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EVENT-DRIVEN ℓ_1-GAIN ASYNCHRONOUS FILTER OF POSITIVE MARKOV JUMP SYSTEMS†

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***Abstract*— This paper investigates the event-driven asynchronous filter for positive Markov jump systems (PMJSs) by employing hidden Markov model. Based on the output of sensor measurement, a weighted event-driven threshold is established in the form of 1-norm. The stability of the corresponding augmented system is achieved by transforming the error signal into interval uncertain form. Under the established driven condition, an event-driven positive ℓ_1-gain asynchronous filter is constructed for PMJSs. Then, the asynchronous filter design for PMJSs with partial information of hidden Markov model is further addressed. All conditions are described in the linear programming (LP) form. Finally, one example is given to illustrate the validity of the proposed design.**

I. INTRODUCTION

In the real world, there are some important dynamic processes which can be described by positive systems [1] such as formation flying, electronic circuits, urban water management, and so on. PMJSs composed of several positive subsystems and a Markov process have attracted much attention in the control filed. Different from Markov jump systems [2]-[4], PMJSs have distinct research approaches. In the literature [5], 1-moment stability, exponential mean and almost-sure stability of PMJSs were addressed. Stochastic stability and control synthesis of PMJSs were investigated by virtue of an LP-based framework [6]. In [7], the issue of ℓ_1-gain stability was explored and a positive ℓ_1-gain filter was constructed using some special properties of the system matrices of positive systems. The literature [8] considered mean stability of PMJSs. More achievements on PMJSs can be found in [9]-[14].

Event-driven mechanism is one of significant strategies in practice. In [15], a distributed event-driven framework was proposed for dealing with the cooperative control of large scale multi-agents. A dynamic triggered threshold was also introduced in [16]. The research status of event-driven approach was further explored in [17], [18]. The literature [19] presented a so-called hybrid-driven mechanism for the stability of networked control systems. The hybrid-driven mechanism introduces a stochastic signal that orchestrates when and how the state/output and the corresponding sampling signal switch. It balances the time-triggered and event-driven mechanisms by virtue of the stochastic signal. Hybrid-driven mechanism was developed to investigate the guaranteed cost control of networked systems in [20]-[22]. Some results have also been published for the event-driven topics of positive systems [23]-[26]. It should be pointed out that the error term between sampled state and real-time state was assumed to keep decreasing in [24], [25] and the weight of every sensor output was assumed to be the same in [26]. However, few results on PMJSs with event-driven mechanism are reported. This paper is to construct a new framework on event-driven filter of PMJSs.

This paper is concerned with the ℓ_1-gain asynchronous filter of PMJSs. The main contributions are as follows: (i) A weighted event-driven linear threshold is established for the systems, where the weighted coefficients of the threshold are different for different sensors, (ii) An event-driven asynchronous filter is proposed for PMJSs using a filter matrix decomposition technique, and (iii) A hidden Markov chain-based asynchronous filter is proposed for PMJSs. The remainder of this paper is structured as follows: Some preliminaries are given in Section II, main results are presented in Section III, Section IV provides one example, and Section V concludes this paper.

Notation: The symbols $\succeq 0, \succ 0, \preceq, \prec$ hold for components. $A^\top$ represents the transpose of A. I stands for the identity matrix. $\mathrm{diag}(x_1, x_2, \ldots, x_n)$ represents a diagonal matrix with diagonal elements $x_1, x_2, \cdots, x_n$. $\mathbf{1}_n$ is an n-dimensional vector whose elements are all 1. $\mathbf{1}_n^{(\imath)}$ is a vector whose $\imath$th element is 1 and other elements are 0. $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operation.

II. PRELIMINARIES

Consider the Markov jump system:

$$\begin{aligned} x(k+1) &= A_{r_k}x(k) + B_{r_k}w(k), \\ y(k) &= C_{r_k}x(k) + D_{r_k}w(k), \\ z(k) &= E_{r_k}x(k) + F_{r_k}w(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n, z(k) \in \mathbb{R}^g, y(k) \in \mathbb{R}^h, w(k) \in \mathbb{R}_+^m$ are the state, the controlled output, the measurable output, and the exogenous disturbance, respectively; $r_k = i \in \mathcal{S}_1 = \{1, 2, \ldots, N\}$ is a homogeneous Markov process with the transition probability: $\Pr(r_{k+1} = j \mid r_k = i) = \pi_{ij}$, where $0 \leq \pi_{ij} \leq 1$ with $\sum_{j=1}^{N} \pi_{ij} = 1$ for $i, j \in S_1$; $A_{r_k}, B_{r_k}, C_{r_k}, D_{r_k}, E_{r_k}$, and F_{r_k} are system matrices with appropriate dimensions. Let $A_i = A_{r_k}$, $B_i = B_{r_k}$, $C_i = C_{r_k}$, $D_i = D_{r_k}$, $E_i = E_{r_k}$, and $F_i = F_{r_k}$ for $r_k = i$. In the paper, it is assumed that $A_i \succeq 0, B_i \succeq 0, C_i \succeq 0, D_i \succeq$

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$0, E_i \succeq 0$, and $F_i \succeq 0$.

Definition 1 ([1]) A system is positive if all its states and outputs are nonnegative whenever initial condition and input are nonnegative.

Lemma 1 ([1]) A system

$$\begin{aligned}x(k+1) &= Ax(k) + Bw(k),\\ y(k) &= Cx(k) + Dw(k),\\ z(k) &= Ex(k) + Fw(k),\end{aligned}$$

is positive iff $A \succeq 0$, $B \succeq 0$, $C \succeq 0$, $D \succeq 0$, $E \succeq 0$, and $F \succeq 0$.

By Lemma 1, it follows that the system (1) is a positive Markov jump system.

Lemma 2 ([1]) Given a positive system $x(k+1) = Ax(k)$, the Schur property of A is equivalent to $(A-I)^{\top} v \prec 0$ for $v \succ 0$.

By Lemma 1 and the assumptions on the system matrices, the system (1) is a positive Markov jump system.

Definition 2 ([9]) The system (1) with $w(k) = 0$ is stochastically stable if for any initial mode $r_0 \in \{1, 2, \cdots, N\}$ and initial state $x(0) \succeq 0$ it holds that:

$$\lim_{k\to\infty} \mathcal{E}\big\{ \textstyle\sum_{t=0}^{k} \|x(t)\|_1 \mid x(0), r_0 \big\} \leq x^{\top}(0)\varrho, \tag{2}$$

where $\varrho \succ 0$ with $\varrho \in \mathbb{R}^n$.

