2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Analysis and Synthesis of Switched Linear Systems with Random Mode-Dependent Sojourn-Time

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Abstract— **This paper concentrates on the issues of stability and stabilization for a class of switched linear systems with a new class of switching signals in discrete-time domain. The considered switching signals are of general random mode-dependent sojourn-time (RMST) property. Compared with the dwell-time switching or Markov chain often studied in the literature, it is capable of describing the mode-dependent sojourn-time consisting of a fixed part and a random part with the known expectation. By fully considering the characteristics of RMST, the criteria of stability and stabilization are derived for the underlying systems. Further, the results are extended to the asynchronously switched stabilizing control for RMST switched systems. A numerical example is provided to demonstrate the effectiveness and potential of the theoretical results.**

I. INTRODUCTION

The past decades have witnessed increasing attention in the era of nondeterministic switching [1]–[3], which is motivated by their powerful capability in modeling complex switching dynamics. The sequence of mode switching and the switching instants in nondeterministic switched systems are usually unknown. As a typical class of nondeterministic switching, the dwell-time (DT) switching [4] have been extensively studied in recent years [5]. An important subject on them is to determine minimum admissible DT [6]. Once the switching conflicts with the minimum value, the resulting closed-loop system stability cannot be guaranteed anymore. Therefore, for a given set of subsystems, it is significant to further obtain more switching signals that ensure the system stability even if the constraint on DT is not satisfied.

Unlike the DT switching, if the nondeterministic switching is attached to a stochastic process, it can be regarded as stochastic switching, i.e., Markov chain. However, as pointed out in [7], the Markov chain is not suitable to describe the cases where the mode switching occurs at least a fixed period of time after the previous switching, while the cases can be encountered in many applications [8], [9], etc. To generalize the scope of stochastic switching, in [7], the sojourn-time of each subsystem is divided into a fixed part and a random part, on which the switching is described by transition probabilities. It greatly impulses the studies on switched systems with such switching signals [10]–[14]. However, a more practical case is that the random part of the sojourn-time is available in the form of mathematical expectation, which covers the switching above as a special case. Unfortunately, the problems of stability and stabilization have not been fully investigated for the switched system with sojourn-time expectation, which motivates us for this work.

Moreover, on control synthesis problems of switched systems, the switchings of system modes and controller modes are commonly assumed to be synchronous [4-15]. However, it is not practical due to the fact that it takes time to identify the system mode and activate the corresponding controller, which results in a time lag between the switchings of system modes and controllers, i.e., asynchronous switching [15]. The mode mismatch between the system and the controller may degrade the system stability [16], [17]. Thus, studies on the switched systems in the presence of asynchronous switching with diverse switching signals have been available in [18], [19]. However, it is worth mentioning that the problems of asynchronously switched control for switched systems with persistent sojourn-time expectation have almost not been investigated so far.

In this paper, we are interested in dealing with the problems of stability and stabilization for discrete-time switched systems with a new class of switching signals. The main contributions lie in the following several aspects:

1) The concept of random mode-dependent sojourn-time (RMST) is proposed such that the restrictions of DT and the Markov chain are relaxed.
2) The stability analysis is carried out for the underlying systems, upon which the conditions on the existence of the stabilizing controller are obtained.
3) The results are extended to RMST switched system with asynchronous switching.

The remainder of the paper is organized as follows. Section II represents the formulation of the considered systems and the definition of RMST. In section III, the stability and stabilization conditions are given for the switched systems with RMST and asynchronous RMST switching, respectively. An illustrative example is given in Section IV. Section V concludes this paper.

Notations: In this paper, $\mathbb{R}^n$ refers to the n-dimensional Euclidean space; $\mathbb{Z}$ and $\mathbb{Z}_+$ denote the set of integers and non-negative integers, respectively; $\mathbb{Z}_{[a,b]}$ denotes the set $\{k \in \mathbb{Z} | a \le k \le b\}$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation. For the complete probability space $(\Omega, \mathcal{F}, \Pr)$, Ω denotes the sample space, $\mathcal{F}$ represents the σ-algebra of subsets of the sample space, and $\Pr$ is the probability measure on $\mathcal{F}$. $P \succ 0 (\succeq 0)$ means that P is

