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Event-Based Admission Control Over Multi-Hop Networks With Self-Interference

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Abstract— **In this work, we investigate the application of event-triggering in a multi-hop networked control scenario with interference constraints. In particular, we consider a line network comprised of H nodes with neighboring nodes affecting the reliability of each other, hence, introducing packet loss and non-negligible end-to-end latency. Having the practical feasibility in mind, we focus on admission control mechanisms at the sensor without assuming a centralized scheduling entity that has the perfect and global knowledge of the entire network. We demonstrate that, if the limitations of the network are neglected, the event-triggering mechanism may lead to low end-to-end reliability causing a significant degradation of the control performance. As a solution, we propose two novel admission control policies that aim to find a minimum inter-event time (MIET) in order to prevent a network congestion followed by a control performance deterioration. While the first policy follows an analytical approach combining the core principles of event-triggering and congestion control, the second policy learns the MIET adaptively without the knowledge of the network model. We show through numerical evaluation that the proposed strategies improve the control performance by more than 20% if the event criterion is selected appropriately.**

I. INTRODUCTION

The recent advancements in sensing, communications, and computing have rendered networked control and industrial networks an essential part of our societies and future technologies. Such systems comprise sensors and controllers exchanging information over a communication network to accomplish a particular control task. Process control, smart agriculture, and telerobotics are some of the most prominent examples of such systems. Generally speaking, the components of *networked control systems (NCSs)* can be scattered in a large area. Thus, the data packets that are essential for their operation may have to traverse multiple wireless communication links, i.e., a *multi-hop* network. As the network resources are limited and wireless links are unreliable by nature, the efficient utilization of the available resources becomes essential in order to minimize the adverse effects of the network on control performance.

The resource efficiency in NCSs has been addressed by the *event-triggering (ET)* concept, which aims to reduce the network utilization by transmitting only a subset of measurements that fulfill a certain ET condition on the system state [1], [2], [3]. Despite extensive research aiming at developing new ET algorithms, the questions of integrating these approaches with the actual communication stack in practice and demonstrating their theoretically stated benefits are remaining vague [4]. The implementation of ET policies becomes particularly challenging in a multi-hop setting, in which the network is subject to non-negligible end-to-end delays, the communication links are unreliable, the acknowledgments are not instantaneous. Moreover, if the network resources are shared among the network nodes between the source and destination, a simultaneous medium access by multiple transceivers causes interference [5], [6], leading to a possible network congestion, while at the same time rendering the ET mechanism sub-optimal. Therefore, an admission control policy that introduces network-awareness to the ET mechanism is essential to sustain fast and regular information flow through the network.

In this work, we consider a feedback control loop closed over a multi-hop wireless network with interference. We are interested in finding a packet admission policy at the source, i.e., sensor, the goal of which is not only preventing the congestion in the network, but filtering the admitted packets according to its content. While the former is achieved by considering the network topology, delay- and interference model, the latter is addressed by the ET mechanism. To that end, we propose two such policies that aim to find the *minimum inter-event time (MIET)*. The first policy follows an analytical approach by calculating the probability of the latest admitted packet being in the non-interfering region of the topology, which is a novel approach applicable to scenarios with known or measurable link reliabilities. The second one is model-free and learns the MIET based on the acknowledgment packets provided by the communication stack, hence, targeting those scenarios where link reliabilities are unknown to the ET mechanism. In order to evaluate the performance of our proposed policies w.r.t. control cost, we compare them to the network-unaware ET mechanism as well as the interference-aware communication protocols from the existing literature. The key difference of our approach to the existing methods is that our protocol design neither assumes perfect global knowledge of the network nor an unrealistic system model with negligible delays or losses, rendering it well-applicable and beneficial in practical scenarios.

Section I-A discusses a selection of related work on event-triggering and multi-hop wireless networks. In section II, we introduce the considered control and network model, discuss the core problem that we aim to solve including the most relevant challenges in terms of feasibility. In section III, we introduce two novel admission control policies and conduct a numerical evaluation based on Monte Carlo simulations. Finally, section IV summarizes and concludes this work.

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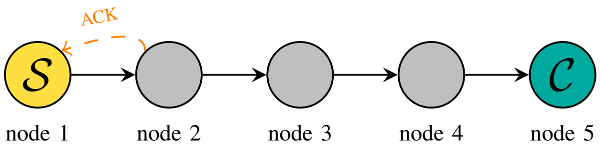

Fig. 1. An example four-hop line network, i.e., $H = 4$. Status update packets are sent from the sensor node, i.e., $\mathcal{S}$ to the controller node, i.e., $\mathcal{C}$ via three additional intermediate relay nodes.

A. *Related Work*

Event-triggering is a well-studied concept in the control theory literature, including but not limited to [1], [2], [3], [7], [8]. A great portion of the existing works employ the ET technique in the presence of a communication network to reduce the number of transmissions, therewith, increasing the resource and energy efficiency. It is quite common in the literature that the transmissions are either perfect in case of a positive admission decision, e.g., [8], or they are instantaneous after the contention resolution, e.g., [2], [3]. Such a model may be seen as a close approximation of a single-hop communication network, but they do not capture the key characteristics and challenges of a multi-hop scenario.