Definition 3 The system (1) is ℓ_1-gain stable, if it is stochastically stable for $w(k) = 0$ and

$$\|y(k)\|_{\ell_1} < \gamma \|w(k)\|_{\ell_1} \tag{3}$$

holds under the zero initial condition and $w(k) \neq 0$, where $\gamma > 0$ is the gain value.

III. Main results

In this section, a positive event-driven ℓ_1-gain asynchronous filter is designed for PMJSs such that the corresponding error system is positive and stochastically stable with an ℓ_1-gain performance. Then, hybrid-driven mechanism is introduced into PMJSs to consider the issue of ℓ_1-gain asynchronous filtering. Based on the hidden Markov model, the positive asynchronous ℓ_1-gain filter design will be investigated. For convenience of later development, denote $\overline{\pi}_j = \max\{\pi_{ij}, i \in \mathcal{S}_1\}$. Define a piecewise vector-valued function $\overline{y}(k)$, where $\overline{y}(k) = y(k_l)$, $k \in [k_l, k_{l+1})$, $l \in \mathbb{N}$, and k_l is the lth event-driven time instant. Denote the error $y_e(k) = \overline{y}(k) - y(k)$. The event-driven condition can be established as:

$$\|\Lambda y_e(k)\|_1 > \beta \|\Lambda y(k)\|_1, \tag{4}$$

where $0 < \beta < 1$, $\Lambda = \mathrm{diag}(\epsilon_1, \epsilon_2, \ldots, \epsilon_h)$, and $\epsilon_1, \epsilon_2, \cdots, \epsilon_h$ are positive known constants.

Remark 1 In (4), a linear weighted event-driven condition is constructed by 1-norm. In [26], a linear triggering condition was also established, where the weight coefficients of different sensor outputs were the same. This implies that the triggering framework in [26] is a special case of (4). It is also clear that one can replace (4) via a more general case $\|\Lambda_1 y_e(k)\|_1 > \beta \|\Lambda_2 y(k)\|_1$, where Λ_1 may not be equal to Λ_2. The event-driven filter based on (4) to be proposed later is easy to be developed for the corresponding filter based on the more general case. Therefore, we only consider the case (4).

In this paper, we establish an asynchronous filter framework on PMJSs. A hidden Markov model approach is employed to describe the phenomena between the filter and the original system. An event-driven asynchronous filter is constructed as:

$$\begin{aligned}x_f(k+1) &= A_{f\delta_k} x_f(k) + B_{f\delta_k}\tilde{y}(k),\\ z_f(k) &= E_{f\delta_k} x_f(k) + F_{f\delta_k}\tilde{y}(k),\end{aligned} \tag{5}$$

where the matrices $A_{f\delta_k}$, $B_{f\delta_k}$, $E_{f\delta_k}$, and $F_{f\delta_k}$ are gain matrices, $x_f(k)$ is the filter state, $z_f(k)$ is the estimation of $z(k)$, and $\tilde{y}(k) = \overline{y}(k)$. The term $\delta_k = \rho \in \mathcal{S}_2 = \{1, 2, \ldots, M\}$ is to describe the asynchronous phenomena. It satisfies the conditional probability $\Pr\{\delta_k = \jmath \mid r_k = j\} = \lambda_{j\jmath}$, which is subject to a hidden Markov model and follows $\sum_{\jmath=1}^{M} \lambda_{j\jmath} = 1$, $0 \leq \lambda_{j\jmath} \leq 1$. Let $\widetilde{x}(k) = (x^{\top}(k) \;\; x_f^{\top}(k))^{\top}$ and $e(k) = z_f(k) - z(k)$. Then, the corresponding error system can be given as:

$$\begin{aligned}\widetilde{x}(k+1) &= \widetilde{A}_{r_k,\delta_k}\widetilde{x}(k) + \widetilde{B}_{r_k,\delta_k} w(k) + \widetilde{B}_{f\delta_k} y_e(k),\\ e(k) &= \widetilde{E}_{r_k,\delta_k}\widetilde{x}(k) + \widetilde{F}_{r_k,\delta_k} w(k) + F_{f\delta_k} y_e(k),\end{aligned} \tag{6}$$

where

$$\begin{aligned}\widetilde{A}_{r_k,\delta_k} &= \begin{pmatrix} A_{r_k} & 0 \\ B_{f\delta_k} C_{r_k} & A_{f\delta_k} \end{pmatrix}, \widetilde{B}_{f\delta_k} = \begin{pmatrix} 0 \\ B_{f\delta_k} \end{pmatrix},\\ \widetilde{B}_{r_k,\delta_k} &= \begin{pmatrix} B_{r_k} \\ B_{f\delta_k} D_{r_k} \end{pmatrix}, \widetilde{F}_{r_k,\delta_k} = F_{f\delta_k} D_{r_k} - F_{r_k},\\ \widetilde{E}_{r_k,\delta_k} &= \Big(F_{f\delta_k} C_{r_k} - E_{r_k} \;\; E_{f\delta_k} \Big).\end{aligned}$$

For convenience, denote $A_{f\rho} = A_{f\delta_k}$, $B_{f\rho} = B_{f\delta_k}$, $E_{f\rho} = E_{f\delta_k}$, and $F_{f\rho} = F_{f\delta_k}$ for $\delta_k = \rho$. Let $\Theta^i = \{\pi_{ij} \mid j \in \mathcal{S}_1\}, \Omega^i = \{\lambda_{i\jmath} \mid \jmath \in \mathcal{S}_2\}$.

Theorem 1 If there exist constants $\tau > 0$ and $\gamma > 0$, $\mathbb{R}^n$ vectors $v_1^{(i)} \succ 0$, $v_2^{(\rho)} \succ 0$, $\varphi_{\rho l} \succeq 0$, $\vartheta_{\rho l} \succeq 0$, $\varphi_{\rho} \succeq 0$, $\vartheta_{\rho} \succeq 0$, and $\mathbb{R}^h$ vectors $\phi_{\rho l} \succeq 0$, $\psi_{\rho l} \succeq 0$, $\phi_{\rho} \succeq 0$, $\psi_{\rho} \succeq 0$ such that

$$\textstyle\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \psi_{\rho l}^{\top} \big(I - \beta \Lambda^{-1} \mathbf{1}_h \epsilon^{\top}\big) C_i \succeq 0, \tag{7a}$$

$$\textstyle\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \psi_{\rho l}^{\top} \big(I - \beta \Lambda^{-1} \mathbf{1}_h \epsilon^{\top}\big) D_i \succeq 0, \tag{7b}$$

$$\textstyle\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \phi_{\rho l}^{\top} \big(I - \beta \Lambda^{-1} \mathbf{1}_h \epsilon^{\top}\big) C_i - g E_i \succeq 0, \tag{7c}$$