This work was supported in part by NSFC, China under Grant 62003117, in part by Natural Science Foundation of Heilongjiang Province, China under Grant YQ2022F013, in part by Postdoctoral Science Foundation, China under Grant 2020M681096, and in part by Postdoctoral Science Foundation of Heilongjiang Province, China under Grant LBH-Z20140 and LBH-TZ2201.
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real symmetric and positive definite (semi-positive definite). $diag\{a_1, a_2, \cdots, a_n\}$ represents the diagonal matrix with diagonal elements $a_1, a_2, \cdots, a_n$. We use $*$ as an ellipsis for the terms that are induced by symmetry. The minimum eigenvalue of positive definite matrix M is denoted by $\varphi_{\min}(M)$. $\Xi(U, V, \rho, X, Y, Z) = \begin{bmatrix} -X & UY + VZ \\ * & -\rho X \end{bmatrix}$ is a matrix function, where U, V, X, Y, Z are the constant matrices with compatible dimensions and ρ is a non-zero constant. The notation $\max(b_1, b_2, \cdots)$ defines a function that returns the largest value from the provided terms $b_1, b_2, \cdots$.

II. Preliminaries and Problem Formulation

Fix the complete probability space $(\Omega, \mathcal{F}, \Pr)$ and consider a family of discrete-time switched linear systems:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) \tag{1}$$

where the vectors $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ are the system state and the control input, respectively; $\sigma(k)$ is a piecewise constant switching signal taking values from the index set $\mathcal{L} = \{1, \cdots, N\}$ with N being the number of candidate subsystems. Throughout the paper, it is assumed that the switching signal σ is unknown *a priori* to the controller but its instantaneous value is available in real time.

In this paper, the switching signal is considered to be of RMST property. To facilitate understanding the concept, the following definition of mode-dependent dwell-time (MDT) is first provided.

Definition 1: [5] Consider the switching instants $k_0, k_1, k_2, \cdots, k_s, \cdots$ with $0 = k_0 < k_1 < k_2 < \cdots < k_s < \cdots$. The positive constant τ_i is called the mode-dependent dwell-time associated with the ith mode if for all $s \geq 0$ such that $\sigma(k) = i$ for $k \in [k_s, k_{s+1})$, $k_{s+1} - k_s \geq \tau_i$.

Unlike "dwell-time", which describes the lower bound of the running time of the subsystem, "sojourn-time" directly defines the actual time interval between two consecutive switchings. Taking the difference into account, the definition of RMST is given as below.

Definition 2: Let $k_0, k_1, k_2, \cdots, k_s, \cdots$ with $0 = k_0 < k_1 < k_2 < \cdots < k_s < \cdots$ denote the switching instants. The positive constant f_i^d and non-negative random variable $r_i^d \sim X_i^d$ are called the fixed mode-dependent sojourn-time and the random mode-dependent sojourn-time associated with the ith mode, respectively, if for all $s \geq 0$ such that $\sigma(k) = i$ for $k \in [k_s, k_{s+1})$, $k_{s+1} - k_s = f_i^d + r_i^d$.

Later development requires the definition of stability, and the following definition of stochastic stability is adopted in this paper.

Definition 3: [20] The discrete-time switched system (1) is said to be stochastically stable (SS) if

$$\mathbb{E}\{\sum_{k=0}^{\infty} \|x(k)\|^2 \,|x(0), \sigma(0)\} < \infty \tag{2}$$

holds for all initial conditions $x(0) \in \mathbb{R}^n$ and $\sigma(0) \in \mathcal{L}$.

The mode-dependent control pattern formulated as $u(k) = K_{\sigma(k)}x(k)$ is used to stabilize the switched system (1), which results the closed-loop system:

$$x(k+1) = (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x(k) \tag{3}$$

Since the switchings of system modes and controllers is difficult to be synchronous in practice, the phenomena of asynchronous switching is considered in controller design scheme. Let Γ_i, $i \in \mathcal{L}$ denote the time lag of asynchronous switching, and the resulting closed-loop switched system is given as below.

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x_k + B_{\sigma(k)}K_{\sigma\left(k-\Gamma_{\sigma(k)}\right)}x(k) \\ &= (A_{\sigma(k)} + B_{\sigma(k)}K_{\widehat{\sigma}(k)})x(k) \end{aligned} \tag{4}$$

where $\widehat{\sigma}(k)$ is the asynchronous switching signal governing the switching of candidate controllers. Fig. 1. presents the illustration of the asynchronous RMST switching signal.

Then, the objectives in this paper are to design a set of mode-dependent state feedback stabilizing controllers such that closed-loop systems (3) and (4) are SS under admissible RMST and asynchronous RMST switching signals, respectively.