Despite being less common, the existing literature contains works that study ET in a multi-hop scenario [1], [7]. In [7], the authors consider a multi-hop line (i.e., relay) network with each wireless link being subject to packet loss. However, in their considered model, the network does not introduce any delay between the source and the destination. In other words, if the transmission on every link along the path is successful, the destination obtains the actual state. This contradicts with the nature of a multi-hop wireless network, as also stated by the authors in their conclusions section.

[1] and [9] consider a multi-hop setting, in which the sensors flood the network with status update packets. These works are remarkable in the way how the communication stack is tailored to the applications of control over a wireless sensors network. [1] considers an ET-based design to reduce the number of transmissions for *slow* control applications, i.e., the sampling period is more than one second. Such a design demonstrates the actual practical gains of ET in terms of energy savings. However, their approach is not applicable to faster control dynamics. On the contrary, [9] addresses the network resource management for time-critical control applications, where the offline schedules are distributed prior to operation. Such a design, however, lacks the flexibility required in those scenarios with time-varying control traffic. In particular, a fixed communication schedule hampers the benefits of ET, as the network resources are already reserved for specific users and cannot dynamically assigned to sporadic transmissions peculiar to ET. To the best of our knowledge, there are no works that have developed flexible ET-empowered mechanisms for communication over a multi-hop network for control systems with possible demand on high update rates.

B. *Notations*

Throughout this paper $\boldsymbol{v}^T$ and $\boldsymbol{M}^T$ stand for the transpose of a vector $\boldsymbol{v}$ and a matrix $\boldsymbol{M}$, respectively. The expected value of a random variable X is denoted by $\mathbb{E}[X]$. The normal distribution with mean μ and standard deviation σ is denoted by $\mathcal{N}(\mu, \sigma^2)$. $\mathbb{N}_0$ denotes the natural numbers including zero, i.e., $\mathbb{N}_0 = \{0, 1, \dots\}$. In addition, $\boldsymbol{M}^p$ and $\boldsymbol{M}^{-1}$ denote, respectively, the p-th power and the inverse of a matrix $\boldsymbol{M}$.

II. SYSTEM MODEL

We consider an NCS consisting of a plant $\mathcal{P}$, a sensor $\mathcal{S}$, and a controller $\mathcal{C}$. We assume that the plant and the controller are co-located, hence closed over an ideal $\mathcal{C}$-to-$\mathcal{P}$ link. However, the sensor operates remotely and transmits the state observations via a multi-hop wireless communication network. That is, the sensor-controller pair is H wireless transmissions away from each other, whereas the network contains $H + 1$ nodes, i.e., the sensor node, the controller node, and $H - 1$ relay nodes. The relay nodes are responsible for forwarding the traffic generated by $\mathcal{S}$. Such a network is often referred as a *line network* in the literature [10] and is encountered in static environments once the shortest path between the source and the destination has been established. Fig. 1 depicts the considered network topology for a four-hop network, i.e., $H = 4$, including the sensor, the controller, and relay nodes. We assume periodic sampling, i.e., the observation of the system state occurs with constant time intervals called the *sampling period*.

A. *Control Model*

The behavior of the control system is represented by the following discrete time linear time-invariant (LTI) stochastic difference equation:

$$\boldsymbol{x}[k+1] = \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}\boldsymbol{u}[k] + \boldsymbol{w}[k]. \tag{1}$$

Here, $\boldsymbol{x}[k] \in \mathbb{R}^n$ is a column vector denoting the system state at time-step k. The time-invariant matrices $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ and $\boldsymbol{B} \in \mathbb{R}^{n \times m}$ are the system and input matrices, respectively. The vector $\boldsymbol{u}[k] \in \mathbb{R}^m$ is the control input at time-step k. Moreover, the noise sequence $\boldsymbol{w}[k] \in \mathbb{R}^n$ is considered independent and identically distributed (i.i.d) according to a zero-mean Gaussian distribution with diagonal covariance matrix $\boldsymbol{\Sigma}$, i.e., $\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$. The system state $\boldsymbol{x}[k]$ with $\boldsymbol{x}[0] = \boldsymbol{w}[0]$ is measurable by $\mathcal{S}$.

We consider a certainty equivalence controller that obtains the control input according to the following control law:

$$\boldsymbol{u}[k] = -\boldsymbol{L}\, \mathbb{E}\left[\boldsymbol{x}[k] \mid \boldsymbol{x}[k - \Delta[k]]\right], \tag{2}$$

with the feedback gain matrix $\boldsymbol{L} \in \mathbb{R}^{m \times n}$. Additionally, $\boldsymbol{x}[k - \Delta[k]]$ denotes the most recent information about the system state that the controller has acquired before the beginning of the sampling period k. In other words, the freshest information available at the controller is $\Delta[k]$ sampling periods old. The variable $\Delta[k] \in \mathbb{N}_0$, which quantifies the freshness aspect of a piece of information, has already been defined as the *age of information (AoI)* in the literature and used in the context of NCSs [11], [12]. The estimated state $\hat{\boldsymbol{x}}[k] \in \mathbb{R}^n$ is obtained by taking the conditional expectation:

$$\hat{\boldsymbol{x}}[k] \triangleq \mathbb{E}\left[\boldsymbol{x}[k] \mid \boldsymbol{x}[k-\Delta[k]]\right] \tag{3a}$$

$$= \boldsymbol{A}^{\Delta[k]}\boldsymbol{x}[k] + \sum_{l=1}^{\Delta[k]} \boldsymbol{A}^{l-1}\boldsymbol{B}\boldsymbol{u}[k-l]. \tag{3b}$$