$$\textstyle\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \phi_{\rho l}^{\top} \big(I - \beta \Lambda^{-1} \mathbf{1}_h \epsilon^{\top}\big) D_i - g F_i \succeq 0, \tag{7d}$$

$$\varphi_\rho + \vartheta_\rho + \tau \mathbf{1}_n - v_2^{(\rho)} \prec 0, \tag{7e}$$

$$\begin{aligned}C_i^{\top} \big(I + \beta \epsilon \mathbf{1}_h^{\top} \Lambda^{-1}\big)(\psi_\rho + \phi_\rho) - E_i^{\top} \mathbf{1}_g + \tau \mathbf{1}_n\\ + A_i^{\top} \textstyle\sum_{j=1}^{N} \pi_{ij} v_1^{(j)} - v_1^{(i)} \prec 0,\end{aligned} \tag{7f}$$

$$\begin{aligned}D_i^{\top} \big(I + \beta \epsilon \mathbf{1}_h^{\top} \Lambda^{-1}\big)(\psi_\rho + \phi_\rho) - F_i^{\top} \mathbf{1}_g\\ + B_i^{\top} \textstyle\sum_{j=1}^{N} \pi_{ij} v_1^{(j)} - \gamma \mathbf{1}_m \preceq 0,\end{aligned} \tag{7g}$$

$$\varphi_{\rho l} \preceq \varphi_\rho, \;\; \psi_{\rho l} \preceq \psi_\rho, \quad l = 1, 2, \cdots, n, \tag{7h}$$

$$\vartheta_{\rho l} \preceq \vartheta_\rho, \;\; \phi_{\rho l} \preceq \phi_\rho, \quad l = 1, 2, \cdots, g, \tag{7i}$$

hold for $i \in \mathcal{S}_1$ and $\rho \in \mathcal{S}_2$, then a positive event-driven ℓ_1-gain asynchronous filter can be designed as (5) with

$$\begin{aligned} A_{f\rho} &= \frac{\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \varphi_{\rho l}^\top}{\sum_{j=1}^{N} \overline{\pi}_j \sum_{j=1}^{M} \lambda_{jj} \mathbf{1}_n^\top v_2^{(j)}}, \\ B_{f\rho} &= \frac{\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \psi_{\rho l}^\top}{\sum_{j=1}^{N} \overline{\pi}_j \sum_{j=1}^{M} \lambda_{jj} \mathbf{1}_n^\top v_2^{(j)}}, \\ E_{f\rho} &= \frac{\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \vartheta_{\rho l}^\top}{g},\ F_{f\rho} = \frac{\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \phi_{\rho l}^\top}{g}, \end{aligned} \tag{8}$$

and the system (6) is positive and stochastically stable.

Proof Give $x(0) \succeq 0$ and $x_f(0) \succeq 0$, that is, $\widetilde{x}(0) \succeq 0$. By Lemma 1, $y(k) \succeq 0$. From the event-driven condition (4), it follows $-\beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top y(0) \preceq y_e(0) \preceq \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top y(0)$. Then, the system (6) can be transformed into

$$\begin{aligned} \widetilde{x}(1) &\succeq \underline{A}_{i,\rho}\widetilde{x}(0) + \underline{B}_{i,\rho}w(0), \\ e(0) &\succeq \underline{E}_{i,\rho}\widetilde{x}(0) + \underline{F}_{i,\rho}w(0), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \underline{A}_{i,\rho} &= \begin{pmatrix} A_i & 0 \\ B_{f\rho}\left(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)C_i & A_{f\rho} \end{pmatrix}, \\ \underline{B}_{i,\rho} &= \begin{pmatrix} B_i \\ B_{f\rho}\left(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)D_i \end{pmatrix}, \\ \underline{E}_{i,\rho} &= \left(F_{f\rho}\left(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)C_i - E_i \quad E_{f\rho}\right), \\ \underline{F}_{i,\rho} &= \left(F_{f\rho}\left(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)D_i - F_i\right). \end{aligned}$$

Using (7h), (7i), and (8), it gives $A_{f\rho} \succeq 0$, $B_{f\rho} \succeq 0$, $E_{f\rho} \succeq 0$, and $F_{f\rho} \succeq 0$. By (7a), (7b), (7c), (7d), and (8), it can be obtained that $\underline{A}_{i,\rho} \succeq 0$, $\underline{B}_{i,\rho} \succeq 0$, $\underline{E}_{i,\rho} \succeq 0$, and $\underline{F}_{i,\rho} \succeq 0$. According to Lemma 1, we have $\widetilde{x}(1) \succ 0$ and $e(0) \succ 0$. By mathematical production, we obtain that $\widetilde{x}(k) \succeq 0$ and $e(k) \succeq 0$ hold for $k = 0, 1, 2, \cdots$. This implies that the system (6) is positive.

Construct a linear stochastic Lyapunov function as

$$V(\widetilde{x}(k), r_k = i, \delta_k = \rho) = \widetilde{x}^\top(k)v^{(i)}, \tag{10}$$

where $v^{(i)} = \left(v_1^{(i)\top} \ v_2^{(\rho)\top}\right)^\top$. Using (4) gives

$$\begin{aligned} \widetilde{x}(k+1) &\preceq \overline{A}_{i,\rho}\widetilde{x}(k) + \overline{B}_{i,\rho}w(k), \\ e(k) &\preceq \overline{E}_{i,\rho}\widetilde{x}(k) + \overline{F}_{i,\rho}w(k), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \overline{A}_{i,\rho} &= \begin{pmatrix} A_i & 0 \\ B_{f\rho}\left(I + \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)C_i & A_{f\rho} \end{pmatrix}, \\ \overline{B}_{i,\rho} &= \begin{pmatrix} B_i \\ B_{f\rho}\left(I + \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)D_i \end{pmatrix}, \\ \overline{E}_{i,\rho} &= \left(F_{f\rho}\left(I + \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)C_i - E_i \quad E_{f\rho}\right), \\ \overline{F}_{i,\rho} &= \left(F_{f\rho}\left(I + \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\right)D_i - F_i\right). \end{aligned}$$

Then,

$$\begin{aligned} &\mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\} \\ \leq &\mathcal{E}\{\widetilde{x}^\top(k)\left(\overline{A}_{i,\rho}^\top v^{(r_{k+1})} + \overline{E}_{i,\rho}^\top \mathbf{1}_g\right) \\ &+ w^\top(k)\left(\overline{F}_{i,\rho}^\top \mathbf{1}_g + \overline{B}_{i,\rho}^\top v^{(r_{k+1})}\right) \\ &- e^\top(k)\mathbf{1}_g \mid r_k = i, \delta_k = \rho\} - \widetilde{x}^\top(k)v^{(i)}. \end{aligned} \tag{12}$$