III. Main Results

In this section, the stability analysis of RMST switched systems is first carried out. Then, the existence conditions of the stabilizing controller are given. In the presence of asynchronous switching, the results are further extended to the switched system with RMST switching.

A. Stability and stabilization of RMST switched systems

Lemma 1: Consider the discrete-time switched system $x(k+1) = f_{\sigma(k)}(x(k))$, and $0 < \alpha_i < 1$, $\mu_i \geq 1$ are given constants. For the prescribed fixed mode-dependent sojourn-time f_i^d and random mode-dependent sojourn-time $r_i^d \sim X_i^d$, suppose that there exists a family of functions $V_{\sigma(k)}(x(k)) : \mathbb{R}^{n_x} \to \mathbb{R}$, $\sigma(k) \in \mathcal{L}$ and a constant $\varepsilon > 0$ such that $\forall \sigma(k) = i \in \mathcal{L}$,

$$V_i(x(k)) \geq \varepsilon \|x(k)\|^2 \tag{5}$$

$$V_i(x(k+1)) - \alpha_i V_i(x(k)) \leq 0 \tag{6}$$

and $\forall(\sigma(k_s - 1) = j, \sigma(k_s) = i) \in \mathcal{L} \times \mathcal{L}$, $i \neq j$

$$V_j(x(k_s)) - \mu_i V_i(x(k_s)) \leq 0. \tag{7}$$

Then the switched system is SS for RMST switching signals satisfying

$$\mu_i \alpha_i^{f_i^d} \mathbb{E}_{r_i^d \sim X_i^d}(\alpha_i^{r_i^d}) < 1, \forall i \in \mathcal{L}. \tag{8}$$

Proof: From (6) and (7), we obtain that

$$\begin{aligned} &\Delta V_{\sigma(k_s)}(x(k_s)) \\ =\ & \mathbb{E}_{r_i^d \sim X_i^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} - V_{\sigma(k_s)}(x(k_s)) \\ \leq\ & \mathbb{E}_{r_i^d \sim X_i^d}\left\{\mu_i V_{\sigma(k_s)}(x(k_{s+1}))\right\} - V_{\sigma(k_s)}(x(k_s)) \\ \leq\ & \mathbb{E}_{r_i^d \sim X_i^d}\{\mu_i \alpha_i^{r_i^d + f_i^d} V_{\sigma(k_s)}(x(k_s))\} - V_{\sigma(k_s)}(x(k_s)) \\ =\ & (\mu_i \alpha_i^{f_i^d} \mathbb{E}_{r_i^d \sim X_i^d}\{\alpha_i^{r_i^d}\} - 1)V_{\sigma(k_s)}(x(k_s)). \end{aligned} \tag{9}$$

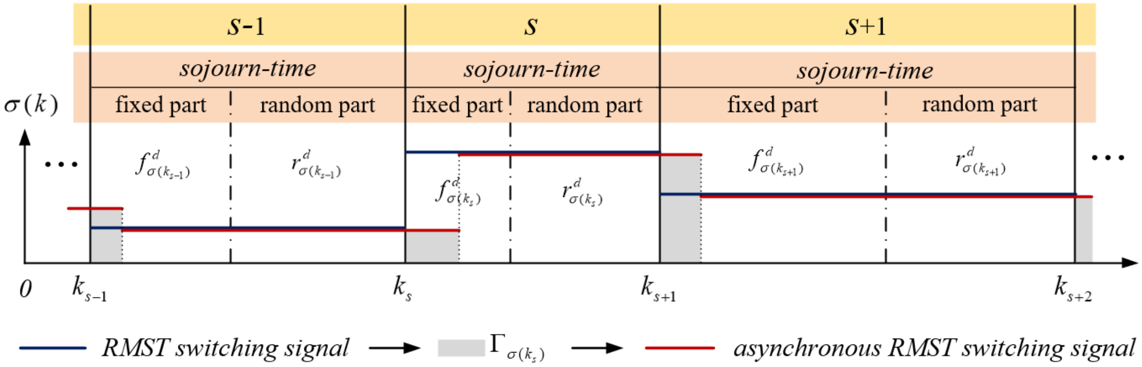

Fig. 1: Illustration of the asynchronous RMST switching signal. The blue and red lines denote the synchronous and asynchronous switching signals, respectively, and gray regions represent asynchronous intervals.

