# Resilient integral control for regulating systems with convex input constraints

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Abstract—In this paper, a novel integral control that can maintain the control input vector trajectory of a generic ISS linear or nonlinear plant within a prescribed compact and convex set is proposed. During normal operating conditions, the proposed controller can regulate the plant to the desired setpoint, while in the case of abnormal conditions, e.g. sensor faults, unrealistic reference input command, the controller introduces an inherent resilience property by maintaining the entire control input vector of the plant within a desired convex set. The boundedness of the control input vector is analytically proven using invariant set theory and vector field analysis (Nagumo's theorem). Opposed to conventional and more advanced integral controllers that either restrict each element of the control input vector independently or bound its Euclidean norm, in this paper, a detailed methodology for designing a resilient integral control to guarantee a generic compact and convex input constraint for a plant with unknown structure or dynamics is presented for the first time. A practical example of an underwater vehicle is investigated to validate the efficiency and resilience of the proposed controller under changes of the reference signal and under sensor faults.

#### I. INTRODUCTION

The majority of regulation problems in the industry has been addressed via control schemes that employ an integral control (IC) action. However, particularly for a nonlinear plant, maintaining closed-loop system stability with IC is a challenging task, especially when particular input constraints affect the plant operation [1], [2]. These input constraints can be either introduced by physical constraints of the plant (actuator constraints) or are important for the stable and reliable operation of the system (e.g. input-to-state stability - ISS, strong iISS properties [3]).

Typical constraints at the input vector elements are often accomplished by the addition of saturation units at the IC output; however, depending on the plant, this can lead to integrator windup and instability. Anti-windup methods can be employed in the control design to address this problem and have been extensively studied in the literature, see for example [4], [5]. Recently, a saturating integrator that overcomes the integrator windup problem and maintains closed-loop system stability for a class of nonlinear systems has been proposed in [6], while a low-gain IC for multi-input multi-output (MIMO) systems has been introduced in [7]– [9]. However, modern anti-windup control methods require knowledge of the plant structure, dynamics and often modify the conventional and widely-adopted IC scheme [10], while conditional integrators fail to facilitate a rigorous closedloop system stability analysis. This becomes even more challenging under abnormal conditions, e.g. unrealistic setpoint variations/attacks, sensor faults, that may shift the desired equilibrium point outside the operating range introduced by the input constraints.

For single-input systems, the bounded integral control (BIC) has been proposed in [11] to regulate any ISS plant and retain the ISS property for the closed-loop system. This is accomplished through its inherent zero-gain property, i.e. the controller output is bounded independently of the input signal, leading to the ISS proof of the closed-loop system based on the generalised small-gain theorem [12]. The BIC was further enhanced in [13] and was extended to bound the Euclidean norm of the control input in [14]-[16], as required in several applications in power and energy systems. However, in applications where more complicated input constraints are introduced for the plant input vector that differ from the Euclidean norm bound, neither the original BIC nor its extensions can be utilised. Recently in [17], a PI control scheme that is able to handle more complicated input constraints has been presented using projected dynamical systems theory. Nevertheless, it requires modification of the control dynamics using the directional derivative when the control input reaches the boundary of the set; thus complicating its implementation. Hence, according to the authors' knowledge, the design of an IC that does not require modification of its continuous-time dynamics, in order to facilitate closed-loop system stability, and can accomplish a generic compact and convex input constraint has not been developed yet. In addition, maintaining the desired constraint under abnormal conditions, i.e. significant changes of the regulation scenario or measurement/sensor faults that can lead the desired equilibrium point outside the desired operating range of the control input is still an open problem.

