2024 IEEE 63rd Conference on Decision and Control (CDC)
December 16-19, 2024. MiCo, Milan, Italy

A blended physics-based and black-box identification approach for spacecraft inertia estimation

Martina Mammarella[1], Cesare Donati[2,1,⋆], Fabrizio Dabbene[1], Carlo Novara[2], Constantino Lagoa[3]

Abstract— **In this paper, the problem of identifying inertial characteristics of a generic space vehicle relying on the physical and structural insights of the dynamical system is presented. To this aim, we exploit a recently introduced framework for the identification of physical parameters directly feeding the measurements into a backpropagation-like learning algorithm. In particular, this paper extends this approach by introducing a recursive algorithm that combines physics-based and black-box techniques to enhance accuracy and reliability in estimating spacecraft inertia. We demonstrate through numerical results that, relying on the derived algorithm to identify the inertia tensor of a nanosatellite, we can achieve improved estimation accuracy and robustness, by integrating physical constraints and leveraging partial knowledge of the system dynamics. In particular, we show how it is possible to enhance the convergence of the physics-based algorithm to the true values by either overparametrization or introducing a black-box term that captures the unmodelled dynamics related to the off-diagonal components.**

Index Terms— **Aerospace, Nonlinear systems identification, Grey-box modeling, Physics-based neural networks**

I. Introduction

The knowledge of a spacecraft inertia tensor is crucial to the performance of the satellite itself. Typically, the inertia is calculated before launch by tabulating the masses and locations of spacecraft components, but the accuracy of such estimates is rather limited. Moreover, a spacecraft inertia may change in unpredictable ways while operated on orbit. This can be due, for instance, to fuel consumption or satellite damage, or in specific scenarios, be a consequence of a mating maneuver leading to a combined system, characterized by new inertial properties that need to be estimated to properly control it. This may happen during on-orbit service missions such as refueling, infrastructures upgrade, and active debris removal. As a result, the ability to estimate a spacecraft inertia during its operative lifetime would allow to largely increase robustness to modeling uncertainty and component failures, thus resulting in improved pointing performance.

Several algorithms for estimating a spacecraft inertia parameters from telemetry data have been proposed in the literature. The majority is based on least squares (LS) methods mainly for *on-ground* estimation, see e.g., [1], [2], [3]. On the other hand, the ability to estimate a spacecraft inertia *on orbit* can offer improved pointing performance and robustness to modeling uncertainty and component failures. In this framework, Kalman Filters have shown better performance than LS methods as presented, e.g., in [4] with an extended Kalman filter for estimating parameters in the case of a gyroless satellite and in [5] with an unscented Kalman filter. In [6], a recursive algorithm for on-orbit estimation of a spacecraft inertia tensor is presented. The estimation problem is formulated as a semi-definite program with explicit enforcement of positive-definiteness and other constraints on the elements of the inertia tensor, which enhances convergence and guarantees that a physically valid result is returned.

However, when dealing with complex systems as space vehicles, relying on purely physics-based identification techniques may be not sufficient. This is especially true in the frequent situation in which the available physical model is only partially known: the presence of unmodeled components tends to hinder the estimation accuracy of the interpretable physical parameters. Indeed, in such cases, the identification algorithm tries to capture and encompass in the identified physical parameter value also the residual dynamical effects caused by the unmodelled part, with the aim of minimizing the prediction error. This comes at the cost of wrong and unreliably identified physical values.

In this paper, we show how it is possible to exploit a purely physics-based identification approach, as the one first proposed in [7], and still achieve a reasonable accuracy of the estimation properly enforcing physical constraints [7] and relying on the concept of "benign over-fitting" [8], [9]. Moreover, differently from what it can be done when exploiting standard neural networks approaches, we show how the proposed method allows to enforce the physics of the system through physical constraints in the optimization problem. In the specific case under analysis, we account for the physical properties of the components of the inertia tensor, i.e., symmetry, positive definiteness, and the triangle inequality constraint to retrace the main features of any inertia matrix.

On the other hand, the use of purely black-box based techniques might be able to capture the entire dynamics of the system at the cost of leading to non interpretable parameters. In the framework of neural-network based estimation, [10] exploits a recurrent neural network to estimate the moment-of-inertia (MOI) of a satellite. More specifically, they showed

C. Donati acknowledges support from PRIN project TECHIE "A control and network-based approach for fostering the adoption of new technologies in the ecological transition", Cod. 2022KPHA24 CUP: D53D23001320006.

⋆corresponding author cesare.donati@polito.it.

[1]M. Mammarella, C. Donati, and F. Dabbene are with the CNR-IEIIT, c/o Politecnico di Torino, Torino, Italy, martina.mammarella@cnr.it, fabrizio.dabbene@cnr.it

[2]C. Donati and C. Novara are with the Department of Electronics and Telecommunications, Politecnico di Torino, Torino, Italy, cesare.donati@polito.it, carlo.novara@polito.it

[3]C. Lagoa is with the School of Electrical Engineering and Computer Science, The Pennsylvania State University, University Park, PA, USA, cml18@psu.edu

that while it is not possible to determine distinctively the absolute MOI but rather the relative MOI ratios, on the other hand, when a (partial) knowledge of the disturbance torque is provided to the neural network, the accuracy estimation error is significantly reduced, suggesting that the neural network is able to predict the MOI.