Define the coefficient

$$\lambda_i \triangleq \mu_i \alpha_i^{f_i^d} \mathbb{E}_{r_i^d \sim X_i^d}\{\alpha_i^{r_i^d}\} - 1, \forall i \in \mathcal{L}.$$

Taking the mathematical expectation of both sides and iterating, we obtain

$$\begin{aligned}
&\mathbb{E}_{r_n^d \sim X_n^d, n=1,\dots,N}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\}\\
&\le V_{\sigma(k_0)}(x(k_0)) + \sum_{h=0}^{s} \mathbb{E}_{r_n^d \sim X_n^d, n=1,\dots,N}\{\lambda_{\sigma(k_h)} V_{\sigma(k_h)}(x(k_h))\}\\
&\le V_{\sigma(0)}(x(0)) + \lambda_{max} \sum_{h=0}^{s} \mathbb{E}_{r_n^d \sim X_n^d, n=1,\dots,N}\{V_{\sigma(k_h)}(x(k_h))\}
\end{aligned}$$

where $\lambda_{max} \triangleq \max_{i\in\mathcal{L}} \lambda_i$. Denoting $\mathbb{E}_{r_n^d \sim X_n^d, n=1,\dots,N}\{\cdot\}$ by $\mathbb{E}_{r^d}^{X^d}\{\cdot\}$, it is straightforward from (8) and (9) that $\Delta V_{\sigma(k_s)}(x(k_s)) < 0$ and

$$\lim_{s\to\infty} \mathbb{E}_{r^d}^{X^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} < \infty.$$

Then, one has $\sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_s)}(x(k_s))\} \le \frac{1}{\lambda_{max}}(\lim_{s\to\infty} \mathbb{E}_{r^d}^{X^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} - V_{\sigma(0)}(x(0))) < \infty$. With the aid of (6), it holds that

$$V_{\sigma(k_s)}(x(k_s)) = \max_{k\in[k_s,k_{s+1})} V_{\sigma(k)}(x(k))$$

and

$$\begin{aligned}
&\sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k)}(x(k))\}\\
&< \sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{(f_{\sigma(k_s)}^d + r_{\sigma(k_s)}^d) V_{\sigma(k_s)}(x(k_s))\}\\
&< \sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{f_{\sigma(k_s)}^d + r_{\sigma(k_s)}^d\} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_s)}(x(k_s))\}\\
&= \max_{i\in\mathcal{L}} \mathbb{E}_{r^d}^{X^d}\{f_i^d + r_i^d\} \sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_s)}(x(k_s))\} < \infty.
\end{aligned}$$

Further, it implies from (5) that

$$\sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{\varepsilon \|x(k)\|^2\} \le \sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k)}(x(k))\} < \infty.$$

Therefore, the stochastic stability of the switched system with the RMST switching signal satisfying (8) can be concluded from Definition 5. ■

Based on Lemma 1, the following theorem gives existence conditions of the mode-dependent stabilizing controllers for the underlying system (1) with the RMST switching signal.

Theorem 1: Consider the discrete-time switched linear system (1) and let $0 < \alpha_i < 1$, $\mu_i \ge 1$ be given constants. For the prescribed fixed mode-dependent sojourn-time f_i^d and random mode-dependent sojourn-time $r_i^d \sim X_i^d$, suppose there exist matrices $S_i \succ 0$, R_i, $\forall i \in \mathcal{L}$, such that $\forall i \in \mathcal{L}$

$$\Xi(A_i, B_i, \alpha_i, S_i, S_i, R_i) \preceq 0 \tag{10}$$

and $\forall (i \times j) \in \mathcal{L}\times\mathcal{L}$, $i \ne j$,

$$S_j - \mu_j S_i \preceq 0 \tag{11}$$

Then the closed-loop switched system (3) is SS for any RMST switching signal satisfying (8). The stabilizing controller gain is given by $K_i = R_i S_i^{-1}$.

Proof: Let $P_i \triangleq S_i^{-1}, \forall i \in \mathcal{L}$. Performing congruence transformation to (10) via $diag\{P_i, P_i\}$ and considering $K_i = R_i S_i^{-1}$, one has

$$\begin{aligned}
&\begin{bmatrix} -P_i & P_iA_i + P_iB_iR_iS_i^{-1}\\ * & -\alpha_iP_i \end{bmatrix}\\
=&\begin{bmatrix} -P_i & P_iA_i + P_iB_iK_i\\ * & -\alpha_iP_i \end{bmatrix} \preceq 0.
\end{aligned}$$

