2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Data-driven Self-triggered Control for Linear Networked Control Systems

Xin Wang, Yifei Li, Jian Sun, Gang Wang, Jie Chen, and Lihua Dou

Abstract— **This paper considers data-driven control of unknown linear discrete-time systems under a self-triggered transmission scheme. While self-triggered control has received much attention in the literature, its design and implementation typically require explicit model knowledge. Due to the difficulties in obtaining accurate models and the abundance of data in applications, this paper proposes a novel data-driven self-triggered control scheme for unknown systems. To this end, we begin by presenting a model-based self-triggered scheme (STS) in form of quadratic matrix inequalities, on the basis of an equivalent switched system representation. Combining the model-based triggering law and a data-based system representation, a data-driven STS is developed leveraging pre-collected input-state data for predicting the next transmission instant while ensuring system stability. A data-based method for co-designing the controller gain and the triggering matrix is then provided. Finally, a numerical simulation showcases the efficacy of STS in reducing transmissions as well as practicality of the proposed co-design methods.**

I. INTRODUCTION

In recent year, much attention has been paid to the study of sampled-data control systems, in which the digital communication network has limited bandwidth. Traditional time-triggered scheme, where the information is transmitted in a periodic way, may incur a number of "redundant" transmissions [1], [2]. Recently, self-triggered scheme (STS) has gained an increasing interest for effectively saving communication resources while maintaining acceptable performance.

The core idea of STS is to predict the next triggering instant based on a function constructed using the current sampled information as well as the system knowledge. Most existing STSs [3], [4] are model-based, in the sense that they require explicit knowledge of the system model. Nonetheless, obtaining accurate system models can be computationally demanding and oftentimes impossible in real-world applications. It is also hard to provide an accurate model with guaranteed uncertainty bounds from limited and noisy data [5]. Naturally, an interesting question is *how to design an STS without explicit knowledge of the system model*. A promising solution, known as data-driven control, is aimed at learning control laws directly from data without resorting to any prior system identification steps [6], [7]. Model-free self-triggered schemes based on reinforcement learning (RL) have been well-studied in [8], [9] without explicit system models, while RL performs a good behavior in learning the dynamics of the plant but requires a larger amount of data. Recently, an excellent work [10] provides a data-driven design method of state feedback controller for linear systems on the basis of Willems' Fundamental Lemma [11]. Data-driven analysis and design of discrete-time linear systems with periodic sampling has been studied in [12]. In [13], a data-driven event-triggered scheme by filtering out redundant sampled data after periodic detection of current system states has been recently investigated based on S-lemma [14]. Extension works on data-driven event-triggered control of discrete-time centralized and distributed systems have been given in [15], [16], [17], [18]. It remains an untapped topic to design a data-driven STS which pre-computes the next execution time for the sensor and controller using only data, avoiding equipping extra hardware for system detection.

These developments have motivated our work to investigate data-driven control of discrete-time sampled-data systems under STS. Firstly, a model-based discrete-time STS is designed. Then, we rewrite the discrete-time sampled-data system as a switched system. Using the data-driven parametrization of switched systems in [12], a data-driven algorithm for pre-computing the next transmission instant is derived, which does not require explicit model knowledge. Subsequently, the co-designed controller and triggering matrix under the STS is employed to guarantee the stability of the system. In a nutshell, the main contributions of the present paper are summarized as follows:

- **c1)** A model-based STS to predict the next transmission instant, and a data-driven STS using only pre-collected data from the system.
- **c2)** Model- and data-based stability conditions for self-triggered discrete-time systems along with co-design methods of the controller and the triggering matrix.

Notation. Throughout this paper, $\mathbb{N}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denote the sets of all non-negative integers, n-dimensional real vectors, and $n\times m$ real matrices, respectively. Then, we define $\mathbb{N}_{[a,b]} := \mathbb{N}\cap[a,b]$, $a,b\in\mathbb{N}$. Sym$\{P\}$ represents the sum of $P^\top$ and P. We write $[\cdot]$ if elements in the matrix can be inferred from symmetry. Notation '$*$' represents the symmetric term in (block) symmetric matrices.

II. PRELIMINARIES

This section formulates the data-driven self-triggered control problems. We consider the networked control system

This work was supported in part by the National Natural Science Foundation of China under Grants 61925303, 62173034, 62088101.

X. Wang, Y. Li, J. Sun, G. Wang, and L. Dou are with the National Key Lab of Autonomous Intelligent Unmanned Systems, School of Automation, Beijing Institute of Technology, Beijing 100081, China, and also with the Beijing Institute of Technology Chongqing Innovation Center, Chongqing 401120, China xinwang@bit.edu.cn; liyifei@bit.edu.cn; sunjian@bit.edu.cn; gangwang@bit.edu.cn; doulihua@bit.edu.cn

J. Chen is with the Department of Control Science and Engineering, Tongji University, Shanghai 201804, China, and also with the National Key Lab of Autonomous Intelligent Unmanned Systems, Beijing Institute of Technology, Beijing 100081, China chenjie@bit.edu.cn

depicted in Fig. 1. The unknown plant is assumed to be a discrete-time linear time-invariant system as follows

$$x(t+1) = Ax(t) + Bu(t),\ \forall t \in \mathbb{N},\ x(0) \in \mathbb{R}^n \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is control input, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are the system matrices. In this paper the system matrices A and B are *unknown*, but some pre-collected state-input measurements $\{x(T)\}$ and $\{u(T)\}$ $(T \in \mathbb{N})$ satisfying the following dynamics

$$x(T+1) = Ax(T) + Bu(T) + B_w w(T) \tag{2}$$

are available in open-loop experiment at discrete time instants $T \in \mathbb{N}$. Here, $B_w \in \mathbb{R}^{n\times n_w}$ is a known matrix, which has full column rank and models the influence of the disturbance on the collected data. The measured data are corrupted by an *unknown* noise (perturbation) sequence $\{w(T)\}$, where $w(T) \in \mathbb{R}^{n_w}$ captures, e.g., process noise or unmodeled system dynamics. This noise only affects the data generated for the controller design and will be neglected in the closed-loop operation, but we note that an extension in this direction is straightforward.