By (7h), (7i) and (8), the following relations hold:

$$\begin{aligned} &\mathcal{E}\{\overline{A}_{i,\rho}^\top v^{(r_{k+1})} \mid r_k = i, \delta_k = \rho\} \\ \preceq &\begin{pmatrix} A_i^\top \sum_{j=1}^{N} \pi_{ij} v_1^{(j)} + C_i^\top\left(I + \beta\epsilon\mathbf{1}_h\Lambda^{-1}\right)\varphi_\rho \\ \varphi_\rho \end{pmatrix}, \\ &\mathcal{E}\{\overline{B}_{i,\rho}^\top v^{(r_{k+1})} \mid r_k = i, \delta_k = \rho\} \\ \preceq &\left(B_i^\top \sum_{j=1}^{N} \pi_{ij} v_1^{(j)} + D_i^\top\left(I + \beta\epsilon\mathbf{1}_h^\top\Lambda^{-1}\right)\psi_\rho\right), \\ &\mathcal{E}\{\overline{E}_{i,\rho}^\top \mathbf{1}_g \mid r_k = i, \delta_k = \rho\} \\ \preceq &\begin{pmatrix} C_i^\top\left(I + \beta\epsilon\mathbf{1}_h^\top\Lambda^{-1}\right)\phi_\rho \\ \vartheta_\rho \end{pmatrix}, \\ &\mathcal{E}\{\overline{F}_{i,\rho}^\top \mathbf{1}_g \mid r_k = i, \delta_k = \rho\} \\ \preceq &D_i^\top\left(I + \beta\epsilon\mathbf{1}_h^\top\Lambda^{-1}\right)\phi_\rho - F_i^\top\mathbf{1}_g. \end{aligned}$$

Then, it gives

$$\begin{aligned} &\mathcal{E}\{\overline{A}_{i,\rho}^\top v^{(r_{k+1})} + \overline{E}_{i,\rho}^\top \mathbf{1}_g \mid r_k = i, \delta_k = \rho\} \\ \preceq &\begin{pmatrix} A_i^\top \sum_{j=1}^{N} \pi_{ij} v_1^{(j)} + C_i^\top\left(I + \beta\epsilon\mathbf{1}_h\Lambda^{-1}\right)(\varphi_\rho + \phi_\rho) \\ \varphi_\rho + \vartheta_\rho \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} &\mathcal{E}\{\overline{B}_{i,\rho}^\top v^{(r_{k+1})} + \overline{F}_{i,\rho}^\top \mathbf{1}_g \mid r_k = i, \delta_k = \rho\} \\ \preceq\ &D_i^\top\left(I + \beta\epsilon\mathbf{1}_h^\top\Lambda^{-1}\right)(\psi_\rho + \psi_\rho) \\ &+ B_i^\top \sum_{j=1}^{N} \pi_{ij} v_1^{(j)} - F_i^\top \mathbf{1}_g. \end{aligned}$$

By (7g), we have

$$\mathcal{E}\{\overline{B}_{i,\rho}^\top v^{(r_{k+1})} + \overline{F}_{i,\rho}^\top \mathbf{1}_g \mid r_k = i, \delta_k = \rho\} \preceq \gamma\mathbf{1}_m. \tag{13}$$

Therefore, (12) can be transformed into (14). Using (7e) and (7f), it yields that

$$\begin{aligned} &\mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\} \\ \leq &-\tau\widetilde{x}^\top(k)\mathbf{1}_{2n} + \gamma w^\top(k)\mathbf{1}_m - e^\top(k)\mathbf{1}_g. \end{aligned} \tag{15}$$

Then,

$$\begin{aligned} &\sum_{k=0}^{\infty} \mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\} \\ \leq &\sum_{k=0}^{\infty} \mathcal{E}\{-\tau\widetilde{x}^\top(k)\mathbf{1}_{2n} + \gamma w^\top(k)\mathbf{1}_m - e^\top(k)\mathbf{1}_g\}. \end{aligned}$$

Together with $w(k) = 0$ and $e(k) \succeq 0$ derives that

$$\begin{aligned} &\sum_{k=0}^{\infty} \mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\} \\ &\leq -\tau \sum_{k=0}^{\infty} \mathcal{E}\{\widetilde{x}^\top(k)\mathbf{1}_{2n}\}, \end{aligned}$$

which implies that $\sum_{t=0}^{\infty} \|\widetilde{x}(k)\|_1 \leq \frac{1}{\tau}\widetilde{x}^\top(0)v^{(r_0)}$. By Definition 2, the system (6) is stochastically stable. When $w(k) \neq 0$ and $\widetilde{x}(0) = 0$, we can obtain

$$\begin{aligned} \sum_{k=0}^{\infty} e^\top(k)\mathbf{1}_g \quad &\leq \sum_{k=0}^{\infty} \{\gamma w^\top(k)\mathbf{1}_m - \tau\widetilde{x}^\top(k)\mathbf{1}_{2n} \\ &\quad - \mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\}\} \\ &\leq \gamma \sum_{k=0}^{\infty} w^\top(k)\mathbf{1}_m. \end{aligned} \tag{16}$$

By Definition 3, γ is the ℓ_1-gain performance value of the system (6). ■

Remark 2 A time-triggering strategy was employed for designing the filter and controller in [14]. Theorem 1 addresses a linear approach to the asynchronous filter of PMJSs. The linear approach refers to linear copositive Lyapunov functions and LP computation conditions. It has been stated in the literature [1], [6], [7], [10]-[12] that such a linear

$$\mathcal{E}\{\Delta V(\widetilde{x}(k), r_k=i, \delta_k=\rho)\} \leq \widetilde{x}^\top(k) \begin{pmatrix} A_i^\top \sum_{j=1}^N \pi_{ij} v_1^{(j)} + C_i^\top \big(I + \beta\epsilon \mathbf{1}_h \Lambda^{-1}\big)(\varphi_\rho + \phi_\rho) - v_1^{(i)} \\ \varphi_\rho + \vartheta_\rho - v_2^{(\rho)} \end{pmatrix} + \gamma w^\top(k)\mathbf{1}_m - e^\top(k)\mathbf{1}_g. \tag{14}$$

approach is more suitable for dealing with the issues of positive systems. Moreover, Theorem 1 constructs a new filter framework (5). Under such a framework, the filtering issues of PMJSs are easily solved. It should be pointed out that the framework (5) can be directly applied for the filter design of other hybrid positive systems. Let $\beta = 0$, which implies that the even-driven condition (4) is changed as time-driven one. The design in Theorem 1 is reduced to the corresponding design in the literature.