Another crucial aspect of models derived only from fundamental physical principles is that they may fail to capture the complexity of the actual system dynamics. A potential solution is the use of a physics-informed, or gray-box model, that extends a physics-based model with a data-driven part. In [11] the proposed approach is based on a physics-informed online learning of gray-box models, which does not require any prior knowledge than the white-box sub-model.

In this paper, we propose a *blended* approach, where we combine the multi-step identification method we introduced in [7], based on the closed-form computation of the gradient of a *multi-step* loss function and where the physical insights are reflected directly into a backpropagation-like learning algorithm, with a black-box term, which aims at compensating (possible) unmodeled dynamics in the physical model. The proposed method uses partial knowledge of the nonlinear system's physical description and combines it with black-box basis functions to define an NN-like structure to counterbalance the lack of a reliable dynamical model. In particular, the black-box component works as a "safety net", capturing the unforeseen dynamics and possible model oversimplification, and improving the overall accuracy and reliability of the model. In the numerical results, we show how the regularized black-box contribution is able to capture the unmodelled dynamics related to the off-diagonal elements of the inertia matrix, thus leading to faster convergence of the parameters to the true values and higher accuracy of the estimates.

The remainder of the paper is structured as follows. In Section II, we define the considered framework, introducing the main features of the considered system dynamics and of the estimation model. Section III is dedicated to the definition of physics constraints to be enforced into the optimization problem while the extension to a blended identification approach including a black-box term is described in Section IV. Simulation results obtained with the proposed approach are discussed in Section V and main conclusions are drawn in Section VI.

Notation Given a vector v, we denote by $\mathbf{v}_{1:T} \doteq \{v_k\}_{k=1}^{T}$ the set of vectors $\{v_1, \dots, v_T\}$. Given integers $a \leq b$, we denote by $[a, b]$ the set of integers $\{a, \dots, b\}$. The Jacobian matrix of α_k with respect to β_k is denoted as $\mathcal{J}_k^{\alpha/\beta} \in \mathbb{R}^{n_\alpha \times n_\beta}$, i.e., $\frac{\partial \alpha_k}{\partial \beta_k}$. Similarly, $\mathcal{J}_k^{\alpha/\alpha} \in \mathbb{R}^{n_\alpha \times n_\alpha}$ is the Jacobian matrix of α_k with respect to α_{k-1}, i.e., $\frac{\partial \alpha_k}{\partial \alpha_{k-1}}$.

II. Framework definition

In this section, we briefly recall the main results of [7], related to a physics-based framework for the identification of dynamical systems, in which the physical and structural insights are reflected directly into a backpropagation-like learning algorithm.

A. Problem setup and modelling

We consider a nonlinear, time-invariant system that captures the physical interaction between variables. In the specific framework of inertial parameter estimation considered in this paper, the system is a satellite, whose rotational dynamics can be modeled in continuous time exploiting the standard Euler equations

$$\mathcal{S}: \quad \dot{\omega} = J_C^{-1}\Big(M - \omega \times J_C \omega\Big), \tag{1}$$

where $\omega = [\omega_x \, \omega_y \, \omega_z]^\top$ is the satellite angular velocity expressed in the body frame, M represents the sum of external input, and $J_C = [J_{C,ij}] \in \mathbb{R}^{3\times 3}$ is the inertia matrix to be identified. Moreover, we assume that for the case under analysis, we have the observations $\widetilde{z} = \omega + \eta^z$, with η^z the observation noise.

The aim is to estimate the physical parameters θ and initial condition x_0 from T-step measured, input sequence $\widetilde{\mathbf{u}}_{0:T-1}$ and the corresponding T collected observations $\widetilde{\mathbf{z}}_{0:T-1}$, leading to a model $\widehat{\mathcal{S}}$ of (1) of the form

$$\widehat{\mathcal{S}}: \quad \begin{aligned} \hat{x}_{k+1} &= f(\hat{x}_k, u_k, \hat{\theta}), \\ \hat{z}_k &= g(\hat{x}_k), \end{aligned} \tag{2}$$

where $\hat{x}_k$ and $\hat{z}_k$ are the estimated state and output at time k, respectively, such that $\widehat{\mathcal{S}}$ is the best approximation of $\mathcal{S}$, given its underlying physical structure and the measured data $\{\widetilde{\mathbf{u}}_{0:T-1}, \widetilde{\mathbf{z}}_{0:T-1}\}$. Clearly, the setup in (2) can be recast from (1) setting $x \doteq \omega$, $u \doteq M$, and letting θ denotes the elements of the inertial tensor. Hence, f can be immediately obtained by discretization of (1).