We refer to [11] for the proof of (3b). The optimal feedback gain matrix $\boldsymbol{L}$ minimizing the linear-quadratic-Gaussian (LQG) cost function, i.e.:

$$\mathcal{J} \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} (\boldsymbol{x}[k])^T \boldsymbol{Q}\boldsymbol{x}[k] + (\boldsymbol{u}[k])^T \boldsymbol{R}\boldsymbol{u}[k], \tag{4}$$

is determined by solving the discrete time algebraic Riccati equation given as:

$$\boldsymbol{P} = \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} - (\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{B})(\boldsymbol{R} + \boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B})^{-1}(\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{A}) + \boldsymbol{Q}. \tag{5}$$

Here, $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ is a positive semi-definite symmetric matrix solving (5). $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{R}^{m\times m}$ are positive semi-definite symmetric matrices that weight the state error and control effort, respectively. As a result, the optimal $\boldsymbol{L}$ can be calculated as:

$$\boldsymbol{L} = (\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B} + \boldsymbol{R})^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{A}. \tag{6}$$

As it has been shown in [13], the optimal feedback gain matrix $\boldsymbol{L}$ minimizing the standard LQG problem is also optimal in the presence of delays and losses for the considered model. As a result, the network-induced imperfections are reflected only in the estimation process in the form of an estimation error defined as:

$$\boldsymbol{e}[k] \triangleq \boldsymbol{x}[k] - \hat{\boldsymbol{x}}[k]. \tag{7}$$

B. Network Model

The network time is divided into equally long slots. Each time slot can accommodate a single data packet followed by an instantaneous acknowledgment (ACK) packet from the neighboring node, as shown in Fig. 1[1]. We assume that each data packet contains a single state measurement and multiple of such measurements cannot be concatenated into a single piece of information. Moreover, to simplify the following analysis, we assume that the duration of a time slot corresponds to the sampling period of the control system. Throughout the paper, we use k to index both the k-th sampling period, as well as the k-th time slot, interchangeably.

We consider packet erasure channel model, according to which, a transmission on the link $h \in \{1, \ldots, H\}$ in time slot k is successful with a certain probability $p_h[k] \in [0,1], \forall h$. Analogously, the probability that the data packet cannot be decoded by the receiver is $1 - p_h[k]$. The packet erasure channel has been employed in the literature for multi-hop settings, where simultaneous transmissions on subsequent links do not have any impact on each other's packet success probability [10], [15], [16]. However, as it has been shown in [5] through real-world testbed, the simultaneous transmissions over the links that are geographically close to each other lead to interference that decreases the corresponding packet failure probabilities significantly. As [5] was able to identify as a more accurate representation, we consider an *interference radius* according to the following rule: a pair of links interfere with each other if the sender of one link is within two hops of the other link's receiver. For instance, in Fig. 1 the data transmission from node 1 to node 2 can only interfere with a simultaneous transmission from node 4 to 5. In the following analysis, we consider the interference of data packets but not of ACK packets.

Accordingly, we indicate an interference-free transmission on link h with $\psi_h[k] = 0$ and $\psi_h[k] = 1$ represents a transmission that is subject to interference. We select the following behavior for the packet erasure channel:

$$p_h[k] = \begin{cases} p^{hi} & \text{, if } \psi_h[k] = 0, \\ p^{lo} & \text{, if } \psi_h[k] = 1, \end{cases} \tag{8}$$

with $p^{lo} < p^{hi}$ and $\psi_h[k] \in \{0,1\}, \forall h$.

In order to cover general multi-hop settings, we assume that the network nodes (users) are scattered over a large area, therefore, it is practically not feasible to implement a centralized network manager that schedules the users in a dynamic fashion. Therefore, we allow either contention-based medium access or offline scheduling policies. In the contention-based case, simultaneous transmissions using the same network resources are possible, whereas for the scheduled access the network resources are distributed among users prior to the deployment and cannot be changed during run-time.

Furthermore, we consider a preemptive last come first serve (PLCFS) queue at each node with the following policy: any packet that is stored in the transmission queue is discarded upon the reception of a more recent packet. It has been shown in [17] that the PLCFS policy is optimal w.r.t. information freshness, i.e., *age-optimal*. Therefore, we do not consider first come first serve (FCFS) queueing in the remainder of this work, as it would lead to additional queueing delays and inefficient utilization of the network resources for transmitting outdated packets. Nevertheless, if a network node detects that its adjacent node, i.e., next hop, does not have the most recent state information due to a previous unsuccessful transmission, it keeps transmitting the data packet containing the freshest information that it has received before the beginning of a time slot. Note that each node knows which information is available to its next hop through the instantaneous ACK packets.

At the beginning of each sampling period, the sensor decides either to admit the newly generated packet containing $\boldsymbol{x}[k]$ into the network or discard without further consideration. Let the binary variable $\delta[k] \in \{0,1\}$ denote the sensor's decision for the k-th packet, i.e., $\boldsymbol{x}[k]$, where $\delta[k] = 1$ represents a positive admission decision and $\delta[k] = 0$

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[1] The timing model is in alignment with the WirelessHART standard [14], according to which a ten milliseconds long time slot supports a packet transmission immediately followed by the ACK. Node that this is different than the end-to-end acknowledgment mechanism between $\mathcal{S}$ and $\mathcal{C}$.

corresponds to a discard event.