In this paper, a new resilient IC structure is proposed to regulate a generic ISS plant and maintain the control input vector trajectory within a desired compact and convex set for the first time. Given this predefined set, the controller dynamics are first formulated and then using set invariance properties and vector field analysis, it is analytically proven that the control input trajectory of a generic multi-input nonlinear plant can be restricted within a desired compact and convex set that satisfies some mild conditions. This is accomplished using the generalised version of Nagumo's theorem [18], while simultaneously retaining the bounded-

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ness property for the closed-loop system. The resilience property of the proposed controller under setpoint attacks, i.e. unrealistic changes in the regulating function, or sensor faults/drifts, is also investigated, demonstrating the existence of equilibrium points within the input constraint set. Both the stability and resilience properties are accomplished without the use of any saturation units, leading to an IC structure with continuous dynamics that ensures the control input operation within a desired compact convex set, opposed to existing IC approaches that require modification of the control structure and more complicated implementation. The proposed controller performance and desired properties (input constraint satisfaction, resilience) are demonstrated on an underwater vehicle application during normal operation as well as during scenarios that include large setpoint values at the regulating function or when sensor drifts are introduced at the measurements of the angular velocity (e.g. when computed via integrating an accelerometer measurement).

### II. RESILIENT IC DESIGN AND ANALYSIS

#### A. Problem formulation

Consider the nonlinear plant system of the form

$$\dot{x} = f(x, u),\tag{1}$$

with  $f: D \times D_u \to \mathbb{R}^n$  being locally Lipschitz in x and u, where  $D \subset \mathbb{R}^n$ ,  $D_u \subset \mathbb{R}^m$  are open neighborhoods of the origin. Let the following assumption for the plant stability property.

Assumption 1: System (1) is ISS with respect to the control input u.

Regarding the input of the nonlinear plant (1), consider a conventional and widely adopted IC structure, which takes the form

$$u = \sigma \tag{2}$$

$$\dot{\sigma} = h(x, \sigma),\tag{3}$$

where  $h: D_x \times D_u \to \mathbb{R}^m$  is locally Lipschitz in x and  $\sigma$ , and represents a vector of functions  $h_1(x, \sigma), ...h_m(x, \sigma)$  required to be regulated to zero at the steady state, i.e.  $\lim_{k \to \infty} h(x(t), \sigma(t)) = 0.$ 

Although IC has been widely used in practice to achieve the above regulation scenario, it is not designed to guarantee specific input constraints for the plant, especially when these constraints are more complex than a typical Euclidean norm bound, hyperplane, etc. To further justify this, consider the following assumption for a set S that encapsulates the plant input constraints.

Assumption 2: Let  $S \subset D_u$  be a non-empty compact and convex set defined as  $S = S_1 \cap S_2 \cap ... \cap S_l$  containing the origin, where  $l \ge 1 \in \mathbb{N}$  and

$$S_i = \{ u \in D_u : g_i(u) \ge 0 \}, \ \forall i = 1, ..., l$$
(4)

describes a convex set with  $g_i : D_u \to \mathbb{R}$  being locally Lipschitz in u, where there exists i = 1, ..., l such that  $g_i(u) = 0, \forall u \in \partial S.$  Note that Assumption 2 describes a set S that represents either typical input constraints, i.e. bounding every input vector element independently, introducing a Euclidean norm bound, a hyperplane or more complicated constraints that formulate a compact and convex set containing the origin. When the input constraint becomes more complex, conventional IC of the form of (2)-(3), equipped with saturation units can no longer be applied, while depending on the application and the plant dynamics, constrained optimisation methods complicate the control design and change the wellaccepted and continuous-time form of the IC. Towards this direction, a controller that maintains the original IC concept and accomplishes the desired input constraint is introduced in the sequel.