During online closed-loop operation, the system state is sampled by the sensor and transmitted to the controller at time $t_k \in \mathbb{N}$, where $t_0 = 0$, $t_{k+1} - t_k \geq 1$, $k \in \mathbb{N}$. In the controller, the sampled state $x(t_k)$ is available, and the control input is computed via the linear state-feedback law $u(t_k) = Kx(t_k)$, where K is the controller gain matrix to be designed. Then, the control input is held constant until $t_{k+1} - 1$ by zero order holder (ZOH), and broadcasted to the actuator. The true plant (1) under the closed-loop sampled-data control can be written as follows

$$x(t+1) = Ax(t) + BKx(t_k), \quad t \in \mathbb{N}_{[t_k, t_{k+1}-1]}. \tag{3}$$

Traditional periodic transmission schemes updates the control input periodically [19], and have been extended to data-driven cases, e.g., by [12], which determine the maximum sampling interval for which stability can be guaranteed. Avoiding "redundant" transmissions in networks, an STS has been employed by [3] to adaptively determine the transmission instant t_k to save communication resources. The idea of the STS is to build a function $\Gamma(x(t_k))$ for computing the next transmission instant t_{k+1} based on the current state $x(t_k)$ of system (3)

$$t_{k+1} = t_k + \Gamma(x(t_k), s_k). \tag{4}$$

However, most existing STSs [3], [4] depend on a precise system model. The main goals of this paper are to 1) develop a data-driven STS with *unknown* matrices A and B, as well as 2) a data-driven design method for computing the controller gain guaranteeing a desired performance under the data-driven STS. See Fig. 1 for an illustration of the setup.

III. MAIN RESULTS

For the first goal, the challenge is to predict the next transmission instant using the already transmitted system states and historical noisy measurements (i.e., $\{x(T)\}$, $\{u(T)\}$) without explicit knowledge of the system matrices A and B. To this end, we begin by designing a model-based STS.

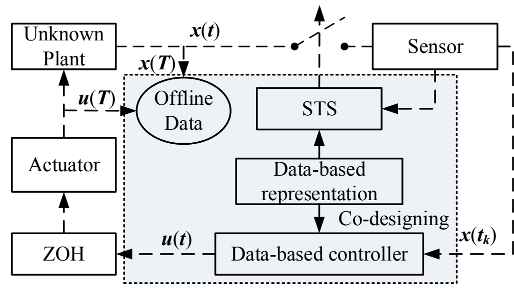

Fig. 1. Structure of data-driven discrete-time systems under STS.

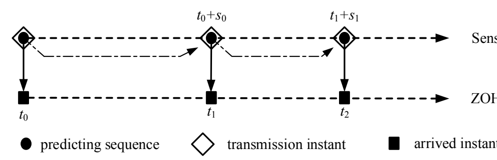

Fig. 2. Evolution of transmission series.

A. Model-based STS

We first need to define a lifted version of the original system (3) as suggested in [12]. To this end, let us define for $s > 0$, $s \in \mathbb{N}$

$$\underline{B}^s := \begin{bmatrix} A^{s-1}B & A^{s-2}B & \cdots & B \end{bmatrix},$$

$$\underline{K}^s := \Big[\underbrace{K^\top \ K^\top \ \cdots \ K^\top}_{s \text{ times}} \Big]^\top.$$

The discrete-time sampled-data systems in (3) can be viewed as switched systems, which is a well-known fact in the literature [20]

$$x(t_k + s_k) = (A^{s_k} + \underline{B}^{s_k}\underline{K}^{s_k})x(t_k), \quad s_k \in \mathbb{N}_{[1,\bar{s}]} \tag{5}$$

where $s_k = t_{k+1} - t_k$ and $\bar{s} > 1 \in \mathbb{N}$.

We employ the following condition to find $\Gamma(x(t_k), s_k)$

$$\sigma_1 x^\top(t_k+s_k)\Omega x(t_k+s_k) + \sigma_2 x^\top(t_k)\Omega x(t_k) - e^\top(s_k)\Omega e(s_k) \geq 0 \tag{6}$$

where $\Omega \succ 0$ is some weight matrix; σ_1 and σ_2 are parameters to be designed; $e(s_k) := x(t_k+s_k) - x(t_k)$ denotes the error between the sampled signals $x(t_k)$ at the latest transmission instant and $x(t_k+s_k)$ at time t_k+s_k. According to (5), the condition (6) can be reformulated in the form of a quadratic matrix inequality (QMI)

$$\mathscr{Q}(x(t_k), s_k) = \begin{bmatrix} (A^{s_k} + \underline{B}^{s_k}\underline{K}^{s_k})x(t_k) \\ x(t_k) \end{bmatrix}^\top \times \begin{bmatrix} (\sigma_1 - 1)\Omega & \Omega \\ * & (\sigma_2 - 1)\Omega \end{bmatrix} [\cdot] \geq 0. \tag{7}$$

