2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Event-triggered control for Hamiltonian-based flexible-joints robots

Qi Zhang, Weiwei Sun, Lusong Ding and Yongshu Li

***Abstract*— This paper focuses on the event-triggered tracking control problem for Hamiltonian-based flexible-joints robots. Based on Hamiltonian theory and the delay system approach, an event-triggered modular control strategy is proposed to guarantee that the link and motor generalized position can track the target signal asymptotically while reducing the waste of transmission resources. The modular controller designed in this paper takes fully into account the structural characteristics of the flexible-joints robots. Moreover, the difficulties of tracking control and event-triggered control caused by the high coupling of Hamiltonian systems are overcome in the design process. This extends the existing theoretical results for flexible-joints robots and effectively improves energy efficiency. The validity of the proposed strategy is verified by a simulation example of a flexible joint robot.**

***Index Terms*— Flexible-joints robots, event-triggered control, Hamiltonian systems, tracking control.**

I. INTRODUCTION

With rapidly developing computers and automation, more and more concerns have been given to robotic systems, which have been widely used in the military, medicine, industry and other areas [1]–[3]. Compared with rigid robots, flexible-joints robots have the advantages of high speed, high precision and lightweight. In recent years, many meaningful results on flexible-joints robots have been presented and successfully applied, such as fuzzy control [4], sliding mode control [5], and passivity-based control [6], just to name a few. Currently, flexible-joints robots are mainly studied using the Euler-Lagrange system, which often requires finding a suitable Lyapunov function. As an alternative, Hamiltonian systems can avoid this problem since its energy function is considered to be available as a Lyapunov function representing dissipativity [7]. Based on this idea, [8] and [9] provided two Hamiltonian realization methods for converting Euler-Lagrange systems into Hamiltonian systems. [10] studied the tracking control problem of a rigid robot based on the Hamiltonian framework. [11] provided a contraction control method for Hamiltonian-based flexible-joints robots. Although Hamiltonian systems have unique advantages in stability analysis due to their dissipative property, their nonlinear coupling increases some obstacles to the treatment of tracking control problems.

Besides, in practical applications, communication resources are limited, so it is necessary to design control strategies that reduce redundancy and useless transmissions. Event-triggered (ET) control can guarantee the system desired performance while reducing data transmission, which shows a unique advantage in saving resources. ET control has received plenty of attention since it was proposed [12]–[15]. Based on a wavelet neural network, a distributed ET control strategy for networked manipulators was proposed in [12]. [13] proposed an ET control scheme by adopting a sliding-mode-based time-delay control strategy to realize angle adjustment and vibration suppression while achieving network channel burden reduction. [14] considered multiple manipulators with external disturbance and studied their ET cooperative control problem. However, so far, there has been little research on the ET control problem of Hamiltonian systems because the high coupling of Hamiltonian systems also plagues the ET control.

We investigate the ET control problem for flexible-joints robots in this paper. Based on the Hamiltonian framework and delay system approach, a Hamiltonian-based ET controller is proposed. Compared with the existing related results, the main contributions of this paper are reflected in the following aspects:

(1) To achieve accurate position tracking, the flexibility of the joints is often not negligible. Compared with the result of [10], this paper extends the Hamiltonian framework to flexible-joints robots. The flexible-joints robot is modeled and synthesized based on Hamiltonian theory to meet the requirements of high accuracy and lightweight in practical applications of robots.

(2) A new control strategy is designed for Hamiltonian-based flexible-joints robots, which overcomes the difficulties of tracking control and ET control caused by the high coupling of the Hamiltonian system. Compared with [11], [16], the dissipative property of Hamiltonian systems is more fully utilized in the design process, and the proposed strategy improves the utilization rate of valuable communication resources.

(3) The modeling methodology and control strategy developed in the Hamiltonian framework take into account the structural characteristics of the flexible-joints robots and preserve the modularity of Hamiltonian systems. Moreover, the Lyapunov-Krasovskii functional is constructed based on the Hamiltonian function, which makes the stability analysis simpler compared with [12]–[14].

Notations: $\mathbb{R}^n$ refers to the n-dimensional Euclidean space. $\mathbb{N}$ refers to the set of natural numbers. $I_{n\times n}$ denotes the $n \times n$ dimensioned identity matrix. $\mathrm{col}\{x_1, \cdots, x_n\}$ marks the column vector $[x_1^{\mathrm{T}}, \cdots, x_n^{\mathrm{T}}]^{\mathrm{T}}$, $\mathrm{row}\{x_1, \cdots, x_n\}$ marks the row vector $[x_1, \cdots, x_n]$, $\mathrm{diag}\{x_1, \cdots, x_n\}$ marks

This work was supported in part by the National Natural Science Foundation of China under Grant 62073189 and Grant 62173207, in part by the Taishan Scholar Project of Shandong Province, China under Grant tsqn202211129.

The authors are with the Institute of Automation, Qufu Normal University, Qufu, P. R. China QiZhangsr@outlook.com, wwsun@hotmail.com, dingls2022@163.com, liyongshuknk@163.com

the diagonal matrix with diagonal elements of $x_1, \cdots, x_n$. In the symmetric matrix, "$*$" indicates the entries that symmetry implies. For vector $x \in \mathbb{R}^n$, $\nabla_x f(x) = [\frac{\partial f(x)}{\partial x_1}, \cdots, \frac{\partial f(x)}{\partial x_n}]^{\mathrm{T}}$ denotes the gradient and $\mathrm{Hess}(f(x)) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1^2} & \cdots & \frac{\partial^2 f(x)}{\partial x_1 \partial x_n} \\ \vdots & & \vdots \\ \frac{\partial^2 f(x)}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f(x)}{\partial x_n^2} \end{bmatrix}$ denotes the Hessian matrix of $f(x)$. $\lambda_{\max}(\cdot)$ represents the maximum eigenvalue of a symmetric matrix. $\|\cdot\|$ represents the Euclidean norm.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following n degree-of-freedom (dof) flexible-joints robot as a Hamiltonian system [11]:

$$\begin{aligned} \begin{bmatrix} \dot{q}_l \\ \dot{q}_m \\ \dot{p}_l \\ \dot{p}_m \end{bmatrix} &= \begin{bmatrix} 0_{l\times l} & 0_{l\times m} & I_l & 0_{l\times m} \\ 0_{m\times l} & 0_{m\times m} & 0_{m\times l} & I_m \\ -I_l & 0_{l\times m} & -D_l(q_l,p_l) & 0_{l\times m} \\ 0_{m\times l} & -I_m & 0_{m\times l} & -D_m(q_m,p_m) \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \nabla_{q_l}\mathcal{H}(q,p) \\ \nabla_{q_m}\mathcal{H}(q,p) \\ \nabla_{p_l}\mathcal{H}(q,p) \\ \nabla_{q_m}\mathcal{H}(q,p) \end{bmatrix} + \begin{bmatrix} 0_{l\times m} \\ 0_{m\times m} \\ 0_{l\times m} \\ B_m(q_m) \end{bmatrix} u, \\ y &= B_m^{\mathrm{T}}(q_m)\nabla_{p_m}\mathcal{H}(q,p), \end{aligned} \tag{1}$$

where $q_l \in \mathbb{R}^{n_l}$ is link position, $q_m \in \mathbb{R}^{n_m}$ is motor shaft position, $q = [q_l^{\mathrm{T}}, q_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^n$ with $n = n_l + n_m$. $\mathcal{H}(q,p) = \frac{1}{2}pM^{-1}(q)p + \mathcal{P}(q)$ is a Hamiltonian function, inertia matrix $M(q) = \mathrm{diag}\{M_l(q_l), M_m(q_m)\}$ is symmetric and positive definitive, $M_l(q_l)$ is the link inertia matrix and $M_m(q_m)$ is the motor inertia matrix. $p = [p_l^{\mathrm{T}}, p_m^{\mathrm{T}}]^{\mathrm{T}}$, $p_l = M_l(q_l)\dot{q}_l$, $p_m = M_m(q_m)\dot{q}_m$. $\mathcal{P}(q) = \mathcal{P}_l(q_l) + \mathcal{P}_m(q_m) + \mathcal{P}_\rho(\rho)$ is the potential energy where $\mathcal{P}_l(q_l)$ is the link potential energy, $\mathcal{P}_m(q_m)$ is the motor potential energy, and $\mathcal{P}_\rho(\rho) = \frac{1}{2}\rho^{\mathrm{T}}K\rho$ is the coupling potential energy with $\rho := q_m - q_l$ and the symmetric and positive definitive matrix $K \in \mathbb{R}^{n\times n}$. Damping matrix $D(q,p) = D^{\mathrm{T}}(q,p) \succeq 0_{n\times n}$ is partitioned into $\mathrm{diag}\{D_l(q_l,p_l), D_m(q_m,p_m)\}$ with the link damping matrix $D_l(q_l,p_l)$ and the motor damping matrix $D_m(q_m,p_m)$. I_l and I_m are the $n_l \times n_l$ and $n_m \times n_m$ identity matrices, respectively. Similarly $0_{l\times m}$ and $0_{l\times l}$ are the $n_l \times n_m$ and $n_l \times n_m$ zero matrices, $B_m(q_m) \in \mathbb{R}^{n_m\times n_m}$.

We follow the standard modeling assumptions of flexible-joints robot [17]:

- Each spring has a small enough deflection/extension ρ that a linear model can represent.
- The i-link is driven by the i-th motor installed on the $(i-1)$-link.
- The center of mass of each motor is located on the axis of rotation.

The control aim of this paper is to construct an ET control strategy for the system (1) such that the link and motor position q can precisely track the target signal q_d. For this purpose, the following system modification is required.

Let $x := [q^{\mathrm{T}}, p^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2n}$, the system (1) can be rewritten as the following alternative model:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\nabla_x\mathcal{H}(x) + G(x)u, \\ y &= G^{\mathrm{T}}(x)\nabla_x\mathcal{H}(x), \end{aligned} \tag{2}$$

where

$$J(x) = \begin{bmatrix} 0_n & I_n \\ -I_n & 0_n \end{bmatrix}, R(x) = \begin{bmatrix} 0_n & 0_n \\ 0_n & D(x) \end{bmatrix},$$
$$G(x) = \mathrm{col}\{0_{l\times m}, 0_{m\times m}, 0_{l\times m}, B_m(q_m)\}.$$

III. MAIN RESULTS

In this section, we design an ET Hamiltonian-based controller that takes full advantage of the modular nature of the Hamiltonian-based flexible-joints robot and constructs asymptotic tracking conditions.

A. Hamiltonian-based ET controller design

Define $\xi = x - x_d$, where $x_d \in \mathbb{R}^{2n}$ is a desired equilibrium. Given a desired energy function $\mathcal{H}_d(\xi) > 0$ satisfying $\mathcal{H}_d(0) = 0$ and the following assumption:

Assumption 1: ([18]) The desired Hamiltonian $\mathcal{H}_d(\xi)$, its gradient $\nabla_\xi\mathcal{H}_d(\xi)$ and its Hessian matrix $\mathrm{Hess}(\mathcal{H}_d(\xi))$ satisfy

1) $\varsigma_1(\|\xi(t)\|) \le \mathcal{H}_d(\xi) \le \varsigma_2(\|\xi(t)\|)$;
2) $\varsigma_3(\|\xi(t)\|) \le \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)\cdot\nabla_\xi\mathcal{H}_d(\xi) \le \varsigma_4(\|\xi(t)\|)$;
3) $\|\mathrm{Hess}(\mathcal{H}_d(\xi))\cdot\mathrm{Hess}^{\mathrm{T}}(\mathcal{H}_d(\xi))\| \le \lambda^2$,

where λ is positive scalar, $\varsigma_1, \varsigma_2, \varsigma_3$ and ς_4 are class-$\mathcal{K}$ functions.