Given the dynamical model $\mathcal{S}$, we propagate each state variable x_i, $i \in [1, n_x]$ over a desired horizon T. Then, given the output predictions $\hat{z}$ and the true measurements $\widetilde{z}$ sampled over the horizon, we define the multi-step prediction error at time k as $e_k \doteq \hat{z}_k - \widetilde{z}_k$, and the local loss at time k as the weighted norm of the error, i.e.,

$$\mathcal{L}(e_k, \theta) \doteq \frac{1}{T}\|e_k\|_{\mathcal{Q}}^2 \doteq \frac{1}{T} e_k^\top \mathcal{Q} e_k, \tag{3}$$

with $\mathcal{Q} \succeq 0$. Then, we consider the minimization of the *multi-step regression cost* $\mathcal{C}$, defined as the sum of local losses over the prediction horizon T, i.e.,

$$\mathcal{C}(e_k, \theta) = \sum_{k=0}^{T-1} \mathcal{L}(e_k, \theta) \doteq \sum_{k=0}^{T-1} \mathcal{L}_k. \tag{4}$$

Since the overall objective function in (4) is (in general) *non-convex*, due to the nonlinearity[1] in θ and x_k of $f(x_k, u_k, \theta)$ and $g(x_k)$ in (2), we can rely on gradient-based algorithms (see, e.g., [12]) to address the optimization problem, aiming to reach some (local) minima, and eventually, compute a (sub)optimal estimation of θ and x_0.

[1]Note that the problem would be still be nonconvex if the systems dynamics were linear, due to the multi-step cost.

Specifically, relying on the similarity between the multi-step dynamics propagation and the well-known structure of neural networks, we have that each time step k is seen as a "*layer*" composed by n_x "*neurons*". Hence, the interconnection links between layers and neurons, are activated or deactivated according to the system dynamical structure defined in $\mathcal{S}$. The peculiarity of the proposed approach is that, unlike neural network backpropagation, we have the *same weights* θ and the *same functions* in all layers, which allows to derive a closed-form of the gradient of $\mathcal{C}(e_k, \theta)$ with respect to θ and x_0, i.e., $\nabla\mathcal{C} = [\nabla_\theta \mathcal{C},\ \nabla_{x_0}\mathcal{C}]$.

In the following, we briefly recall the backpropagation-based algorithm introduced in [7] to compute the gradient in closed-form, which will also be extended to the identification problem discussed in Section IV. The interested reader is referred to [7] for additional details.

B. Closed-form gradient computation

First, let us derive the gradient of the cost function $\mathcal{C}$ at epoch ℓ with respect to θ, i.e., $\nabla_\theta \mathcal{C}^{(\ell)}$, as the product of some intermediate partial derivatives. Specifically, we can define $\nabla_\theta \mathcal{C}^{(\ell)}$ by considering the effect of the current (in terms of epochs) estimate $\hat{\theta}$ *for each time step* k on the cost $\mathcal{C}$, i.e.,

$$\nabla_\theta \mathcal{C}^{(\ell)} = \sum_{k=1}^{T-1} \frac{\mathrm{d}\mathcal{C}^{(\ell)}}{\mathrm{d}\theta}\bigg|_k = \sum_{k=1}^{T-1}\left(\frac{\partial \mathcal{C}^{(\ell)}}{\partial \theta}\bigg|_{k|k} + \sum_{\tau=k+1}^{T-1} \frac{\mathrm{d}\mathcal{C}^{(\ell)}}{\mathrm{d}\theta}\bigg|_{\tau|k} \right). \tag{5}$$

Applying the chain rule of differentiation, as typically done in classical backpropagation, in (5), the closed-form of $\nabla_\theta \mathcal{C}^{(\ell)}$ is given by

$$\begin{aligned} \nabla_\theta \mathcal{C}^{(\ell)} &= \sum_{k=1}^{T-1} \gamma_k + \sum_{k=1}^{T-1} \Gamma_k \mathcal{J}_k^{x/\theta} \\ &+ \sum_{k=1}^{T-1} \sum_{\tau=k+1}^{T-1} \left(\Gamma_\tau \prod_{c=0}^{\tau-k-1} \mathcal{J}_{\tau-c}^{x/x} \right) \mathcal{J}_k^{x/\theta}. \end{aligned} \tag{6}$$

Then, at each epoch ℓ, the gradient can be simply *evaluated* at the current value of $\hat{\theta}^{(\ell)}, \hat{x}_0^{(\ell)}$, and the ensuing predictions $\hat{\mathbf{x}}_{1:T}^{(\ell)}$ and $\hat{\mathbf{z}}_{0:T-1}^{(\ell)}$.

Analogously, we can derive in closed-form the gradient with respect to the initial condition $\nabla_{x_0} \mathcal{C}^{(\ell)}$ obtaining

$$\nabla_{x_0} \mathcal{C}^{(\ell)} = \sum_{k=1}^{T-1} \Gamma_k \prod_{c=0}^{k-1} \mathcal{J}_{k-c}^{x/x}. \tag{7}$$

Once the gradients $\nabla_\theta \mathcal{C}^{(\ell)}$ and $\nabla_{x_0} \mathcal{C}^{(\ell)}$ are computed, it is possible to apply a gradient-based algorithm to solve the optimization problem (4), such that the estimate of θ and x_0 are updated at each epoch ℓ.