## B. Proposed resilient IC

In order to accomplish the desired input constraint, i.e.  $u(t) \in S, \forall t \ge 0$ , where S is defined in Assumption 2, the following IC is proposed:

$$u = \sigma$$
 (5)

$$\dot{\sigma} = h(x,\sigma) \prod_{i=1}^{l} g_i(\sigma) - k\sigma, \tag{6}$$

where  $\prod_{i=1}^{l} g_i(\sigma)$  is a scalar term describing the multiplication of the functions  $g_i(\cdot)$  used in (4) to formulate the convex sets  $S_i$ , k is an arbitrarily small positive constant and  $z(x,\sigma) = h(x,\sigma) \prod_{i=1}^{l} g_i(\sigma) - k\sigma : D_x \times D_u \to \mathbb{R}^m$ is locally Lipschitz in x and  $\sigma$ . The aim of the proposed controller is to ensure that  $u(t) = \sigma(t) \in S, \forall t \ge 0$  and that the solution  $(x(t), \sigma(t))$  of the closed-loop system (1), (5)-(6) is bounded. This is captured in the following theorem.

Theorem 1: Consider the closed-loop system dynamics (1), (5)-(6) resulting from the feedback interconnection of the plant (1), satisfying Assumption 1, and the resilient IC (5)-(6), with  $D_x = \mathbb{R}^n$  and  $D_u = \mathbb{R}^m$ . Then for any initial condition x(0) and  $\sigma(0) \in S$ , with S being any non-empty compact and convex set satisfying Assumption 2, the solution  $(x(t), \sigma(t))$  of the closed-loop system is bounded with  $\sigma(t) \in S$  for all  $t \geq 0$ .

**Proof:** The proof follows by utilising the ISS property of the plant (1) and the generalisation of Nagumo's theorem [18, Theorem 4.10]. The solution of the closed-loop system exists due to the Lipschitz properties of functions f and z. Consider now a non-empty compact set  $\mathcal{X} \subset D_x$ . Since  $\mathcal{X}$  and S are compact subsets of  $D_x$  and  $D_u$ , respectively, then functions f and z are also locally Lipschitz on  $\mathcal{X}$  and S. Let us investigate the velocity vector of the proposed IC dynamics at the boundary of S, i.e.  $z(x, \sigma)$ ,  $\forall \sigma \in \partial S$ . Since  $S = S_1 \cap S_2 \cap ... \cap S_l$ , where  $S_i$  is given from (4), then according to Assumption 2, at the boundary of S there exists  $i \in 1, ..., l$  such that  $g_i(\sigma) = 0$ . Hence the velocity vector at the boundary of S becomes

$$z(x,\sigma) = -k\sigma \in \partial S. \tag{7}$$

Expression (7) clearly describes a vector pointing from the boundary of S towards origin. Hence, at every point on the



Fig. 1: Example of a compact and convex set S satisfying Assumption 2, i.e.  $S = S_1 \cap S_2 \cap S_3 \cap S_4$ , and the vector field of the proposed IC dynamics (two-dimensional control system) at the boundary of S.

boundary of the compact and convex set S, the vector field will point towards the origin, as it can be seen in Fig. 1, where a typical example of a two-dimensional system is depicted. Since S is a compact and convex set and the origin belongs in the interior of S, then from the definition of convex sets, the velocity vector (7) belongs in the tangent cone  $\mathcal{T}_S(\sigma)$ , as described in [18, Definition 4.6], for all  $\sigma \in \partial S$ , independently of  $x \in \mathcal{X}$ , i.e.

$$z(x,\sigma) = -k\sigma \in \mathcal{T}_S(\sigma), \text{ for all } x \in \mathcal{X}.$$
 (8)

In addition, it obviously holds that for any  $\sigma \in \inf \{S\}$  there is  $\mathcal{T}_S(\sigma) = \mathbb{R}^m$  and therefore property (8) holds for all  $\sigma \in$ S, i.e. both at the boundary and in the interior of S. Then, according to the generalised version of Nagumo's theorem [18, Theorem 4.10], the set S is robustly positively invariant with respect to (6), i.e.  $\sigma(t) \in S$  for all  $t \ge 0$  given that  $\sigma(0) \in S$ . Due to the ISS property of the plant dynamics (1) (Assumption 1), then for any initial condition x(0) and  $\sigma(0) \in S$ , the solution x(t) is also bounded and  $\sigma(t) \in S$ for all  $t \ge 0$ , which completes the proof.

