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Concurrent evacuation planning and adaptive spatial field estimation using field-dependent guidance with on-the-fly trajectory reconfiguration

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Abstract— **This work examines an on-the-fly trajectory reconfiguration for the human evacuation in indoor environments for two different situations. When the spatial field representing carbon monoxide concentration is known but time varying, the evacuation planning uses a level-set guidance to reposition the evacuee towards an exit that will result in the smallest amount of the hazardous substance accumulated in the lungs. As the level-set guidance is based on the time-invariant spatial field, a discrete time snapshot of the time-varying field is used to generate viable escape trajectories. The escape trajectories are recalculated when new spatial field knowledge is updated thus leading to continuous trajectory reconfiguration. The other situation involves an unknown spatial field that is either constant in time or slowly time-varying. The level-set guidance is based on a current estimate of the field furnished via an adaptive spatial field estimation. Such an adaptive estimate is made possible due to the motion of the evacuee which is capable of inducing persistence of excitation and thus yielding convergence of the spatial field estimation. A planning stage is added to the duration of one cycle, consisting now of both a planning stage in which the agent is immobile and computing, and a travelling stage in which the agent is moving towards the currently-declared viable exit. During both stages, the agent is able to adaptively estimate the spatial field, but using the frozen-in-time (snapshots) knowledge of the spatial field's adaptive estimate that is produced at the end of the planning stage. Numerical studies for both cases are included to provide insights on the effects of accumulated amounts of hazardous environments on the escape trajectories in human evacuation.**

I. Introduction

This work combines earlier efforts on level-set evacuation guidance for a single evacuee in indoor environments. The evacuation scheme proposed in [1], [2] was executed over a single cycle meaning that the evacuee had to find the optimal trajectory for each of the available exits and then select the one yielding the smallest accumulated amount of the hazardous substance inhaled. Such level-set guidance was an extension of earlier works [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] on the use of level-sets for human navigation with the added element of minimizing the accumulated amount over a selected trajectory. Since the spatial field is unknown, a way to reconstruct it and use its estimate for the level-set based evacuation guidance is to use the adaptive spatial field estimation scheme by mobile agents presented in [13].

This paper examines two different situations regarding the hazardous field depicting the concentration of carbon monoxide in indoor environments. The first one considers a spatiotemporally varying field, often described by advection-diffusion PDE [14], that is known to the agent. This perception of the environment is available to the agent at discrete time instances (snapshots). The other one considers a spatially varying field that is described by a Poisson-type PDE (elliptic PDE) that is time invariant; that is, constant-in-time, but varying-in-space. For the latter, the agent is implementing a learning scheme based on adaptive estimation, in order to have an understanding of the hazardous environment (situation). Such perceived knowledge of the environment represents the estimated concentration which is time varying. It is noted that while the true environment is only spatially varying, its adaptive estimate is time varying. This estimate of the spatial field (snapshot) is similarly made available to the agent at discrete time instances.

In both situations, the agent is using the snapshots of the environment in order to plan a trajectory that takes into account reachability constraints and has a finite time horizon. The escape trajectory in this case is only projected locally into the future that has a duration limited by the planning-travel cycle. Thus one has situational awareness of the agent as it encompasses the three levels of *perception-comprehension-projection*. In the perception level, the agent acquires knowledge of the environment, or generates its own knowledge of the environment via adaptive learning. In the comprehension level, the agent uses the knowledge of the environment to generate candidate trajectories to any of the escape exits that result in minimum levels of the accumulated amount of the hazardous substance inhaled. In the third level, the agent projects into the future by examining the candidate escape trajectories and follows the optimal escape trajectory.

Figure 1 depicts an animation for the proposed on-the-fly trajectory reconfiguration. In a given cycle, the agent is acquiring the intermittent information of the environment and computes the optimal trajectory for each escape exit. Then it selects the one that results in the smallest projected accumulated amount of the hazardous substance and follows that trajectory. In the first cycle depicted in Figure 1a, the agent acquires the initial snapshot of the spatial field and generates three trajectories for the three escape exits that each yield the smallest possible accumulated amount of the hazardous field. It selects the one with the smallest amount which corresponds to exit 2 and follows it for one cycle (yellow line). As new information of the environment is received, see Figure 1b, it generates three new escape trajectories and follows the one that predicts minimum accumulated amount at the exit. In Cycle 3, Figure 1c, it follows the new trajectory (cyan line) for exit 3 and subsequently plans the new escape

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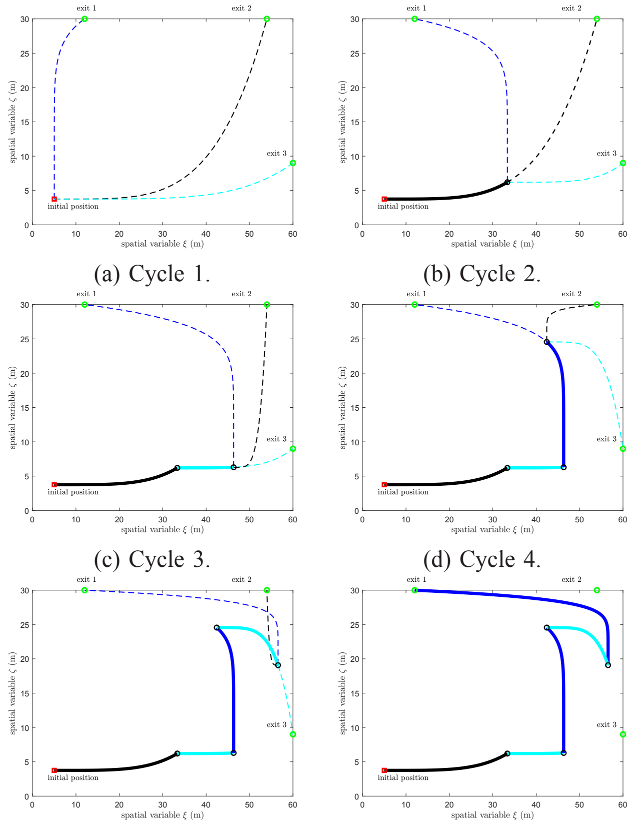


(a) Cycle 1. (b) Cycle 2.