Suppose that there exist an assigned skew-symmetric matrix J_a, an assigned semi-definite matrix R_a and an assigned Hamiltonian function such that

$$H_d(\xi) = H_a(x) + H(\xi), \tag{3}$$

$$J_d(\xi) = J_a + J(\xi), \tag{4}$$

$$J(\xi + x_d) = J(\xi) + J(x_d), \tag{5}$$

$$R_d(\xi) = R_a + R(\xi) \succeq 0, \tag{6}$$

then the state error system model is described as follows [19], [20]:

$$\begin{aligned} \dot{\xi} &= [J_d(\xi) - R_d(\xi) - J_a + R_a + J(x_d)]\nabla_\xi\mathcal{H}_d(\xi) \\ &\quad + J(\xi)\nabla_{x_d}\mathcal{H}(x_d) + G(x)u(t). \end{aligned} \tag{7}$$

Inspired by [7], consider the following equation:

$$\begin{aligned} &[J_d(\xi) - R_d(\xi)]\nabla_\xi\mathcal{H}_d(\xi) = G(x)u(t) + J(\xi)\nabla_{x_d}\mathcal{H}(x_d) \\ &+ [J_d(\xi) - J_a - R_d(\xi) + R_a + F_d + J(x_d)]\nabla_\xi\mathcal{H}_d(\xi), \end{aligned} \tag{8}$$

where F_d is a positive definite matrix. By solving the equation (8), we can arrive at the Hamiltonian-based controller

$$\begin{aligned} u(t) &= G^\dagger(x)[(J_a - R_a - F_d - J(x_d))\nabla_\xi\mathcal{H}_d(\xi) \\ &\quad - J(\xi)\nabla_{x_d}\mathcal{H}(x_d)], \end{aligned} \tag{9}$$

where $G^\dagger(x) = (G^{\mathrm{T}}(x)G(x))^{-1}G^{\mathrm{T}}(x)$.

To reduce the burden of network transmission, an ET strategy that can overcome the coupling of the Hamiltonian system and simplify the performance analysis is designed below. As shown in Fig. 1, the networked system consists of a controlled system, a sampler, an event trigger, a ZOH (zero-order holder), an actuator and a data network.

Assume that the sampling period of the sampler is h, i.e., sampling at $t = jh$, where $j \in \mathbb{N}$. The state $\xi(t)$ is monitored periodically and the set of sampled data is denoted as $\{\xi(jh)|j \in \mathbb{N}\}$. The periodic sampling data $\xi(jh)$ is then entered into the event trigger for further filtering. The event trigger relies on the following condition to determine whether the current sampling state needs to be transmitted:

$$\begin{aligned} t_{r+1}h = t_r h + \min_{v\in\mathbb{N}}\{vh|\bar{\nabla}_{\mathcal{H}}^{\mathrm{T}}(\xi(t_r^v h))\Psi\bar{\nabla}_{\mathcal{H}}(\xi(t_r^v h)) \\ \geq \eta\nabla_{\xi}^{\mathrm{T}}\mathcal{H}_d(\xi(t_r h))\Psi\nabla_{\xi}\mathcal{H}_d(\xi(t_r h))\}, \quad (10)\end{aligned}$$

where $t_r h$, $t_{r+1}h$ are r-th and $r+1$-th triggering instants, respectively. $t_r^v h := (t_r + v)h \in [t_r h, t_{r+1}h)$ is the sampling instant, $\bar{\nabla}_{\mathcal{H}}(\xi(t_r^v h)) = \nabla_{\xi}\mathcal{H}_d(\xi(t_r^v h)) - \nabla_{\xi}\mathcal{H}_d(\xi(t_r h))$. $\eta \geq 0$ is a triggering threshold and $\Psi \succ 0$ is a triggering weight matrix. $r \in \mathbb{N}$ and $v \in \mathbb{N}$ denote the number of triggering point sequences and sampling point sequences, respectively.

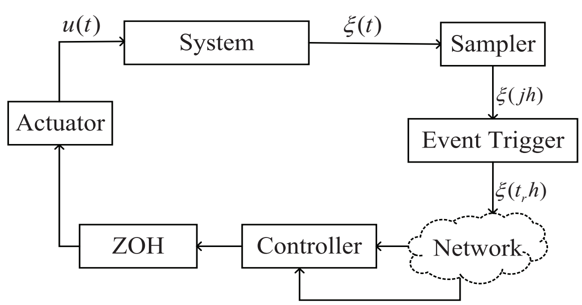

Fig. 1. Diagram of an ET control loop.

Remark 1: Note that Hamiltonian systems are highly coupled. The premise of using traditional ET mechanisms is that the Hamiltonian system is subject to certain restrictions, which reduces the scope of the application of the control strategy. In addition, this will destroy the modularity of the original system, leading to a very complex performance analysis. We introduce the Hamiltonian function in the ET mechanism (10) to make it modular like a Hamiltonian system, thus avoiding the need to deal with complex coupling problems.

Remark 2: In ET control, an inevitable question is whether there is a minimum triggering interval. If it does not exist, it will be sampled an infinite number of times over a finite period of time, which is called the Zeno phenomenon. This behavior cannot be realized by physical devices, so the ET mechanism must be designed to avoid this situation. Since the states are first discretely sampled using a sampler, there must be a minimum trigger interval h, i.e., a sampling period, for the states entering the trigger. This naturally avoids the occurrence of the Zeno phenomenon and requires no additional theoretical proof.