C. Information Model

Now, let us introduce how the outcome of transmissions affect the information flow along the path. To that end, let $\nu_h[k] \in \mathbb{N}_0$ denote the generation time of the most recent information that has been received by the h-th node with $h = 1$ being $\mathcal{S}$, which transmits on the first link and $H+1$-th node being $\mathcal{C}$, which receives on the H-th link. Since the sensor is the information source, $\nu_1[k]$ depends only on the sensor's admission decision as[2]:

$$\nu_1[k] = \begin{cases} k & \text{, if } \delta[k] = 1, \\ \nu_1[k-1] & \text{, if } \delta[k] = 0. \end{cases} \tag{9}$$

For all other nodes $h \in \{2, \ldots, H\}$, the most recent information depends on the transmission outcome on the previous link, i.e.:

$$\nu_h[k] = \begin{cases} \nu_{h-1}[k-1] & \text{, if } \gamma_{h-1}[k-1] = 1, \\ \nu_h[k-1] & \text{, if } \gamma_{h-1}[k-1] = 0, \end{cases} \tag{10}$$

where $\gamma_h[k] \in \{0, 1\}$ indicates a successful reception on the h-th link. If a node $h+1$ has already received the most recent information available to its predecessor, the h-th node does not transmit rendering the h-th link idle, i.e., $\Pr[\gamma_h[k] = 0 \mid \nu_h[k] = \nu_{h+1}[k]] = 1$. Otherwise, the transmission is successful with probability $p_h[k]$, i.e., $\Pr[\gamma_h[k] = 1 \mid \nu_h[k] > \nu_{h+1}[k]] = p_h[k]$. Analogously, $\Pr[\gamma_h[k] = 0 \mid \nu_h[k] > \nu_{h+1}[k]] = 1 - p_h[k]$.

D. Problem Statement

Given the scenario and system model described in the previous section, the question we raise is the following: *How should the sensor decide which packets to admit into the network and which packets to discard?* We focus on the following two major considerations that flow into the decision-making:

I) The *importance* of the currently generated information for the considered (control) application;

II) The effect of an admission on the network, particularly, on the network congestion and reliability of the subsequent links.

The aspect I) has been tackled in the literature through the *event-triggering* (ET) concept. Typically, the ET employs a threshold-based policy depending on the network-induced estimation error in the form of a squared two-norm, i.e., $\|\boldsymbol{e}[\boldsymbol{k}]\|_2^2 > \theta$ as in [2], [18], [19]. However, having the practical feasibility in mind, this would contradict with our generalized multi-hop scenario, as the transmissions are not instantaneous and it is practically not feasible for the sensor to know the exact value of the estimated state at the controller without a significant delay. Therefore, we define an event as the deviation of the weighted system state above a certain threshold value $\theta \geq 0$ as:

$$(\boldsymbol{x}[k])^T \boldsymbol{\Lambda} \boldsymbol{x}[k] > \theta, \tag{11}$$

with the square matrix $\boldsymbol{\Lambda} \in \mathbb{R}^{n \times n}$. The quadratic form of the state was considered as one of the alternatives to estimation error based triggering in [20]. Before delving into the second aspect, let us first consider an admission policy based on ET as:

$$\delta[k] = \begin{cases} 1 & \text{, if } (\boldsymbol{x}[k])^T \boldsymbol{\Lambda} \boldsymbol{x}[k] > \theta, \\ 0 & \text{, otherwise.} \end{cases} \tag{12}$$

It is evident from the equation above that the ET mechanism is agnostic to the capabilities of the underlying network. In other words, the admission decision solely depends on the current system state and whether the resulting admission policy leads to a congestion is entirely irrelevant for the algorithm given in (12). More importantly, (12) leads to "bursty" admission patterns, meaning that consecutive packets are more likely to be injected into the network, irrespective of their innovation relative to each other. The bursty admission pattern follows from the fact that once the system state exceeds a certain threshold, it is expected that the event condition remains satisfied until the appropriate actuation is performed. Note that this feature of the ET mechanism is particularly relevant for multi-hop networks, in which the end-to-end latency is non-negligible. As we are going to show later in the following section, the ET mechanism delivers inadequate performance under certain conditions, if designed entirely independent of the network characteristics.

One way to tackle this issue, which is related to aspect II), is the utilization of MIET, which introduces an additional rule into ET enforcing a minimum time distance between two consecutive admission events [21]. The MIET does not only reduce possible adverse effects of an admission on the network but also prevent consecutive packets from being transmitted. However, the determination of the optimal MIET is not a trivial task as it depends on many considerations and design choices that characterize the operation of the communication stack's lower layers. To name a few, the packet loss and interference model, number of hops between $\mathcal{S}$ and $\mathcal{C}$, the selection of the medium access control strategy, and the underlying packet queueing strategy. In the next section, we propose two strategies that aim to improve the achieved control performance by combining the idea of event-triggering and MIET concept. While one of them utilizes the network topology and interference model to determine the MIET, the second one is an adaptive strategy that tries to learn the best possible MIET through the provided information during runtime.

III. MAIN RESULTS

A. The Interference-Aware Event-Triggering (IAET) Policy

The first proposed policy is a model-based policy that is aware of the network topology, the interference model as well as the state measurements. It consists of two main blocks. Firstly, it evaluates the occurrence of an event according to the condition from (12) for a given θ and $\boldsymbol{\Lambda}$. To obtain the second block, which dictates the MIET, let us introduce a new variable $\tau[k]$ denoting the elapsed time since the latest

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[2] Note that (9) implies that any admitted packet at the sensor replaces the older packet in its transmission queue due to PLCFS strategy.

positive admission decision until k, i.e., $\tau[k] = \inf\ \{k - t : t \leq k,\ \delta[t] = 1\}$.

