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Asymptotic Stabilization of Uncertain Systems with Prescribed Transient Response via Smooth Barrier Integral Control

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Abstract— **This paper considers the problem of *asymptotic* stabilization of a class of MIMO control-affine systems with *unknown* nonlinear terms subject to prescribed transient constraints. We propose a novel control methodology, Barrier Integral Control (BRIC), that achieves asymptotic results while complying with the aforementioned constraints. BRIC relies on a novel integration of reciprocal barrier functions, commonly used in funnel-constrained control, and error-integral terms. The proposed methodology does not use any information from the model's dynamic terms and, unlike previous works in the related literature, consists of *smooth* feedback. Theoretical guarantees are provided for three different classes of control-affine nonlinear systems, without adopting boundedness assumptions, growth conditions, or control-gain tuning. Simulation results verify the theoretical findings.**

I. INTRODUCTION

Control of systems with uncertain dynamics has been a central research focus for decades, with applications in areas like autonomous robots, self-driving cars, and biological networks, where systems face modelling uncertainties and unknown disturbances

Robust and adaptive control has been the primary approach for handling systems with uncertain dynamics [1]. However, most results offer "practical stability", where errors converge to a residual set near zero [2]–[4], with the set's size dependent on gain selection and system dynamics, requiring large gains to minimize. In contrast, asymptotic stability — where errors converge to zero — is more desirable but more challenging to achieve, particularly for systems with uncertain dynamics. Asymptotic stability guarantees often requires conservative assumptions like linear parametrizations of the dynamics, where the uncertainty is restricted to constant terms [5], gain tuning with known parts or bounds of the dynamics [6]–[8], growth conditions [6], or a priori available data [9]. Works avoiding these assumptions typically provide only local results, with the region of attraction limited by the system's dynamics [10].

An essential property of control systems is ensuring compliance with predefined trajectory specifications. Methods like Prescribed Performance Control (PPC) [11] and funnel control [12] guarantee the confinement of the tracking errors within a pre-specified funnel independently of control gains or the system dynamics. However, these approaches often fail to achieve asymptotic stability unless the funnel converges to zero [13], which can lead to numerical issues or impractically large inputs. Other methods that achieve asymptotic stability typically focus on linear systems [14], limit uncertainties to unknown parameters [15], or rely on neural-network approximation [16] that yield local results. The works [17]–[19] have removed these assumptions but require discontinuous control laws, which are not applicable in practice.

This paper considers the problem of asymptotic stabilization with predefined funnel-type transient specifications for control-affine nonlinear MIMO systems with entirely unknown dynamics. We introduce Barrier Integral Control (BRIC), which ensures that the system's stabilization error evolves within a predefined funnel and converges asymptotically to zero. BRIC is based on a novel integration of reciprocal barrier functions and the integral of the error's squared norm. It does not employ any information from the system's dynamics and does not require any gain tuning, growth/boundedness conditions or approximation schemes for the the dynamic terms. Furthermore and unlike [17]–[19], it is a *smooth*-feedback controller. The BRIC guarantees are established for three different kinds of nonlinear MIMO systems, based on the structure of the control-input matrix and the drift term. Such guarantees are practically global, in the sense that the initial conditions need only comply with the funnel-based transient specifications. Finally, we illustrate the effectiveness of BRIC via comparative simulations.

The rest of the paper is organized as follows. Section II provides notation and preliminary background, and Section III gives the main results of the paper. Section IV is devoted to simulation examples and Section V concludes the paper.

II. PRELIMINARIES

Notation: The sets of real, positive real, and non-negative real numbers are denoted by $\mathbb{R}$, $\mathbb{R}_{>0}$, and $\mathbb{R}_{\geq 0}$, respectively; $\|\cdot\|$ denotes the vector 2-norm. The derivative of a function $f:\mathbb{R}^n \to \mathbb{R}^m$ is denoted by f' and we use $\mathcal{N} := \{1,\dots,n\}$.

Lemma 1 ([20]): Let $x = [x_1,\dots,x_n]^\top \in \mathbb{R}^n$ and $\bar{x}_i = [x_1,\dots,x_i]^\top$ for $i \in \{1,2,\dots,n-1\}$. Further let $a(t)$ and $b(t)$ be continuous scalar functions, and $\phi:\mathbb{R}^n \to \mathbb{R}$ a continuous function satisfying $0 < a_m \leq \phi(x) \leq a_M$ for all $x \in \mathbb{R}^n$ and for positive constants a_m, a_M. Define $P(t) := \int_0^{a(t)} s\phi(\bar{x}_{n-1}, s+b(t))\mathrm{d}s$. Then it holds that

$$\frac{1}{2}a_m a(t)^2 \leq P(t) \leq \frac{1}{2}a_M a(t)^2 \tag{1a}$$

$$\dot{P}(t) = a(t)\phi(x)\dot{a}(t) - \dot{b}(t)a(t)\int_0^1 \phi(\bar{x}_{n-1}, \theta a(t)+b(t))\mathrm{d}\theta + a(t)\phi(x)\dot{b}(t) + a(t)^2\sum_{i=1}^{n-1}\dot{x}_i\int_0^1 \theta\frac{\partial\phi(\bar{x}_{n-1},\theta a(t)+b(t))}{\partial x_i}\mathrm{d}\theta. \tag{1b}$$

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Lemma 2: It holds that $\frac{1}{2}y\ln\left(\frac{1+y}{1-y}\right) \geq y^2$, $\forall y \in (-1,1)$.