Remark 3: The ET condition clearly needs to ensure that there is a uniform minimum time between two transmissions to meet inherent hardware constraints. Therefore, according to [21], it may be assumed that the jumps in the ET mechanism (10) are spaced at least by $T > 0$, i.e. $T := \inf\{t|\bar{\nabla}_{\mathcal{H}}^{\mathrm{T}}(\xi(t))\Psi\bar{\nabla}_{\mathcal{H}}(\xi(t)) \geq \eta\nabla_{\xi}^{\mathrm{T}}\mathcal{H}_d(\xi(t_r h))\Psi\nabla_{\xi}\mathcal{H}_d(\xi(t_r h))\}$. Therefore, it is reasonable to select the sampling period that is smaller than the minimum inter-transmission time of the ET mechanism to avoid reducing the system to a traditional discrete time implementation.

Remark 4: The ZOH in Fig.1 ensures that the input signal to the actuator is continuous. During the signaling process, the ZOH keeps the control signal of the r-th triggering until the moment before the $r+1$-th triggering. As soon as the event trigger releases a new triggering signal, ZOH will immediately update the transmission signal and transmit the new control signal to the actuator until the next transmission signal is received.

Remark 5: The triggering threshold η can regulate the intensity of resource saving by changing the amount of data transmitted by the network. The threshold η and the amount of data transmitted by the network are negatively correlated, i.e., the amount of data transmitted increases as the threshold decreases. When $\eta = 0$, the ET mechanism is converted into a time-triggered mechanism.

For all $t \in [t_r h, t_{r+1}h)$, the Hamiltonian-based controller (9) can be designed as

$$\begin{aligned} u(t) &= G^{\dagger}(x)[(J_a - R_a - F_d - J(x_d))\nabla_{\xi}\mathcal{H}_d(\xi(t_r h)) \\ &\quad -J(\xi)\nabla_{x_d}\mathcal{H}(x_d)], \quad (11)\end{aligned}$$

Divide the interval $[t_r h, t_{r+1}h)$ into the following subintervals:

$$[t_r h, t_{r+1}h) = \cup_{v=0}^{\delta_r}[t_r^v h, t_r^{v+1}h) := \cup_{v=0}^{\delta_r}\Lambda, \quad (12)$$

where $\delta_r = t_{r+1} - t_r - 1$. Define

$$\tau(t) = t - t_r^v h, t \in [t_r^v h, t_r^{v+1}h) \quad (13)$$

satisfying $0 \leq \tau(t) \leq h$. For $t \in [t_r^v h, t_r^{v+1}h)$, the ET control input (11) is described as

$$\begin{aligned} u(t) &= G^{\dagger}(x)[(J_a - R_a - F_d - J(x_d)) \\ &\quad \cdot(\nabla_{\xi}\mathcal{H}_d(\xi(t-\tau(t))) - \bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t)))) \\ &\quad -J(\xi)\nabla_{x_d}\mathcal{H}(x_d)]. \quad (14)\end{aligned}$$

The corresponding closed-loop error system obtained based on the system (7) is

$$\begin{aligned} \dot{\xi} &= [J_d(\xi) - J_a - R_d(\xi) + R_a + J(x_d)]\nabla_{\xi}\mathcal{H}_d(\xi) \\ &\quad +(J_a - R_a - F_d - J(x_d))(\nabla_{\xi}\mathcal{H}_d(\xi(t-\tau(t))) \\ &\quad -\bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t)))), \\ \xi(\theta) &= \phi(\theta), \theta \in [-h, 0] \quad (15)\end{aligned}$$

with the initial function $\phi(\theta)$.

B. Performance analysis

In the following, sufficient conditions are established for the system (1) to implement ET asymptotic tracking.

Theorem 1: For the system (1), given positive constants η, λ, a scale κ and matrices W, J_a, R_a with appropriate dimensions. If there exist positive definite matrices Q, Z, Ψ,

F_d and a desired energy function $\mathcal{H}_d(\xi)$ satisfying Assumption 1, such that the following matching equality and matrix inequality hold:

$$\begin{aligned} & G^{\perp}(\xi)J(\xi)\nabla_{x_d}\mathcal{H}(x_d) \\ = \ & G^{\perp}(\xi)[(J_a - R_a - J(x_d))\nabla_\xi \mathcal{H}_d(\xi)], \end{aligned} \tag{16}$$

$$\begin{bmatrix} \Pi & hW^{\mathrm{T}} & h\lambda\mathcal{S}^{\mathrm{T}} \\ & -Z & 0 \\ & * & -2\kappa I + \kappa^2 Z \end{bmatrix} \prec 0, \tag{17}$$

where $G^{\perp}(\xi)$ satisfying $G^{\perp}(\xi)G(\xi) = 0$ is left annihilator of $G(\xi)$,

$$\Pi = \begin{bmatrix} Q & 0 & 0 \\ * & \eta\Psi - Q & 0 \\ * & * & -\Psi \end{bmatrix} + \mathcal{I}_1^{\mathrm{T}}\mathcal{S} - 2hW^{\mathrm{T}}\mathcal{I}_1 + 2hW^{\mathrm{T}}\mathcal{I}_2,$$

$\mathcal{S} = \mathrm{row}\{J_d(\xi) - J_a - R_d(\xi) + R_a + J(x_d), J_a - R_a - J(x_d) - F_d, -J_a + R_a + J(x_d) + F_d\}$, $\mathcal{I}_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}$, $\mathcal{I}_2 = \begin{bmatrix} 0 & I & 0 \end{bmatrix}$, $J_d(\xi) = J(\xi) + J_a$, $R_d(\xi) = R(\xi) + R_a \succeq 0$, then the position q can asymptotically track the target signal q_d under the ET mechanism (10) and the Hamiltonian-based controller (11).