Next, let us define a function $f_h(\tau[k])$ that provides the probability of a packet being currently located (i.e., served) at the h-th node. For instance, if the latest packet was admitted at time step $k-1$, then the probability that the packet is currently in service at the third node along the path is zero, i.e., $f_3(\tau[k]) = 0$, whereas $f_1(\tau[k]) + f_2(\tau[k]) = 1$. We can calculate the probability that the packet is still to be served by the sensor node as:

$$f_1(\tau[k]) = (1 - p_1)^{\tau[k]}, \tag{13}$$

for a constant reliability p_1 of the first link. As more time passes without any further admission, i.e., as $\tau[k]$ grows, the probability that the packet has traveled further along the path increases, while $f_1(\tau[k])$ decreases. Similarly, the probability that the packet is currently waiting to be transmitted on the second link is given as:

$$f_2(\tau[k]) = \sum_{a=1}^{\tau[k]} (1 - p_1)^{a-1} p_1 (1 - p_2)^{\tau[k]-a} \tag{14}$$

for $\tau[k] \geq 1$ and zero otherwise. Here, the packet has been retransmitted $a - 1$ times on the first link before it has been successfully received by the second node. $\tau[k] - a$ denotes the number of consecutive failures on the second link. Analogously, we can write f_3 and f_4 as follows:

$$f_3(\tau[k]) = \sum_{a_1=1}^{\tau[k]-1} \sum_{a_2=1}^{\tau[k]-a_1} g_3(a_1, a_2, \boldsymbol{p}), \tag{15}$$

$$g_3(a_1, a_2, \boldsymbol{p}) \triangleq \prod_{i=1}^{2} \big(\alpha(p_i, a_i)\big) \beta(p_3, \tau[k] - a_1 - a_2),$$

and:

$$f_4(\tau[k]) = \sum_{a=1}^{\tau[k]-2} \sum_{b=1}^{\tau[k]-a-1} \sum_{c=1}^{\tau[k]-a-b} g_4(a, b, c, \boldsymbol{p}), \tag{16}$$

$$g_4(a_1, a_2, a_3, \boldsymbol{p}) \triangleq \prod_{i=1}^{3} \big(\alpha(p_i, a_i)\big) \beta(p_4, \tau[k] - a_1 - a_2 - a_3),$$

with $\alpha(x, y) \triangleq x(1 - x)^{y-1}$ and $\beta(x, y) \triangleq (1 - x)^y$. Note that $f_3(\tau[k])$ and $f_4(\tau[k])$ apply for $\tau[k] \geq 2$ and $\tau[k] \geq 3$, respectively. Otherwise, the functions return zero. We stop at f_4 because the packet leaves the interference radius of $\mathcal{S}$ after $h = 4$.

What equations (13) through (16) provide us is the probability of the latest admitted packet to be at a specific location, if the transmission success probability of the first four hops were constant over time. However, according to our system model, in particular, (8), the individual link reliabilities alternate between p^{hi} and p^{lo} depending on the experienced interference. Nevertheless, if the MIET is selected large enough, hence, leaving enough distance between two consecutive packets, the chance of a self-interference along the path can be significantly reduced. To that end, we provide the probability that a previously admitted packet has left the interference radius of $\mathcal{S}$:

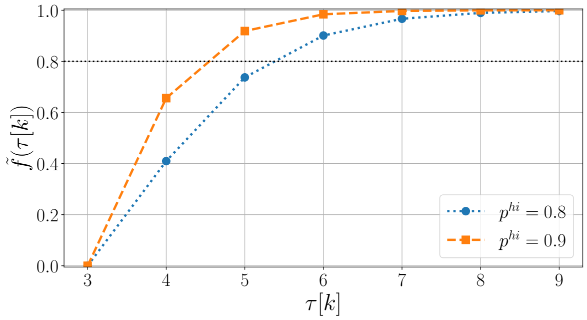

Fig. 2. The probability that a previously admitted packet has left the interference radius for $p^{hi} \in \{0.8, 0.9\}$ and $H = 5$.

$$\tilde{f}(\tau[k]) \triangleq 1 - \sum_{i=1}^{4} f_i(\tau[k]). \tag{17}$$

Fig. 2 presents two example curves for $\tilde{f}$ for a five hop network.

As the last missing step to characterize the admission control policy, we define the decision rule of the first proposed *interference-aware event-triggering (IAET)* policy:

$$\delta[k] = \begin{cases} 1 & \text{, if } (\boldsymbol{x}[k])^T \boldsymbol{\Lambda} \boldsymbol{x}[k] > \theta \text{ and } \tilde{f}(\tau[k]) > \lambda, \\ 0 & \text{, otherwise.} \end{cases} \tag{18}$$

The parameter $\lambda \in (0, 1)$ is a design parameter determining the value of the MIET. The parameter λ can also be interpreted as the minimum required confidence level to allow the admission of the next packet. For instance, according to Fig. 2, the selection of $\lambda = 0.8$ implies an MIET of five for $p^{hi} = 0.9$ and an MIET of six $p^{hi} = 0.8$. As λ increases beyond $\lambda = 0.8$, the MIET increases further.