In other words, the proposed IC introduces a zero-gain property, as its output  $\sigma(t)$  remains bounded within S independently of the input signal x(t) (independently of the function  $h(x, \sigma)$ ), leading to the boundedness of the solution for the feedback interconnection of (1) and (5)-(6) in accordance to the generalised small-gain theorem [12].

Consider now an equilibrium point for the closed-loop system  $(x_e, \sigma_e)$ , where  $\sigma_e \in \text{int} \{S\}$ , obtained from (1) and (6) at the steady state. Then

$$f(x_e, \sigma_e) = 0 \text{ and } h(x_e, \sigma_e) = \frac{k\sigma_e}{\prod_{i=1}^l g_i(\sigma_e)}, \qquad (9)$$

where  $\prod_{i=1}^{l} g_i(\sigma_e) \neq 0$  given that  $\sigma_e \in \inf \{S\}$ . From (9), one can realise that in order to accomplish the desired regulation scenario, i.e.  $h(x_e, \sigma_e) = 0$ , the controller gain k should be equal to zero. In fact, if k = 0, the analysis in Theorem 1 will still be valid with the only difference that the velocity vector (7) will be equal to zero at the

boundary of S but will still belong in the tangent cone  $\mathcal{T}_S(\sigma)$ . Nevertheless, in the proposed scheme, it is suggested that k is chosen as an arbitrarily small positive constant which, although it cannot precisely guarantee the desired regulation scenario, i.e.  $h(x_e, \sigma_e) = 0$ , it offers significant resilience properties for the controller, and consequently for the closed-loop system as it will be explained in the remarks that follow. It should be noted, however, that since k can be selected as an arbitrarily small positive constant, then  $h(x_e, \sigma_e) \approx 0$ , which is acceptable in many practical applications, especially when the proposed IC is applied as an inner controller in a cascaded control loop (the outer loop achieves the desired regulation and the inner loop accomplishes the expression (9)).

Remark 1: During abnormal conditions when either the plant dynamics or the regulating function  $h(\cdot)$  change (e.g. due to a faulty sensor, loss of communication, unrealistic regulation demand etc.), it is possible that there does not exist a pair  $(x_e, \sigma_e)$  that sets  $h(x_e, \sigma_e) = 0$  with  $\sigma_e \in S$ , i.e. inside the desired plant input constraint set. For example, consider the scenario where there exists an equilibrium point  $(x_e, \sigma_e)$  for the plant (1) with the original IC (3), i.e.  $f(x_e, \sigma_e) = 0$  and  $h(x_e, \sigma_e) = 0$  where  $\sigma_e \notin S$ . In this case, the conventional IC applied to a plant with actuator limits or combined with saturation units can lead to integrator windup and might eventually become unstable. Although anti-windup methods can be utilised, conditional integrators or clamping often cannot facilitate a rigorous stability analysis [19], [20], whereas modern anti-windup techniques require knowledge of the plant dynamics and modify the well-known IC structure [4], [10]. On the other hand, the proposed resilient IC can result in the existence of an equilibrium point  $(x_e^*, \sigma_e^*)$  satisfying (9) where  $\sigma_e^* \in S$ since the denominator in (9) may become arbitrarily small (i.e.  $\sigma_e^*$  gets close to the boundary of S) to ensure that (9) holds. In other words, the proposed IC can realise an equilibrium point within the desired plant input constraint set.