Proof: Let $f : (-1,1) \to \mathbb{R}$, with $f(y) := \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right) - y$. The derivative of f is $f'(y) = \frac{1}{1-y^2} - 1$, which satisfies $f'(0) = 0$ and $f'(y) > 0$, for all $y \in (-1,1)\backslash\{0\}$, implying that $f(y)$ is increasing in $(-1,1)$. Hence, we conclude that $y \geq 0 \Rightarrow f(y) \geq f(0) = 0 \Rightarrow yf(y) \geq 0$ and $y \leq 0 \Rightarrow f(y) \leq f(0) = 0 \Rightarrow yf(y) \geq 0$. ■

III. MAIN RESULTS

A. Problem Formulation

Consider the nonlinear MIMO system

$$\dot{x}_i = f_i(x,t) + g_i(x_i)u_i, \ \forall i \in \mathcal{N}, \tag{2}$$

where $x_i, u_i \in \mathbb{R}$ are the system's ith state and input, respectively, and $f_i : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, $g_i : \mathbb{R} \to \mathbb{R}$ are *unknown* nonlinear functions, for all $i \in \mathcal{N}$. By defining $x := [x_1, \dots, x_n]^\top \in \mathbb{R}^n$, $u := [u_1, \dots, u_n]^\top \in \mathbb{R}^n$, and $f := [f_1, \dots, f_n] \in \mathbb{R}^n$, $g := \text{diag}\{g_1, \dots, g_n\} \in \mathbb{R}^{n\times n}$, we can write (2) as

$$\dot{x} = f(x,t) + g(x)u. \tag{3}$$

The problem we consider is the design of a *smooth* feedback control algorithm $u(x,t)$ that stabilizes *asymptotically* the state x to a configuration $x_\text{d} := [x_{\text{d}_1}, \dots, x_{\text{d}_n}]^\top \in \mathbb{R}^n$ despite the unknown terms $f(\cdot)$ and $g(\cdot)$, while at the same time establishing prescribed specifications on the evolution of $x(t)$. We consider the following assumptions:

Assumption 1: The maps $x \mapsto f(x,t) : \mathbb{R}^n \to \mathbb{R}^n$ and $x \mapsto g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $i \in \mathcal{N}$, are locally Lipschitz for each fixed $t \geq 0$, and the map $t \mapsto f(x,t) : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is continuous and bounded for each fixed $x \in \mathbb{R}^n$.

Assumption 2: There exist positive constants $\underline{g}_i$, $\bar{g}_i$ such that $0 < \underline{g}_i \leq g_i(x_i) \leq \bar{g}_i < \infty$, for all $x_i \in \mathbb{R}$ and $i \in \mathcal{N}$.

Assumption 3: It holds that $f(x_\text{d}, t) = 0$ for all $t \geq 0$.

Assumption 1 provides mild regularity conditions for the solution of (2). Assumption 2 is a sufficient controllability condition, also encountered in numerous works (e.g., [11], [13]; the constants $\underline{g}_i$ and $\bar{g}_i$ are assumed to be unknown. Finally, Assumption 3 is necessary for (3) to have an equilibrium point at x_d. The diagonality of $g(x)$ and Assumpt. 3 are relaxed in Sections III-C and III-D, respectively.

As mentioned before, the control objective is the design of a smooth control algorithm that achieves *asymptotic* stabilization to x_d. Moreover, as discussed in Section I, we aim at imposing a certain predefined behavior for the transient response of the system. More specifically, motivated by funnel control techniques, given n predefined funnels, described by the smooth functions (also called performance functions in [11]) $\rho_i : \mathbb{R}_{\geq 0} \to [\underline{\rho}_i, \bar{\rho}_i] \subset \mathbb{R}_{>0}$, where $\underline{\rho}_i$, $\bar{\rho}_i \in \mathbb{R}_{>0}$ are positive lower and upper bounds, respectively, we aim at guaranteeing that $-\rho_i(t) < x_i(t) - x_{\text{d}_i} < \rho_i(t)$, for all $t \geq 0$, given that $-\rho_i(0) < x_i(0) - x_{\text{d}_i} < \rho_i(0)$, $i \in \mathcal{N}$. We further require $\rho_i(t)$ to have bounded first derivatives $\dot{\rho}_i$, $i \in \mathcal{N}$. These functions can encode maximum overshoot or convergence rate properties. Note that, compared to the majority of the related works on funnel control (e.g., [11]–[13]), we do not require arbitrarily small final values $\lim_{t\to\infty}\rho_i(t)$, which would achieve convergence of $x_i(t) - x_{\text{d}_i}$ arbitrarily close to zero, since one of the objectives is actual *asymptotic stability*. Formally, the problem statement is as follows:

Problem 1: Consider the system (2) and let $x_\text{d} \in \mathbb{R}^n$ as well as n prescribed funnels, described by $\rho_i(t)$, $\forall i \in \mathcal{N}$, and satisfying $-\rho_i(0) > x_i(0) - x_{i,\text{d}} > \rho_i(0)$, $i \in \mathcal{N}$. Design a *smooth* control protocol $u : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that

1) $\lim_{t\to\infty}(x_i(t) - x_{\text{d}_i}) = 0$,
2) $-\rho_i(t) < x_i(t) - x_{\text{d}_i} < \rho_i(t)$, $\forall t \geq 0$,

for all $i \in \mathcal{N}$, and all closed loop signals remain bounded.

B. Barrier Integral Control

We introduce Barrier Integral Control (BRIC), which consists of two main elements: A reciprocal barrier term, which establishes the boundedness of $x(t)$ within the $\rho_i(t)$ funnel, and an integral term that guarantees asymptotic convergence to x_d. BRIC is considered to establish semi-global results, since its region of attraction is constrained in the set $(-\rho_1(0), -\rho_1(0)) \times \cdots \times (-\rho_n(0), -\rho_n(0))$. However, we claim that it achieves "practically global" results since, given the initial error $x_i(t) - x_{\text{d}_i}$, one can always choose $\rho_i(t)$ such that $\rho_i(0) > |x_i(t) - x_{\text{d}_i}|$, without relying on the system dynamics or necessitating tuning of the control gains.

We begin by defining the stabilization and normalized errors, where we denote $\rho := \text{diag}\{\rho_1, \dots, \rho_n\}$,

$$e := [e_1, \dots, e_n]^\top := x - x_\text{d} \tag{4a}$$

$$\xi := [\xi_1, \dots, \xi_n]^\top := \rho(t)^{-1}e, \tag{4b}$$

Consider now the transformation $\chi := [\chi_1, \dots, \chi_N]^\top$, with

$$\chi_i(\xi) := \mathbb{T}(\xi_i), \tag{5}$$

for $i \in \mathcal{N}$, where $\mathbb{T} : (-1,1) \to \mathbb{R}$ is the smooth, increasing map $\mathbb{T}(*) := \frac{1}{2}\ln\left(\frac{1+*}{1-*}\right)$, with inverse $\mathbb{T}^{-1}(*) = \tanh(*)$.