Proof: A Lyapunov-Krasovskii functional is selected as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{18}$$

where

$$\begin{aligned} V_1(t) &= \mathcal{H}_d(\xi), \\ V_2(t) &= \int_{t-\tau(t)}^{t} \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(\vartheta))Q\nabla_\xi\mathcal{H}_d(\xi(\vartheta))\mathrm{d}\vartheta, \\ V_3(t) &= h\int_{-h}^{0}\int_{t+\sigma}^{t} \left(\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(\vartheta))\right)' Z \\ & \quad \cdot \left(\nabla_\xi\mathcal{H}_d(\xi(\vartheta))\right)' \mathrm{d}\vartheta\mathrm{d}\sigma. \end{aligned}$$

According to Assumption 1, we have

$$\varsigma_1(\|\xi\|) \le V(t) \le \varsigma(\|\xi\|), \tag{19}$$

where $\varsigma(\|\xi\|) = \varsigma_2(\|\xi\|) + h\lambda_{\max}(Q)\varsigma_4(\|\xi\|) + h\lambda^2\lambda_{\max}(Z)\int_{-h}^{0}\int_{t+\sigma}^{t}\dot{\xi}^{\mathrm{T}}(\vartheta)\dot{\xi}(\vartheta)\mathrm{d}\vartheta\mathrm{d}\sigma$. Taking the derivative of $V(t)$ along the trajectory of the system (15) yields

$$\begin{aligned} \dot{V}(t) \le \ & \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)\dot{\xi} + \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)Q\nabla_\xi\mathcal{H}_d(\xi) \\ & + h^2\dot{\xi}^{\mathrm{T}}(t)\cdot\mathrm{Hess}\left(\mathcal{H}_d(\xi)\right)\cdot Z\cdot\mathrm{Hess}^{\mathrm{T}}\left(\mathcal{H}_d(\xi)\right)\cdot\dot{\xi}(t) \\ & - h\int_{t-\tau(t)}^{t}\left(\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(\vartheta))\right)' Z\left(\nabla_\xi\mathcal{H}_d(\xi(\vartheta))\right)'\mathrm{d}\vartheta \\ & - \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t)))Q\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))). \end{aligned} \tag{20}$$

Based on Jensen's inequality and Newton-Leibniz formula, the following inequality is obtained:

$$\begin{aligned} & -h\int_{t-\tau(t)}^{t}\left(\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(\vartheta))\right)' Z\left(\nabla_\xi\mathcal{H}_d(\xi(\vartheta))\right)'\mathrm{d}\vartheta \\ \le \ & -2h\varpi^{\mathrm{T}}W^{\mathrm{T}}\left(\nabla_\xi\mathcal{H}_d(\xi) - \nabla_\xi\mathcal{H}_d(\xi(t-\tau(t)))\right) \\ & + \tau(t)h\varpi^{\mathrm{T}}W^{\mathrm{T}}Z^{-1}W\varpi, \end{aligned} \tag{21}$$

where $\varpi = \mathrm{col}\{\nabla_\xi\mathcal{H}_d(\xi), \nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))), \bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t)))\}$. It can be learned from the ET transmission condition (10) that

$$\begin{aligned} & \bar{\nabla}_{\mathcal{H}}^{\mathrm{T}}(\xi(t-\tau(t)))\Psi\bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))) \\ < \ & \eta\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t))\Psi\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))). \end{aligned} \tag{22}$$

Then substituting (21)–(22) into (20), one gets

$$\begin{aligned} \dot{V}(t) \le \ & \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)\dot{\xi} + \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)Q\nabla_\xi\mathcal{H}_d(\xi) \\ & + h^2\dot{\xi}^{\mathrm{T}}(t)\cdot\mathrm{Hess}\left(\mathcal{H}_d(\xi)\right)\cdot Z\cdot\mathrm{Hess}^{\mathrm{T}}\left(\mathcal{H}_d(\xi)\right)\cdot\dot{\xi}(t) \\ & - 2h\varpi^{\mathrm{T}}W^{\mathrm{T}}\left(\nabla_\xi\mathcal{H}_d(\xi) - \nabla_\xi\mathcal{H}_d(\xi(t-\tau(t)))\right) \\ & - \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t)))Q\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & - \bar{\nabla}_{\mathcal{H}}^{\mathrm{T}}(\xi(t-\tau(t)))\Psi\bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))) \\ & + \eta\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t))\Psi\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & + \tau(t)h\varpi^{\mathrm{T}}W^{\mathrm{T}}Z^{-1}W\varpi. \end{aligned} \tag{23}$$

By (15) and Assumption 1, we further have

$$\begin{aligned} \dot{V}(t) \le \ & \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)[(J_d(\xi) - J_a - R_d(\xi) + R_a + J(x_d)) \\ & \cdot\nabla_\xi\mathcal{H}_d(\xi) + \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi)Q\nabla_\xi\mathcal{H}_d(\xi) + (J_a - R_a \\ & - J(x_d) - F_d)(\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & - \bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))))] + h^2\lambda^2[(J_d(\xi) - R_d(\xi) \\ & - J_a + R_a + J(x_d))\nabla_\xi\mathcal{H}_d(\xi) + (J_a - R_a \\ & - J(x_d) - F_d)(\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & - \bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))))]^{\mathrm{T}}Z[(J_d(\xi) - J_a - R_d(\xi) \\ & + R_a + J(x_d))\nabla_\xi\mathcal{H}_d(\xi) + (J_a - R_a - J(x_d) \\ & - F_d)(\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) - \bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))))] \\ & - 2h\varpi^{\mathrm{T}}W^{\mathrm{T}}\left(\nabla_\xi\mathcal{H}_d(\xi) - \nabla_\xi\mathcal{H}_d(\xi(t-\tau(t)))\right) \\ & - \nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t)))Q\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & - \bar{\nabla}_{\mathcal{H}}^{\mathrm{T}}(\xi(t-\tau(t)))\Psi\bar{\nabla}_{\mathcal{H}}(\xi(t-\tau(t))) \\ & + \eta\nabla_\xi^{\mathrm{T}}\mathcal{H}_d(\xi(t-\tau(t))\Psi\nabla_\xi\mathcal{H}_d(\xi(t-\tau(t))) \\ & + \tau(t)h\varpi^{\mathrm{T}}W^{\mathrm{T}}Z^{-1}W\varpi. \end{aligned} \tag{24}$$