By the Schur complement, we obtain

$$(P_iA_i + P_iB_iK_i)^T P_i^{-1} (P_iA_i + P_iB_iK_i) - \alpha_iP_i \preceq 0$$

which yields that

$$\begin{aligned}
&x^T(k)A_i^TP_iA_ix(k) + 2x^T(k)A_i^TP_iB_iK_ix(k)\\
&+x^T(k)K_i^TB_i^TP_iB_iK_ix(k) - \alpha_ix^T(k)P_ix(k) \preceq 0.
\end{aligned}$$

Now consider the Lyapunov function with the quadratic form:

$$V_{\sigma(k)}(x(k)) = x^T(k)P_{\sigma(k)}x(k), \forall\sigma(k) = i \in \mathcal{L}. \tag{12}$$

Then, inequation (6) in Lemma 1 can be derived. It follows from (11) that $-P_i^{-1} - I^T(-\mu_jP_j)^{-1}I \preceq 0$, and by Schur complement, it holds that $\left[-P_j^{-1}\ I; I\ -\mu_iP_i\right] \preceq 0$. By considering the Schur complement again to the term $-\mu_i P_i$, one has $P_j - \mu_i P_i \preceq 0$, together with (12), which implies (7) in Lemma 1. Further, selecting $\varepsilon \triangleq \min_{i\in\mathcal{L}}\{\varphi_{\min}(P_i)\}$, the stochastic stability of the resulting closed-loop switched system (3) can be concluded by Lemma 1. ■

B. Stability and stabilization of RMST asynchronously switched systems

Lemma 2: Consider the discrete-time switched system $x(k+1) = f_{\sigma(k)}(x(k))$, $\sigma(k) \in \mathcal{L}$. $0 < \alpha_i < 1$, $\beta_i > 0$, $\mu_i \geq 1$, $i \in \mathcal{L}$ are given constants. For the prescribed fixed mode-dependent sojourn-time f_i^d, random mode-dependent sojourn-time $r_i^d \sim X_i^d$ and time lag Γ_i, suppose that there exists a family of functions $V_{\sigma(k)}(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{L}$ and a constant $\varepsilon > 0$ such that $\forall \sigma(k) = i \in \mathcal{L}$, $\forall(\sigma(k_s - 1) = j, \sigma(k_s) = i) \in \mathcal{L}\times\mathcal{L}$, $i \neq j$, (5) and (7) are satisfied, and $\forall\sigma(k) = i \in \mathcal{L}$,

$$V_i(x_{k+1}) - \theta_i V_i(x_k) \leq 0 \quad (13)$$

hold, where

$$\theta_i = \begin{cases} \beta_i, & \forall k \in [k_s, k_s + \Gamma_i) \\ \alpha_i, & \forall k \in [k_s + \Gamma_i, k_{s+1}) \end{cases}.$$

Then the switched system is SS for asynchronous RMST switching signals satisfying

$$\mu_i \beta_i^{\Gamma_i} \alpha_i^{f_i^d - \Gamma_i} \mathbb{E}_{r_i^d \sim X_i^d}(\alpha_i^{r_i^d}) < 1, \forall i \in \mathcal{L}. \quad (14)$$

Proof: From (7) and (13), we obtain that

$$\begin{aligned} &\Delta V_{\sigma(k_s)}(x(k_s)) \\ =\ &\mathbb{E}_{r_i^d \sim X_i^d}\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\} - V_{\sigma(k_s)}(x(k_s)) \\ \leq\ &\mathbb{E}_{r_i^d \sim X_i^d}\{\mu_i \beta_i^{\Gamma_i} \alpha_i^{f_i^d - \Gamma_i} \mathbb{E}_{r_i^d \sim X_i^d}(\alpha_i^{r_i^d}) V_{\sigma(k_s)}(x(k_{s+1}))\} \\ -\ &V_{\sigma(k_s)}(x(k_s)) \\ =\ &(\mu_i \beta_i^{\Gamma_i} \alpha_i^{f_i^d - \Gamma_i} \mathbb{E}_{r_i^d \sim X_i^d}(\alpha_i^{r_i^d}) - 1) V_{\sigma(k_s)}(x(k_s)). \quad (15) \end{aligned}$$

Define the coefficient

$$\lambda_i^{[\Gamma_i]} \triangleq \mu_i \beta_i^{\Gamma_i} \alpha_i^{f_i^d - \Gamma_i} \mathbb{E}_{r_i^d \sim X_i^d}(\alpha_i^{r_i^d}) - 1, \forall i \in \mathcal{L}.$$