If (7) is satisfied, the time $t_k + s_k$ is declared to be the next transmission instant, i.e., $t_{k+1} = t_k + s_k$. When the sampled state $x(t_k + s_k)$ is transmitted to the controller, the ZOH is used to maintain it within the interval $[t_{k+1}, t_{k+2} - 1]$. Simultaneously, the self-triggering module is updated and employed to predict the next transmission instant t_{k+2}. Consequently, the function $\Gamma(x(t_k))$ is designed to be

$$\Gamma(x(t_k)) = \max_{s_k \in \mathbb{N}} \Big\{ s_k > 0 \Big| \mathscr{Q}(x(t_k), s_k) \geq 0 \Big\}. \tag{8}$$

In Fig. 2, an example illustrating the STS is given. The next transmission instant is predicted using the current transmitted measurement; that is, t_{k+1} is determined by the system state at time t_k.

B. Data-driven STS

In this part, a data-based discrete-time STS is proposed based on the model-based function in (8) and the measurements $\{x(T)\}_{T=0}^{\rho+s}$, $\{u(T)\}_{T=0}^{\rho+s-1}$ $(T,\rho,s\in\mathbb{N},\rho>0,s>0)$ of the perturbed system (2). Our idea is to rebuild a self-triggering function using the data $\{x(T)\}_{T=0}^{\rho+s}$, $\{u(T)\}_{T=0}^{\rho+s-1}$ to replace the $[A^s\ \underline{B}^s]$-based representation (8). To that end, we recall the data-driven parametrization of the lifted matrix $[A^s\ \underline{B}^s]$ in [12]. Similar to the system expression in (5), we firstly re-write the perturbed system (2) as follows

$$x(T+s)=A^s x(T)+\underline{B}^s\begin{bmatrix} u(T)\\ \vdots \\ u(T+s-1)\end{bmatrix} + \begin{bmatrix} A^{s-1}B_w & \cdots & B_w\end{bmatrix}\begin{bmatrix} w(T)\\ \vdots \\ w(T+s-1)\end{bmatrix}. \quad (9)$$

Recall that the measured data $\{x(T)\}_{T=0}^{\rho+s-1}$, $\{u(T)\}_{T=0}^{\rho+s-2}$ are corrupted by the *unknown* noise $\{w(T)\}_{T=0}^{\rho+s-2}$. Let us define the following matrices containing the measurements

$$X_+^s := \begin{bmatrix} x(s) & x(1+s) & \cdots & x(\rho+s-1)\end{bmatrix}$$
$$U^s := \begin{bmatrix} u(0) & u(1) & \cdots & u(\rho-1)\\ \vdots & \vdots & & \vdots \\ u(s-1) & u(s) & \cdots & u(\rho+s-2)\end{bmatrix}.$$

We further define the following lifted disturbance

$$W^1 := \begin{bmatrix} w(0) & w(1) & \cdots & w(\rho-1)\end{bmatrix}$$
$$\underline{W}^s := \begin{bmatrix} w(0) & \cdots & w(\rho-1)\\ \vdots & & \vdots \\ w(s-1) & \cdots & w(\rho+s-2)\end{bmatrix}$$
$$W^s := \begin{bmatrix} A^{s-1}B_w & \cdots & B_w\end{bmatrix}\underline{W}^s,\quad \text{for } s>1.$$

Then, it is clear that

$$X_+^s = A^s X + \underline{B}^s U^s + B_w^s W^s \quad (10)$$

where $B_w^1 := B_w$, $B_w^s := I$ for $s>1$. For the proposed approach, the required data are collected offline and can be transmitted once by the sensor for the controller design step. Therefore, we can neglect network effects on these data and we can assume that they are collected with a sampling period. In practice, the noise is typically bounded. We make the following standing assumption on the noise.

Assumption 1 (Lifted noise bound): The noise sequence $\{w(t)\}_{t=0}^{\rho+s-2}$ in the matrix W^s satisfies $W^s\in\mathscr{W}^s$ with

$$\mathscr{W}^s=\left\{W^s\in\mathbb{R}^{n_w^s\times\rho}\,\middle|\,\begin{bmatrix} W^{s\top}\\ I\end{bmatrix}^\top\begin{bmatrix} Q_d^s & S_d^s\\ * & R_d^s\end{bmatrix}\begin{bmatrix} W^{s\top}\\ I\end{bmatrix}\succeq 0\right\}$$

for some known matrices $Q_d^s\prec 0\in\mathbb{R}^{\rho\times\rho}$, $S_d^s\in\mathbb{R}^{\rho\times n_w^s}$, and $R_d^s=R_d^{s\top}\in\mathbb{R}^{n_w^s\times n_w^s}$, where $n_w^1:=n_w$, $n_w^s:=n$ for $s>1$.