III. Physics-based constraints and overparametrization

In order to ensure that the estimated parameters align with the physics of the phenomena under investigation, it is necessary to take into account the specific behavior of the system. This means that the identified model shall comply with the fundamental laws and be consistent with the physical principles. This aspect is formally embedded into the cost $\mathcal{C}$ by means of *penalty terms* that introduce physical constraints of the form

$$\begin{aligned} h(\hat{x}_k, \theta) &\leq 0, \qquad \forall k \in [0, T], \\ q(\hat{x}_k, \theta) &= 0, \qquad \forall k \in [0, T], \end{aligned}$$

with $h, q : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \to \mathbb{R}$ time-invariant, differentiable functions with continuous derivatives. Specifically, the general cost $\mathcal{C}$ is modified as follows

$$\mathcal{C} = \sum_{k=0}^{T-1} \mathcal{L}_k + \lambda h(\hat{x}_k, \theta) + \nu q(\hat{x}_k, \theta)^2,$$

where $\lambda \in \mathbb{R}$ and $\nu \in \mathbb{R}$ control the relevance of the physical inequality constraint, i.e., $h(\hat{x}_k, \theta)$, and of the physical equality constraint, i.e., $q(\hat{x}_k, \theta)$, such that the higher the violation of the physical properties in the predicted states and weights is, the larger the associated loss value will be.

For the system under analysis, three physical constraints on the elements of the inertia matrix are accounted, i.e., symmetry, positive definiteness, and the triangle inequality [6].

A. Symmetry

1) By design: The symmetry of the identified matrix can be imposed by identifying the lower-triangular matrix and forcing the matrix to be symmetric in the model. In particular, the inertia matrix is parametrized as

$$\hat{J}_C \doteq \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_4 \\ \hat{\theta}_2 & \hat{\theta}_3 & \hat{\theta}_5 \\ \hat{\theta}_4 & \hat{\theta}_5 & \hat{\theta}_6 \end{bmatrix}.$$

2) Via overparametrization: The second approach to enforce symmetry relies on the recent findings in the field of neural networks related to the concept of "benign overfitting" [8], [9]. This method allows to achieve more accurate estimates and faster convergence to the true values by the overparametrization of the vector of parameters $\hat{\theta} \in \mathbb{R}^9$. In this case, the symmetry of the overparametrized inertia matrix

$$\hat{J}_C \doteq \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 \\ \hat{\theta}_4 & \hat{\theta}_5 & \hat{\theta}_6 \\ \hat{\theta}_7 & \hat{\theta}_8 & \hat{\theta}_9 \end{bmatrix},$$

is imposed by enforcing the following set of equality constraints

$$\begin{aligned} q_1(\hat{x}_k, \theta) &\doteq \hat{\theta}_2 - \hat{\theta}_4 = 0, \\ q_2(\hat{x}_k, \theta) &\doteq \hat{\theta}_3 - \hat{\theta}_7 = 0, \\ q_3(\hat{x}_k, \theta) &\doteq \hat{\theta}_6 - \hat{\theta}_8 = 0. \end{aligned}$$

B. Positive semi-definiteness

The positive semi-definiteness of the inertia matrix is enforced by imposing the matrix to be diagonal dominant with real non-negative diagonal entries. Hence, the following inequality constraints are defined

$$\begin{aligned} h_1(\hat{x}_k, \theta) &\doteq |\hat{J}_{C,12}| + |\hat{J}_{C,13}| - |J_{C,11}| \leq 0 \\ h_2(\hat{x}_k, \theta) &\doteq |\hat{J}_{C,21}| + |\hat{J}_{C,23}| - |J_{C,22}| \leq 0 \\ h_3(\hat{x}_k, \theta) &\doteq |\hat{J}_{C,31}| + |\hat{J}_{C,32}| - |J_{C,33}| \leq 0. \end{aligned}$$

Such non-differentiable constraints are effectively handled by projecting the parameters into the feasible parameter set defined by the inequalities whenever a violation occurs.

C. Triangle inequality

Last, we rely on the physical property of an inertia tensor defined by the triangle inequality. Indeed, as detailed in [6], if a symmetric, positive semi-definite real matrix does not satisfy the triangle inequality, it doesn't represent a physically possible distribution of mass. Hence, triangle inequality constraints are imposed on the diagonal entries of the inertia matrix to prevent nonphysical predictions. These constraints are defined as follows

$$\begin{aligned} h_4(\hat{x}_k, \theta) &\doteq \hat{J}_{C,11} - \hat{J}_{C,22} - \hat{J}_{C,33} \leq 0, \\ h_5(\hat{x}_k, \theta) &\doteq \hat{J}_{C,22} - \hat{J}_{C,11} - \hat{J}_{C,33} \leq 0, \\ h_6(\hat{x}_k, \theta) &\doteq \hat{J}_{C,33} - \hat{J}_{C,11} - \hat{J}_{C,22} \leq 0. \end{aligned}$$

IV. BLACK-BOX MODEL EXTENSION

In this section, we show how we can include into the previous identification approach a black-box term that counterbalances the unmodelled dynamics and improves the accuracy and the convergence rate of the estimated parameters to their true values.