*Remark 2:* The above remark describes the resilience property of the proposed controller. This can be further extended in the case where an equilibrium point does not exist, e.g. in the case where there is a sensor drift fault, and the measurement signal which is inherited in  $h(x, \sigma)$ continuously increases or decreases. As it has been analytically shown in Theorem 1, the trajectory of the plant input (controller state)  $u(t) = \sigma(t)$  will always remain within *S* independently of  $h(x, \sigma)$ . Hence, even if  $h(x, \sigma)$  continuously changes due to a sensor drift fault, the trajectory of  $u(t) = \sigma(t)$  will approach the boundary of *S* by continuously slowing down its velocity from (6) but will never reach the boundary as visually demonstrated in Fig. 1. The resilience property of the proposed IC will become more clear in the simulation example that follows in the next section.

#### III. APPLICATION ON UNDERWATER ROBOTIC VEHICLE

In this section, the regulation problem of an underwater robotic vehicle will be investigated. We consider as an example the 4-thruster configuration of the Seabotix LBV150 [21] underwater vehicle, depicted in Fig. 2. By taking into account some particular assumptions and simplifications on the model for this specific vehicle (interested readers are referred to [21] for details), and ignoring the motion along the z-axis for simplicity, as the heave motion is decoupled by the planar motion of the vehicle, we investigate the case where the robot does not change its depth level but can move forward and turn based on the port (p), starboard (s) and lateral (l) thrusters. Hence, the planar motion follows the equations of motion:

$$m_x \dot{u} - m_y vr - X_u u - X_{u|u|} u |u| = \tau_x$$
(10)

$$m_y \dot{v} + m_x ur - Y_v v - Y_{v|v|} v |v| = \tau_y$$
(11)

$$I_{z}\dot{r} - m_{x}uv + m_{y}uv - N_{r}r - N_{r|r|}r|r| = \tau_{N}, \quad (12)$$

where u, v and  $m_x, m_y$  are the velocities and added masses with respect to the surge and sway axes, r and  $I_z$  are the angular velocity and moment of inertia with respect to the z-axis,  $X_u$ ,  $Y_v$ ,  $N_r$  and  $X_{u|u|}$ ,  $Y_{v|v|}$ ,  $N_{r|r|}$  are the first- and second-order hydrodynamic drag coefficients. The force/torque inputs  $\tau_x$ ,  $\tau_y$  and  $\tau_N$  are linked to the thruster inputs via the following thruster allocation matrix:

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_N \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0.0475 & -0.0475 & 0 \end{bmatrix} \begin{bmatrix} \tau_p \\ \tau_s \\ \tau_v \end{bmatrix}.$$
 (13)

Each thruster input  $\tau_i$  for i = p, s, v is given as:

$$\tau_i = 14.34\omega_i |\omega_i| + 8.83\omega_i$$
, for  $i = p, s, v$ 

where  $\omega_i$  is the normalised rotor speed of each thruster with  $-1 \leq \omega_i \leq 1$ . Therefore, the constraint for each thruster input is translated to

$$-23.17 \le \tau_i \le 23.17$$
, for  $i = p, s, v$ . (14)

From (13), it yields that

$$\tau_p = \frac{\tau_x + \frac{\tau_N}{0.0475}}{2},$$
$$\tau_s = \frac{\tau_x - \frac{\tau_N}{0.0475}}{2},$$
$$\tau_v = \tau_u$$



Fig. 2: Schematic diagram of Seabotix LVB thrusters [21]

TABLE I: System parameters

Parameters	Value	Parameter	Value
$m_x$	9.7532	$m_y$	8.6636
$X_u$	-8.6040	$Y_v$	-18.1106
$X_{ u u}$	-17.8534	$Y_{ v v}$	-1.0594
$I_z$	0.1589	$N_r$	-1.4146
$N_{ r r}$	-10.3483	$k_x$	5e-6
$k_y$	0.1	$k_N, k$	1e-4

leading to the following coupled input constraints for the plant (10)-(12):

$$-46.34 \le \tau_x + \frac{\tau_N}{0.0475} \le 46.34$$
$$-46.34 \le \tau_x - \frac{\tau_N}{0.0475} \le 46.34$$
$$-23.17 \le \tau_y \le 23.17.$$