Define the set of all pairs $[A^s\ \underline{B}^s]$ consistent with the model (10) and Assumption 1 as the same as [12]

$$\Sigma_{AB}^s := \{[A^s\ \underline{B}^s]\in\mathbb{R}^{n\times(n+sm)}\mid X_+^s = A^s X+\underline{B}^s U^s + B_w^s W^s,\ W^s\in\mathscr{W}^s\}. \quad (11)$$

Then, we obtain the following equivalent expression of Σ_{AB}^s in the form of a QMI

$$\Sigma_{AB}^s = \left\{[A^s\ \underline{B}^s]\,\middle|\,\begin{bmatrix} [A^s\ \underline{B}^s]^\top\\ I\end{bmatrix}^\top \Theta_{AB}^s[\cdot]\succeq 0\right\} \quad (12)$$

where

$$\Theta_{AB}^s=\begin{bmatrix} Q_c^s & S_c^s\\ * & R_c^s\end{bmatrix} := \begin{bmatrix} -X & 0\\ -U^s & 0\\ \hline X_+^s & B_w^s\end{bmatrix}\begin{bmatrix} Q_d^s & S_d^s\\ * & R_d^s\end{bmatrix}[\cdot]^\top.$$

Having obtained a data-based representation of system (5), we can now translate the model-based self-triggering function (8) that depends on $[A^s\ \underline{B}^s]$ to a data-based one. The following technical assumption on the matrix Θ_{AB}^s is required for the subsequent derivation.

Assumption 2: The matrix Θ_{AB}^s is invertible and has n_w positive eigenvalues.

In practice, Assumptions 2 is satisfied when the data are sufficiently rich and B_w is invertible [12]. Based on Assumption 2, we have the following theorem.

Theorem 1 (Data-driven self-triggering condition): For given scalars $\sigma_1\geq 0$, $\sigma_2\geq 0$, matrix $\Omega\succ 0$, controller gain K, and $x(t_k)$ from system (5), $\mathscr{Q}(x(t_k),s)$ in (7) satisfies

$$\mathscr{Q}(x(t_k),s)\geq 0 \quad (13)$$

for any $[A^s\ \underline{B}^s]\in\Sigma_{AB}^s$, if there exists a scalar $\gamma>0$, such that the following LMI holds for some $s\in\mathbb{N}, s\geq 1$

$$\tilde{\mathscr{Q}}(x(t_k))-\gamma\tilde{\mathscr{G}}^s(x(t_k))\succeq 0 \quad (14)$$

where

$$\tilde{\mathscr{Q}}(x(t_k)) := \begin{bmatrix} I & 0\\ 0 & x^\top(t_k)\end{bmatrix}\begin{bmatrix} (\sigma_1-1)\Omega & \Omega\\ * & (\sigma_2-1)\Omega\end{bmatrix}[\cdot]^\top$$
$$\tilde{\mathscr{G}}^s(x(t_k)) := \begin{bmatrix} I & 0 & 0\\ 0 & x^\top(t_k) & x^\top(t_k)\underline{K}^{s\top}\end{bmatrix}\tilde{\Theta}_{AB}^s[\cdot]^\top$$
$$\tilde{\Theta}_{AB}^s := \begin{bmatrix} -\tilde{R}_c^s & \tilde{S}_c^{s\top}\\ * & -\tilde{Q}_c^s\end{bmatrix},\ \begin{bmatrix} \tilde{Q}_c^s & \tilde{S}_c^s\\ * & \tilde{R}_c^s\end{bmatrix} := \begin{bmatrix} Q_c^s & S_c^s\\ * & R_c^s\end{bmatrix}^{-1}.$$

Proof: The matrix $\mathscr{Q}(x(t_k),s)$ in (7) is rewritten as

$$\mathscr{Q}(x(t_k),s)=\begin{bmatrix} (A^s+\underline{B}^s\underline{K}^s)x(t_k)\\ I\end{bmatrix}^\top\tilde{\mathscr{Q}}(x(t_k))[\cdot]. \quad (15)$$

Applying the dualization lemma [21, Lemma 4.9] to the system representation in (12) under Assumption 2, it can be proven that $[A^s\ \underline{B}^s]\in\Sigma_{AB}^s$ if and only if

$$\begin{bmatrix} [A^s\ \underline{B}^s]\\ I\end{bmatrix}^\top\tilde{\Theta}_{AB}^s\begin{bmatrix} [A^s\ \underline{B}^s]\\ I\end{bmatrix}\succeq 0. \quad (16)$$

Immediately, through the full-block S-procedure, we have $\tilde{Q}(x(t_k))\succeq 0$ for any $[A^s\ \underline{B}^s]\in\Sigma_{AB}^s$ if there exists a scalar $\gamma>0$ such that the LMI (14) holds. End the proof. ■

Remark 1 (Explanation of the data-driven STS condition): Theorem 1 offers a data-driven triggering condition based on the model-based one in (8). The key idea is that we leverage the data-based representation in (12) to robustly verify the STS condition for all $[A^s\ \underline{B}^s]$ consistent with the data. As a result, the model-based triggering STS function in (8) is translated into a data-driven one as follows

$$\bar{\Gamma}(x(t_k),s_k) = \max_{s_k\in\mathbb{N}} \left\{ s_k \geq 1 \middle| \tilde{\mathscr{Q}}(x(t_k)) - \gamma \tilde{\mathscr{G}}^{s_k}(x(t_k)) \succeq 0 \right\}. \tag{17}$$

Overall, our data-driven STS is given by

$$t_{k+1} = t_k + \bar{\Gamma}(x(t_k),s_k) \tag{18}$$

under Assumptions 1 and 2 for system (5). Note that, in Theorem 1, the data-driven condition $\tilde{\mathscr{Q}}(x(t_k)) - \gamma\tilde{\mathscr{G}}^s(x(t_k)) \succeq 0$ sufficiently guarantees the model-based one $\mathscr{Q}(x(t_k),s) \geq 0$ in (8) that is consistent with system stability characteristics. A smaller triggering interval may be produced by (17), since (17) only provides a sufficient condition for (8).