In industrial applications, the probability parameters π_{ij} and $\lambda_{\jmath\jmath}$ are usually partly known. The following theorem will develop the filter framework (5) for the system (1) with partly known probabilities. Denote $\Theta_1 = \{\pi_{ij} \mid r_k = i, r_{k+1} = j \in \mathcal{S}_1, \pi_{ij} \text{ is known}\}$, $\Theta_2 = \{\pi_{ij} \mid r_k = i, r_{k+1} = j \in \mathcal{S}_1, \pi_{ij} \text{ is unknown}\}$, $\Omega_1 = \{\lambda_{\jmath\jmath} \mid r_k = j, \delta_k = \jmath, \jmath \in \mathcal{S}_2, \lambda_{\jmath\jmath} \text{ is known}\}$, and $\Omega_2 = \{\lambda_{\jmath\jmath} \mid r_k = j, \delta_k = \jmath, \jmath \in \mathcal{S}_2, \lambda_{\jmath\jmath} \text{ is unknown}\}$. This reveals that $\Theta_1 \cup \Theta_2 = \Theta$ and $\Omega_1 \cup \Omega_2 = \Omega$. Let $\overline{\pi}_{ij} = \begin{cases} \pi_{ij}, \text{ if } \pi_{ij} \in \Theta_1, \\ 1 - \sum_{l=1}^N \pi_{il}, \ \pi_{il} \in \Theta_1, \text{if } \pi_{ij} \in \Theta_2, \end{cases}$ $\overline{\lambda}_{\jmath\jmath} = \begin{cases} \lambda_{\jmath\jmath}, \text{ if } \lambda_{\jmath\jmath} \in \Omega_1, \\ 1 - \sum_{l=1}^M \lambda_{\jmath l}, \ \lambda_{\jmath l} \in \Omega_1, \text{if } \lambda_{\jmath\jmath} \in \Omega_2. \end{cases}$ Set $\widetilde{\pi}_j = \max\{\overline{\pi}_{ij}, i \in \mathcal{S}_1\}$.

Theorem 2 If there exist constants $\gamma > 0$, $\varepsilon > 0$, $\mathbb{R}^n$ vectors $v_1^{(i)} \succ 0$, $v_2^{(\rho)} \succ 0$, $u_1 \succ 0$, $u_2 \succ 0$, $u_3 \succ 0$, $\varphi_{\rho l} \succeq 0$, $\vartheta_{\rho l} \succeq 0$, $\varphi_\rho \succeq 0$, $\vartheta_\rho \succeq 0$, and $\mathbb{R}^h$ vectors $\phi_{\rho l} \succeq 0$, $\psi_{\rho l} \succeq 0$, $\phi_\rho \succeq 0$, $\psi_\rho \succeq 0$ such that

$$\textstyle\sum_{l=1}^g \mathbf{1}_g^{(l)} \phi_{\rho l}^\top \big(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\big) C_i - gE_i \succeq 0, \tag{17a}$$

$$\textstyle\sum_{l=1}^n \mathbf{1}_n^{(l)} \psi_{\rho l}^\top \big(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\big) C_i \succeq 0, \tag{17b}$$

$$\textstyle\sum_{l=1}^g \mathbf{1}_g^{(l)} \phi_{\rho l}^\top \big(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\big) D_i - gF_i \succeq 0, \tag{17c}$$

$$\textstyle\sum_{l=1}^n \mathbf{1}_n^{(l)} \psi_{\rho l}^\top \big(I - \beta\Lambda^{-1}\mathbf{1}_h\epsilon^\top\big) D_i \succeq 0, \tag{17d}$$

$$\varphi_\rho + \vartheta_\rho + \tau\mathbf{1}_n - v_2^{(\rho)} \prec 0, \tag{17e}$$

$$\begin{aligned} A_i^\top \big(\textstyle\sum_{j\in\Theta_1} \pi_{ij} v_1^{(j)} + (1 - \sum_{j\in\Theta_1} \pi_{ij}) u_1 \big) \\ + C_i^\top \big(I + \beta\epsilon\mathbf{1}_h^\top \Lambda^{-1}\big)(\psi_\rho + \phi_\rho) \\ - E_i^\top \mathbf{1}_g - v_1^{(i)} + \tau\mathbf{1}_n \prec 0, \end{aligned} \tag{17f}$$

$$\begin{aligned} B_i^\top \big(\textstyle\sum_{j\in\Theta_1} \pi_{ij} v_1^{(j)} + (1 - \sum_{j\in\Theta_1} \pi_{ij}) u_1 \big) \\ + D_i^\top \big(I + \beta\epsilon\mathbf{1}_h^\top \Lambda^{-1}\big)(\psi_\rho + \phi_\rho) \\ - F_i^\top \mathbf{1}_g - \gamma\mathbf{1}_m \prec 0, \end{aligned} \tag{17g}$$

$$\begin{aligned} v_1^{(i)} \preceq u_1, \quad v_2^{(\rho)} \preceq u_2, \quad \textstyle\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath} v_2^{(\jmath)} \\ + (1 - \textstyle\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath}) u_2 \preceq u_3, \end{aligned} \tag{17h}$$

$$0 \preceq \varphi_{\rho l} \preceq \varphi_\rho, 0 \preceq \psi_{\rho l} \preceq \psi_\rho, l = 1, 2, \cdots, n, \tag{17i}$$

$$0 \preceq \vartheta_{\rho l} \preceq \vartheta_\rho, 0 \preceq \phi_{\rho l} \preceq \phi_\rho, l = 1, 2, \cdots, g, \tag{17j}$$

hold for $i \in \mathcal{S}_1$, $\rho \in \mathcal{S}_2$, then a positive event-driven asynchronous filter can be constructed as (5) with (18) and the system (6) is positive and stochastically ℓ_1-gain stable.

Proof The positivity can be obtained by Theorem 1. Choose a linear Lyapunov function as (10). Then,

$$\begin{aligned} &\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho) \\ &\leq \widetilde{x}^\top(k) \begin{pmatrix} A_i^\top & C_i^\top (I + \beta\epsilon\mathbf{1}_h^\top \Lambda^{-1}) B_{f\rho}^\top \\ 0 & A_{f\rho}^\top \end{pmatrix} \\ &\quad \times \begin{pmatrix} v_1^{(r_{k+1})} \\ v_2^{(\delta_{k+1})} \end{pmatrix} - \widetilde{x}^\top(k) \begin{pmatrix} v_1^{(i)} \\ v_2^{(\rho)} \end{pmatrix} \\ &\quad + w^\top(k) (B_i^\top \quad D_i^\top (I + \beta\epsilon\mathbf{1}_h^\top \Lambda^{-1}) B_{f\rho}^\top) \begin{pmatrix} v_1^{(r_{k+1})} \\ v_2^{(\delta_{k+1})} \end{pmatrix}. \end{aligned} \tag{19}$$