These input constraints can be expressed as  $(\tau_x, \tau_y, \tau_N) \in S$ where  $S = S_1 \cap S_2 \cap ... \cap S_6$  with

$$S_{1} = \left\{ g_{1}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = \tau_{x} + \frac{\tau_{N}}{0.0475} + 46.34 \ge 0 \right\}$$

$$S_{2} = \left\{ g_{2}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = -\tau_{x} - \frac{\tau_{N}}{0.0475} + 46.34 \ge 0 \right\}$$

$$S_{3} = \left\{ g_{3}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = \tau_{x} - \frac{\tau_{N}}{0.0475} + 46.34 \ge 0 \right\}$$

$$S_{4} = \left\{ g_{4}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = -\tau_{x} + \frac{\tau_{N}}{0.0475} + 46.34 \ge 0 \right\}$$

$$S_{5} = \left\{ g_{5}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = \tau_{y} + 23.17 \ge 0 \right\}$$

$$S_{6} = \left\{ g_{6}\left(\tau_{x}, \tau_{y}, \tau_{N}\right) = -\tau_{y} + 23.17 \ge 0 \right\}.$$

One can notice that S represents a compact and convex set containing the origin, thus satisfying Assumption 2.

The aim of the controller design is to regulate the velocities to some constant references at the steady state, in order for the underwater vehicle to move with constant vertical and angular velocities, i.e. achieve  $u = u_{ref}$ ,  $v = v_{ref}$ and  $r = r_{ref}$  at the steady state, while ensuring that  $(\tau_x(t), \tau_y(t), \tau_N(t)) \in S$  at all times. Hence, the proposed IC can be designed according to (5)-(6) as

$$\begin{aligned} \tau_x &= \sigma_x \\ \tau_y &= \sigma_y \\ \tau_N &= \sigma_N \\ \dot{\sigma}_x &= k_x (u_{ref} - u) \cdot g_1(\sigma_x, \sigma_y, \sigma_N) \cdot \dots \cdot g_6(\sigma_x, \sigma_y, \sigma_N) - k\sigma_x \\ \dot{\sigma}_y &= k_y (v_{ref} - v) \cdot g_1(\sigma_x, \sigma_y, \sigma_N) \cdot \dots \cdot g_6(\sigma_x, \sigma_y, \sigma_N) - k\sigma_y \\ \dot{\sigma}_N &= k_N (r_{ref} - r) \cdot g_1(\sigma_x, \sigma_y, \sigma_N) \cdot \dots \cdot g_6(\sigma_x, \sigma_y, \sigma_N) - k\sigma_N. \end{aligned}$$

The system and controller parameters are shown in Table I. In order to validate the resilience property of the proposed approach, the following two scenarios are investigated.

# Scenario 1 Operation under changes of the reference (setpoint) signals

Starting from zero initial conditions and reference values, at the time instant t = 1 s, the reference  $u_{ref}$  is set to 0.2 m/s, while at t = 10 s, the reference  $r_{ref}$  increases to  $\pi/10 rad/s$ . As it can be seen from Figs 3a, 3b and 3c, the proposed IC can successfully regulate the system states at

the desired setpoint values after a short transient. However, at t = 15 s, the reference signal  $r_{ref}$  increases even more to  $\pi/5 rad/s$  in order to test the controller operation under unrealistic setpoint values that force the control inputs to violate the desired constraint. In this case, the proposed IC automatically regulates the robotic vehicle angular velocity r to a lower value (see Fig. 3c) in order to satisfy the desired constraint  $(\tau_x, \tau_y, \tau_N) \in S$ . The time response of the control inputs is depicted in Fig. 3d, while the control input vector trajectory  $(\tau_x(t), \tau_y(t), \tau_N(t))$  is shown in Fig. 3e in the 3-dimensional  $\tau_x - \tau_y - \tau_N$  space, where it is clearly illustrated that the desired input constraints are satisfied at all times.