(c) Cycle 3. (d) Cycle 4.

(e) Cycle 5. (f) Cycle 6.

Fig. 1: On-the-fly reconfiguration of escape trajectories; dashed lines represent candidate trajectories corresponding to each escape exit for the local horizon $[t_k, \infty)$ and solid lines represent trajectory taken in the local interval $[t_{k-1}, t_k]$.

trajectories. In cycle 4, Figure 1d, it follows the trajectory leading to exit 3 and projects into the near future the new escape trajectories. In cycle 5 it follows the trajectory for exit 3 (cyan line) and at cycle 6 it follows the trajectory for escape exit 1. In this case, the sequence of reconfigured trajectories is $2 \to 3 \to 1 \to 3 \to 1$.

It is envisioned that such a trajectory reconfiguration can be computed in real-time using hand-held devices by human evacuees which can also steer them towards the most viable escape exit. For sensing agents onboard mobile platforms exploring hazardous spatial fields [15] that have accumulated effects on the platform hardware and agent's mission, the computational time and planing stage can be essentially zero with the mission reconfiguration occurring instantaneously.

Contribution. This work extends the earlier work [16] on the intermittent adaptive spatial field estimation for a single escape exit to multiple escape exits with the added task on the real-time trajectory reconfiguration which allows for mid-flight escape exit reclassification.

The problem formulation is presented in Section II. The modified level-set guidance for known time varying fields is presented in Section III and the corresponding guidance for unknown spatially varying fields with adaptive estimation is given in Section IV. Numerical results are discussed in Section V with conclusions following in Section VI.

II. Mathematical Formulation

Various components of the proposed evacuation planning are summarized in a self-contained and modular fashion.

A. Modeling of hazardous field

The hazardous field representing the concentration of carbon monoxide or any other harmful substance, is mathematically represented by a PDE over a 2D spatial domain. When the spatial field is assumed known, it is modelled by a 2D advection-diffusion PDE given by

$$\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (uc) + f, \tag{1}$$

defined over a rectangular domain $\Omega = [0, L_\xi] \times [0, L_\zeta]$. The solution to (1) is denoted by $c(t, \xi, \zeta)$ and is assumed to be available to the agent only at discrete time instances. In other words, it is the function $c(t_k, \xi, \zeta)$ for $t_k \in \{t_1, t_2, t_3, \ldots, \}$ and all $(\xi, \zeta) \in \Omega$ that is available to the agent and *not* $c(t, \xi, \zeta)$ for all $t \in \mathbb{R}^+$ and all $(\xi, \zeta) \in \Omega$. Associated with (1) are the boundary conditions and the initial conditions. For boundary conditions, it is assumed that for part of the boundary $\partial\Omega$, Dirichlet conditions are assumed and for the other part, Neumann conditions are assumed with $\Gamma_D \cup \Gamma_N = \partial\Omega$. The initial conditions are $c(0, \xi, \zeta) = c_0(\xi, \zeta)$. The function $f(t, \xi, \zeta)$ denotes the source term and represents the spatial and temporal component of various sources of the hazardous substance in the domain Ω, e.g. locations as doors, windows and input signals such as mass rates.

When the spatial field is not known, then based on the current schemes for on-line field reconstruction, it is assumed to be a function $c(\xi, \zeta)$ which is taken to be the solution to a Poisson-type elliptic PDE and viewed as the steady-state solution to (1). In this case, it is governed by

$$0 = \nabla \cdot (D \nabla c) - \nabla \cdot (uc) + f, \tag{2}$$

where the source term is time-invariant with $f = f(\xi, \zeta)$ and with boundary conditions similar to the unsteady case in (1).

B. Equations of pedestrian motion

Following the earlier work [1], [2], a simple model for an evacuee motion in a 2D indoor environment is given by

$$\begin{aligned} \dot{x}_1(t) &= \upsilon(t)\cos(\theta), \\ \dot{x}_2(t) &= \upsilon(t)\sin(\theta), \end{aligned} \qquad (x_1(0), x_2(0)) = (x_{10}, x_{20}), \tag{3}$$

where $(x_1(t), x_2(t)) \in \Omega$ are the agent's coordinates in the spatial domain Ω, θ is the angle between the motion direction and the horizontal axis ξ, and $\upsilon(t)$ is the evacuee speed. In this paper, it is taken as a constant when the evacuee has not inhaled "too much" carbon monoxide, and becomes equal to zero when the accumulated amount inhaled is above a given threshold. This is a reasonable assumption to make as a first level approximation.[1] In this case, we model the speed as

$$\upsilon(t) = \upsilon_{max} - H(t - t_{thr})\upsilon_{max} \tag{4}$$

[1] Ideally one assumes the agent starts with zero speed, accelerates to reach a steady-state maximum speed with the speed eventually decreasing due to fatigue or other physiological factors. In the context of evacuation, the speed can be decreasing from the maximum speed in proportion to the accumulated amount of carbon monoxide inhaled in the lungs, see [2].

where $H(t)$ is the Heaviside step function and t_{thr} denotes the instance the accumulated amount of carbon monoxide inhaled in the lungs exceeds the limit of carbon monoxide poisoning. The initial conditions $(x_1(0), x_2(0))$ represent the agent's coordinates at initial time. The angle θ is the control variable which will be designed using the level-set guidance. This will take the form of time histories of the angle which the agent implements over a given time interval. Thus, we associate θ with a trajectory selected for a given time interval.