For $r_k = i$, we have

$$\begin{aligned} &\mathcal{E}\{v_1^{(r_{k+1})} \mid r_k = i\} \\ &= \textstyle\sum_{j\in\Theta_1} \pi_{ij} (\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath} v_2^{(\jmath)} + \sum_{\jmath\in\Omega_2} \lambda_{\jmath\jmath} v_2^{(\jmath)}) \\ &\quad + \textstyle\sum_{j\in\Theta_2} \pi_{ij} (\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath} v_2^{(\jmath)} + \sum_{\jmath\in\Omega_2} \lambda_{\jmath\jmath} v_2^{(\jmath)}). \end{aligned}$$

By (17h), $\sum_{j\in\Theta_2} \pi_{ij} = 1 - \sum_{j\in\Theta_1} \pi_{ij}$, and $\sum_{\jmath\in\Omega_2} \lambda_{\jmath\jmath} = 1 - \sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath}$, it yields that

$$\begin{aligned} &\mathcal{E}\{v_1^{(r_{k+1})} \mid r_k = i\} \\ &\preceq \textstyle\sum_{j\in\Theta_1} \pi_{ij} v_1^{(j)} + (1 - \sum_{j\in\Theta_1} \pi_{ij}) u_1, \\ &\mathcal{E}\{v_2^{(\delta_{k+1})} \mid r_k = i\} \\ &\preceq \textstyle\sum_{j\in\Theta_1} \pi_{ij} (\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath} v_2^{(\jmath)} \\ &\quad + (1 - \textstyle\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath}) u_2) + (1 - \sum_{j\in\Theta_1} \pi_{ij}) u_3. \end{aligned} \tag{20}$$

Denote $\xi_1 = \sum_{j\in\Theta_1} \pi_{ij} v_1^{(j)} + (1 - \sum_{j\in\Theta_1} \pi_{ij}) u_1$ and $\xi_2 = \sum_{j=1}^N \widetilde{\pi}_j \big(\sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath} \mathbf{1}_n^\top v_2^{(\jmath)} + (1 - \sum_{\jmath\in\Omega_1} \lambda_{\jmath\jmath}) \mathbf{1}_n^\top u_2 + \mathbf{1}_n^\top u_3 \big)$. Combining (19) and (20), it derives (21). By (17i) and (17j), we obtain

$$\begin{aligned} &\frac{\sum_{l=1}^n \psi_{\rho l} \mathbf{1}_n^{(l)\top}}{\mathbf{1}_n^\top \xi_2} \xi_2 \preceq \psi_\rho + \phi_\rho - \phi_\rho \\ &\preceq \psi_\rho + \phi_\rho - \frac{\sum_{l=1}^g \phi_{\rho l} \mathbf{1}_g^{(l)\top} \mathbf{1}_g}{g}, \\ &\frac{\sum_{l=1}^n \varphi_{\rho l} \mathbf{1}_n^{(l)\top}}{\mathbf{1}_n^\top \xi_2} \xi_2 \preceq \varphi_\rho + \vartheta_\rho - \vartheta_\rho \\ &\preceq \varphi_\rho + \vartheta_\rho - \frac{\sum_{l=1}^g \vartheta_{\rho l} \mathbf{1}_g^{(l)\top} \mathbf{1}_g}{g}. \end{aligned} \tag{22}$$

Together with (17e), (17f), and (17g) follows that $\mathcal{E}\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\} \leq -\tau \widetilde{x}^\top(k)\mathbf{1}_{2n} - e^\top(k)\mathbf{1}_g + \gamma w^\top(k)\mathbf{1}_m$. The remainder of the proof can be obtained by Theorem 1 and is omitted. ■

$$A_{f\rho} = \frac{\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \varphi_{\rho l}^\top}{\sum_{j=1,\widetilde{\pi}_j \in \Theta_1}^{N} \widetilde{\pi}_j \big(\sum_{\jmath \in \Omega_1} \lambda_{j\jmath} \mathbf{1}_n^\top v_2^{(\jmath)} + (1 - \sum_{\jmath \in \Omega_1} \lambda_{j\jmath}) \mathbf{1}_n^\top u_2 \big) + \sum_{j=1,\widetilde{\pi}_j \in \Theta_2}^{N} \widetilde{\pi}_j \mathbf{1}_n^\top u_3}, \; E_{f\rho} = \frac{\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \vartheta_{\rho l}^\top}{g},$$
$$B_{f\rho} = \frac{\sum_{l=1}^{n} \mathbf{1}_n^{(l)} \psi_{\rho l}^\top}{\sum_{j=1,\widetilde{\pi}_j \in \Theta_1}^{N} \widetilde{\pi}_j \big(\sum_{\jmath \in \Omega_1} \lambda_{j\jmath} \mathbf{1}_n^\top v_2^{(\jmath)} + (1 - \sum_{\jmath \in \Omega_1} \lambda_{j\jmath}) \mathbf{1}_n^\top u_2 \big) + \sum_{j=1,\widetilde{\pi}_j \in \Theta_2}^{N} \widetilde{\pi}_j \mathbf{1}_n^\top u_3}, \; F_{f\rho} = \frac{\sum_{l=1}^{g} \mathbf{1}_g^{(l)} \phi_{\rho l}^\top}{g}. \tag{18}$$

$$\begin{aligned} \mathcal{E}\left\{\Delta V(\widetilde{x}(k), r_k = i, \delta_k = \rho)\right\} \leq & \widetilde{x}^\top(k) \begin{pmatrix} A_i^\top \xi_1 + C_i^\top (I + \beta\epsilon \mathbf{1}_h^\top \Lambda^{-1}) \frac{\sum_{l=1}^{n} \psi_{\rho l} \mathbf{1}_n^{(l)\top}}{\mathbf{1}_n^\top \xi_2} \xi_2 - v_1^{(i)} \\ \frac{\sum_{l=1}^{n} \varphi_{\rho l} \mathbf{1}_n^{(l)\top}}{\mathbf{1}_n^\top \xi_2} \xi_2 - v_2^{(\rho)} \end{pmatrix} \\ & + w^\top(k) \times \left(B_i^\top \xi_1 + D_i^\top (I + \beta\epsilon \mathbf{1}_h^\top \Lambda^{-1}) \frac{\sum_{l=1}^{n} \psi_{\rho l} \mathbf{1}_n^{(l)\top}}{\mathbf{1}_n^\top \xi_2} \xi_2 \right). \end{aligned} \tag{21}$$

IV. Illustrative example

In [27], the traffic in a circle road was modeled via a positive switched system. In the traffic systems, the traffic participants: vehicles and people entering the traffic road are uncertain, abrupt changing, and random. Therefore, it is more effective and practical to describe such a traffic problem in terms of a stochastic system. Here, system (1) is introduced for modeling the traffic problem, where there are three main roads and two branch roads at the intersection. The number of vehicles passing through each black spots is denoted as x_1, x_2, and x_3, respectively. The terms w_1 and w_2 represent the exogenous inputs which may affect the traffic conditions. The ιth row $\wp$th column element of the system matrix A_i stands for the coefficient of the number of vehicles flowing from one road to another one, the ιth row $\Im$th column element of the system matrix B_i is the coefficient of the number of vehicles from one road to one branch, C_i and D_i are the sensor coefficient matrices, and E_i and F_i are the system output matrices.