## Scenario 2 Operation under sensor drifts

In order to further evaluate the resilience property of the proposed controller under sensor faults, a sensor drift of  $-4 \times 10^{-3}$  is considered at all three measurements of the velocities u, v and r, starting at t = 1 s. Although the drift value is higher than normal for demonstration purposes, i.e. to evaluate the controller resilience in a shorter simulation time, this drift often exists if accelerometers are used in the robotic system and its integral values are obtained for the measurement of the velocities u, v and r. At t = 1 s, the value  $u_{ref}$  is set to 0.2 m/s and at t = 10 s, the reference  $r_{ref}$  becomes  $\pi/10 \, rad/s$ , while  $v_{ref}$  remains constant at zero at all times. The actual values of u, v and r as well as their measured values that include the drift and are fed in the proposed IC are depicted in Figs 4a, 4b and 4c. It becomes clear from the overall time response that when the control inputs  $(\tau_x, \tau_y, \tau_N)$  remain in the set S, the proposed controller manages to regulate the measured quantities at the desired references, while when  $(\tau_x, \tau_y, \tau_N)$  approach the boundary of S as the measurement drifts progressively more and more in time, the regulation task is automatically sacrificed to ensure that  $(\tau_x, \tau_y, \tau_N) \in S$  is satisfied at all times, as it becomes clear from Figs 4d and 4e. Although  $\tau_x$ and  $\tau_N$  are regulated at their constant values at the steady state,  $\tau_{y}$  continues to increase as the control input trajectory can still move towards the  $\tau_y$  axis and has not reached the boundaries of  $S_5$  or  $S_6$ . Nevertheless, the trajectory remains within S as clearly shown in Fig. 4e satisfying the resilience property of the proposed IC, thus validating the theoretical analysis developed in the paper.

#### **IV. CONCLUSIONS**

A resilient IC that maintains the control input trajectory of a generic linear/nonlinear plant within a generic compact and convex set, is introduced in this paper for the first time. Given the desired input constraint, the proposed IC takes a particular dynamic form and using invariant set theory, it is rigorously proven that the control input trajectory of the generic plant will satisfy the desired constraint at all times, independently of the regulating function or the plant dynamics, offering a unique resilient property. The controller design and efficiency was validated on an real application involving an underwater robotic vehicle under normal operation and abnormal scenarios that include unrealistic reference signals and sensor faults.



(e) control input trajectory within S

Fig. 3: Simulation results of the proposed IC under changes of the reference signals  $u_{ref}$ ,  $v_{ref}$ ,  $r_{ref}$ .



(e) control input trajectory within S

Fig. 4: Simulation results of the proposed IC under sensor drifts at the measurements u, v, r.

Future research will focus on the design of resilient IC that guarantee specific state constraints for the original plant in addition to the input constraints. In this case, partial information on the plant dynamics/structure is required, but is expected to offer significant advantages in robotics, power systems and electromechanical system applications, compared to the existing widely-applied IC methods, which still dominate the regulating controllers in these areas. REFERENCES