For any scale κ, one has

$$-Z^{-1} \preceq -2\kappa I + \kappa^2 Z. \tag{25}$$

Define $\mathcal{I}_3 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}$. From (25), (24) can be rewritten as

$$\begin{aligned} \dot{V}(t) \le \ & \varpi^{\mathrm{T}}[\mathcal{I}_1^{\mathrm{T}}\mathcal{S} + \mathcal{I}_1^{\mathrm{T}}Q\mathcal{I}_1 + \mathcal{I}_2^{\mathrm{T}}(\eta\Psi - Q)\mathcal{I}_2 - 2hW^{\mathrm{T}}\mathcal{I}_1 \\ & + h^2\lambda^2\mathcal{S}^{\mathrm{T}}(2\kappa I - \kappa^2 Z)^{-1}\mathcal{S} + \tau(t)hW^{\mathrm{T}}Z^{-1}W \\ & - \mathcal{I}_3^{\mathrm{T}}\Psi\mathcal{I}_3 + 2hW^{\mathrm{T}}\mathcal{I}_2]\varpi \\ := \ & \varpi^{\mathrm{T}}\Phi(\tau(t))\varpi. \end{aligned} \tag{26}$$

Combining Schur complement and (16), we have $\Phi(0) \prec 0$ and $\Phi(h) \prec 0$, which implies that $\Phi(\tau(t)) \prec 0$, since $\Phi(\tau(t))$ depends linearly on $\tau(t) \in [0, h)$. Hence, for any $t \in [t_r h, t_{r+1}h)$, the following inequality holds:

$$\dot{V}(t) \le 0. \tag{27}$$

Therefore, according to LaSalle's invariance principle in [22], it can be seen that $\xi = 0$ is the only asymptotically stable equilibrium point of the system (15). Hence, $q(t) \to q_d(t)$ as $t \to +\infty$. The proof is completed. ■

Remark 6: The above analysis shows that the ET mechanism (10) can effectively mitigate the unnecessary waste of resources while maintaining the desired tracking performance. The condition (17) in Theorem 1 is not rigorous, because the appropriate matrices Q, Z, Ψ, F_d can be easily found by using the linear matrix inequality toolbox in MATLAB. If the traditional time-triggered strategy is used, the controller does not need to satisfy the inequality (17), but the system still needs to satisfy the matching equation (16) since it is an underdriven system. In this way, although a not rigorous condition is reduced, the redundancy and waste of communication resources are inevitable.

Remark 7: Matching equation (16) is a linear partial differential equation, for which powerful solution techniques, in particular the method of characteristics, are available [19].

IV. AN ILLUSTRATIVE EXAMPLE

Consider a flexible-joints robot system which can be modeled as a Hamiltonian-based flexible-joints dynamic model as (1) with $n_l = n_m = 2$. Parameters of the robot are shown in Table I.

TABLE I
THE PARAMETER VALUES OF THE FLEXIBLE-JOINTS ROBOT.

Parameter	Description	Value
m_{l1}	Mass of link 1	1.510kg
m_{l2}	Mass of link 2	0.873kg
m_{m1}	Mass of motor 1	0.230kg
m_{m2}	Mass of motor 2	0.010kg
I_{l1}	Inertia of link 1	$0.0392\text{kg}\cdot\text{m}^2$
I_{l2}	Inertia of link 2	$0.00808\text{kg}\cdot\text{m}^2$
r_{l1}	Distance from the joint to the center of gravity of the link 1	0.159m
r_{l2}	Distance from the joint to the center of gravity of the link 2	$0.055\text{kg}\cdot\text{m}$
l_{l1}	Length of link 1	0.343m
l_{l2}	Length of link 2	0.267m

The link and motor inertia matrices are

$$M_l(q_l) = \begin{bmatrix} \beta_1+\beta_2+2\alpha\cos(q_{l2}) & \beta_2+\alpha\cos(q_{l2}) \\ \beta_2+\alpha\cos(q_{l2}) & \beta_2 \end{bmatrix}, \tag{28}$$

and

$$M_m(q_m) = \begin{bmatrix} m_{m1} & 0 \\ 0 & m_{m2} \end{bmatrix}, \tag{29}$$

where $\beta_1 = m_{l1}r_{l1}^2 + m_{l2}l_{l1}^2 + I_{l1}$, $\beta_2 = m_{l_2}r_{l2}^2 + I_{l2}$, $\alpha = m_{l2}l_{l1}r_{l2}$. The link damping matrix, motor damping matrix and stiffness coefficients matrix are

$$D_l = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.55 \end{bmatrix}, D_m = \begin{bmatrix} 0.2 & 0 \\ 0 & 90 \end{bmatrix}, K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{30}$$

The Hamiltonian function is

$$\mathcal{H}(q,p) = \frac{1}{2}p^{\mathrm{T}}M^{-1}(q)p + \frac{1}{2}(q-q_d)^{\mathrm{T}}(q-q_d) + \frac{1}{2}\rho^{\mathrm{T}}K\rho. \tag{31}$$

In the following, the desired equilibrium configuration is constructed by Hamiltonian function shaping and desired damping injection. In this process, the Hamiltonian-based ET controller is obtained and the condition of asymptotic tracking is realized.