Remark 2 (Summary of data-driven STS algorithm): According to (18), the next transmission instant t_{k+1} of system (3) can be computed using only collected data $\{x(T)\}_{T=0}^{\rho+s-1}$ and $\{u(T)\}_{T=0}^{\rho+s-2}$. Note that the matrix $\tilde{\Theta}_{AB}^s$ in (18) needs to be determined in advance from the given noise bound. To that end, we recall [12, Algorithm 1], which can be used to construct a lifted noise bound as in Assumption 1 based on pointwise bound $||w(T)||_2 \leq \bar{w}$ for all $T = 0,\ldots,\rho+s-2$ with some $\bar{w} > 0$. This leads to a lifted system parametrization as in (12). Then, we continuously check the data-driven self-triggering condition using the matrices $\tilde{\Theta}_{AB}^s$ from [12, Algorithm 1]. The next triggering instant can be determined by checking the LMI (14) as soon as the current transmission instant and state become available.

C. Stability analysis and controller design

In the following, we address the second goal that is co-designing the triggering matrix Ω and the controller gain K, such that the *unknown* system (3) under the transmission scheme (18) is stable. To this end, a model-based stability condition for system (3) under the STS (6) is set up first.

Theorem 2 (Model-based condition): For given scalars $\sigma_1 > 0$ and $\sigma_2 > 0$, system (3) is asymptotically stable under the triggering condition (6), if there exist matrices $P \succ 0$, $\Omega \succ 0$, and F, such that the following LMI holds

$$\mathscr{H} + \mathscr{J} \prec 0 \tag{19}$$

where $E_i := \left[0_{n\times(i-1)n},\ I_n,\ 0_{n\times(3-i)n}\right]$, $(i = 1,2,3)$, and

$$\begin{aligned}
\mathscr{H} &:= E_2^\top P E_2 - E_1^\top P E_1\\
\mathscr{J} &:= \mathrm{Sym}\left\{F(AE_1 + BKE_3 - E_2)\right\} + \sigma_1 E_1^\top \Omega E_1\\
&\quad + \sigma_2 E_3^\top \Omega E_3 - (E_1 - E_3)^\top \Omega (E_1 - E_3)
\end{aligned}$$

Proof: Choose the following functional for system (3)

$$V(t) = x^\top(t) P x(t),\ P \succ 0,\ t \in \mathbb{N}_{[t_k,t_{k+1}-1]}. \tag{20}$$

Calculating the forward difference of $V(t)$ yields

$$\Delta V(t) = \zeta^\top(x,t)\Big(E_2^\top P E_2 - E_1^\top P E_1\Big)\zeta(x,t) \tag{21}$$

with $\zeta(x,t) := \left[x^\top(t), x^\top(t+1), x^\top(t_k)\right]^T$.

Through the descriptor method, the system representation (3) can be written as

$$0 = 2\zeta^\top(x,t)F\big(AE_1 + BKE_3 - E_2\big)\zeta(x,t) \tag{22}$$

for $t \in \mathbb{N}_{[t_k,t_{k+1}-1]}$, where F is a matrix of dimensions $3n\times n$.

In spirit of (6), when the current system state is not transmitted, the following inequality is true

$$\begin{aligned}
&\xi^\top(x,t)\Big[\sigma_1 E_1^\top \Omega E_1 + \sigma_2 E_3^\top \Omega E_3\\
&\qquad - (E_1 - E_3)^\top \Omega (E_1 - E_3)\Big]\xi(x,t) \geq 0.
\end{aligned} \tag{23}$$

Summing up (21)-(23), we arrive at

$$\Delta V(t) \leq \zeta^\top(x,t)\,(\mathscr{H} + \mathscr{J})\,\zeta(x,t). \tag{24}$$

Finally, $\mathscr{H} + \mathscr{J} \prec 0$ implies that $\Delta V(t) < 0$ whenever $\zeta \neq 0$. We conclude that system (3) is asymptotically stable under the triggering condition (6) if the LMI (19) holds. This concludes the proof. ■

We now derive a data-based stability certificate for the self-triggered control system (3). Motivated by [12], the main idea is to employ a system expression using the data $\{x(T)\}$, $\{u(T)\}$ to replace the system matrices $[A\ B]$ in Theorem 2. We begin with an algebraically equivalent system to (3).

Assume that $G \in \mathbb{R}^{n\times n}$ is nonsingular, and let $x(t) = Gz(t)$. The system (3) is restructured as follows

$$z(t+1) = G^{-1}AGz(t) + G^{-1}BK_c z(t_k) \tag{25}$$

for $t \in \mathbb{N}_{[t_k,t_{k+1}-1]}$, where $K_c := KG$. The system (25) exhibits the same stability behavior as (3), and the triggering condition (6) remains effective. Based on Theorem 2, we have the following theoretical result.