Suppose that the physical model f in (2) is not completely available. Instead, we have access to an approximated physical model $\widehat{f}$ that does not capture entirely the physics of the phenomenon, leading to an unmodeled dynamics that is too relevant to allow an accurate identification of the physical parameters θ simply relying on $\widehat{f}$ and a pure physics-based identification procedure. Then, it is possible to extend the model $\widehat{\mathcal{S}}$ introducing a black-box component to the physics-based one to capture the information that remains unaccounted by the physical priors. The extended model dynamics can be defined as

$$\begin{aligned} \hat{x}_{k+1} &= \widehat{f}(\hat{x}_k, u_k, \hat{\theta}) + \psi(\hat{x}_k, \Omega, W, B), \\ \hat{z}_k &= g(\hat{x}_k), \end{aligned} \tag{8}$$

where $\psi(\hat{x}_k, \Omega, W, B) = [\psi_1, \ldots, \psi_{n_x}]^\top$, and $\Omega = [\omega_{\iota j}] \in \mathbb{R}^{n_x, m}$, $W = [W^{(1)}, \ldots, W^{(n_x)}] \in \mathbb{R}^{n_x, n_x m}$ with $W^{(\iota)} = [w_{ij}^{(\iota)}] \in \mathbb{R}^{n_x, m}$, and $B = [b_{\iota j}] \in \mathbb{R}^{n_x, m}$ are the black-box additional weights to be learned during the training procedure, without any physical interpretation. Let us define $\varphi \in \mathbb{R}^m$, i.e., the vector of basis functions $\varphi(\hat{x}_k) = [\varphi_1(\hat{x}_k), \ldots, \varphi_m(\hat{x}_k)]^\top$, with $\varphi_j : \mathbb{R}^{n_x} \to \mathbb{R}$. Here, the term ψ captures the discrepancy between the true system and the incomplete model $\widehat{f}$, and its ι-th element is defined as

$$\psi_\iota = \sum_{j=1}^{m} \omega_{\iota j} \varphi_j \left(\sum_{i=1}^{n_x} w_{ij}^{(\iota)} \hat{x}_{i,k} + b_{\iota j} \right), \quad \iota \in [1, n_x]. \tag{9}$$

Hence, let us consider the modified *multi-step regression cost* $\mathcal{C}$ as the sum of local losses over the prediction horizon T accounting for the black-box term, i.e.,

$$\mathcal{C}(e_k, \theta, \Omega) = \sum_{k=0}^{T-1} \mathcal{L}(e_k, \theta, \Omega) \doteq \sum_{k=0}^{T-1} \mathcal{L}_k^{\Omega}. \tag{10}$$

The local loss at time k is defined by the weighted norm of the error (3), plus a regularization term, i.e.,

$$\mathcal{L}_k^{\Omega} \doteq \frac{1}{T} \|e_k\|_{\mathcal{Q}}^2 + \frac{\rho}{T} \sum_{\iota=1}^{n_x} \|\omega_\iota^\top\|_1,$$

with $\omega_\iota^\top$ the ι-th row of Ω, and $\rho \in \mathbb{R}$ penalizing the regularization term such that the lower the sparsification of Ω rows is, the larger the associated black-box loss value will be. Notice that, since the ℓ_1-norm is not differentiable, it must be replaced with a differentiable approximation of the absolute value to allow gradient computation. In this work, we exploit the softplus approximation of the absolute value proposed in [13], resulting in the following definition of the differentiable local loss operator

$$\begin{aligned} \mathcal{L}_k^{\Omega} &\doteq \frac{1}{T} \|e_k\|_{\mathcal{Q}}^2 + \frac{\rho}{T} \|\Omega\|_{1\mu}, \\ \|\Omega\|_{1\mu} &\doteq \frac{1}{\mu} \sum_{i=1}^{n_x} \sum_{j=1}^{m} \left[\log(1+\mathrm{e}^{-\mu\omega_{ij}}) + \log(1+\mathrm{e}^{\mu\omega_{ij}}) \right], \end{aligned} \tag{11}$$

where the tunable parameter μ controls the level of the ℓ_1-norm approximation. Last, we define the novel hybrid identification problem as

$$(\hat{\theta}^\star, \hat{x}_0^\star, \Omega^\star, W^\star, B^\star) \doteq \arg \min_{\theta, x_0, \Omega, W, B} \mathcal{C}_T,$$

in which we aim to minimize the mean squared error over the sampled measurements to obtain an estimate of θ and x_0.