In order to derive (17), we have assumed that there are at least five hops in the network. If this is not the case, the probability of leaving the interference radius $\tilde{f}(\tau[k])$ can be replaced by the probability of being successfully received by the controller.

B. Model-free Adaptive Admission Control (MFAC) Policy

As a model-free adaptive alternative to the IAET policy, we introduce a second enhancement of the event-triggering mechanism that learns the MIET by observing the ACK packets. We consider that the *model-free adaptive admission control (MFAC)* policy is provided with two types of ACK packets, i.e., the *local ACK* informing the sensor node $\mathcal{S}$ about a successful reception by its direct neighbor and a *global ACK* generated by $\mathcal{C}$ that acknowledges each successfully received data packet[3]. To bring the system closer to reality, we assume that unlike the local ACKs, the delivery of the global ACK packets is not instantaneous but subject to a certain delay that is uniformly distributed between H and $2H$ time slots, i.e., $\mathcal{U}(H, 2H)$[4].

[3] The global ACK is an additional end-to-end acknowledgment mechanism, which has been left out in Fig. 1 to avoid confusion.

[4] We do not allow out-of-order delivery of the ACK packets. The sensor detects a missing ACK if it receives an ACK containing a greater sequence number than the first unacknowledged data packet.

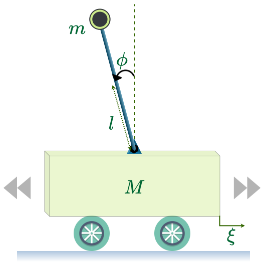

Fig. 3. Illustration of an inverted pendulum application. ϕ is the angular deviation of the pendulum from the equilibrium point in radians. ξ denotes the cart position in meters.

Similar to the IAET policy, the decision on the occurrence of an event is done similar to (12), i.e., whether $(\boldsymbol{x}[k])^T\boldsymbol{\Lambda}\boldsymbol{x}[k] > \theta$. In contrast to the IAET policy, the MFAC does not consider the topology knowledge or the interference model. Instead, the algorithm relies solely on the observation of the local and global ACKs and adjusts the MIET dynamically according to Algorithm 1. The MFAC algorithm requires two step size parameters, $\alpha_l \geq 0$ and $\alpha_g \geq 0$, which determine the magnitude of a decrease in the MIET upon the reception of a new local and global ACK, respectively. In the case of a failed transmission on the first link, i.e., $\delta[k] = 1$ and $\gamma_1[k] = 0$, then the algorithm tries to decrease the sending rate, by increasing the MIET[5]. Similarly, for the global ACK packets, the algorithm strives to increase the sending rate with every successfully received ACK packet. On the other hand, if a data packet was admitted by the sensor but its global ACK is missing, the sending interval is increased by α_g, since such a situation is an indicator of insufficient time distance between two adjacent admission decisions.

The core idea behind the MFAC policy can be explained as follows. On the one hand, it probes the multi-hop network and always tries to increase the admission rate to prevent under-utilization. On the other, it aims to find the maximum sending rate that does not cause self-interference along the path. In contrast to recent works [22], [23], which propose adaptive admission policies primarily tailored for queueing systems, that is, trying to keep the number of backlogged packets in the system under control, the MFAC algorithm exploits the PLCFS queueing strategy while taking lower layer considerations of a wireless multi-hop network into account such as the interference radius. The key difference of the *zero-wait event-triggering (ZWET)* strategy from [23] to the MFAC algorithm is that the ZWET allows a single packet into the network until the corresponding global ACK is received. Note that this would lead to an under-utilization of the network resources as the highest possible MIET would be $2H$ in our considered network[6].

Algorithm 1 The MFAC Algorithm

```
Require: $\alpha_l, \alpha_g, \theta \geq 0$, $\boldsymbol{\Lambda}$
  $k \leftarrow 0, b \leftarrow 1$
  repeat every sampling period
    $MIET[k] \leftarrow \lceil b \rceil$                         ▷ Update the MIET
    if $\tau[k] \geq MIET[k]$ and $(\boldsymbol{x}[k])^T\boldsymbol{\Lambda}\boldsymbol{x}[k] > \theta$ then
      $\delta[k] \leftarrow 1$                         ▷ Admit the data packet
    else
      $\delta[k] \leftarrow 0$                         ▷ Discard the data packet
    end if
    /* $k$-th sampling period (i.e., time slot) passes */
    if $\delta[k] = 1$ and $\gamma_1[k] = 1$ then
      $b \leftarrow \max(1, b - \alpha_l)$                         ▷ local ACK
    else if $\delta[k] = 1$ and $\gamma_1[k] = 0$ then
      $b \leftarrow b + \alpha_l$                         ▷ missing local ACK
    end if
    for all received global ACKs during $k$ do
      $b \leftarrow \max(1, b - \alpha_g)$
    end for
    for all missed global ACKs do
      $b \leftarrow b + \alpha_g$
    end for
    $k \leftarrow k + 1$
  until termination
```