Taking the mathematical expectation of both sides and iterating, we obtain

$$\begin{aligned} &\mathbb{E}_{r^d}^{X^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} \\ \leq &V_{\sigma(k_0)}(x(k_0)) + \sum_{h=0}^{s} \mathbb{E}_{r^d}^{X^d}\{\lambda_{\sigma(k_h)}^{[\Gamma_{\sigma(k_h)}]} V_{\sigma(k_h)}(x(k_h))\} \\ \leq &V_{\sigma(0)}(x(0)) + \lambda_{max}^{[\Gamma]} \sum_{h=0}^{s} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_h)}(x(k_h))\} \end{aligned}$$

where $\lambda_{max}^{[\Gamma]} \triangleq \max_{i\in\mathcal{L}} \lambda_i^{[\Gamma_i]}$. It is straightforward from (14) and (15) that $\Delta V_{\sigma(k_s)}(x(k_s)) < 0$ and

$$\lim_{s\to\infty} \mathbb{E}_{r^d}^{X^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} < \infty.$$

Then, one has $\sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_s)}(x(k_s))\} \leq \frac{1}{\lambda_{max}^{[\Gamma]}}(\lim_{s\to\infty} \mathbb{E}_{r^d}^{X^d}\left\{V_{\sigma(k_{s+1})}(x(k_{s+1}))\right\} - V_{\sigma(0)}(x(0))) < \infty$. Consider cases $0 < \beta < 1$ and $\beta > 1$, it holds from (13) that $\max_{k\in[k_s,k_{s+1})} V_{\sigma(k)}(x(k)) \leq \max(V_{\sigma(k_s)}(x(k_s)), \beta_i^{f_i^d} V_{\sigma(k_s)}(x(k_s)))$ and

$$\begin{aligned} &\sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k)}(x(k))\} \\ < &\sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{(f_{\sigma(k_s)}^d + r_{\sigma(k_s)}^d)\max(1, \beta_{\sigma(k_s)}^{f_{\sigma(k_s)}^d}) V_{\sigma(k_s)}(x(k_s))\} \\ < &\max_{i\in\mathcal{L}} \mathbb{E}_{r^p}^{X^p}\{(f_i^d + r_i^d)\max(1, \beta_i^{f_i^d})\} \\ \times &\sum_{s=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k_s)}(x(k_s))\} < \infty. \end{aligned}$$

It follows from (5) that

$$\sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{\varepsilon \|x(k)\|^2\} \leq \sum_{k=0}^{\infty} \mathbb{E}_{r^d}^{X^d}\{V_{\sigma(k)}(x(k))\} < \infty.$$

Therefore, the stochastic stability of the switched system with the asynchronous RMST switching signal satisfying (14) is concluded from Definition 5. ■

Now, we are in a position to derive the existence conditions of the controller such that the asynchronously switched system (4) with RMST switching is SS.

Theorem 2: Consider discrete-time switched linear system (1) and let $0 < \alpha_i < 1$, $\beta_i > 0$, $\mu_i \geq 1$, $i \in \mathcal{L}$ be given constants. For the prescribed fixed mode-dependent sojourn-time f_i^d, random mode-dependent sojourn-time $r_i^p \sim X_i^d$ and time lag Γ_i, $i \in \mathcal{L}$, suppose there exist matrices $S_i \succ 0$, R_i, $\forall i \in \mathcal{L}$, such that $\forall(i \times j) \in \mathcal{L}\times\mathcal{L}$, $i \neq j$, (10) and (11) are satisfied, and

$$\Xi(A_i, B_i, -\beta_i, S_i, S_j, R_j) - diag\{0, \beta_i(S_j + S_j^T)\} \preceq 0 \quad (16)$$

Then the closed-loop switched system (4) is SS for any RMST switching signal satisfying (14). The stabilizing controller gain is given by $K_j = R_j S_j^{-1}$.

Proof: In a similar vein to the proof of Theorem 1, the one for Theorem 2 can be obtained and thus is omitted here. ■

Likewise, the following corollary is given for the case of random mode-independent sojourn-time switching with mode-independent delay.