We now design the BRIC law as

$$u = -k_1\chi(\xi) - k_2\int_0^t \left\|e(\tau)\right\|^2 \text{d}\tau \ \chi(\xi) \tag{6}$$

where k_1 and k_2 are positive constant gains.

The first term of (6) constitutes the barrier term that aims to establish the boundedness of $x(t)$ and retain the errors e_i within the prescribed funnel formed by ρ_i, $i \in \mathcal{N}$. The second term of (6) aims to compensate for the uncertainties in $f(\cdot)$ and $g(\cdot)$ and accomplish $\lim_{t\to\infty} e(t) = 0$.

Remark 1: The control law (6) does not involve any information from $f(\cdot)$ and $g(\cdot)$ or any approximation schemes. The main difference with respect to standard funnel-type controllers [11], [12] is the incorporation of the integral term $\int_0^t \|e(\tau)\|^2 \text{d}\tau$, which enforces asymptotic convergence of the errors to zero. A similar term was used in our previous work [18], coupled, however, with a discontinuous signal, and in the PI funnel control of [21], without, however, accommodating the uncertainty considered here. Finally, the proposed control algorithm can further handle stable *internal* dynamics in (3), by following the techniques of [18].

The theoretical properties of (6) are given as follows.

Theorem 1: Consider system (3), Assumption 1-3, and functions $\rho_i(t)$ satisfying $-\rho_i(0) < e_i(0) < \rho_i(0)$ for all $i \in \mathcal{N}$. The control law (6) guarantees that $-\rho_i(t) < e_i(t) < \rho_i(t)$, for all $t \geq 0$, and $\lim_{t\to\infty} e(t) = 0$, as well as the boundedness of all closed-loop signals, for all $t \geq 0$.

Proof: The proof proceeds in two parts; first, we establish that $x_i(t) - x_{\mathrm{d}_i}$ evolves within $(-\rho_i(t), \rho_i(t))$, which implies the boundedness of $x(t)$; second, we show the asymptotic convergence of $e(t)$ to zero.

We first compute the dynamics of the error $e = x - x_{\mathrm{d}}$:

$$\dot{e} = f(e + x_{\mathrm{d}}, t) + g(e + x_{\mathrm{d}})u =: f_e(e,t) + g_e(e)u, \quad (7)$$

with $f_e(e,t) = [f_{e_1}(e,t), \dots, f_{e_n}(e,t)]^\top$, $g_e(e,t) = \mathrm{diag}\{g_{e_1}(e_1), \dots, g_{e_n}(e_n)\}$, and $f_{e_i}(e,t) = f_i(e + x_{\mathrm{d}}, t)$, $g_{e_i}(e_i) = g_i(e_i + x_{\mathrm{d}_i})$, for all $i \in \mathcal{N}$.

A) Funnel confinement: Consider first the open set $\Omega := (-1,1)^n$. Since $|e_i(0)| < \rho_i(0)$ for all $i \in \{1, \dots, n\}$, Ω is non-empty. Furthermore, by differentiating (4b) and using (3) and (6), we obtain $\dot{\xi} = f_\xi(\xi, t)$, where $f_\xi : (-1,1)^n \times \mathbb{R}_{\geq 0}$ is is a locally Lipschitz map in ξ in Ω for each fixed $t \geq 0$ and continuous in t in $\mathbb{R}_{\geq 0}$ for each fixed $\xi \in \Omega$. Therefore, according to [22, Theorem 54], we conclude the existence of a unique maximal solution $\xi : [0, t_{\max}) \to \Omega$ that satisfies $\xi(t) \in \Omega$, for all $t \in [0, t_{\max})$, where $t_{\max}$ is a positive constant. Note that, for all $t \in [0, t_{\max})$,

$$\|e(t)\| \leq \sqrt{n} \max_{i\in\mathcal{N}} \bar{\rho}_i. \quad (8)$$

Consider now the positive definite function $W(\chi) := \frac{1}{2}\|\chi\|^2 = \frac{1}{2}\sum_{i\in\mathcal{N}} \chi_i^2$, which is well-defined for $t \in [0, t_{\max})$. Differentiating W and substituting (6) yields

$$\begin{aligned}\dot{W} = &-\sum_{i\in\mathcal{N}} \left(k_1 + k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau\right) \frac{J_{T_i}(\xi_i)}{\rho_i(t)} g_{e_i}(e_i)\chi_i(\xi_i)^2 \\ &+ \sum_{i\in\mathcal{N}} \chi_i(\xi_i) \frac{J_{T_i}(\xi_i)}{\rho_i(t)} \left(f_{e_i}(e_i,t)) - \dot{\rho}_i(t)\xi_i\right) \quad (9)\end{aligned}$$

where $J_{T_i}(\xi_i) := \frac{\mathrm{d}\chi_i(\xi_i)}{\mathrm{d}\xi_i} = \frac{1}{1-\xi_i^2} \geq 1$, for $t \in [0, t_{\max})$, $i \in \mathcal{N}$. Further, we observe that $|f_{e_i}(e_i(t),t) - \dot{\rho}_i(t)\xi_i(t)| \leq \bar{F}$, for all $i \in \mathcal{N}$, due to Assumption 1, (8), and the boundedness of $\dot{\rho}_i(t)$. Note that $\bar{F}$ is independent of $t_{\max}$. By further noting that $\int_0^t \|e(\tau)\|^2 \mathrm{d}\tau \geq 0$, $g_{e_i} > 0$, $J_{T_i}(\xi_i) > 0$, $\rho_i(t) > 0$, $i \in \mathcal{N}$, and completing the squares, we obtain

$$\dot{W} \leq -\sum_{i\in\mathcal{N}} R_i(\xi_i, t) \left\{\kappa \chi_i(\xi_i)^2 - \frac{\bar{F}}{2\alpha}\right\},$$

