (19) force $\gamma_s^{b,\mathrm{arr}}(h)$ to 0 and $\gamma_s^{b,\mathrm{dep}}(h)$ to 1. Both variables $\gamma_s^{b,\mathrm{arr}}(h)$ and $\gamma_s^{b,\mathrm{dep}}(h)$ are fixed to 0 if $w_s^b(h) = w_s^b(h+1)$. Constraints (20) impose that the sum of arrivals and departures of the bus in each stop cannot be greater than 2.

Constraints (21)-(22) are used to correctly define the variables $z_i^b(h)$ and, analogously, (23)-(24) to define the variables $y_i^b(h)$, depending on the bus position $p^b(h)$. Moreover, constraints (25)-(27) define $\lambda_i^b(h)$ on the basis of $z_i^b(h)$ and $y_i^b(h)$, i.e. $\lambda_i^b(h) = 1$ if $z_i^b(h) = y_i^b(h) = 1$. Similarly, constraints (28)-(29) are used to properly fix $\zeta^{b,\mathrm{fin}}$ on the basis of the desired final energy $\bar{e}^{b,\mathrm{fin}}$in the bus battery [kWh].

Constraints (30)-(31) impose positivity and upper bounds for the bus speed, respectively considering the predicted traffic flow speed $v_i(h)$ and the maximum allowed speeds $v^{b,\max}$. Note that (31) also imposes that the speed is null if the bus is at a stop. Finally, constraints (32) guarantee that the speed variation between two consecutive time steps cannot exceed $\overline{\Delta V}$ [km/h], while (33) impose lower and upper bounds for the energy stored in the batteries, respectively given by $e^{b,\min}$ and $e^{b,\max}$ [kWh].

B. The Lexicographic Approach

In order to solve Problem 1, that is a multi-objective optimal control problem, scalarization techniques can be applied, transforming a multi-objective optimal control problem into a sequence of single-objective problems. In case there is a prioritization among the objectives, lexicographic approaches can be applied. Generally speaking, their application consists in solving a sequence of single-objective problems, where new constraints are added to each subsequent problem so as to fix the already considered objectives equal to the previously found optimal values.

For the specific case considered in this paper, Problem 1 can be solved with a lexicographic approach in two ways, depending on the priority given to the two objectives F_1 and F_2. In particular, if the priority is given to F_1, i.e. to the compliance with the bus timetable, the following two problems must be solved:

Problem 2: Lexicographic with priority to F_1 - Step a

$$\min F_1 \tag{34}$$

where F_1 is given by (8), subject to (10)-(33).

Problem 3: Lexicographic with priority to F_1 - Step b

$$\min F_2 \tag{35}$$

where F_2 is given by (9), subject to (10)-(33) and

$$F_1 = F_1^* \tag{36}$$

where F_1^* is the optimal value of F_1 found by solving Problem 2.

If, instead, the priority is given to F_2, i.e. to the final energy level, the following two problems must be solved:

Problem 4: Lexicographic with priority to F_2 - Step a

$$\min F_2 \tag{37}$$

where F_2 is given by (9), subject to (10)-(33).

Problem 5: Lexicographic with priority to F_2 - Step b

$$\min F_1 \tag{38}$$

where F_1 is given by (8), subject to (10)-(33) and

$$F_2 = F_2^* \tag{39}$$

where F_2^* is the optimal value of F_2 found by solving Problem 4.

Algorithm 1 summarizes the whole procedure to be applied at time t_s^b for bus b.

Algorithm 1: Control scheme at time t_s^b for bus b

Predict the traffic state evolution for time steps $k \in \mathcal{K}(t_s^b)$ with model (1)-(3)
Predict the bus trajectory for time steps $k \in \mathcal{K}(t_s^b)$ with model (4)-(5)
if $t_{s+1}^{b,\text{pred}} > \bar{t}_{s+1}^b$ **then**
  Apply the lexicographic scheme with priority to F_1, i.e. solve Problem 2 and Problem 3 for time steps $h \in \mathcal{H}(t_s^b)$
end
else
  Apply the lexicographic scheme with priority to F_2, i.e. solve Problem 4 and Problem 5 for time steps $h \in \mathcal{H}(t_s^b)$
end
Update the control variables $v^b(h)$, $w_s^b(h)$, $c_s^b(h)$, $h \in \mathcal{H}^b(t_s^b)$, $s \in \mathcal{S}(t_s^b)$, and apply them until the bus reaches the next stop at time t_{s+1}^b

V. Case Study Application

The considered case study is an intercity bus line located in Italy, that is 26.3 kilometers long and includes eight stops. The effectiveness of the lexicographic approach has been tested considering a bus with departure at the Savona stop scheduled at 07:13 a.m. and with departure from the Finalborgo stop scheduled at 8:10 a.m. The bus has to meet the timetable shown in Table I considering a 1-minute stop for each intermediate stop between Savona and Finalborgo.