Theorem 3 (Data-driven co-design): For given scalars $\sigma_1 > 0$, $\sigma_2 > 0$, α, there exists a controller gain K such that system (3) is asymptotically stable under the triggering condition (18) for any $[A\ B] \in \Sigma_{AB}^1$, if there exist a scalar $\varepsilon > 0$, and matrices $P \succ 0$, $\Omega_z \succ 0$, G, K_c such that the following LMI holds

$$\begin{bmatrix} \mathscr{Y}_1 & \mathscr{Y}_2 + \mathscr{K} \\ * & \mathscr{H} + \bar{\mathscr{J}} + \mathscr{Y}_3 \end{bmatrix} \prec 0 \tag{26}$$

where $\mathscr{Z}_1 := \begin{bmatrix} I & 0 \end{bmatrix}$, $\mathscr{Z}_2 := \begin{bmatrix} 0 & \mathscr{L} \end{bmatrix}$,

$$\begin{aligned}
\bar{\mathscr{J}} &:= \mathrm{Sym}\left\{-\mathscr{L}GE_2\right\} + \sigma_1 E_1^\top \Omega_z E_1 + \sigma_2 E_3^\top \Omega_z E_3\\
&\quad - (E_1 - E_3)^\top \Omega_z (E_1 - E_3)\\
\mathscr{L} &:= (E_1 + \alpha E_2)^T,\ \mathscr{K} := \left[E_1^\top G^\top, E_3^\top K_c^\top\right]^\top\\
\mathscr{Y}_1 &:= \varepsilon \mathscr{Z}_1 \Theta_{AB}^1 \mathscr{Z}_1^\top, \mathscr{Y}_2 := \varepsilon \mathscr{Z}_1 \Theta_{AB}^1 \mathscr{Z}_2^\top, \mathscr{Y}_3 := \varepsilon \mathscr{Z}_2 \Theta_{AB}^1 \mathscr{Z}_2^\top.
\end{aligned}$$

Moreover, $K = K_c G^{-1}$ and $\Omega = {G^{-1}}^\top \Omega_z G^{-1}$ are desired the controller gain K and triggering matrix.

Proof: It is known from Theorem 1 that data-driven STS (18) implies $\mathscr{Q}(x(t_k),s_k) \geq 0$ in (13), which is equal to (23). Then, choose a functional $V(z,t)$ for the system (25), where $V(z,t)$ comes from (20). Similar to the proof of Theorem 2, it can be obtained that

$$\Delta V(z,t) \leq \zeta^{\top}(z,t)\,(\mathscr{H}+\mathscr{J})\,\zeta(z,t) \tag{27}$$

where $\zeta(z,t) := \left[z^{\top}(t), z^{\top}(t+1), z^{\top}(t_k)\right]^{\top}$.

We can use the descriptor method as in (22) to rewrite the system description (25) as follows

$$0 = 2\zeta^{\top}(z,t)\mathscr{L}(AGE_1 + BK_cE_3 - GE_2)\zeta(z,t) \tag{28}$$

where α is a given constant. Replacing the equality (22) by (28) in the proof of Theorem 2, (27) becomes

$$\Delta V(z,t) \leq \zeta^{\top}(z,t)\mathscr{U}\zeta(z,t). \tag{29}$$

where $\mathscr{U} := \mathrm{Sym}\{\mathscr{L}(AGE_1 + BK_cE_3)\} + \mathscr{H} + \bar{\mathscr{J}}$.

Now, the matrix $\mathscr{U}$ can be defined as

$$\mathscr{U} := \begin{bmatrix} [\mathscr{L}A\ \mathscr{L}B]^{\top} \\ I \end{bmatrix}^{\top} \begin{bmatrix} 0 & \mathscr{K} \\ * & \mathscr{H}+\bar{\mathscr{J}} \end{bmatrix} \begin{bmatrix} [\mathscr{L}A\ \mathscr{L}B]^{\top} \\ I \end{bmatrix}.$$

According to the data-based representation in (12) with $s=1$, it holds for any $[A\ B] \in \Sigma_{AB}^{1}$ that

$$\begin{bmatrix} [A\ B]^{\top} \\ I \end{bmatrix}^{\top} \Theta_{AB}^{1} \begin{bmatrix} [A\ B]^{\top} \\ I \end{bmatrix} \succeq 0. \tag{30}$$

By the full-block S-procedure, we have $\mathscr{U} \prec 0$ for any $[A\ B] \in \Sigma_{AB}^{1}$ if there exists a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} 0 & \mathscr{K} \\ * & \mathscr{H}+\bar{\mathscr{J}} \end{bmatrix} + \varepsilon \begin{bmatrix} \mathscr{Z}_1\Theta_{AB}^{1}\mathscr{Z}_1^{\top} & \mathscr{Z}_1\Theta_{AB}^{1}\mathscr{Z}_2^{\top} \\ * & \mathscr{Z}_2\Theta_{AB}^{1}\mathscr{Z}_2^{\top} \end{bmatrix} \prec 0.$$

Finally, by Schur Complement Lemma, the LMI (26) is a sufficient stability condition for the system (25) under the triggering condition (18) for any $[A\ B] \in \Sigma_{AB}^{1}$. Since G is nonsingular, the system (25) exhibits the same stability characteristics as (3). ■

Remark 3 (Model-based design under STS): A model-based co-design method under the STS (8) can be derived analogously by replacing (26) by the condition $\mathscr{U} \prec 0$.

IV. Example and Simulation

In this section, one numerical example from [22] is employed to certificate the effectiveness and merits of our proposed methods. All numerical computations were performed using Matlab, together with the SeDuMi toolbox.