for all $t \in [0, t_{\max})$, where $\underline{g} := \min_{i\in\mathcal{N}}\{\underline{g}_i\}$, α is a positive constant satisfying $\alpha < 2\underline{g}k_1$, $R_i(\xi_i, t) := \frac{J_{T_i}(\xi_i)}{\rho_i(t)} > 0$, $i \in \mathcal{N}$, and $\kappa := \underline{g}\left(k_1 - \frac{\alpha}{2\underline{g}}\right) > 0$. Therefore, we conclude that $\dot{W} < 0$ when $|\chi_i(\xi)| > \sqrt{\frac{\bar{F}}{2\alpha\kappa}}$ for at least one $i \in \mathcal{N}$. Hence, we conclude that $\chi(\xi(t))$ is bounded as $\|\chi(\xi(t))\| \leq \bar{\chi}$, for all $t \in [0, t_{\max})$, where $\bar{\chi}$ is a positive constant. Consequently, by inverting $\mathbb{T}$, as defined in (5), we obtain that $|\xi_i(t)| = |\tanh(\chi_i)| \leq \bar{\xi} := |\tanh(\bar{\chi})| < 1$, for all $i \in \mathcal{N}$ and $t \in [0, t_{\max})$. Finally, note that $\xi(t) \in \Omega' := [-\bar{\xi}, \bar{\xi}]^n \subset (-1,1)^n$ for all $t \in [0, t_{\max})$. Therefore, [22, Prop. C.3.6] dictates that $t_{\max} = \infty$. Hence, it holds that $-\rho_i(t) < e_i(t) = x_i(t) - x_{\mathrm{d}_i} < \rho_i(t)$, for $t \geq 0$ and $i \in \mathcal{N}$.

B) Asymptotic convergence: We define first $h_e(e,t) := g_e(e)^{-1} f_e(e,t)$. From the previous analysis, we obtain that $e(t) \in (-\max_{i\in\mathcal{N}}\{\bar{\rho}_i\}, \max_{i\in\mathcal{N}}\{\bar{\rho}_i\})^n$, for all $t \geq 0$. Hence, by further using the Lipschitz property and time-boundedness from Assumption 1, the positive definiteness of $g_e(\cdot)$ from Assumption 2, and the fact that $f(x_{\mathrm{d}}, t) = 0$ - and hence $h_e(0,t) = 0$ - from Assumption 3, we obtain

$$\|h_e(e(t),t)\| = \|h_e(e(t),t) - h_e(0,t)\| \leq L\|e(t)\|, \quad (10)$$

for all $t \geq 0$ and a positive constant L. Additionally, let $\widetilde{\ell}(t) := k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau - L$ and consider the function

$$V(e, \widetilde{\ell}) = \sum_{i\in\mathcal{N}} \int_0^{e_i} \frac{1}{g_{e_i}(\mathsf{s})} \mathsf{s} \; \mathsf{ds} + \frac{1}{2k_2}\widetilde{\ell}^2$$

In view of Assumption 2, it holds that $0 < g_m \leq \frac{1}{g_{e_i}(e)} \leq g_M$ for some positive constants g_m and g_M and all $i \in \mathcal{N}$. Hence, according to Lemma 1, it holds that $\frac{1}{2}g_m e_i^2 \leq \int_0^{e_i} \frac{1}{g_{e_i}(\mathsf{s})} s \leq \frac{1}{2}g_M e_i^2$, for all $i \in \mathcal{N}$, and therefore

$$\frac{1}{2}g_m\|e\|^2 + \frac{1}{2k_2}\widetilde{\ell}^2 \leq V(e, \widetilde{\ell}) \leq \frac{1}{2}g_M\|e\|^2 + \frac{1}{2k_2}\widetilde{\ell}^2$$

By differentiating V, using Lemma 1 with $b(t) = 0$ and $a(t) = e_i$, and using (7), (10), and (6), we obtain

$$\dot{V} \leq L\|e\|^2 - k_1 e^\top \chi(\xi) - k_2 \int_0^t \|e(\tau)\|^2 \; \mathrm{d}\tau \; e^\top \chi(\xi) + \widetilde{\ell}\|e\|^2.$$

Next, in view of (4b), it holds that $e_i\chi_i(\xi_i) = \rho_i(t)\xi_i\chi_i(\xi_i)$, which, according to (5), Lemma 2 and the positiveness of $\rho_i(t)$, is greater than or equal to $\rho_i(t)\xi_i^2 = \rho_i(t)^{-1}e_i^2$, $i \in \mathcal{N}$. Therefore it holds that $-e^\top\chi(\xi) \leq -\widetilde{\rho}\|e\|^2$ and $\dot{V}$ becomes

$$\dot{V} \leq -k_1\widetilde{\rho}\|e\|^2 - \left(k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau - L\right)\|e\|^2 + \widetilde{\ell}\|e\|^2$$

where $\widetilde{\rho} := \min_{i\in\mathcal{N}}\{\bar{\rho}_i^{-1}\}$. Finally, by using $\widetilde{\ell}(t) := k_2 \int_0^t \|e(\tau)\|^2 - L$, $\dot{V}$ becomes $\dot{V} \leq -k_1\|e\|^2 \leq 0$, i.e., V is non-increasing and that $V(e(t), \widetilde{\ell}(t)) \leq V(e(0), \widetilde{\ell}(0))$, for all $t \geq 0$, implying that V has a finite limit as $t \to \infty$. Further note that $\widetilde{\ell}(t)$ and $\int_0^t \|e(\tau)\|^2$ remains bounded for all $t \geq 0$. By differentiating $\dot{V}$ and using the boundedness of $\chi(t)$, $\widetilde{\ell}(t)$, we conclude that $\ddot{V}(e(t), \widetilde{\ell}(t))$ remains bounded for all $t \geq 0$, which implies the uniform continuity of $\dot{V}$. Therefore, by invoking Barbalat's Lemma ([23, Lemma 8.2]), we conclude that $\lim_{t\to\infty} \dot{V}(e(t), \widetilde{\ell}(t)) = 0$, which implies that $\lim_{t\to\infty} e(t) = 0$, leading to the conclusion of the proof. ■

C. Extension to non-diagonal g(x)

In this section, we extend the results to systems of the form (3) but with a more general control-input matrix $g(x)$, not requiring it to be diagonal. We do need, however, the following additional assumptions concerning $g(x)$ and the vector $q(x) := [q_1(x), \dots, q_n(x)]^\top := g(x)^{-1}x$:

Assumption 4: The matrix $g(x)$ is symmetric and there exist constants $\underline{g}$ and $\bar{g}$ such that $0 < \underline{g}I_n \leq g(x) \leq \bar{g}I_n < \infty$, for all $x \in \bar{\mathbb{R}}^n$.