C. Spatial field measurement and inhalation model

The measurement obtained by the mobile agent is dependent on its position $(x_1(t), x_2(t)) \in \Omega$ and is given by

$$y(t) = \int_{\Omega} \delta(\xi - x_1(t))\delta(\zeta - x_2(t))c(t,\xi,\zeta)\,\mathrm{d}\xi\mathrm{d}\zeta, \quad (5)$$

where $\delta(\cdot)$ denotes the Dirac delta function. Thus, the measurement is equal to the concentration evaluated at the current agent position $(x_1(t), x_2(t))$ i.e., $y(t) = c(t, x_1(t), x_2(t))$. Since the agent is mobile, it renders the output $y(t)$ *position-dependent*. For the case of an unknown time-invariant field $c(\xi,\zeta)$, the measurements of the mobile agent are given by

$$y(t) = c(x_1(t), x_2(t)). \quad (6)$$

The amount of the hazardous substance (carbon monoxide) accumulated in the agent's lungs due to inhalation is calculated via the line integral of the species concentration $c(\xi,\zeta)$ or $c(t,\xi,\zeta)$ along the yet-to-be-determined path towards an escape exit. Following [1], [2], we summarize the θ-dependent accumulated amount due to an inhale-exhale cycle via the total amount inhaled

$$J(\mathcal{C}_\theta) = \frac{1}{2}\int_{\mathcal{C}_{\theta(t)}} c(\mathbf{r})\,\mathrm{d}s,$$

where $\mathbf{r}(t) = (x_1(t), x_2(t))$ and $\mathcal{C}_\theta$ is the θ-dependent trajectory. The coefficient $1/2$ in front of the integral designates the ratio between the time of inhalation and the total time for a breath cycle, which here assumes equal time of inhaling and exhaling.

The concentration at each position is denoted by $c(x_1(t), x_2(t))$ or $c(t, x_1(t), x_2(t))$ along a trajectory $\mathcal{C}_{\theta(t)}$. Use of the line integral [17] leads to $\mathrm{d}s = \upsilon(t)\mathrm{d}t$ which produces

$$J(0,t,\mathcal{C}_{\theta(t)}) = \frac{1}{2}\int_0^t \upsilon(\tau)c(x_1(\tau), x_2(\tau))\,\mathrm{d}\tau, \quad (7)$$

for the case of time-invariant and unknown $c(\xi,\zeta)$, and

$$J(0,t,\mathcal{C}_{\theta(t)}) = \frac{1}{2}\int_0^t \upsilon(\tau)c(t, x_1(\tau), x_2(\tau))\,\mathrm{d}\tau, \quad (8)$$

for the case of known and time-varying $c(t,\xi,\zeta)$. For a constant speed υ, the above two expressions simplify to

$$J(0,t,\mathcal{C}_{\theta(t)}) = \frac{\upsilon}{2}\int_0^t c(x_1(\tau), x_2(\tau))\,\mathrm{d}\tau, \quad (9)$$

and

$$J(0,t,\mathcal{C}_{\theta(t)}) = \frac{\upsilon}{2}\int_0^t c(t, x_1(\tau), x_2(\tau))\,\mathrm{d}\tau. \quad (10)$$

Remark 1: Please note that for constant speed, with the aid of the assumed measurement models (5), (6), the expressions (9), (10) simplify to

$$J(0,t,\mathcal{C}_{\theta(t)}) = \frac{\upsilon}{2}\int_0^t y(\tau)\,\mathrm{d}\tau.$$

Further, when the guidance follows level-sets with $c(t, x_1(\tau), x_2(\tau)) = m$, m a constant, then $J(0,t,\mathcal{C}_{\theta(t)}) = \upsilon m t/2$.

D. Adaptive estimation of a time-invariant field

When the field is unknown and given by the solution to the PDE (2), then the agent must generate its adaptive estimate. As was presented in [13], this can be accomplished via an adaptive scheme using the single agent measurements in (6). The unknown $c(\xi,\zeta)$ is assumed to admit

$$c(\xi,\zeta) = \sum_{i=1}^n \alpha_i\phi_i(\xi,\zeta), \quad (\xi,\zeta) \in \Omega, \quad (11)$$

where $\phi_i(\xi,\zeta)$ are known spatial functions and α_i, $i = 1,\ldots,n$ are the unknown coefficients. The adaptive estimation problem is to estimate the unknown coefficients α_i using the scalar measurements (6), i.e., using only the spatially varying field $c(\xi,\zeta)$ evaluated at the current position $(x_1(t), x_2(t))$.