Consider system (1) with three subsystems, where

$$A_1 = \begin{pmatrix} 0.31 & 0.25 & 0.22 \\ 0.15 & 0.33 & 0.26 \\ 0.25 & 0.22 & 0.38 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 0.25 & 0.22 \\ 0.17 & 0.26 \\ 0.24 & 0.20 \end{pmatrix}, C_1 = \begin{pmatrix} 0.60 & 0.64 \\ 0.56 & 0.58 \\ 0.52 & 0.56 \end{pmatrix}^\top,$$
$$D_1 = \begin{pmatrix} 0.49 & 0.51 \\ 0.52 & 0.47 \end{pmatrix}, E_1 = \begin{pmatrix} 0.22 \\ 0.21 \\ 0.19 \end{pmatrix}^\top, F_1 = \begin{pmatrix} 0.17 \\ 0.15 \end{pmatrix}^\top,$$
$$A_2 = \begin{pmatrix} 0.32 & 0.15 & 0.23 \\ 0.25 & 0.43 & 0.26 \\ 0.25 & 0.12 & 0.32 \end{pmatrix},$$
$$B_2 = \begin{pmatrix} 0.16 & 0.16 \\ 0.20 & 0.16 \\ 0.15 & 0.17 \end{pmatrix}, C_2 = \begin{pmatrix} 0.60 & 0.59 \\ 0.65 & 0.59 \\ 0.53 & 0.60 \end{pmatrix}^\top,$$
$$D_2 = \begin{pmatrix} 0.63 & 0.63 \\ 0.60 & 0.61 \end{pmatrix}, E_2 = \begin{pmatrix} 0.20 \\ 0.21 \\ 0.21 \end{pmatrix}^\top, F_2 = \begin{pmatrix} 0.16 \\ 0.18 \end{pmatrix}^\top,$$

and

$$A_3 = \begin{pmatrix} 0.35 & 0.31 & 0.22 \\ 0.25 & 0.33 & 0.26 \\ 0.25 & 0.22 & 0.32 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0.17 & 0.21 \\ 0.15 & 0.18 \\ 0.22 & 0.22 \end{pmatrix}, C_3 = \begin{pmatrix} 0.63 & 0.71 \\ 0.71 & 0.71 \\ 0.63 & 0.61 \end{pmatrix}^\top,$$
$$D_3 = \begin{pmatrix} 0.56 & 0.63 \\ 0.60 & 0.54 \end{pmatrix}, E_3 = \begin{pmatrix} 0.19 \\ 0.21 \\ 0.22 \end{pmatrix}^\top, F_3 = \begin{pmatrix} 0.16 \\ 0.15 \end{pmatrix}^\top.$$

Choose $\beta = 0.11$, $\Lambda = \mathrm{diag}\{0.32, 0.85\}$, and the transition probability and corresponding condition probability matrices:

$$\Pi_1 = \begin{pmatrix} 0.80 & 0.10 & 0.10 \\ 0.10 & 0.70 & 0.20 \\ 0.08 & 0.12 & 0.80 \end{pmatrix}, \Pi_2 = \begin{pmatrix} 0.60 & 0.10 & 0.30 \\ 0.20 & 0.70 & 0.10 \\ 0.10 & 0.10 & 0.80 \end{pmatrix}.$$

By Theorem 1, the filter gain matrices are shown as (23) and $\gamma = 0.8077$. Fig. 1 gives the simulations of the measurable output and the filter output. Fig. 2 gives the event-triggered time instants and release intervals. Fig. 3 provides the simulation results of the measurable output and the filter output under different initial conditions.

V. Conclusion

This paper investigates the event-driven ℓ_1-gain asynchronous filter of PMJSs. Based on 1-norm, a weighted event-triggered threshold is established. The asynchronous filter framework with event-driven mechanism was constructed by transforming the error term into interval uncertainty. Moreover, an asynchronous filter is explored for PMJSs with partial information measurement. All conditions are formulated in the form of LP.

References

[1] L. Farina and S. Rinaldi, Positive Linear Systems: Theory and Applications, John Wiley and Sons 2000.

[2] W. Qi, G. Zong, Y. Hou, M. Chadli, SMC for discrete-time nonlinear semi-Markovian switching systems with partly unknown semi-Markov kernel, IEEE Transactions on Automatic Control, 2023, 68(3): 1855-1861.

$$
\begin{aligned}
A_{f1} &= \begin{pmatrix} 0.0116 & 0.1293 & 0.1281 \\ 0.1165 & 0.0116 & 0.0116 \\ 0.2330 & 0.1281 & 0.1281 \end{pmatrix}, \; B_{f1} = \begin{pmatrix} 0.0118 & 0.0202 \\ 0.1301 & 0.1385 \\ 0.2484 & 0.2568 \end{pmatrix}, E_{f1} = \begin{pmatrix} 0.0010 \\ 0.0010 \\ 0.0160 \end{pmatrix}^{\top}, F_{f1} = \begin{pmatrix} 0.0010 \\ 0.4302 \end{pmatrix}^{\top}, \\
A_{f2} &= \begin{pmatrix} 0.0119 & 0.0119 & 0.2182 \\ 0.1466 & 0.1612 & 0.1343 \\ 0.0119 & 0.0119 & 0.0119 \end{pmatrix}, B_{f2} = \begin{pmatrix} 0.0119 & 0.0204 \\ 0.1610 & 0.1695 \\ 0.3101 & 0.3185 \end{pmatrix}, E_{f2} = \begin{pmatrix} 0.0010 \\ 0.0160 \\ 0.0010 \end{pmatrix}^{\top}, F_{f2} = \begin{pmatrix} 0.0010 \\ 0.4452 \end{pmatrix}^{\top}, \\
A_{f3} &= \begin{pmatrix} 0.1377 & 0.3202 & 0.1653 \\ 0.0103 & 0.1653 & 0.0103 \\ 0.1663 & 0.0103 & 0.1684 \end{pmatrix}, \; B_{f3} = \begin{pmatrix} 0.0103 & 0.1653 \\ 0.2322 & 0.0103 \\ 0.0103 & 0.1653 \end{pmatrix}, E_{f3} = \begin{pmatrix} 0.0010 \\ 0.0310 \\ 0.0010 \end{pmatrix}^{\top}, F_{f3} = \begin{pmatrix} 0.0010 \\ 0.4602 \end{pmatrix}^{\top}.
\end{aligned}
\tag{23}
$$