Assumption 5: It holds that, for all $i, j \in \mathcal{N}$ with $i \neq j$,

$$\frac{\partial q_i(x)}{\partial x_j} = \frac{\partial q_j(x)}{\partial x_i}. \quad (11)$$

Assumption 4 is equivalent to Assumption 2. Assumption 5 implies that $q(x)$ is the gradient of a scalar function, which is needed in the subsequent analysis. We note that such a condition is only sufficient, and not necessary, as verified by the simulations of Sec. IV.

The control law is designed similar to (6), but with the additional gains $J_{T_i}(\xi_i)$ and $\rho(t)^{-1}$:

$$u = -k_1 R(\xi, t)\chi(\xi) - k_2 \int_0^t \left\|e(\tau)\right\|^2 \mathrm{d}\tau \; R(\xi, t)\chi(\xi) \quad (12)$$

where k_1 and k_2 are positive constant gains, and $R(\xi, t) := \rho(t)^{-1} J_T(\xi)$, where $J_T(\xi) := \mathrm{diag}\{[J_{T_i}(\xi_i)]_{i\in\{1,\ldots,n\}}\}$, and $J_{T_i}(\xi_i) = \frac{\mathrm{d}\chi_i(\xi_i)}{\mathrm{d}\xi_i} = \frac{1}{1-\xi_i^2}$ for all $i \in \mathcal{N}$, as defined in (9). We are now ready to state the main results of this section.

Theorem 2: Consider system (3) with a non-diagonal $g(x)$, Assumptions 1, 3, 4, and 5, and functions $\rho_i(t)$ satisfying $-\rho_i(0) < e_i(0) < \rho_i(0)$, $i \in \mathcal{N}$. The control law (12) guarantees $-\rho_i(t) < e_i(t) < \rho_i(t)$, $t \geq 0$, $\lim_{t\to\infty} e(t) = 0$, and boundedness of all closed-loop signals.

Proof: We follow similar steps as in Th. 1.

A) Funnel confinement: Following the proof of Theorem 1, we conclude the existence of a unique maximal solution $\xi(t) \in (-1, 1)^n$ for all $[0, t_{\max})$, implying (8) for $[0, t_{\max})$. By differentiating $W(\chi) = \frac{1}{2}\|\chi\|^2$, we obtain

$$\dot{W} \leq -\left(k_1 + k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau\right) \chi(\xi)^\top R(\xi, t) g(x) R(\xi, t)\chi(\xi) + \|J_T(\xi)\chi(\xi)\|\bar{F}$$

for all $t \in [0, t_{\max})$, where $\bar{F}$ is a positive constant, independent of $t_{\max}$, satisfying $\bar{F} \geq \max_{i\in\mathcal{N}}\{\underline{\rho}_i^{-1}\}\|(f(x(t), t) - \dot{\rho}(t)\xi)\|$, $t \in [0, t_{\max})$. By further using Assumption 4 and $\int_0^t \|e(\tau)\|\mathrm{d}\tau \geq 0$, we obtain

$$\dot{W} \leq -k_1 \underline{g}\widetilde{\rho}\|J_T(\xi)\chi(\xi)\|^2 + \|J_T(\xi)\chi(\xi)\|\bar{F}$$

for all $t \in [0, t_{\max})$, where $\widetilde{\rho} = \min_{i\in\mathcal{N}}\{\bar{\rho}_i^{-1}\}$. Hence, we conclude that $\dot{W} < 0$ when $\|J_T(\xi)\chi(\xi)\| \geq \frac{\bar{F}}{k_1 \underline{g}\widetilde{\rho}}$. Therefore, since $J_{T_i}(\xi_i) \geq 1$, for all $t \in [0, t_{\max})$ and $i \in \mathcal{N}$, we conclude the boundedness of $\chi(\xi(t))$ as $\|\chi(\xi(t))\| \leq \bar{\chi}$, where $\bar{\chi}$ is a positive constant. By following similar steps with the proof of Theorem 1, we conclude that $t_{\max} = \infty$ as well as $-\rho_i(t) < e_i(t) < \rho_i(t)$, for all $t \geq 0$ and $i \in \mathcal{N}$.

Similar to the proof of Theorem 1, we define $h_e(e, t) := g_e(e)^{-1} f_e(e, t)$, where $f_e(e, t) = f(e + x_\mathrm{d}, t)$ and $g_e(e) = g(e + x_\mathrm{d})$. The function $h_e(e, t)$ satisfies $\|h_e(e(t), t)\| \leq L\|e(t)\|$, for all $t \geq 0$ and a positive constant L, as shown in (10). Additionally, let $\widetilde{\ell}(t) := k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau - \widetilde{\rho}^{-2} L$. Next, it can be verified that (11) holds for $q_e(e) := g_e(e)^{-1} e$ as well. Therefore, in view of Assumption 5, the term $q_e(e)$ is the gradient of a scalar function [23, Example 4.4], i.e., there exists a function $V_g : \mathbb{R}^n \to \mathbb{R}$ that satisfies $\mathrm{d}V_g(e)/\mathrm{d}e = g(e)^{-1} e$, i.e., $V_g(e) = \int_0^e g_e(s)^{-1} s \mathrm{d}s$. Since $g_e(e)^{-1} e$ is a gradient vector, its line integral from 0 to e is independent of the path [24]. Let such a parametrized path be $\gamma : [0, 1] \to \mathbb{R}^n$ with $\gamma(*) = *e$, such that $\gamma(0) = 0$ and $\gamma(1) = e$. Then, V_g can be written as

$$V_g = \int_0^1 \gamma(\tau)^\top g_e(\gamma(\tau))^{-1} \gamma(\tau)' \mathrm{d}\tau = \int_0^1 \tau e^\top g_e(\tau e)^{-1} e \; \mathrm{d}\tau$$