The *adaptive estimate* of the unknown $c(\xi,\zeta)$ is given by

$$\widehat{c}(t,\xi,\zeta) = \sum_{i=1}^n \widehat{\alpha}_i(t)\phi_i(\xi,\zeta), \quad (12)$$

where $\widehat{\alpha}_i(t)$, $i = 1,\ldots,n$ are the *adaptive estimates* of the unknown coefficients α_i, $i = 1,\ldots,n$. Central to the extraction of the adaptive laws for $\widehat{\alpha}_i(t)$ is the state estimation error

$$e(t,\xi,\zeta) = \widehat{c}(t,\xi,\zeta) - c(\xi,\zeta) = \sum_{i=1}^n \widetilde{\alpha}_i(t)\phi_i(\xi,\zeta), \quad (13)$$

where $\widetilde{\alpha}_i(t) = \widehat{\alpha}_i(t) - \alpha_i$ are the *adaptive parameter errors*. Using the measurement model (6), the state estimation error at the position $(x_1(t), x_2(t))$ is given by the <u>scalar</u> signal

$$e(t, x_1(t), x_2(t)) = \sum_{i=1}^n \widetilde{\alpha}_i(t)\phi_i(x_1(t), x_2(t)).$$

In compact form it is

$$e(t, x_1(t), x_2(t)) = \Phi^T(x_1(t), x_2(t))\widetilde{\alpha}(t), \quad (14)$$

where the n-dimensional vectors are given by

$$\Phi(x_1(t), x_2(t)) = \begin{bmatrix} \phi_1(x_1(t), x_2(t)) \\ \vdots \\ \phi_n(x_1(t), x_2(t)) \end{bmatrix}, \ \widetilde{\alpha}(t) = \begin{bmatrix} \widetilde{\alpha}_1(t) \\ \vdots \\ \widetilde{\alpha}_n(t) \end{bmatrix}.$$

It should be noted that with the aid of (6), the scalar $e(t, x_1(t), x_2(t))$ is also given by $e(t, x_1(t), x_2(t)) = \widehat{c}(t, x_1(t), x_2(t)) - y(t)$. Following [13], the adaptive laws for $\widehat{\alpha}_i(t)$ along with convergence results are summarized below.

Lemma 1 ([13]): Assume that an unknown time-invariant spatial field satisfies (11) with the functions $\phi_i(\xi,\zeta)$, $i = 1,\ldots,n$, known. The adaptation of the unknown α_i's is

$$\dot{\widetilde{\alpha}}(t) = \dot{\widehat{\alpha}}(t) = -\gamma e(t, x_1(t), x_2(t))\Phi(x_1(t), x_2(t)), \quad (15)$$

where $\gamma > 0$ is the adaptive gain [18]. The convergence of the scalar error $e(t, x_1(t), x_2(t))$ to zero is guaranteed if

$$\Phi^T(x_1(t), x_2(t))\Phi(x_1(t), x_2(t)) > 0, \quad (16)$$

is satisfied uniformly in time. The parameter convergence $\lim_{t\to\infty}\widetilde{\alpha}_i(t)=0$ for all $i=1,\ldots,n$ can be established when the persistence of excitation (PE) condition below is ensured

$$\kappa_1\mathbf{I}\geq\frac{1}{T_0}\int_t^{t+T_0}\Phi(x_1(\tau),x_2(\tau))\Phi^T(x_1(\tau),x_2(\tau))\,d\tau\geq\kappa_0\mathbf{I}\quad(17)$$

Remark 2: As was noted in [13], both the uniform positiveness of the scalar $\Phi^T(x_1(t),x_2(t))\Phi(x_1(t),x_2(t))$ in (16) and the PE condition in (17) are dependent on the agent motion. The PE condition requires the the integral in (17) be uniformly positive definite over any interval $[t,t+T_0]$ despite the fact that the integrant $\Phi(x_1(\tau),x_2(\tau))\Phi^T(x_1(\tau),x_2(\tau))$ is a rank one $n\times n$ matrix for each time. When the agent is not moving, the integrant produces a rank-one constant matrix which violates the PE bound in (17).

Remark 3: As was also discussed in [13], when the guidance $\theta(t)$ in (3) is selected so that the vector $\Phi(\xi,\zeta)$ evaluated at the position $(x_1(t),x_2(t))$ satisfies (16) and (17), then the agent motion becomes both a *necessary* and *sufficient* condition for parameter and functional convergence.

E. Planning and travel stages of a guidance cycle

The agent's guidance may not be performed in a single stage. Instead, it is assumed that during the *planning stage* the agent is not moving and uses this time interval to compute its escape trajectory. The agent can only use the snapshot $c(t_k,\xi,\zeta)$, $\forall(\xi,\zeta)\in\Omega$ of the spatial field at the time instance t_k (the beginning of a new cycle) to generate the escape trajectory. While the field in this case is time-varying $c(t,\xi,\zeta)$, the spatial field information is available to the agent *only* at the discrete time instances t_k, $k=1,2,\ldots$ and at every spatial coordinate (ξ,ζ) in the spatial domain Ω.

In the *travel stage* portion of a single cycle, the mobile agent is executing the escape guidance computed during the planning stage. At the end of this stage, which ends the current cycle, the agent stops and initiates a new cycle. We use τ_{cycle} to denote the duration of a single guidance cycle. The two stages have durations τ_{plan} and τ_{travel} leading to

$$\tau_{cycle}=\tau_{plan}+\tau_{travel}.\qquad(18)$$

By denoting the unknown time since initial time where the agent arrives at any of the escape exits by t_{esc}, the entire interval $[0,t_{esc}]$ is partitioned into N time subintervals, each having a duration τ_{cycle} and initial cycle time instances

$$t_k=(k-1)\tau_{cycle},\qquad k=1,2,\ldots,N.\qquad(19)$$

Of course, an agent does not know t_{esc} and thus one *a priori* decides the duration of a cycle τ_{cycle} and the associated durations of the planning and travel stages. With the length of the subintervals a priori decided, one can then define the instances t_k designating the beginning of a new cycle.