Remark 1 (Gradient computation for the blended model): Notice that the closed-form gradient computation algorithm in [7] is general and it can be used also when a black-box compensation term is included in the loss function. In particular, this can be done redefining the local loss term with a new differentiable function, as the one in (11). In the considered case, the terms in the closed-form gradient formulas (6), (7) are consequently redefined as

$$\begin{aligned} \gamma_k^{\Omega} &\doteq \nabla_\theta \mathcal{L}_k + \frac{\rho}{T} \nabla_\theta \|\Omega\|_{1\mu}, \\ \Gamma_k^{\Omega} &\doteq \nabla_e \mathcal{L}_k \mathcal{J}_k^{e/z} \mathcal{J}_k^{z/x} + \frac{\rho}{T} \nabla_x \|\Omega\|_{1\mu}. \end{aligned}$$

V. NUMERICAL RESULTS

We consider a 3U CubeSat with a mass of 4 kg, a (true) tensor of inertia

$$J_C = \begin{bmatrix} 0.04027 & 0.00312 & 0.000145 \\ 0.00312 & 0.04028 & 0.000971 \\ 0.000145 & 0.000971 & 0.00801 \end{bmatrix},$$

and initial condition $x_0 = [0, -0.0011, 0]^\top$. In this example, we rely on a strategy similar to the one proposed in [14] to generate a random input signal that persistently excites the system and simulates a tumbling motion useful for the identification of the inertia matrix. Hence, we consider $M \sim \mathcal{N}(10^{-5}, \sigma_{M_d})$ with $\sigma_{M_d} = 10^{-7} \frac{\text{rad}}{\text{s}}$. The observation noise is assumed to be a Gaussian white noise $\eta_k^z \sim \mathcal{N}(0, \sigma_{\eta^z})$ with standard deviation $\sigma_{\eta^z} = \left[5 \cdot 10^{-4}, 5 \cdot 10^{-4}, 3 \cdot 10^{-3}\right]^\top$ rad/s [2].

The following analyses involve the identification of the optimal values for the satellite inertia matrix (i.e., the physical parameters $\hat{\theta}$ are the elements of J_C) using the proposed gradient approach and a first-order gradient descent algorithm with an adaptive learning rate. The simulations are run in MATLAB 2021a over an Apple M1 with an 8-core CPU, 8GB of RAM, and a 256 GB SSD unit.

First, we consider the scenario where the physical model fully describes the true system dynamics, that is θ contains all the elements of the inertia matrix. We generate a sequence of $T = 500$ data, integrating (1) with a sampling time $T_s = 0.1$s over a 50s simulation. Hence, we consider the forward Euler discretization of the same model as the physical model for the identification (2), i.e.,

$$\omega_{k+1} = \omega_k + T_s J_C^{-1}\Big(M - \omega_k \times J_C \omega_k\Big).$$

Here, we analyze the results obtained with the identification algorithm proposed in [7] and resumed in Section II-B, observing how the overparametrization affects the identification of the parameters. Following the approach detailed in Section III-A, we first minimize the number of parameters to identify (i.e., $\hat{\theta}$ represent the element in the lower-triangular part of J_C), while forcing symmetry directly into the definition of the estimated inertia matrix. Then, we exploit the concept of "benign over-fitting" increasing the number of parameters to identify (i.e., $\hat{\theta}$ contains all the elements of J_C) and symmetry is forced by additional equality constraints, as described in Section III-A.2.

In Fig. 1 we observe the evolution of the estimated parameters with respect to the algorithm epochs ℓ for $N = 100$ different initial conditions of $\hat{\theta}$, randomly selected as $\hat{\theta}_0 \doteq \theta + \mathcal{N}(0, \theta)$. It can be observed that, although both approaches provide a good mean estimation of the parameters, overparametrization leads to faster convergence and more accurate estimates. This reflects that the majority of the trajectories reach the vicinity of the true parameter value in a shorter time and with a lower estimation error. This behavior is confirmed by the evolution of the loss functions over epoch $\ell \in [0, 250]$ drawn in Fig. 2.

In the second scenario, we consider the same sequence of $T = 500$ data. However, we consider the satellite as a rigid, symmetric body, obtaining a simplified model due to the diagonality of the inertia matrix, leading to the following set of equations, discretized using a forward Euler method

$$\begin{aligned}
\omega_{x,k+1} &= \omega_{x,k} + T_s\left(\frac{M_{x,k}}{J_{C,11}} - \frac{J_{C,33} - J_{C,22}}{J_{C,11}}\omega_{y,k}\omega_{z,k}\right),\\
\omega_{y,k+1} &= \omega_{y,k} + T_s\left(\frac{M_{y,k}}{J_{C,22}} - \frac{J_{C,11} - J_{C,33}}{J_{C,22}}\omega_{x,k}\omega_{z,k}\right),\\
\omega_{z,k+1} &= \omega_{z,k} + T_s\left(\frac{M_{z,k}}{J_{C,33}} - \frac{J_{C,22} - J_{C,11}}{J_{C,33}}\omega_{x,k}\omega_{y,k}\right).
\end{aligned} \tag{12}$$

[2]The noise values are compatible with the case study selected (i.e., around 10% of the state values).