In view of Assumption 4, it holds that $\bar{g}^{-1}\|e\|^2 \leq e(t)^\top g_e(e(t))^{-1} e(t) \leq \underline{g}^{-1}\|e\|^2$ and hence $\frac{1}{2\bar{g}}\|e\|^2 \leq V_g(e) \leq \frac{1}{2\underline{g}}\|e\|^2$. Consider now the positive definite function

$$V = \frac{V_g(e)}{\widetilde{\rho}^2} + \frac{1}{2k_2}\widetilde{\ell}^2$$

where $\widetilde{\rho} = \min_{i\in\mathcal{N}}\{\bar{\rho}_i^{-1}\}$, as defined before. Differentiation of V and use of (7) and (10) yields

$$\dot{V} \leq \widetilde{\rho}^{-2} L\|e\|^2 - \widetilde{\rho}^{-2} k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau \; e^\top R(\xi, t)\chi(\xi) + \widetilde{\ell}\|e\|^2 - k_1 \widetilde{\rho}^{-2} e^\top R(\xi, t)\chi(\xi)$$

In view of Lemma 2 and the positiveness of $\rho_i(t)$, it holds that $e_i \chi_i(\xi_i) = \rho_i(t)\xi_i \chi_i(\chi_i) \geq \rho_i(t)\xi_i^2 = \rho_i(t)^{-1} e_i^2$ for all $i \in \mathcal{N}$. Since $J_{T_i}(\xi_i) \geq 1$, we conclude that $J_{T_i}(\xi_i) e_i \chi_i(\xi_i) \geq e_i \chi_i(\xi_i) \geq \rho_i(t)^{-1} e_i^2$ and hence $\rho_i(t)^{-1} J_{T_i}(\xi_i) e_i \chi_i(\xi_i) \geq \rho_i(t)^{-2} e_i^2 \geq \min_{i\in\mathcal{N}}\{\bar{\rho}_i^{-2}\} e_i^2$, for all $i \in \mathcal{N}$. Hence, we conclude that $e^\top R(\xi, t)\chi(\xi) \geq \widetilde{\rho}^2 \|e\|^2$. Therefore, since $\int_0^t \|e(\tau)\|^2 \mathrm{d}\tau \geq 0$, $\dot{V}$ becomes

$$\dot{V} \leq \widetilde{\rho}^{-2} L\|e\|^2 - k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau \|e\|^2 + \widetilde{\ell}\|e\|^2 - k_1\|e\|^2$$

which, by employing $\widetilde{\ell}(t) := k_2 \int_0^t \|e(\tau)\|^2 \mathrm{d}\tau - \widetilde{\rho}^{-2} L$, finally becomes $\dot{V} \leq -k_1\|e\|^2$. By following similar steps as in the proof of Theorem 1, we conclude that $\lim_{t\to\infty} e(t) = 0$ and the boundedness of all closed-loop signals, for all $t \geq 0$. ■

D. Extension to non-zero $f(x_\mathrm{d}, t)$

We now remove Assumption 3, considering a non-zero $f(x_\mathrm{d}, t)$, requiring it, however, to be constant.

Assumption 6: It holds that $f(x_\mathrm{d}, t) = \mathrm{const}$ for all $t \geq 0$. Furthermore, we relax the transient constraints to encode boundedness by constant terms $\rho_i = \mathrm{const}$, for all $i \in \mathcal{N}$.

The control law is now designed as

$$u = -k_1\chi(\xi) - k_{\ell_1}\left(\int_0^t \|R(\xi(\tau))\chi(\xi(\tau))\|^2 \mathrm{d}\tau\right) R(\xi)\chi(\xi) - k_{\ell_2} \int_0^t R(\xi(\tau))\chi(\xi(\tau))\mathrm{d}\tau \quad (13)$$

where k_1 and k_{ℓ_1}, and k_{ℓ_2} are positive constant gains, and $R(\xi) := \rho^{-1} J_T(\xi)$, as defined in (12). The stabilization properties of (13) are given in the following theorem.

Theorem 3: Consider system (3), Assumptions 1, 2, and 6, and constants ρ_i satisfying $-\rho_i < e_i(0) < \rho_i$ for $i \in \mathcal{N}$. Then, the control law (13) guarantees that $-\rho_i < e_i(t) < \rho_i$, for all $t \geq 0$, and $\lim_{t\to\infty} e(t) = 0$, as well as the boundedness of all closed-loop signals, for all $t \geq 0$.

Proof: First, by following similar steps as in the proofs of Theorems 1 and 2, we conclude the existence of a unique maximal solution $\xi(t) \in (-1,1)^n$, i.e., $|e_i(t)| < \rho_i$ for all $i \in \mathcal{N}$ and $t \in [0, t_{\max})$, where $t_{\max}$ is a positive constant.