The planning stage occurs during the times $t\in[t_k,t_k+\tau_{plan})$ and the learning stage occurs during the times $t\in[t_k+\tau_{plan},t_k+\tau_{cycle})$. Summarizing, we have:

i) Planning stage: For $t\in[t_k,t_k+\tau_{plan})$, agent is not moving but is planning the escape trajectory for the travel stage. It generates $\theta(t;t_k,\infty)$ for $[t_k+\tau_{plan},t_k+\tau_{cycle})$.

ii) Travel stage: For $t\in[t_k+\tau_{plan},t_k+\tau_{cycle})$ the agent executes the trajectory $\theta(t;t_k,\infty)$ obtained during the planning stage, but terminates it at $t=t_k+\tau_{cycle}$, i.e., executes $\theta(t;t_k,t_k+\tau_{cycle})$.

III. Level-set guidance using snapshots $c(t_k,\xi,\zeta)$ of a slowly time-varying field $c(t,\xi,\zeta)$

When the spatial field is the solution $c(t,\xi,\zeta)$ to the unsteady PDE (1), the assumption is that it is available to the agent *only* at the discrete time instances t_k in (19). Thus, the mobile agent has the knowledge of the function

$$c(t_k,\xi,\zeta),\quad\forall(\xi,\zeta)\in\Omega\text{ and }\forall t_k\text{ in (19)}.\qquad(20)$$

The use of the level-set guidance from [1], [2] is modified here in order to account for partial information of the spatial field (20). The modified level-set guidance is as follows:

i) At the planning stage $t\in[t_k,t_k+\tau_{plan})$, the agent is not moving and is using the spatially varying function $c(t_k,\xi,\zeta)$ to compute the escape trajectory that yields the smallest amount of accumulated substance in the lungs. The optimal trajectory $\theta^{opt}(t;t_k,\infty)$ corresponds to the optimization of the accumulated cost J for the interval $t\in[t_k+\tau_{plan},\infty)$, meaning that the agent tries to find the optimal trajectory $\theta^{opt}(t;t_k,\infty)$ that would lead to an escape with the smallest accumulated J.

ii) During the travel stage $t\in[t_k+\tau_{plan},t_k+\tau_{cycle})$, the agent implements the optimal trajectory $\theta^{opt}(t;t_k,\infty)$ but terminates it at $t=t_k+\tau_{cycle}$ till the next cycle and the next updated information $c(t_{k+1},\xi,\zeta)$.

The details of this guidance are presented in Algorithm 1.

IV. Level-set guidance using snapshots $\widehat{c}(t_k,\xi,\zeta)$ of adaptive estimates of a steady field $c(\xi,\zeta)$

Unlike the previous case, the agent can only use the *estimated* field at the discrete times (19). While the estimator generates $\widehat{c}(t,\xi,\zeta)$ via the adaptation (15), it can only provide the estimated field at the cycle times t_k. Another modification to the level-based guidance is applied in this case.

The modified level-set guidance is as follows:

i) At the planning stage $t\in[t_k,t_k+\tau_{plan})$, the agent is not moving and is using the spatially varying function $\widehat{c}(t_k,\xi,\zeta)$ to compute the escape trajectory that yields the smallest amount of accumulated substance in the lungs. While the guidance is only using $\widehat{c}(t_k,\xi,\zeta)$, the adaptive identifier continues to obtain the adaptive estimates of the coefficients α_i using the modified adaptive law

$$\dot{\widehat{\alpha}}(t)=-\gamma e(t,x_1(t),x_2(t))\Phi(x_1(t_k),x_2(t_k)),\qquad(22)$$

for all $t\in[t_k,t_k+\tau_{plan})$, i.e., it uses the arrested adaptation presented in [16]. The output estimation error $e(t,x_1(t),x_2(t))$ continues to change in the interval $[t_k,t_k+\tau_{plan})$ even though the agent is immobile with

$$(x_1(t),x_2(t))=(x_1(t_k),x_2(t_k)),\quad\forall t\in[t_k,t_k+\tau_{plan}).$$

The regressor vector $\Phi(x_1(t_k),x_2(t_k))$ in this case is constant. The optimal trajectory $\theta^{opt}(t;t_k,\infty)$ corresponds to the optimization of the accumulated cost J for the

Algorithm 1 Level-set based evacuation guidance over $[t_k, t_k + \tau_{cycle}[$ using discrete time field information $c(t_k, \xi, \zeta)$ with on-the-fly trajectory recalculation

1: **initialize:** Using sampling constraints, define the instances t_k in (19) and define the cycle duration $\tau_{cycle} = t_{k+1} - t_k$. Using individual agent capacity, select the planning stage τ_{plan} and travel stage τ_{travel} durations in $\tau_{cycle} = \tau_{plan} + \tau_{travel}$. Using initial $(x_1(t_0), x_2(t_0))$ and desired locations (escape exits) (ξ_j^d, ζ_j^d), $j = 1, \ldots, n_{exits}$, determine the first trajectory planning $\theta(t, t_1)$ for $t \in [\tau_{plan}, \infty)$ but implement in $t \in [\tau_{plan}, \tau_{plan} + \tau_{travel})$.
2: **iterate:** $k = 2$
3: **loop**
4: Define next cycle $[t_k, t_k + \tau_{cycle})$ with $t_k = (k-1)\tau_{cycle}$.
5: In the k^{th} *planning stage* of duration τ_{plan} with $t \in [t_k, t_k + \tau_{plan})$, use the most recent snapshot of the field $c(t_k, \xi, \zeta)$ in (20) to plan the trajectory for $t \in [t_k + \tau_{plan}, \infty)$ using the level-set based guidance in [1], [2].
6: **for** $j = 1$ to n_{exits} **do**
7: from current position $(x_1(t_k), x_2(t_k))$, compute the level-set trajectory to each j^{th} exit in $[t_k + \tau_{plan}, \infty)$
8: compute anticipated accumulated amount $J_j(t_k, \infty, \theta(t, t_k))$ for each exit and truncate to $J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k))$
9: select optimal trajectory $\theta_k^{opt}(t)$ using

$$\theta_k^{opt}(t) = \arg\min_j J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k)) \qquad (21)$$