Corollary 1: Consider discrete-time switched linear system (1) and let $0 < \alpha < 1$, $\mu \geq 1$, $\beta > 0$ be given constants. For the fixed sojourn-time f^d, the random mode-dependent sojourn-time $r^d \sim X^d$ and time lag Γ, suppose there exist matrices $S_i \succ 0$, R_i, $\forall i \in \mathcal{L}$, such that $\forall(i \times j) \in \mathcal{L}\times\mathcal{L}$, $i \neq j$,

$$\Xi(A_i, B_i, \alpha, S_i, S_i, R_i) \preceq 0 \quad (17)$$

$$\Xi(A_i, B_i, -\beta, S_i, S_j, R_j) - diag\{0, \beta(S_j + S_j^T)\} \preceq 0 \quad (18)$$

$$S_j - \mu S_i \preceq 0 \quad (19)$$

Then the closed-loop switched system (4) is stochastically stable for the random mode-independent sojourn-time switching signal satisfying

$$\mu \beta^{\Gamma} \alpha^{f^d - \Gamma} \mathbb{E}_{r^d \sim X^d}(\alpha^{r^d}) < 1.$$

The stabilizing controller gain is given by $K_j = R_j S_j^{-1}$.

IV. Numerical examples

In this section, a numerical example is used to verified the developed theoretical results on RMST asynchronously switched systems.

Example 1: Consider the switched linear system (1) with three subsystems:

$$A_i = \begin{bmatrix} 1.5144 & 0.0041 & 0.0029 & -0.0692 \\ 0.0073 & 1.3665 & 3.6480 & -0.6112 \\ 0.0152 & a_{1i} & 1.4125 & a_{2i} \\ 0 & 0 & 0.1520 & 0.1520 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0.0442 & 0.0176 \\ b_i & -0.7592 \\ -0.5520 & 0.4490 \\ 0 & 0 \end{bmatrix}$$

where

$$a_{11} = 0.0560, a_{12} = 0.0101, a_{13} = 0.0822$$
$$a_{21} = 0.2250, a_{22} = 0.0182, a_{23} = 0.3870$$
$$b_1 = 0.3545, b_2 = 0.0978, b_3 = 0.5112$$

$x_0 = [\ 3 \quad -2 \quad 0.5 \quad -2\ ]^T$. Setting

$$\alpha_1 = 0.95, \alpha_2 = 0.97, \alpha_3 = 0.93$$
$$\mu_1 = 1.85, \mu_2 = 1.82, \mu_3 = 1.88$$

and the mode-dependent stabilizing controllers can be obtained by Theorem 1. Let $f_1^p = 6, f_2^p = 5, f_3^p = 7$, $r_i^p = 4, 6, 8$ with given probability distribution $0.24, 0.12, 0.64$, respectively, and generate 80 RMST switching signals randomly. The state responses of the closed-loop switched system with and without asynchronous switching are illustrated in Fig. 2 and Fig. 3, respectively. When the time lag between subsystems and controllers is $\Gamma_1 = 4, \Gamma_2 = 3, \Gamma_3 = 5$, it can be clearly seen that the controllers designed without considering asynchronous switching lead to larger overshoots. One can infer that the system tend to be unstable with the increase of the time lag. Thus, the controller ensuring the system stability is designed below for RMST switching signals in the presence of asynchronous switching.

Then, considering $\alpha_i = 0.835$, $\beta_i = 1.38$, $\mu_i = 1.26$, the minimal MDT $\tau_i^* = 12.4262$, $\forall i \in \mathcal{L}$ can be derived from [15]. Therefore, the system stability is guaranteed for MDT no less than 12. By Lemma 2, it can be checked that the above RMST switching signals ensure the resulting closed-loop system stability. Obviously, under the framework of the RMST switching, the system can remain stable even if the actual running time of the subsystem does not satisfy the minimal MDT constraint. It is concluded that by utilizing statistical information, the set of admissible switching signals ensuring system stability can be expanded.

Finally, by applying the stabilizing controllers derived by Theorem 2, the state responses of the asynchronously switched system with RMST switching are given in Fig. 4. Compared with the case where the asynchronous switching exists but not be considered in Fig. 3, the overshoots of state responses are significantly reduced, which verifies the validity of the theoretical result.

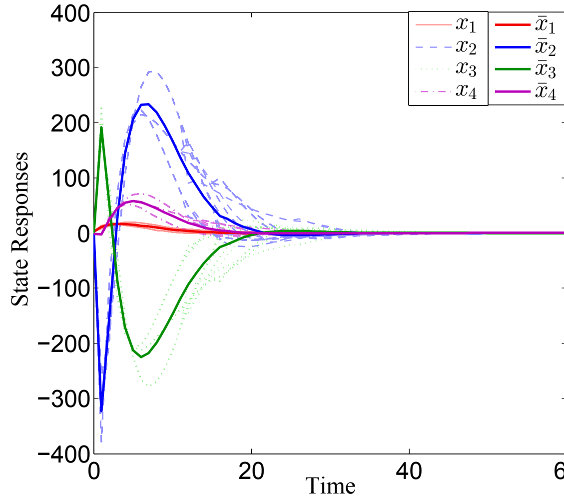

Fig. 2: 80 realizations of state responses of the closed-loop system with RMST switching.