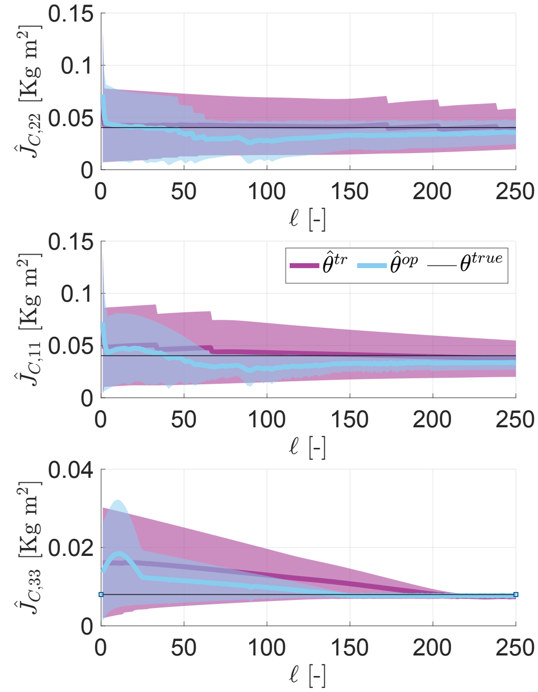

Fig. 1. Comparison between estimated parameters $\hat{\theta}_i$ obtained relying on the complete physical model when symmetry is forced by definition (purple) and when overparametrization is exploited (blue).

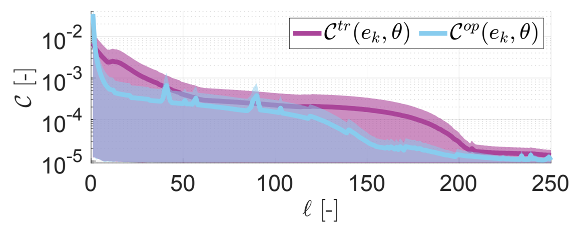

Fig. 2. Evolution of the cost functions for the two approaches.

This choice for $\widehat{S}$ oversimplifies the model, considering only the dynamical terms that result from the diagonal entries of the inertia matrix and neglecting the dynamics arising from the off-diagonal terms. In this case, we compare the performance achievable with the purely physics-based identification algorithm (i.e., (12) directly represents the term f in (2)) and with the blended approach, where we include the black-box term (9), as detailed in Section IV. In the latter case, (12) represents the term $\widehat{f}$ in (8), while the selected vector of basis functions consists of sigmoid, softplus, hyperbolic tangent, and trigonometric functions. The regularization term in (11) is tuned with $\rho = 0.1$ and $\mu = 100$.

Fig. 3 shows the trajectories of the estimated parameters with respect to the algorithm epochs ℓ, calculated over $N = 100$ simulation for different parameter initial conditions. It is worth noting that, when the identification relies on a simplified and incomplete physical model, the parameters converge to incorrect values, "absorbing" the dynamic effects that are not considered. On the other hand, the black-box compensation is able to recover these effects, enabling

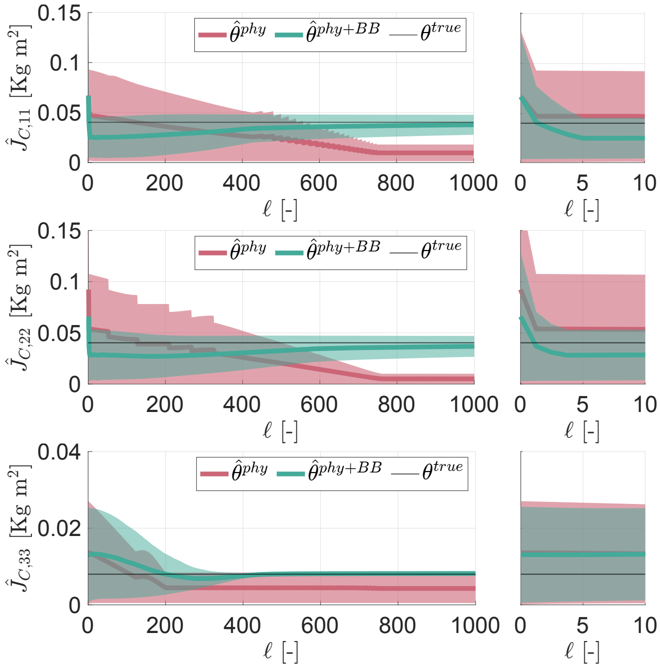

Fig. 3. Comparison between estimated parameters $\hat{\theta}_i$ obtained relying on the given physics only and exploiting a black-box compensation.