10: **end for**
11: In the k^{th} *travel stage* of duration τ_{travel} with $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$, implement the level-set based trajectory planning $\theta_k^{opt}(t)$ developed at the most recent planning stage.
12: At the end of the k^{th} *cycle* $t_{k+1} = t_k + \tau_{cycle}$, update the information of the spatial field $c(t_{k+1}, \xi, \zeta)$
13: **if** $\sqrt{(x_1(t_{k+1}) - \xi_d)^2 + (x_2(t_{k+1}) - \zeta_d)^2} > 0$ **then**
14: $k \leftarrow k + 1$
15: **goto** 2
16: **else**
17: terminate-success: reached a safety exit!
18: **end if**
19: **end loop**

interval $t \in [t_k + \tau_{plan}, \infty)$, meaning that the agent tries to find the optimal trajectory $\theta^{opt}(t; t_k, \infty)$ that would lead to an escape with the smallest accumulated J. The level-set guidance in [1], [2] is also modified since the maximum value of the true field needed for the derivation of the level-set guidance is no longer available. Instead, its estimated value $\widehat{c}(t_k, \xi, \zeta)$ is used to generate the level-set guidance.

ii) During the travel stage $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$, the agent implements the optimal trajectory $\theta^{opt}(t; t_k, \infty)$ but terminates it at $t = t_k + \tau_{cycle}$ till the next cycle and the next updated information $c(t_{k+1}, \xi, \zeta)$. At the same time, the agent is using the standard adaptive law in (15), i.e.,

$$\dot{\widehat{\alpha}}(t) = -\gamma e(t, x_1(t), x_2(t)) \Phi(x_1(t), x_2(t)), \qquad (23)$$

for all $t \in [t_k + t_{plan}, t_k + \tau_{cycle})$.

The details of this guidance are presented in Algorithm 2.

Algorithm 2 Arrested adaptation-based evacuation guidance over $[t_k, t_k + \tau_{cycle}[$ using discrete time estimated field information $\widehat{c}(t_k, \xi, \zeta)$ with on-the-fly trajectory recalculation

1: **initialize:** Using sampling constraints, define the instances t_k in (19) and define the cycle duration $\tau_{cycle} = t_{k+1} - t_k$. Using individual agent capacity, select the planning stage τ_{plan} and travel stage τ_{travel} durations in $\tau_{cycle} = \tau_{plan} + \tau_{travel}$. Using the initial estimates of the parameters $\widehat{\alpha}(t_0)$, calculate the initial estimate $\widehat{c}(t_1, \xi, \zeta)$ of the unknown spatially varying field. Using initial $(x_1(t_0), x_2(t_0))$ and desired locations (escape exits) (ξ_j^d, ζ_j^d), $j = 1, \ldots, n_{exits}$, determine the first trajectory planning $\theta(t, t_1)$ for $t \in [\tau_{plan}, \infty)$. but implement in $t \in [\tau_{plan}, \tau_{plan} + \tau_{travel})$. Obtain sensor measurements in both $t \in [t_0, t_0 + \tau_{plan})$ and $t \in [t_0 + \tau_{plan}, t_0 + \tau_{plan} + \tau_{travel})$ and use them to implement the adaptation (15).
2: **iterate:** $i = 2$
3: **loop**
4: Define next cycle $[t_k, t_k + \tau_{cycle})$ with $t_k = (k-1)\tau_{cycle}$. For each time $t \in [t_k, t_k + \tau_{cycle})$ continue to obtain sensor measurements and implement the arrested adaptation (22) regardless of the agent motion.
5: In the k^{th} *planning stage* of duration τ_{plan} with $t \in [t_i, t_i + \tau_{plan})$, use the most recent arrested estimate of the field $\widehat{c}(t_k, \xi, \zeta)$ to plan the trajectory for $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$ using the level-set based guidance in [1], [2]. Continue the nominal adaptation (23).
6: **for** $j = 1$ to n_{exits} **do**
7: from current position $(x_1(t_k), x_2(t_k))$, compute the level-set trajectory to each j^{th} exit in $[t_k + \tau_{plan}, \infty)$
8: compute anticipated accumulated amount $J_j(t_k, \infty, \theta(t, t_k))$ for each exit and truncate to $J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k))$
9: select optimal trajectory $\theta_k^{opt}(t)$ using

$$\theta_k^{opt}(t) = \arg\min_j J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k)) \qquad (24)$$

10: **end for**
11: In the k^{th} *travel stage* of duration τ_{travel} with $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$, implement the level-set based path planning developed at the most recent planning stage.
12: At the end of the k^{th} *cycle* $t_{k+1} = t_k + \tau_{cycle}$, update the adaptive estimate of the spatial field using $\widehat{\alpha}(t_{k+1})$
13: **if** $\sqrt{(x_1(t_{k+1}) - \xi_d)^2 + (x_2(t_{k+1}) - \zeta_d)^2} > 0$ **then**
14: $k \leftarrow k + 1$
15: **goto** 2
16: **else**
17: terminate-success: reached a safety exit!
18: **end if**
19: **end loop**

V. Numerical Studies

The spatial domain is $\Omega = [0, L_\xi] \times [0, L_\zeta] = [0, 100] \times [0, 30]$m with three escape exits at $(\xi_1^d, \zeta_1^d) = (100, 10)$, $(\xi_2^d, \zeta_2^d) = (100, 20)$ and $(\xi_3^d, \zeta_3^d) = (90, 30)$. The cycle duration is taken as $\tau_{cycle} = 6$s with $\tau_{plan} = 2$s, which means that field information is given to the agent every 6 seconds.