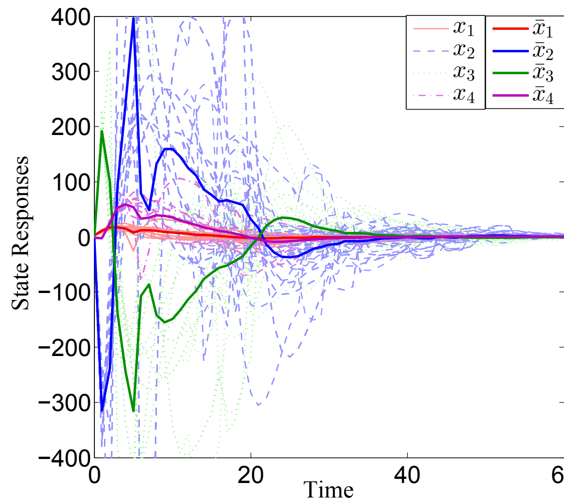

Fig. 3: Illustration of the increase of overshoot and the degradation of convergence caused by asynchronous switching.

V. Conclusion

In this paper, the stability and stabilization problems are addressed for switched systems with sojourn-time expectation. A new concept of RMST switching signals is proposed, and the stability analysis and control synthesis are carried out for the RMST switched system such that more admissible switching signals can be obtained to ensure the system stability. Compared with the existing studies on sojourn-time with fixed and random parts, the restriction of the Markov chain is relaxed. Further, extended results are presented for the switched system with asynchronous RMST switching. Finally, a numerical example is used to illustrate the effectiveness of the proposed methods.

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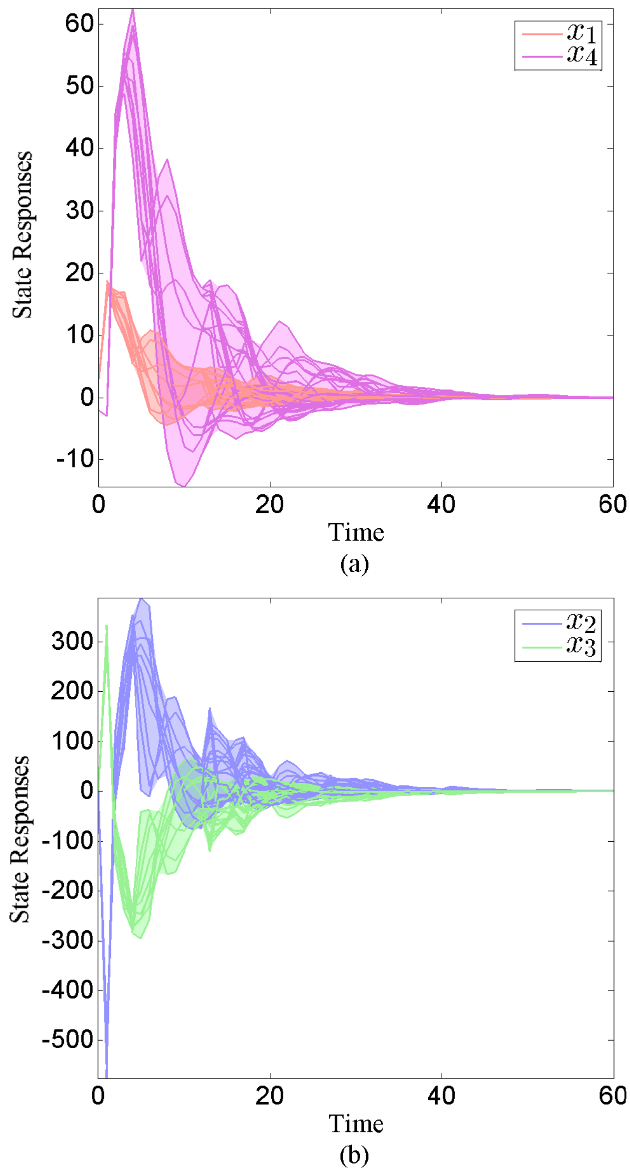

Fig. 4: 80 realizations of state responses of the closed-loop system with asynchronous RMST switching.

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