C. Numerical Results

In order to evaluate the performance of our proposed IAET and MFAC policies, we consider a well known control application, namely, an inverted pendulum. Fig. 3 illustrates an inverted pendulum with the system state $\boldsymbol{x}[k] = [\xi[k] \quad \dot{\xi}[k] \quad \phi[k] \quad \dot{\phi}[k]]^T$. The system parameters as well as the controller design are the same as in [24]. We assume a slot length of 10 ms, which also corresponds to the sampling period of the considered inverted pendulum model. In addition, we select $\boldsymbol{\Lambda} = \text{diag}\{0, 0, (\frac{180}{\pi})^2, 0\}$ that generates an event only depending on the angular deviation of the pendulum from the equilibrium point, i.e., ϕ. We evaluate various values for the event threshold θ chosen from the following set: $\theta \in \{0, 0.04, 0.16, 0.36, 0.64, 1.00, 1.44, 1.96, 2.56, 3.24\}$. In such a setting, we conduct a study based on Monte Carlo simulations, where each $T = 10\ 000$ time steps long simulation run is repeated $C_{rep} = 30$ times. We assume that the success probability in the interference-free case is 90%, i.e., $p^{hi} = 0.9$, whereas the chance of a successful reception drops to 10% otherwise, i.e., $p^{lo} = 0.1$. Additionally, the design of the IAET mechanism, characterized by (18), has been done using $\lambda = 0.95$ and we use $\alpha_l = 0.1$ and $\alpha_g = 0.2$ for the MFAC algorithm.

The control performance per measurement run is captured by the long-term average LQG cost and averaged among all runs, i.e.:

$$\overline{\mathcal{J}} = \frac{1}{T}\frac{1}{C_{rep}}\sum_{rep=1}^{C_{rep}} \mathcal{J}, \tag{19}$$

with $\mathcal{J}$ as in (4). The $\boldsymbol{Q}$ and $\boldsymbol{R}$ weighting matrices are selected as it has been done for the LQR controller design, i.e., $\boldsymbol{Q} = \text{diag}(5000, 0, 100, 0)$ and $\boldsymbol{R} = 1$.

In addition to IAET and MFAC, we include the results for

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[5] The MIET is always a positive integer.

[6] A minimum of H time slots for the $\mathcal{S}$-to-$\mathcal{C}$ path plus a minimum of H time slots to deliver the global ACK from the controller to the sensor.

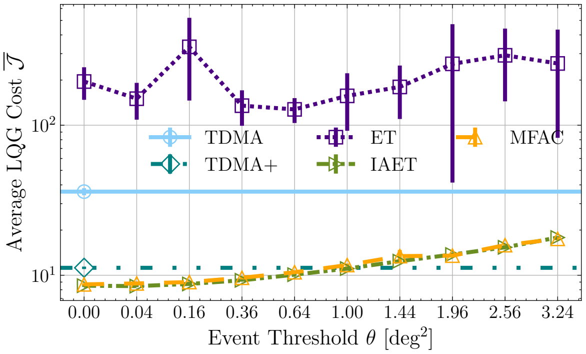

Fig. 4. Average LQG cost representing the achieved control performance for an eight-hop network, i.e., $H = 8$. A lower $\overline{\mathcal{J}}$ indicates a higher performance. If applicable, the selected event threshold θ is given in the lower x-axis. The vertical bars represent 90% confidence intervals.

pure ET, i.e., when the packet admission decision depends solely on the system state according to (12). Moreover, we have implemented an offline medium access control policy for benchmarking purposes, according to which each node has a fixed dedicated transmission slot. The benefit of this policy is that each wireless link operates without any interference from neighboring nodes, thus the success probability of each link is always p^{hi}. We refer to this contention-free policy as *time-division multiple access (TDMA)* in the following evaluation, in compliance with the literature [6].

Fig. 4 shows the achieved $\overline{\mathcal{J}}$ for the considered policies when the communication network consists of eight wireless links, i.e. $H = 8$. We observe that the ET mechanism, which is designed in a network-unaware fashion, is significantly outperformed by the remaining policies. This is already an expected result, since the selected medium access control mechanism is contention-based by nature. As a result, the ET policy suffers from high number of retransmissions on each link causing high end-to-end delays.

If we look at the performance of the proposed IAET and MFAC policies w.r.t. $\overline{\mathcal{J}}$, we can clearly notice that both policies are able to improve the control performance by up to 75% when compared to the TDMA protocol that allows only a single device to transmit at a time[7].

One could argue that the considered TDMA protocol is sub-optimal as multiple nodes could transmit simultaneously without interfering, as also mentioned in [6]. To that end, we also provide the results for the enhanced version of the TDMA protocol, which we call TDMA+ in Fig. 4. In brief words, the TDMA+ protocol allows for the spatial reuse of a time slot by multiple users, if these are in non-conflicting parts of the multi-hop network. Nevertheless, the proposed IAET and MFAC protocols outperform the TDMA+ protocol by more than 20% percent under the condition that θ is selected appropriately. Particularly, for our considered set of candidates for θ, the IAET and MFAC algorithms start falling behind the TDMA+ protocol after a threshold value of one.

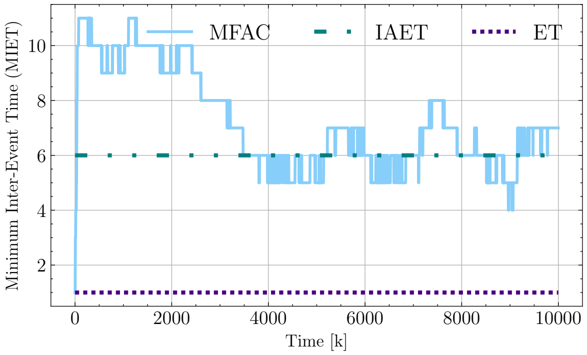

Fig. 5. Minimum inter-event time (MIET) during a randomly selected measurement run for the ET, MFAC, and IAET policies. The network consists of eight hops, i.e., $H = 8$.