In the case of a time-varying field, the true field is

$$c(t_k, \xi, \zeta) = 4100 \exp\left(-{(\xi - 0.7L_\xi)^2}/{2\sigma_\xi^2}\right) \exp\left(-{(\xi - 0.375L_\zeta)^2}/{2\sigma_\zeta^2}\right)$$

with $\sigma_\xi = 0.1L_\xi$, $\sigma_\zeta = \sqrt{L_\zeta}$ for the cycles $k = 1, 2$, i.e., for $t \in [0, 12]$ seconds. For any $t > 2\tau_{cycle}$, the spatial field is

$$c(t_k, \xi, \zeta) = 4100 \exp\left(-{(\xi - 0.7L_\xi)^2}/{2\sigma_\xi^2}\right) \exp\left(-{(\xi - 0.75L_\zeta)^2}/{2\sigma_\zeta^2}\right).$$

In other words, while the spatial field is time-varying, the agent knows that $c(t_1, \xi, \zeta)$ and $c(t_2, \xi, \zeta)$ are given by the first expression and that for any other cycle, the agent knows that $c(t_k, \xi, \zeta)$, for $k \geq 3$ is given by the second expression.

In the case of a time-invariant field, as used in the adaptive case, the true and unknown field is given by

$$c(\xi, \zeta) = 4100 \exp\left(-{(\xi - 0.7L_\xi)^2}/{2\sigma_\xi^2}\right) \exp\left(-{(\xi - 0.75L_\zeta)^2}/{2\sigma_\zeta^2}\right).$$

The agent starts inside the spatial domain at the initial location $(x_1(0), x_2(0)) = ({L_\xi}/{3}, {L_\zeta}/{3})$ and has a speed $\upsilon = 3$m/s.

A. Known time-varying spatial field

The trajectory along with the intermediate escape exit selections is depicted in Figure 2. It is observed that in cycle 1, the agent selects exit #3 and after re-calculation in cycle #2 also selects exit #3. In cycle #3 it changes to exit #2 and retains this escape path for exit #2 in cycles #4, #5 and #6, i.e. it follows the sequence of exits $3 \rightarrow 3 \rightarrow 2 \rightarrow 2 \rightarrow 2 \rightarrow 2$. This agrees with the intermittent knowledge of the spatial field: for the first two cycles it has a peak in the bottom right of Ω and the agent is trying to avoid its large values. After the second cycle, the new information on the spatial field shows that it has a peak value at the top right of Ω and the agent is trying to avoid exit#3 since it would result in higher instantaneous and accumulated values of the spatial field. This is compared to the case where the agent is unaware of the spatial field and elects to follow the path with the shortest distance. Table I summarizes the times to exits and the associated accumulated amounts inhaled. The agnostic agent selects the trajectory that yields the smallest time (smallest distance) without any regards on the effects of the hazardous field. When the decision is based on shortest distance the agent selects exit #3 which turns out to be the worst escape trajectory as it would yield an accumulated amount above the threshold of 25,000ppm, thus rendering such an escapee unconscious before reaching the escape exit. As a comparison, the time-to-escape and accumulated amount for the agent implementing the proposed scheme is included in Table I. When the effects of the accumulated amount are included in trajectory planning, the agent makes it to exit #2 with an accumulated amount below the threshold.

B. Adaptively estimated unknown time-invariant spatial field

The adaptive scheme is used to provide the snapshots of the *estimated* field at the beginning of a new cycle. While

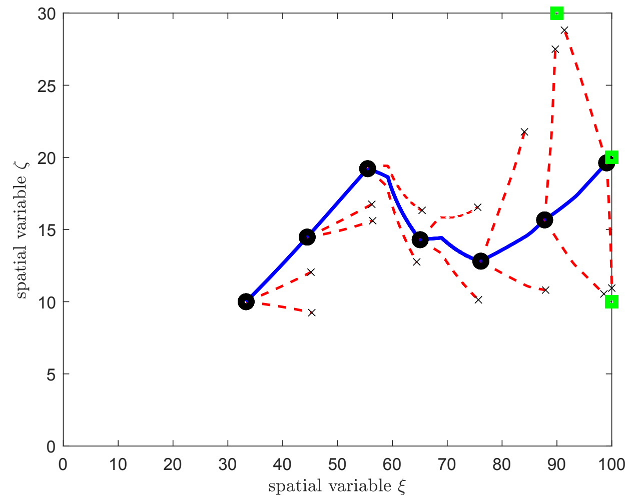


Fig. 2: Example 1: Trajectory defined over different cycles with intermediate trajectory recalculations.