Similar to the proof of Theorem 1, we use $h_e(e,t) := g_e(e)^{-1} f_e(e,t)$, with $f_e(e,t) = f(e + x_{\mathrm{d}}, t)$ and $g_e(e) = g(e + x_{\mathrm{d}})$. In view of Assumptions 1, 2, 6, and the fact that $|e_i(t)| < \rho_i$ for $i \in \mathcal{N}$ and $t \in [0, t_{\max})$, $h_e(e,t)$ satisfies

$$\|h_e(e(t),t) - h_e(0,t)\| \leq L\|e(t)\| \tag{14}$$

for $t \in [0, t_{\max})$ and a positive constant L independent of $t_{\max}$. In view of Lemma 2 and by following similar steps with the proofs of Th. 1 and 2, we conclude that $\|e(t)\| \leq \max_{i\in\mathcal{N}}\{\bar{\rho}^2\}\|R(\xi(t))\chi(\xi(t))\|$, for $t \in [0, t_{\max})$. Further let

$$\widetilde{\ell}_1(t) := k_{\ell_1} \int_0^t \|R(\xi(\tau))\chi(\xi(\tau))\|^2 \mathrm{d}\tau - \max_{i\in\mathcal{N}}\{\bar{\rho}^2\} L \tag{15a}$$

$$\widetilde{\ell}_2(t) := k_{\ell_2} \int_0^t R(\xi(\tau))\chi(\xi(\tau)) \mathrm{d}\tau - h_e(0,t) \tag{15b}$$

Note that $h_e(0,t) = g(x_{\mathrm{d}})^{-1} f(x_{\mathrm{d}}, t)$ is constant due to Assumption 6. Let now the function $V(\chi, \ell_1, \ell_2, t)$, with

$$V = \sum_{i\in\mathcal{N}} \int_0^{\chi_i} \frac{1}{g_{e_i}(\rho_i \tanh(\mathsf{s}))} \mathsf{s}\mathrm{d}\mathsf{s} + \frac{1}{2k_{\ell_1}} \widetilde{\ell}_1^2 + \frac{1}{2k_{\ell_2}} \|\widetilde{\ell}_2\|^2$$

Setting $\mathsf{s} = \theta\chi_i$ and using (5) and Assumption 2 yields

$$\int_0^{\chi_i} \frac{1}{g_{e_i}(\rho_i \tanh(\mathsf{s}))} \mathsf{s}\mathrm{d}\mathsf{s} = \chi_i^2 \int_0^1 \frac{1}{g_{e_i}(e_i)} \mathrm{d}\theta \in \left[\frac{1}{2\bar{g}_i}\chi_i^2, \frac{1}{2\underline{g}_i}\chi_i^2\right],$$

for all $i \in \mathcal{N}$, implying the positive definiteness of $V(\chi, \ell_1, \ell_2, t)$. Differentiation of V leads to

$$\dot{V} = \chi(\xi)^\top R(\xi)(h_e(e,t) + u) + \widetilde{\ell}_1 \|R(\xi), \chi(\xi)\|^2 + \widetilde{\ell}_2^\top R(\xi)\chi(\xi)$$

By adding and subtracting $\chi(\xi)^\top R(\xi) h_e(0,t)$ and using (13), (14), and (15) and the fact that $R(\xi) \geq \min_{i\in\mathcal{N}}\{\rho_i^{-1}\} I_n$, it can be concluded that, for $t \in [0, t_{\max})$,

$$\dot{V} \leq -k_1 \chi(\xi) R(\xi)\chi(\xi) \leq -k_1 \min_{i\in\mathcal{N}}\{\rho_i^{-1}\}\|\chi(\xi)\|^2$$

Therefore, $\|\chi(t)\|$, $|\widetilde{\ell}_1(t)|$ and $\|\widetilde{\ell}_2(t)\|$ are bounded by $V_0 := V(\chi(\xi(0), \ell_1(0), \ell_2(0), 0)$, implying the boundedness of $u(t)$ for $t \in [0, t_{\max})$. Hence, $|e_i(t)| \leq \rho_i \tanh(V_0) < \rho_i$ for $t \in [0, t_{\max})$ and $i \in \mathcal{N}$, which means that $\xi(t)$ evolves in a compact subset of $(-1,1)^n$, leading to the conclusion that $t_{\max} = \infty$. Finally, by using similar arguments as in the proof of Theorem 1, we conclude via Barbalat's Lemma [23, Lemma 8.2] that $\lim_{t\to\infty} \chi(\xi(t)) = 0$ and $\lim_{t\to\infty} e(t) = 0$. ■

IV. Simulation Results

We present three simulation examples to illustrate the control design of Sec. III-B, III-C, and III-D, respectively.

Case I: We consider first a system of the form (2) with $n = 2$, $x(0) = [0,0]^\top$, $x_{\mathrm{d}} = [4,-5]^\top$, and

$$f(x,t) = \begin{bmatrix} e_2 e_1^2 + \cos(t)\|e\|^2 \\ \sin(t) e_2^2 - (2 + \sin(t - \pi/5)) e_1 e_2 \end{bmatrix}$$

$$g_i(x_i) = x_i^2 + 1, \quad i \in \{1,2\}$$

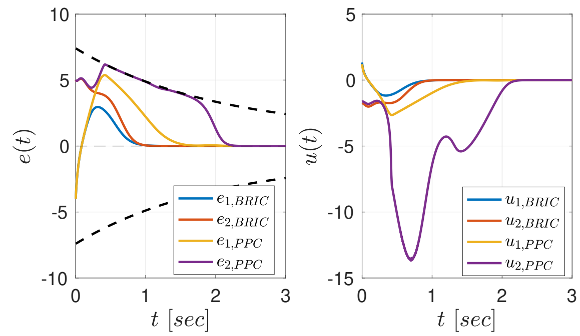

Fig. 1. Simulation results for Case I for BRIC and PPC; Left: error $e(t)$, along with funnels $\pm\rho_i(t)$ (dashed black lines); Right: control input $u(t)$.

We impose the transient behaviour dictated by $\rho_1(t) = \rho_2(t) = \|e(0)\| \exp^{-0.5t} + 1$. We apply the BRIC control law (6) with $k_1 = 1$, $k_2 = 0.1$ and we further compare with the standard PPC $u = -k_1\chi(\xi)$. The results are depicted in Fig. 1 for 3 seconds, depicting $e(t)$ and $u(t)$ for both BRIC and PPC. Note that $e(t)$ converges to zero while remaining inside the funnel in both schemes. However, the PPC performance is significantly deteriorated since $e_2(t)$ approaches the upper funnel boundary causing a spike in $u_2(t)$.