Example 1: Consider the linear system used in [22]

$$\dot{x}(\nu) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(\nu) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(\nu),\ \ \nu \geq 0.$$

Through system discretization with an interval $T_k > 0$, the system with the linear sampled-data state-feedback controller $u(t) = Kx(t_k)$ can be written as

$$x(t+1) = A(T_k)x(t) + B(T_k)Kx(t_k),\ t \in \mathbb{N}_{[t_k, t_{k+1}-1]} \tag{31}$$

where

$$A(T_k) := e^{\begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} T_k}, B(T_k) := \int_0^{T_k} e^{A(s)} \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} ds.$$

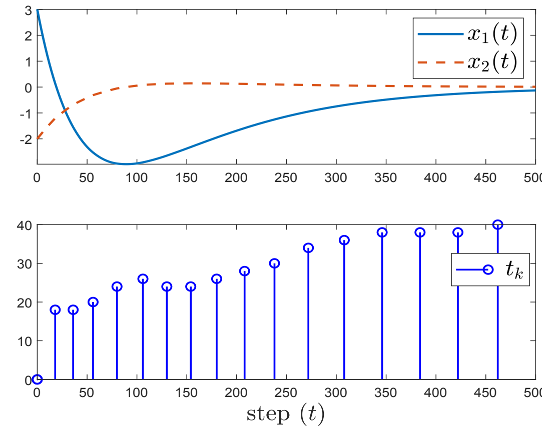

Fig. 3. Trajectories of system (3) under data-driven STS (18) with the initial condition $x(0) = [3\ \ -2]^{\top}$.

In the following, the proposed data-driven STS (18) is applied for system (31)

(*Data-driven STS control*) Assume that the matrices A and B are *unknown*. We set the discretization interval as $T_k = 0.2$ and generated measurements $\{x(T)\}_{T=0}^{800}$, $\{u(T)\}_{T=0}^{800}$, where the data-generating input was sampled uniformly from $u(T) \in [-1,\ 1]$. The measured data were perturbed by a disturbance distributed uniformly over $w(T) \in [-0.001, 0.001]^2$. Such disturbance $w(T)$ fulfills Assumption 1. The matrix B_w was taken as $B_w = 0.01I$, which has full column rank. According to [12, Algorithm 1], the bound on the lifted disturbance in Assumption 1 can be computed by using above measurements with $\rho = 750$ and $\bar{s} = 50$. Then, it is straightforward to obtain the data-based matrices $\tilde{\mathscr{G}}^{s}(x(t_k))$ for $s \in \mathbb{N}_{[1,\bar{s}]}$ in (14) based on the matrix Θ_{AB}^{s} from the bound in Assumption 1. Solving the data-based LMIs (26) in Theorem 3 with $\sigma_1 = \sigma_2 = 0.9$, $\alpha = 2$, the controller and triggering matrices are co-designed as follows

$$K = \begin{bmatrix} -0.1537 & -1.6465 \end{bmatrix},\ \Omega = \begin{bmatrix} 0.0001 & 0.0008 \\ 0.0008 & 0.0091 \end{bmatrix}.$$

We simulate the system state trajectory under the data-based STS (18), as depicted in Fig. 3 for $t \in [0,\ 500]$. All states converge to the origin, thereby validating the practicality of our proposed co-design method and STS. Note that, in Fig. 3, *only* 17 out of 250 samples were transmitted to the controller. This illustrates the usefulness of the STS in reducing transmissions while ensuring stability.

(*Compared with model-based STS control*) We compare the data-driven approach described above to an alternative approach, consisting of least-squares identification of the system matrices and subsequent model-based STS (8). In the system identification step, the following least-squares problem is considered

$$[\hat{A}\ \hat{B}]^{\top} = \arg\min_{(A\ \ B)} \sum_{T=0}^{\rho} \|x(T+1) - Ax(T) - Bu(T)]\|_2^2.$$

Using the least-squares approach and the same measurements

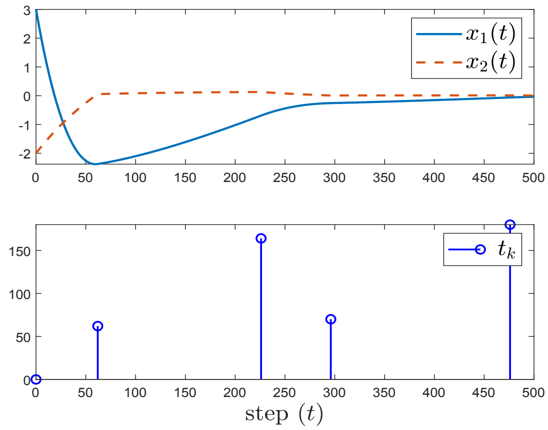

Fig. 4. Trajectories of system (3) under identification-based STS (8) with the initial condition $x(0) = [3 \ \ -2]^\top$.

as in the data-driven control, the system matrices are

$$\hat{A} = \begin{bmatrix} 1.0000 & 0.0995 \\ 0.0000 & 0.9900 \end{bmatrix}, \ \hat{B} = \begin{bmatrix} 0.0005 \\ 0.0100 \end{bmatrix}.$$

Subsequently, by solving the model-based approach in Remark 3 with the same parameters as in data-driven control, the controller and the triggering matrices were computed as

$$K = [\ -0.0944 \quad -1.3662\ ], \ \Omega = \begin{bmatrix} 0.0001 & 0.0006 \\ 0.0006 & 0.0092 \end{bmatrix}.$$

The trajectories of system (31) under the identification-based STS (8) were depicted in Fig. 4 over $t \in [0, 500]$. The simulation results show that $x(t)$ approaches to zero as $t \to \infty$ under the identification-based STS with the above designed K and Ω. The comparison between Figs. 3 and 4 certificates the effectiveness of our data-driven STS in guaranteeing stability characteristics at the approximate level to model-based one. However, in contrast to the direct data-driven design, the identification-based STS approaches do not provide stability guarantees.