TABLE I: Example 1: escape times and accumulated J for the field-agnostic agent. Trajectory with minimum time is marked with **red** and results in accumulated J above the threshold 25,000. The agnostic agent unaware of the field, selects the wrong exit even though exit#1, while taking longer to complete, results in the smallest possible accumulated J. The time and accumulated J for the agent with the proposed adaptive scheme is marked with **green**.

agent	exits	time to exit (sec)	J at exit
agnostic	# 1	22.22	5,222
agnostic	# 2	22.47	24,630
agnostic	**# 3**	**20.03**	**25,076**
adaptive	**# 2**	**38.29**	**22,567**

the true field in the expansion (11) with $n = 6$ uses

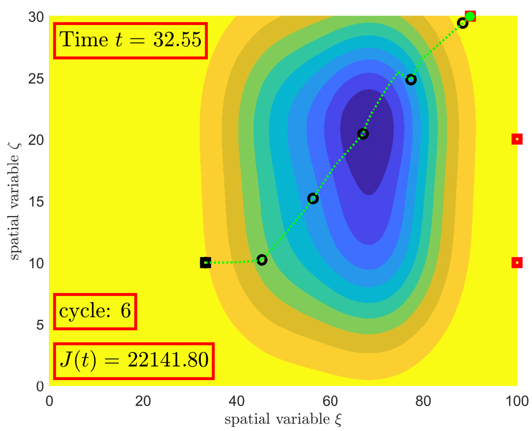
$$\phi_i(\xi, \zeta) = 4100 \exp\left(-{(\xi - \mu_\xi^i)^2}/{2\sigma_\xi^2}\right) \exp\left(-{(\xi - \mu_\zeta^i)^2}/{2\sigma_\zeta^2}\right),$$

where $\mu_\xi^1 = 0.3L_\xi = \mu_\xi^2$, $\mu_\xi^3 = 0.5L_\xi = \mu_\xi^4$, $\mu_\xi^5 = 0.7L_\xi = \mu_\xi^6$, $\mu_\zeta^1 = 0.375L_\zeta = \mu_\zeta^3 = \mu_\zeta^5$ and $\mu_\zeta^2 = 0.75L_\zeta = \mu_\zeta^4 = \mu_\zeta^6$ with $\alpha_6 = 1$ and all other $\alpha_i = 0$. The adaptive scheme uses the initial guess $\widehat{\alpha}_4(0) = 0.3$, $\alpha_5(0) = 0.6$, $\alpha_6(0) = 0.1$, and $\alpha_1(0) = \alpha_2(0) = \alpha_3(0) = 0$, i.e., it overparametrizes the unknown field. The adaptive gain in (15) was $\gamma = 10^{-6}$.

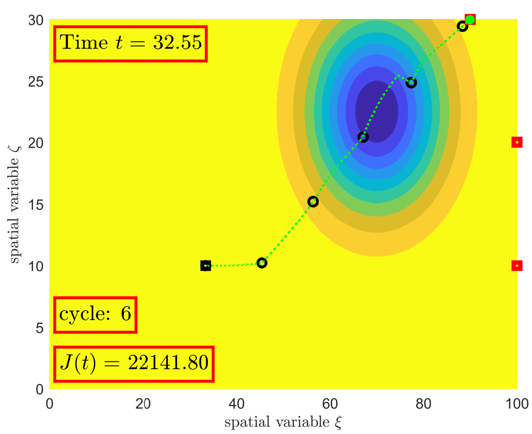
TABLE II: Example 2: escape times and accumulated J for the adaptive guidance and the field-agnostic agent.

case	time to exit (sec)	J at exit
adaptive	32.55	22,142
agnostic	20.03	25,105

Table II summarizes the results for the proposed adaptive estimation based level-set guidance and that of an agnostic agent. Both select exit #3 to escape. The field-agnostic agent follows a straight path from initial position to exit #3 without any considerations for the effects of accumulated amounts of the hazardous field. Without the presence of the field, the agnostic agent would reach exit #3 in 20.03 seconds. However, at the time instance $t = 13.18$s the accumulated



(a) Adaptively estimated field.



(b) True spatial field.

Fig. 3: Example 2: Evacuation guidance based on snapshots of adaptively estimated constant spatial field.

amount exceeds the threshold of $25,000$ppm and the agent is incapacitated at the location $(70.64, 23.17)$ having only covered a distance of 39.55m from the initial position. The field-agnostic agent at that time has an additional distance of 20.55m to cover to the escape exit. The agent with the proposed adaptive estimation-based guidance reaches the safety exit at the later time of 32.55s but manages to escape with the accumulated amount in the lungs at the value of $22,142$ppm (barely escaping).

To further delve into the results of the proposed adaptive estimation based level-set guidance, Figure 3 depicts the escape trajectory with the true and unknown spatial field (Fig 3b). Since the agent does not have access to the true field, but only at its adaptive estimate at the beginning of a new cycle , it uses the estimate $\widehat{c}(t_k, \xi, \zeta)$ to generate the escape trajectory to exit #3, see Figure 3a. Had the agent known the true field, it would have chosen a straight path to exit #1 in 22.2 seconds and having an accumulated $J = 3,795$ppm which is far below the threshold of $25,000$ppm.

VI. Conclusions

This paper presented an extension of earlier level-set based evacuation guidance over indoor environments. The additional modifications included a planning period, in which the agent was computing viable escape trajectories using partial spatial field information (snapshots), and a travel period in which the agent was executing the optimal trajectory for a given time cycle. Combining earlier works on the use of adaptive estimation of spatial fields, the other extension included the use of adaptive estimates of the spatial field to generate viable escape trajectories during the planning period and executing optimal trajectory during the travel period.

Extensive numerical studies presented both guidance modifications and highlighted the improvements of the modified level set guidance with intermittent field information.

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