The main parameters related to the bus and the bus stops are the following: $e^{b,\min} = 50$ [kWh], $e^{b,\max} = 300$ [kWh], $e^b(0) = 160$ [kWh], which corresponds to a State of Charge (SOC) of about 53%, $\bar{e}^{b,\text{fin}} = 200$ [kWh], corresponding to a SOC of about 67%, $\delta^b = 1.16$ [kWh/km], $\beta^b = 3$ [kWh], $v^{b,\max} = 70$ [km/h], $\varsigma_s = 150$ [kW], $s \in \mathcal{S}$. The reference values $\bar{w}_s^b(h)$ have been set according to the timetable given in Table I, while the weights $\alpha_s^b(h)$ are set to penalize late departures more than early departures from a stop and to strongly penalize non-compliance with the timetable.

TABLE I: Bus timetable in the Savona-Finalborgo line.

0	*Savona*	07:13:00
1	*Vado Ligure*	07:25:00
2	*Bergeggi*	07:30:00
3	*Spotorno*	07:35:00
4	*Noli*	07:40:00
5	*Varigotti*	07:45:00
6	*Finalpia*	07:52:00
7	*Finalmarina*	07:55:00
8	*Finalborgo*	08:00:00

The lexicographic approach implies the application of Algorithm 1 eight times, i.e. for each bus stop from Savona to Finalmarina. The optimization problems present in Algorithm 1 have been solved in Matlab using YALMIP [14] and adopting Gurobi 9 as solver. The time t_s^b, the predicted departure time $t_{s+1}^{b,\text{pred}}$, the chosen priority and the computational times for each run of Algorithm 1 are summarized in Table II.

For each run of Algorithm 1, the prediction of traffic conditions along the bus line is performed using the model described in Section III. For example, Fig. 2 shows the profile, in space and time, of the average traffic speed predicted at t_0^b, showing that the bus travels in free-flow conditions from Savona to Spotorno, while it finds congestion from

TABLE II: Application of Algorithm 1 to the case study.

s	t_s^b	$t_{s+1}^{b,\text{pred}}$	*Priority*	*Computational time [s]*
0	07:13:00	07:19:00	F_2	1003.07
1	07:25:00	07:29:00	F_2	45.51
2	07:30:00	07:34:00	F_2	20.51
3	07:35:00	07:40:00	F_1	1.45
4	07:40:00	07:46:00	F_1	0.47
5	07:46:00	07:53:00	F_1	0.23
6	07:53:00	07:56:00	F_1	0.09
7	07:56:00	08:00:00	F_1	0.04

Spotorno to the final destination. Figure 3 shows the values of all the optimal control variables $w_s^b(h)$ obtained in the eight executions of the Algorithm 1. Note that the values of $w_s^b(h)$ coincide with the values of $c_s^b(h)$ for this case study.

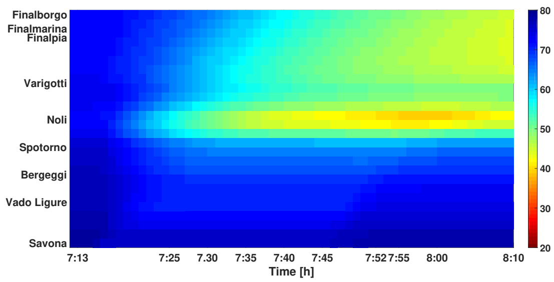

Fig. 2: Average speed [km/h] predicted at $t_0^b = 1$.

As it is show in Table II, for the first three runs of Algorithm 1 the bus speed and the charging and dwell times at stops are found by prioritizing F_2, i.e. the objective related to the final level of energy. Indeed, by observing Fig. 3, it is possible to notice that the dwell and charging times, at the first three stops, have durations which are longer than those expected from the timetable. From the Spotorno stop and for all the subsequent stops, Algorithm 1 gives priority to objective F_1, i.e., adherence to the bus timetable, and thus reducing the dwell and charging time at stops. Applying the control approach proposed in this paper, the final energy level achieved by the bus is about 191 [kWh], that corresponds to a SOC of approximately 64%, while the maximum delay at the stops is 1 minute with respect to the timetable but within the tolerance windows defined for each stop (red dashed line in Fig. 3).

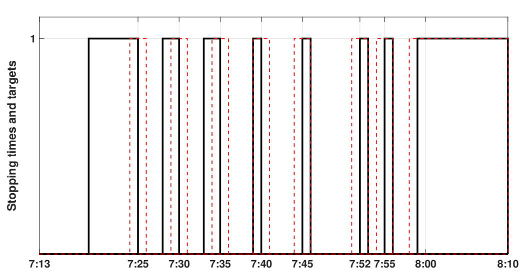

Fig. 3: Control variables $w_s^b(h)$ (black line) and target values $\bar{w}_s^b(h)$ (red dashed line).

As for the computational effort shown in Table II, it can be stated that the proposed methodology can be implemented in a real-world application. In fact, the greatest computational effort (16 minutes) is required at the first run of the algorithm, which in a real-world application can also be performed off line; for the other runs of the algorithm the required execution times are lower than 1 minute, that is, they are shorter than the minimum bus stop time.

VI. Conclusions

This paper deals with automated electric buses that have to follow a given route in inter-urban roads. The proposed control scheme is devised to define the speed profile of the buses and the time spent at the stops for waiting passengers and for charging the battery. Two main objectives are considered, i.e. the minimization of the deviations from the timetable and the minimization of the energy lack at the end of the bus route. Since the priority between them can change in real time, the proposed control scheme switches between two lexicographic algorithms. The application of this control strategy to a real case study shows the effectiveness of the time-varying objective prioritization.

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