Case II: Next, we consider a robotic manipulator with 2 rotational degrees of freedom $q = [q_1, q_2]^\top \in [-\pi, \pi]^2$, $q(0) = [0,0]^\top$, and Lagrangian dynamics $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u$, where $M(q) = [M_{ij}(q)]_{i,j\in\{1,2\}} \in \mathbb{R}^{2\times 2}$ is the positive definite inertia matrix

$$M_{11} = I_{Z_1} + I_{Z_2} + m_1 \frac{l_1^2}{4} + m_2\left(l_1^2 + \frac{l_2^2}{4} + l_1 l_2 c_2\right)$$

$$M_{12} = M_{21} = I_{Z_2} + m_2\left(\frac{l_2^2}{4} + \frac{1}{2} l_1 l_2 c_2\right), \; M_{22} = I_{Z_2} + m_2 \frac{l_2^2}{4}$$

$C(q,\dot{q})$ is the Coriolis matrix $C(q,\dot{q}) = \begin{bmatrix} c_a \dot{q}_2 + 1 & c_a(\dot{q}_1 + \dot{q}_2) \\ c_a \dot{q}_1 & 1 \end{bmatrix}$ with $c_a = -\frac{1}{2} m_2 l_1 l_2 s_2$, and $G(q) = [\frac{1}{2} m_1 g_r l_1 c_1 + m_2 g_r (l_1 c_1 + \frac{1}{2} l_2 c_{12}), \frac{1}{2} m_2 g_r l_2 c_{12}]^\top$ is the gravity vector where $c_1 = \cos(q_1)$, $c_2 = \cos(q_2)$, $s_2 = \sin(s_2)$, $c_{12} = \cos(q_1 + q_2)$, and $I_{Z_1} = 0.96$, $I_{Z_2} = 0.81$, $m_1 = 3.2$, $m_2 = 2$, $l_1 = 0.5$, $l_2 = 0.4$, $g_r = 9.81$ are geometrical and inertial parameters. To bring the system to the first-order form of (3), we use the transformation $x = q + \dot{q}$, whose double differentiation leads to form (3), appended with the internal dynamics of $\dot{q} = x - q$. We aim to stabilize the manipulator to the upright configuration $q_{\mathrm{d}} = [\frac{\pi}{2}, 0]^\top$ with zero velocity. Hence, we set $x_{\mathrm{d}} = [\frac{\pi}{2}, 0]^\top$, which complies with Assumption 3. Note that $x = q + \dot{q}$ creates a stable filter with output q and input x. Therefore, stabilization of $x(t)$ to x_{d} translates to the boundedness of $q(t)$. We impose the transient behaviour of the system via the functions $\rho_1(t) = \rho_2(t) = \|e(0)\| \exp^{-0.5t} + 1$. We apply the BRIC control law (12) with $k_1 = 3$ and $k_2 = 2$ and we further compare with the standard PPC scheme $u = -k_1 R(\xi, t)\chi(\xi)$. The results are depicted in Fig. 2 for 10 seconds, depicting $e(t)$ and $u(t)$ for both BRIC and PPC. Note that $e(t)$ remains within the funnel while converging to

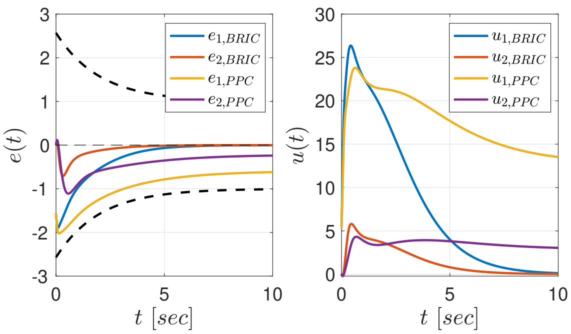

Fig. 2. Simulation results for Case II for BRIC and PPC; Left: error $e(t)$, along with funnels $\pm\rho_i(t)$ (dashed black lines); Right: control input $u(t)$.

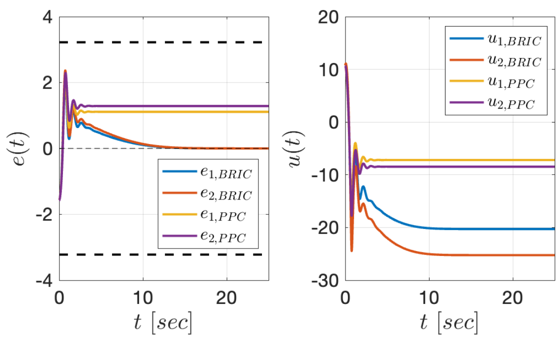

Fig. 3. Simulation results for Case II for BRIC and PPC; Left: error $e(t)$, along with bounds $\pm\rho_i$ (dashed black lines); Right: control input $u(t)$.

zero only for the BRIC scheme. Further notice that $M(q)q$ does not satisfy Assumption 5, implying it is not necessary for the convergence results.

Case III: Finally, to illustrate the BRIC of Section III-D, we consider a 2nd-order system of two inverted pendulums connected by a spring and a damper, with angles $\theta = [\theta_1, \theta_2]^\top \in [-\pi, \pi]^2$, $\theta(0) = [0, 0]^\top$ - the dynamic equations and parameters can be found in [18].

Similarly to the previous section, we use the transformation $x = \theta + \dot{\theta}$ to bring the system to the form (3) with the internal dynamics $\dot{\theta} = x - \theta$. We set the desired configuration at $\theta_{\rm d} = x_{\rm d} = [\frac{\pi}{2}, \frac{\pi}{2}]^\top$, and impose a maximum error overshoot via $\rho_1 = \rho_2 = \|e(0)\| + 1$. We further set the control gains $k_1 = 10$, $k_{\ell_1} = 1$, $k_{\ell_2} = 10$, and we compare the performance with the standard PPC $u = -k_1\chi(\xi)$. The results are depicted in Fig. 3 for 25 seconds, depicting $e(t)$ and $u(t)$ for both BRIC and PPC, showing that $e(t)$ converges to zero only for the BRIC scheme.

V. Conclusion and Future Work

This paper presents Barrier Integral Control, a control scheme that achieves asymptotic stabilization while complying with funnel-type transient-response specifications. The controller is smooth and does not use any information from the system's dynamics. Future directions will consider higher-order systems and relax the respective assumptions.

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