V. Concluding Remarks

In this paper, we proposed a data-based STS for discrete-time systems, as well as a data-driven method for co-designing the controller gain and the triggering matrix for the discrete-time STS systems. Finally, a numerical example was presented to corroborate the role of our triggering schemes in saving communication resources, as well as the merits and effectiveness of our co-designing methods. Our future works will center on extending the data-driven STS to more complicated systems, such as multi-agent systems [23] and complex dynamic networks.

References

[1] M. C. F. Donkers and W. P. M. H. Heemels, "Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralized event-triggering," *IEEE Trans. Autom. Control*, vol. 57, no. 6, pp. 1362–1376, Jun. 2012.

[2] X. Wang, J. Sun, G. Wang, and L. Dou, "A mixed switching event-triggered transmission scheme for networked control systems," *IEEE Trans. Control Netw. Syst.*, vol. 9, no. 1, pp. 390–402, Mar. 2022.

[3] X. Wang and M. D. Lemmon, "Self-triggering under state-independent disturbances," *IEEE Trans. Autom. Control*, vol. 55, no. 6, pp. 1494–1500, Jun. 2010.

[4] N. Strijbosch, G. E. Dullerud, A. R. Teel, and W. Heemels, "$\mathcal{L}_2$-gain analysis of periodic event-triggered control and self-triggered control using lifting," *IEEE Trans. Autom. Control*, Aug. 2021.

[5] N. Matni, A. Proutiere, A. Rantzer, and S. Tu, "From self-tuning regulators to reinforcement learning and back again," in *Proc. Conf. Decision and Control*, Dec. 2019, pp. 3724–3740.

[6] I. Markovsky and F. Dörfler, "Behavioral systems theory in data-driven analysis, signal processing, and control," *Annu. Rev. Control*, vol. 52, pp. 42–64, Jan. 2021.

[7] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel, "Data informativity: A new perspective on data-driven analysis and control," *IEEE Trans. Autom. Control*, vol. 65, no. 11, pp. 4753–4768, Nov. 2020.

[8] K. Hashimoto, Y. Yoshimura, and T. Ushio, "Learning self-triggered controllers with Gaussian processes," *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 6294–6304, Dec. 2021.

[9] A. Acernese, A. Yerudkar, L. Glielmo, and C. Del Vecchio, "Model-free self-triggered control co-design for probabilistic boolean control networks," *IEEE Control Syst. Lett.*, vol. 5, no. 5, pp. 1639–1644, Nov. 2021.

[10] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 909–924, Mar. 2020.

[11] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. De Moor, "A note on persistency of excitation," *Syst. Control Lett.*, vol. 54, no. 4, pp. 325–329, Apr. 2005.

[12] S. Wildhagen, J. Berberich, M. Hertneck, and F. Allgöwer, "Data-driven analysis and controller design for discrete-time systems under aperiodic sampling," *IEEE Trans. Autom. Control*, vol. 68, no. 6, pp. 3210–3225, Jun. 2023.

[13] X. Wang, J. Sun, J. Berberich, G. Wang, F. Allgöwer, and J. Chen, "Data-driven control of dynamic event-triggered systems with delays," *Int. J. Robust Nonlin. Control*, vol. 33, no. 12, pp. 7071–7093, Aug. 2023.

[14] H. J. van Waarde, M. K. Camlibel, and M. Mesbahi, "From noisy data to feedback controllers: Nonconservative design via a matrix s-lemma," *IEEE Trans. Autom. Control*, vol. 67, no. 1, pp. 162–175, Jan. 2022.

[15] Y. Li, X. Wang, J. Sun, G. Wang, and J. Chen, "Data-driven consensus control of fully distributed event-triggered multi-agent systems," *Sci. China Inf. Sci.*, vol. 66, no. 5, p. 152202, Feb. 2023.

[16] X. Wang, J. Berberich, J. Sun, G. Wang, F. Allgöwer, and J. Chen, "Model-based and data-driven control of event- and self-triggered discrete-time linear systems," *IEEE Trans. Cybern.*, vol. 53, no. 9, pp. 6066–6079, Sep. 2023.

[17] X. Wang, J. Sun, G. Wang, F. Allgöwer, and J. Chen, "Data-driven control of distributed event-triggered network systems," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 2, pp. 351–364, Feb. 2023.

[18] W. Liu, J. Sun, G. Wang, F. Bullo, and J. Chen, "Data-driven resilient predictive control under denial-of-service," *IEEE Trans. Autom. Control*, vol. 68, no. 8, pp. 4722–4737, Aug. 2023.

[19] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, Feb. 2010.

[20] M. Yu, L. Wang, T. Chu, and G. Xie, "Stabilization of networked control systems with data packet dropout and network delays via switching system approach," in *Proc. Conf. Decision and Control*, vol. 4, Dec. 2004, pp. 3539–3544.

[21] C. W. Scherer and S. Weiland, *Linear Matrix Inequalities in Control, 3rd ed*, New York: Springer-Verlag, 2000.

[22] W. Zhang, M. Branicky, and S. Phillips, "Stability of networked control systems," *IEEE Contr. Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.

[23] X. Wang, J. Sun, F. Deng, G. Wang, and J. Chen, "Event-triggered consensus control of heterogeneous multi-agent systems: model- and data-based approaches," *SCIENCE CHINA Inf. Sci.*, vol. 66, no. 9, p. 192201, Sept. 2023.