Fig. 5 presents the MIET evolution of the MFAC algorithm together with the ET and IAET algorithms. The ET policy allows the admission of consecutive packets, thus the MIET is always one sampling period. The IAET obtains the (time-invariant) MIET by evaluating the condition $\tilde{f}(\tau[k]) > \lambda$. However, since the MFAC policy is model-free and adapts to network conditions based on the received ACK packets, the selected MIET varies dynamically over time. Note that despite being initialized with an MIET value of one, i.e., $MIET[0] = 1$, the MFAC algorithm is able to learn rapidly through local ACK packets that it should reduce its (maximum allowed) packet admission rate to reduce self-interference. It is important to emphasize once again that the MIET by itself does not directly dictate the triggering rate of the ET, IAET, and MFAC algorithms unless the event threshold θ is zero. It only defines the minimum time distance between two subsequent admissions in case the MFAC and IAET identifies an event according to the condition $(\boldsymbol{x}[k])^T \boldsymbol{\Lambda} \boldsymbol{x}[k] > \theta$. We would like add that when the TDMA and TDMA+ policies were employed, the packet admission rate was eight and four, respectively[8].

Fig. 6 presents the achieved control performance and the evolution of the MIET when the network consists of five hops[9]. Note that the MIET enforced by the IAET algorithm is unaffected by this change, as the IAET algorithm's admission decision solely considers the probability of the recently admitted packet having left the interference radius and not on the total number of hops. The results from Fig. 4 and 6 show that the selection of the event threshold θ plays a key role for our proposed IAET and MFAC policies. In fact, the LQG cost increases in θ, indicating a control performance degradation. This can be explained with the help of Fig. 7, which presents the average number of admitted packets into the network when various policies are utilized. As the figure clearly shows, a higher threshold enforces a stricter admission criteria for a newly generated packet causing a reduced sending rate at the source node. It is worth mentioning that among all measurement runs for a five-hop

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[7] The transmission order of network nodes is the same as their appearance order along the established path. That is, if the sensor node, i.e., $h = 1$, transmits in time slot t, then the next slot $t + 1$ is dedicated to the next user, i.e., $h = 2$.

[8] For the TDMA and TDMA+ mechanisms, the admission rate can also be seen as the cycle time of the first link being scheduled. We exclude TDMA and TDMA+ from Fig. 5, as these policies lack the notion of an "event".

[9] For presentation purposes, we exclude the ET policy from Fig. 6 as it achieves a much higher $\overline{\mathcal{J}}$ than the remaining policies as in Fig. 5.

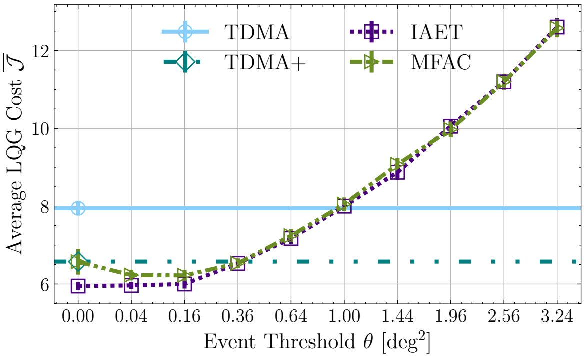

Fig. 6. The average LQG cost representing the achieved control performance when the network is a five-hop wireless communication network, i.e., $H = 5$. The vertical bars represent 90% confidence intervals.

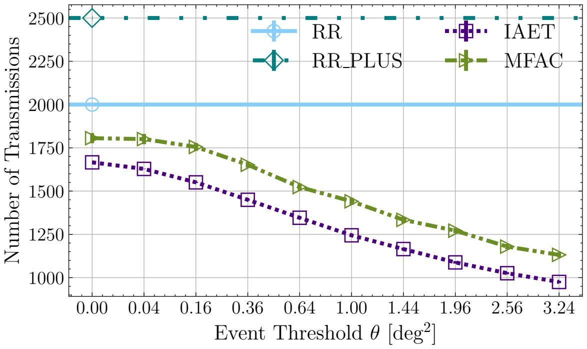

Fig. 7. Number of admitted packets as the event threshold θ increases. The vertical bars represent 90% confidence intervals.

network, the maximum observed angular deviation of the pendulum, i.e., $|\phi|$, was 13.2 degrees for the IAET and 14.4 degrees for the MFAC algorithms. On the other hand, the ET algorithm fails to keep the pendulum in an upright position irrespective of θ. However, we do not provide detailed results on ϕ due to space considerations.

IV. CONCLUSIONS

We propose two admission control policies for networked control over multi-hop wireless networks. The first policy, namely the interference-aware event-triggering (IAET), sets a maximum admission rate by also considering the existence of a threshold-based event criterion. The second policy, the model-free adaptive admission control (MFAC), is a learning-based mechanism that adjusts the minimum time between two consecutive events according to the acknowledgment mechanism provided by the network. Our results reveal that the event-triggering mechanism causes a significant performance deterioration if the chance of a network congestion is neglected. Moreover, communication protocols such as time-division multiple access (TDMA) may lead to inefficiencies and performance degradation at the expense of guaranteeing an interference-free communication.

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