# [1] H. K. Khalil, Nonlinear Systems. Prentice Hall, 2001.

- [2] J. Tsinias, "Sufficient lyapunov-like conditions for stabilization," *Mathematics of Control, Signals and Systems*, vol. 2, no. 4, pp. 343– 357, Dec 1989.
- [3] A. Chaillet, D. Angeli, and H. Ito, "Combining iISS and ISS with respect to small inputs: The strong iISS property," *IEEE Trans. Autom. Control*, vol. 59, no. 9, pp. 2518–2524, Sept 2014.
- [4] L. Zaccarian and A. R. Teel, "Nonlinear scheduled anti-windup design for linear systems," *IEEE Trans. Autom. Control*, vol. 49, no. 11, pp. 2055–2061, Nov 2004.
- [5] J. Choi and S. Lee, "Antiwindup strategy for pi-type speed controller," *IEEE Trans. Ind. Electron.*, vol. 56, no. 6, pp. 2039–2046, June 2009.
- [6] G. Weiss and V. Natarajan, "Stability of the integral control of stable nonlinear systems," in 2016 IEEE International Conference on the Science of Electrical Engineering (ICSEE), Nov 2016, pp. 1–5.
- [7] C. Guiver, H. Logemann, and S. Townley, "Low-gain integral control for multi-input, multi-output linear systems with input nonlinearities," *IEEE Trans. Autom. Control*, 2017.
- [8] J. W. Simpson-Porco, "Analysis and synthesis of low-gain integral controllers for nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 66, no. 9, pp. 4148–4159, 2021.
- [9] —, "Low-gain stability of projected integral control for inputconstrained discrete-time nonlinear systems," *IEEE Control Systems Letters*, vol. 6, pp. 788–793, 2022.
- [10] L. Zaccarian and A. R. Teel, Modern Anti-windup Synthesys: Control Augmentation for Actuator Saturation. Princeton Univ. Press, 2011.
- [11] G. C. Konstantopoulos, Q.-C. Zhong, B. Ren, and M. Krstic, "Bounded integral control of input-to-state practically stable nonlinear systems to guarantee closed-loop stability," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4196–4202, Dec 2016.
- [12] Z.-P. Jiang, A. R. Teel, and L. Praly, "Small-gain theorem for ISS systems and applications," *Mathematics of Control, Signals and Systems*, vol. 7, no. 2, pp. 95–120, 1994.
- [13] G. C. Konstantopoulos, "Enhanced bounded integral control of inputto-state stable nonlinear systems," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 8151 – 8156, 2017, 20th IFAC World Congress.
- [14] Y. Wang, B. Ren, Q.-C. Zhong, and J. Dai, "Bounded integral controller with limited control power for nonlinear multiple-input multiple-output systems," *IEEE Transactions on Control Systems Technology*, vol. 29, no. 3, pp. 1348–1355, 2021.
- [15] G. C. Konstantopoulos and A. T. Alexandridis, "Full-scale modeling, control, and analysis of grid-connected wind turbine induction generators with back-to-back AC/DC/AC converters," *IEEE Journal of Emerg. and Select. Topics in Power Electron.*, vol. 2, no. 4, pp. 739– 748, 2014.
- [16] A. T. Alexandridis, G. C. Konstantopoulos, and Q. C. Zhong, "Advanced integrated modeling and analysis for adjustable speed drives of induction motors operating with minimum losses," *IEEE Transactions* on Energy Conversion, vol. 30, no. 3, pp. 1237–1246, Sept 2015.
- [17] P. Lorenzetti and G. Weiss, "PI control of stable nonlinear plants using projected dynamical systems," *Automatica*, vol. 146, p. 110606, 2022.
- [18] F. Blanchini and S. Miani, Set-Theoretic Methods in Control, 2nd ed., Ed. Birkhäuser Cham, 2015.
- [19] R. S. Baheti, "Simple anti-windup controllers," in 1989 American Control Conference, 1989, pp. 1684–1686.
- [20] M. H. A. Jalil, R. Hamdan, R. Ngadengon, H. M. Shariff, S. Amely Jumaat, and S. Z. Islam, "PID with clamping anti-windup performance on temperature regulation of glycerin bleaching process," in *IEEE 12th Contr. & Syst. Grad. Res. Colloq. (ICSGRC)*, 2021, pp. 127–131.
- [21] G. C. Karras, C. P. Bechlioulis, P. Marantos, S. Heshmati-alamdari, and K. J. Kyriakopoulos, "Sensor-based motion control of autonomous underwater vehicles, part I: modeling and low-complexity state estimation," pp. 15–43, 2020.