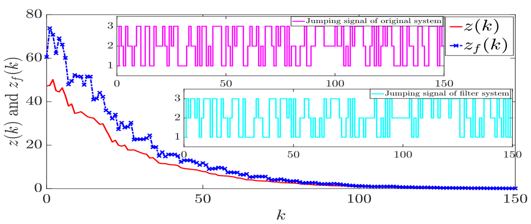

Fig. 1. The simulations of $z(k)$ and $z_f(k)$ with jumping signal

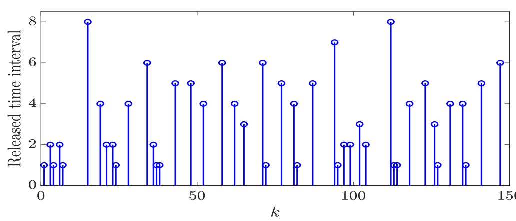

Fig. 2. The event-triggering release intervals and time instants

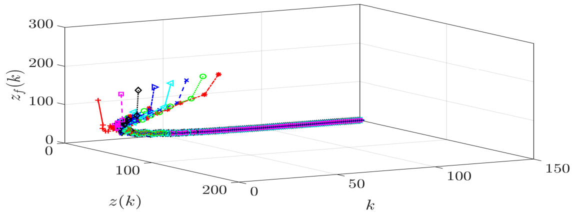

Fig. 3. The simulations of $z(k)$ and $z_f(k)$ under different initial conditions

[3] J. Cheng, Y. Wu, H. Yan, Z.G. Wu, K. Shi, Protocol-based filtering for fuzzy Markov affine systems with switching chain, Automatica, 2022, 141, 110321.

[4] G. Zong, H. Ren, Guaranteed cost finite-time control for semi-Markov jump systems with event-triggered scheme and quantization input, International Journal of Robust and Nonlinear Control, 2019, 29(15): 5251-5273.

[5] P. Bolzern, P. Colaneri, and N. G. De, Stochastic stability of positive Markov jump linear systems, Automatica, 2014, 50(4): 1181-1187.

[6] J. Zhang, G. Zheng, Y. Feng, et al., Event-triggered state-feedback and dynamic output-feedback control of PMJSs with intermittent faults, IEEE Transactions on Automatic Control, 2023, 68(2): 1039-1046.

[7] S. Zhu, Q. Han, and C. Zhang, l_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: A linear programming approach, Automatica, 2014, 50(8): 2098-2107.

[8] J. Lian, J. Liu, and Y. Zhuang, Mean stability of positive Markov jump linear systems with homogeneous and switching transition probabilities, IEEE Transactions on Circuits and Systems II: Express Briefs, 2015, 62(8): 801-805.

[9] W. Qi and X. Gao, State feedback controller design for singular positive Markovian jump systems with partly known transition rates, Applied Mathematics Letters, 2015, 46: 111-116.

[10] D. Zhang, Q. Zhang, and B. Du, L_1 fuzzy observer design for nonlinear positive Markovian jump systems, Nonlinear Analysis: Hybrid Systems, 2018, 27: 271-288.

[11] I. Park, N. Kwon, and P. Park, A linear programming approach for stabilization of positive Markovian jump systems with a saturated single input, Nonlinear Analysis: Hybrid Systems, 2018, 29: 322-332.

[12] J. Zhang, H. Yang, and T. Raïssi, Stability analysis and saturation control for nonlinear positive Markovian jump systems with randomly occurring actuator faults, International Journal of Robust and Nonlinear Control, 2020, 30(13): 5062-5100.

[13] J. Cavalcanti and H. Balakrishnan, Sign-stability of positive Markov jump linear systems, Automatica, 2020, 111: 108638.

[14] K. Yin, D. Yang, J. Liu, et al., Positive l_1-gain asynchronous filter design of positive Markov jump systems, Journal of the Franklin Institute, 2020, 357(15): 11072-11093.

[15] D. Dimarogonas, E. Frazzoli, and K. Johansson, Distributed event-triggered control for multi-agent systems, IEEE Transactions on Automatic Control, 2011. 57(5): 1291-1297.

[16] A. Girard, Dynamic triggering mechanisms for event-triggered control, IEEE Transactions on Automatic Control, 2014, 60(7): 1992-1997.

[17] W. Heemels, and M. Donkers, Model-based periodic event-triggered control for linear systems, Automatica, 2013, 49(3): 698-711.

[18] R. Postoyan, P. Tabuada, D. Nešić, et al., A framework for the event-triggered stabilization of nonlinear systems, IEEE Transactions on Automatic Control, 2014, 60(4): 982-996.

[19] J. Liu, L. Wei, J. Cao, et al., Hybrid-driven H_∞ filter design for T-S fuzzy systems with quantization, Nonlinear Analysis: Hybrid Systems, 2019, 31: 135-152.

[20] J. Wu, C. Peng, J. Zhang, et al., Guaranteed cost control of hybrid-triggered networked systems with stochastic cyber-attacks, ISA Transactions, 2020, 104: 84-92.

[21] J. Liu, Z. G. Wu, D. Yue, et al., Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2019, 51(2): 943-953.

[22] K. Shi, S. Zhong, Y. Tang, et al., Hybrid-driven finite-time H_∞ sampling synchronization control for coupling memory complex networks with stochastic cyber attacks, Neurocomputing, 2020, 387: 241-254.

[23] Y. Yin, Z. Lin, Y. Liu, et al., Event-triggered constrained control of positive systems with input saturation, International Journal of Robust and Nonlinear Control, 2018, 28(11): 3532-3542.

[24] S. Xiao, Y. Zhang, and B. Zhang, Event-triggered network-based state observer design of positive systems, Information Sciences, 2018, 469: 30-43.

[25] S. Xiao, Y. Zhang, Q. Xu, et al., Event-triggered network-based l_1-gain filtering for positive linear systems, International Journal of Systems Science, 2017, 48(6): 1281-1290.

[26] J. Zhang, T. Raïssi, Y. Shao, et al., Event-triggered filter design of positive systems with state saturation, IEEE Systems Journal, 2021, 15(3): 4281-4292.

[27] F. Blanchini, P. Colaneri, and M. Valcher, Co-positive Lyapunov functions for the stabilization of positive switched systems, IEEE Transactions on Automatic Control, 2012, 57(12): 3038-3050.