Let $\xi = x - x_d = [q_\xi^{\mathrm{T}}, p_\xi^{\mathrm{T}}]^{\mathrm{T}}$ be state error, we specify expected Hamiltonian function is

$$\mathcal{H}_d(\xi) = \frac{1}{2}p_\xi^{\mathrm{T}}M_d^{-1}p_\xi + \frac{1}{2}q_\xi^{\mathrm{T}}q_\xi + \frac{1}{2}\rho_\xi^{\mathrm{T}}K_d\rho_\xi, \tag{32}$$

where $\rho_\xi = q_{m\xi} - q_{l\xi}$, $K_d = \mathrm{diag}\{1.5, 1.5\}$,

$$M_d = \begin{bmatrix} \beta_{d1}+\beta_{d2}+2\bar{\alpha} & \beta_{d2}+\bar{\alpha} & 0 & 0 \\ \beta_{d2}+\bar{\alpha} & \beta_{d2} & 0 & 0 \\ 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & \gamma_2 \end{bmatrix},$$

with $\beta_{d1} = m_{l1}r_{l1}^2 + \frac{1}{4}m_{l1}l_{l1}^2 + m_{l2}l_{l1}^2$, $\beta_{d2} = m_{l2}r_{l2}^2 + \frac{1}{4}m_{l2}l_{l2}^2$, $\bar{\alpha} = m_{l2}l_{l1}r_{l2}\cos(q_{l2})$, $\gamma_1 = \frac{1}{2}m_{m1}, \gamma_2 = 2m_{m2}$. Choose J_a and R_a as

$$J_a = \begin{bmatrix} 0 & \bar{J}_a \\ -\bar{J}_a & 0 \end{bmatrix}, R_a = \begin{bmatrix} 0 & 0 \\ 0 & \bar{R}_a \end{bmatrix},$$

where $\bar{J}_a = 0.53I_{4\times4}$, $\bar{R}_a = 0.79I_{4\times4}$. Then desired interconnection and damping matrices are

$$J_d = \begin{bmatrix} 0 & \bar{J}_d \\ -\bar{J}_d & 0 \end{bmatrix}, R_d = \begin{bmatrix} 0 & 0 \\ 0 & \bar{R}_d \end{bmatrix},$$

where $\bar{J}_d = 1.53I_{4\times4}$, $\bar{R}_d = D + \bar{R}_a$.

The sampling period h and the triggering threshold η are set to 0.08 and 0.1, respectively. From Theorem 1 we can get the following gain matrix and triggering weight matrix:

$$F_d = \begin{bmatrix} 6.46I_{4\times4} & 0 \\ 0 & 5.7I_{4\times4} \end{bmatrix}, \Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{bmatrix}, \tag{33}$$

where $\Phi_1 = \mathrm{diag}\{2.6789, 2.3574, 1.7743, 5.6170\}$, $\Phi_2 = \mathrm{diag}\{-0.2527, -0.288, -0.3461, -0.1831\}$, $\Phi_3 = \mathrm{diag}\{4.0879, 3.1496, 1.8626, 157.5559\}$.

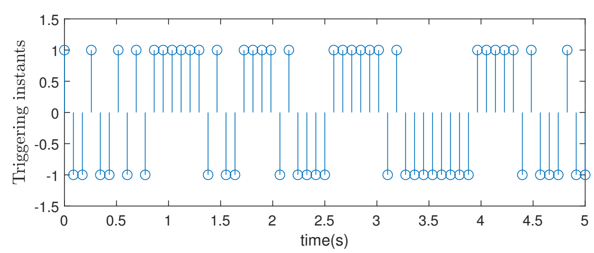

Fig. 2. Triggering instants.

Initial conditions are set as $q_l(0) = [1.6, 1.1]^{\mathrm{T}}$, $q_m(0) = [1.6, 1.1]^{\mathrm{T}}$, $\dot{q}_l = \dot{q}_m(0) = [0, 0]^{\mathrm{T}}$. We require to track the desired trajectory $q_d = [q_{dl}^{\mathrm{T}}, q_{dm}^{\mathrm{T}}]^{\mathrm{T}} = [\sin(3t), 2\cos(t), \sin(3t), 2\cos(t)]^{\mathrm{T}}$. Simulation results are given in Figs. 2-4. Figure 2 illustrates the response during trigger moments, where a vertical value of 1 indicates the trigger moment, otherwise, it is -1. In Fig. 3 and 4, the responses of the manipulator's actual position and the desired position are presented, comparing the proposed control strategy with that from [11]. The results clearly demonstrate the capability of the approach to enable q to closely track the desired trajectory q_d, all while significantly reducing data transmission requirements.

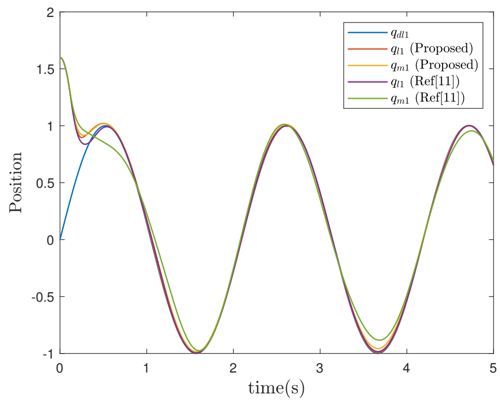

Fig. 3. Position trajectory for link 1 and motor 1.

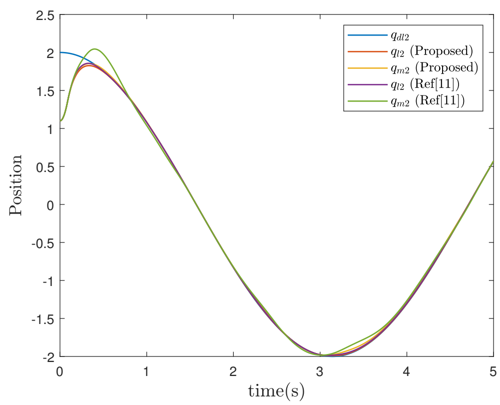

Fig. 4. Position trajectory for link 2 and motor 2.