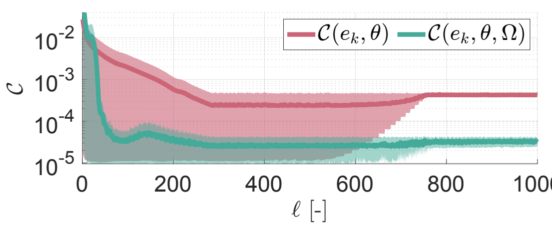

Fig. 4. Evolution of the cost functions for the two approaches.

physical parameters to converge to better estimates in a neighborhood of the true values with a higher convergence rate. Additionally, analyzing the zoomed portion on the right-hand side of Fig. 3, it is possible to notice that the influence of the black-box terms appears to immediately steer the parameter values towards a more favorable region of the parameters space. This, in turn, allows reaching lower values of the cost function and better local minima, as shown in Fig. 4. In particular, it can be noticed that the mean value of the loss functions over all the trajectories is reduced by one order of magnitude when the black-box is exploited to compensate for missing terms in the physical model. However, it is important to remark that having access to the complete physical model results in faster convergence of all the parameters towards the true values ($\sim$250 epochs compared to $\sim$600) with comparable mean squared errors, especially when involving overparameterization. This is shown in Table I, where the average of the mean square error over $N = 100$ simulations is shown for all the proposed approaches.

VI. Conclusions and Future Research

In this paper, we presented an identification algorithm that combines physics-based and black-box techniques. Numerical results showed that either overparametrization or the introduction of a black-box term capturing unmodeled effects can lead to improved accuracy and reliability in estimating the spacecraft inertia matrix. Based on this analysis, we conclude that having access to the complete physical model results in faster convergence of the parameters towards the actual values, leading to lower prediction errors. Hence, it is always optimal to leverage all available physics, but in situations where it is not feasible, black-box compensation plays a crucial role in maintaining accurate identification outcomes.

TABLE I

Average MSE over 100 simulations.

Approach	e_{MSE}^{avg}
Full model	$1.66 \cdot 10^{-4}$
Full model + OP	$1.10 \cdot 10^{-5}$
Simple model	$6.00 \cdot 10^{-3}$
Simple model + BB	$3.70 \cdot 10^{-5}$

References

[1] S. Tanygin and T. Williams, "Mass property estimation using coasting maneuvers," *Journal of Guidance, Control, and Dynamics*, vol. 20, no. 4, pp. 625–632, 1997.

[2] M. L. Psiaki, "Estimation of a spacecraft's attitude dynamics parameters by using flight data," *Journal of Guidance, Control, and Dynamics*, vol. 28, no. 4, pp. 594–603, 2005.

[3] M. C. Norman, M. A. Peck, and D. J. O'shaughnessy, "In-orbit estimation of inertia and momentum-actuator alignment parameters," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 6, pp. 1798–1814, 2011.

[4] H. Yoon, K. M. Riesing, and K. Cahoy, "Kalman filtering for attitude and parameter estimation of nanosatellites without gyroscopes," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 9, pp. 2272–2288, 2017.

[5] G. Magnani, H. Evain, and S. Delavault, "Satellite inertia estimation and observability analysis," in *9^{th} European Conference for Aeronautics and Space Sciences (EUCASS)*, 2022.

[6] Z. R. Manchester and M. A. Peck, "Recursive inertia estimation with semidefinite programming," in *AIAA Guidance, Navigation, and Control Conference*, 2017, p. 1902.

[7] C. Donati, M. Mammarella, F. Dabbene, C. Novara, and C. Lagoa, "One-shot backpropagation for multi-step prediction in physics-based system identification," in *20th IFAC Symposium on System Identification (SYSID)*, 2024.

[8] A. C. B. de Oliveira, M. Siami, and E. D. Sontag, "Dynamics and perturbations of overparameterized linear neural networks," in *2023 62nd IEEE Conference on Decision and Control (CDC)*, 2023.

[9] Z. Xu, H. Min, S. Tarmoun, E. Mallada, and R. Vidal, "Linear convergence of gradient descent for finite width over-parametrized linear networks with general initialization," in *International Conference on Artificial Intelligence and Statistics*, 2023.

[10] N. Enders, J. Curro, J. Hess, and R. Cobb, "Spacecraft moment of inertia estimation via recurrent neural networks," in *AIAA/AAS Astrodynamics Specialist Conference*, 2020.

[11] K. F. Løwenstein, D. Bernardini, L. Fagiano, and A. Bemporad, "Physics-informed online learning of gray-box models by moving horizon estimation," *European Journal of Control*, vol. 74, p. 100861, 2023.

[12] S. Sun, Z. Cao, H. Zhu, and J. Zhao, "A survey of optimization methods from a machine learning perspective," *IEEE Transactions on Cybernetics*, vol. 50(8), pp. 3668–3681, 2019.

[13] M. Schmidt, G. Fung, and R. Rosales, "Fast optimization methods for l1 regularization: A comparative study and two new approaches," in *18th European Conference on Machine Learning*, 2007.

[14] I. A. Lomaka, "A possible approach to the identification of inertial parameters of large-sized space debris using a specialized nanosatellite," in *Journal of Physics: Conference Series*, 2020.