V. Conclusion

In this paper, a Hamiltonian-based ET control scheme for flexible-joints robots has been proposed. Using this control scheme, asymptotic tracking to the desired reference trajectory can be guaranteed. The proposed controller is designed utilizing desired damping injection and desired energy shaping to ensure that the resulting closed-loop system maintains the structure of the Hamiltonian system. By introducing the Hamiltonian function into the ET mechanism, the ET mechanism becomes as modular as the Hamiltonian system, thus avoiding the need to deal with complex coupling problems. The resulting ET mechanism can easily avoid Zeno behavior because of the periodic character of the ET condition. Finally, the simulation result illustrates that the proposed control strategy can guarantee the asymptotic tracking of the robot while reducing the waste of transmission resources.

References

[1] S. Macenski, T. Foote, B. Gerkey, C. Lalancette, and W. Woodall, "Robot operating system 2: Design, architecture, and uses in the wild," *Science Robotics*, vol. 7, no. 66, p. 6074, 2022.

[2] Y. Hu, H. Su, J. Fu, H. R. Karimi, G. Ferrigno, E. D. Momi, and A. Knoll, "Nonlinear model predictive control for mobile medical robot using neural optimization," *IEEE Transactions on Industrial Electronics*, vol. 68, no. 12, pp. 12 636–12 645, 2021.

[3] P. Mesmer, M. Neubauer, A. Lechler, and A. Verl, "Robust design of independent joint control of industrial robots with secondary encoders," *Robotics and Computer-Integrated Manufacturing*, vol. 73, p. 102232, 2022.

[4] S. Ling, H. Wang, and P. X. Liu, "Adaptive fuzzy tracking control of flexible-joint robots based on command filtering," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 5, pp. 4046–4055, 2020.

[5] K. Rsetam, Z. Cao, and Z. Man, "Cascaded-extended-state-observer-based sliding-mode control for underactuated flexible joint robot," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 12, pp. 10 822–10 832, 2020.

[6] E. Spyrakos-Papastavridis and J. S. Dai, "Minimally model-based trajectory tracking and variable impedance control of flexible-joint robots," *IEEE Transactions on Industrial Electronics*, vol. 68, no. 7, pp. 6031–6041, 2021.

[7] R. Ortega, A. van der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.

[8] Y. Wang, D. Cheng, C. Li, and Y. Ge, "Dissipative hamiltonian realization and energy-based L_2-Disturbance attenuation control of multimachine power systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 8, pp. 1428–1433, 2003.

[9] R. Reyes-Báez, A. van der Schaft, and B. Jayawardhana, "Virtual contractivity-based control of fully-actuated mechanical systems in the port-Hamiltonian framework," *Automatica*, vol. 141, p. 110275, 2022.

[10] Y. Ren and W. Sun, "Robust adaptive control for robotic systems with input time-varying delay using Hamiltonian method," *IEEE/CAA Journal of Automatica Sinica*, vol. 5, no. 4, pp. 852–859, 2018.

[11] R. Reyes-Báez, A. van der Schaft, B. Jayawardhana, and L. Pan, "A family of virtual contraction based controllers for tracking of flexible-joints port-Hamiltonian robots: Theory and experiments," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 8, pp. 3269–3295, 2020.

[12] V.-T. Ngo and Y.-C. Liu, "Object transportation with force-sensorless control and event-triggered synchronization for networked uncertain manipulators," *IEEE Transactions on Industrial Electronics*, vol. 68, no. 1, pp. 902–912, 2021.

[13] S. Baek, H. Lee, and S. Han, "Communication-efficient event-triggered time-delay control and its application to robot manipulators," *IEEE Transactions on Industrial Electronics*, vol. 69, no. 9, pp. 9288–9297, 2022.

[14] P. B. G. Dohmann and S. Hirche, "Distributed control for cooperative manipulation with event-triggered communication," *IEEE Transactions on Robotics*, vol. 36, no. 4, pp. 1038–1052, 2020.

[15] T. Kargar Tasooji and H. J. Marquez, "Cooperative localization in mobile robots using event-triggered mechanism: Theory and experiments," *IEEE Transactions on Automation Science and Engineering*, vol. 19, no. 4, pp. 3246–3258, 2022.

[16] Q. Zhang, W. Sun, and C. Qiao, "Event-triggered stabilisation of switched nonlinear systems with actuator saturation: a Hamiltonian approach," *International Journal of Systems Science*, vol. 54, no. 4, pp. 849–866, 2023.

[17] M. W. Spong, "Modeling and control of elastic joint robots," *Journal of Dynamic Systems, Measurement, and Control*, vol. 109, no. 4, pp. 310–318, 1987.

[18] W. Sun, "Stabilization analysis of time-delay Hamiltonian systems in the presence of saturation," *Applied Mathematics and Computation*, vol. 217, no. 23, pp. 9625–9634, 2011.

[19] R. Ortega, A. van der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.

[20] H. Yu, J. Yu, J. Liu, and Q. Song, "Nonlinear control of induction motors based on state error pch and energy-shaping principle," *Nonlinear Dynamics*, vol. 72, pp. 49–59, 2013.

[21] R. Postoyan, A. Anta, W. Heemels, P. Tabuada, and D. Nešić, "Periodic event-triggered control for nonlinear systems," in *52nd IEEE Conference on Decision and Control*, 2013, pp. 7397–7402.

[22] H. K. Khalil, *Nonlinear systems third edition*. New Jersey: Patience Hall, 2002.