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Adaptive Coverage Control for Heterogeneous Mobile Sensor Networks in an Unknown Environment

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Abstract— **This article addresses the coverage control problems for heterogeneous mobile sensor networks (MSNs) in an environment with unknown event density functions. In contrast to existing works, unknown heterogeneous sensing abilities of the mobile sensor network (MSN) are considered by leveraging a weighted Voronoi diagram, namely, the Power diagram. To guarantee that the time-varying Power diagram converges to that defined by the true sensing weights, an online weight learning law is designed. Moreover, to handle certain applications such as forest fire investigation or nuclear radiation leakage mapping where the density information for the events of interest is not known to the MSN, an adaptive law is presented so that the event density approximation of each sensor converges to the real one along its trajectory. In addition, a move-to-centroid control law is proposed to drive the MSN to a near-optimal coverage configuration as time goes to infinity. Finally, the effectiveness of the proposed approach is illustrated by an example.**

I. INTRODUCTION

Coverage control of mobile sensor networks (MSNs) has attracted much attention in recent years due to its wide range of applications, such as 3D structure surveillance, resilient monitoring, and perimeter defense [1]–[3]. In coverage control, a group of mobile sensors is tasked to maximize the likelihood of event detection with respect to their locations. The events of interest represent certain features/phenomena in the environment, such as forest fire temperature, chemical amount, and nuclear radiation quantity. In most of these applications, the density function of the concerned event is not known to the MSNs *a priori*. Thus, considerable effort has recently been devoted to achieving a near-optimal coverage configuration in the presence of unknown event density functions. One approach uses a mixture of Gaussian processes to estimate this function [4], [5]. However, a centralized master computational unit is needed to process all collected samples. Another approach is to estimate the event density function by a linear combination of known basis functions, which is often referred to as adaptive coverage control [6]–[9]. Compared with the first approach, this one has the advantage of being fully distributed and can adapt to a large-scale MSN. It is originally proposed in [6] for sensors with single-integrator dynamics. It is then extended to vehicles with unicycle dynamics in [7]. In [8], adaptive coverage laws are proposed for differential-drive vehicles with not fully known dynamics. It is assumed in [6]–[8] that the event density function can be exactly modeled by a linear combination of basis functions. However, in practice, such an assumption can be violated. For example, the event of interest may be too complex to be modeled exactly by a finite set of basis functions. In this case, a new adaptive law based on the dead-zone function is proposed in [9] to improve the robustness of the MSN.

Another typical assumption in most existing works on coverage control is that each sensor has identical sensing abilities. In practice, there exist different kinds of heterogeneity in sensors, such as service costs [10], [11], footprint size differences [12], energy constraints [13], actuation variations [14], or sensing performances [15]. In [10], [11], multiplicative weighted Voronoi diagrams are proposed to handle different service costs for mobile sensors with integrator and nonholonomic dynamics. In [12], Power diagrams are used to tackle sensor footprint differences. Some results mentioned above assume that the sensing differences in the sensors are previously known to the MSNs [10]–[12]. In real world applications, sensing abilities can degrade during long-term deployment, which can lead to uncertainties of these differences in the MSNs. Hence, it is important for MSNs to learn these differences in an adaptive manner [13]–[15]. In [13], a weight learning law is designed to handle different energy constraints for integrator-type sensors. In [14], actuation heterogeneity is considered with the coverage control problem. In [15], sensing performance degradation is estimated for team-based multiple MSNs. However, few results have been reported to simultaneously consider the MSN's sensing heterogeneity and the event density uncertainty for a coverage control problem.

Motivated by the above-mentioned observations, this work investigates a coverage control problem for heterogeneous mobile sensor networks in an unknown environment. The main contributions of this work can be summarized as follows. First, compared with [10]–[12], where the MSN is assumed to have prior knowledge of their relative sensing abilities, this work relaxes this assumption and an online weight learning law is designed. Second, unlike previous results [6]–[9], where no heterogeneity among sensors is considered, we propose a new adaptive law to compensate for the influence of the time-varying learning weights. Third, a new framework is proposed to deal with heterogeneous sensing abilities and unknown density functions simultaneously. In this framework, the weight learning process is integrated into the basis function approximation scheme so that a move-to-centroid control law can drive the MSN to a near-optimal

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coverage configuration.

Notations:$\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the space of all real n-dimensional vectors and the space of all real $m\times n$ matrices, respectively. For a vector $x\in\mathbb{R}^2$, $||x||$ denotes its Euclidean distance. $\mathbf{0}_n$ and $\mathbf{1}_n\in\mathbb{R}^n$ represent vectors with all their elements being 0 and 1, respectively. $I_m\in\mathbb{R}^{m\times m}$ is the identity matrix. For a matrix $M\in\mathbb{R}^{m\times n}$, M^{T} denotes its transpose. $\mathrm{Diag}\{k_1,\ldots,k_N\}\in\mathbb{R}^{N\times N}$ represents a diagonal matrix, whose diagonal elements are $k_i\in\mathbb{R},\ i=1,\ldots,N$. The symbol $\otimes$ represents the Kronecker product.

The remainder of this article is organized as follows. The preliminaries and problem formulation are introduced in Section II. A weight learning law, a distributed controller, an adaptive law, and analysis of the closed-loop system are presented in Section III. A simulation example is provided in Section IV. Finally, some conclusions are stated in Section V.

II. Preliminaries and Problem Formulation

A. Graph theory

Let $\mathcal{G}^w=\{\mathcal{V},\mathcal{W}\}$ define an undirected weight-balanced communication graph of the MSN, where $\mathcal{V}=\{1,\ldots,N\}$ is a set of nodes that represent the set of N sensors in the network and $\mathcal{W}$ is a symmetric matrix called the weight matrix, such that $w_{ij}\geq 0$ for all $i,j\in\mathcal{V}$, and $w_{ii}=0$ for $i\in\mathcal{V}$. We say that a set (i,j) is an edge if and only if $w_{ij}>0$. The Laplacian matrix L^w of the weighted graph $\mathcal{G}^w$ is a $N\times N$ matrix whose element is defined as follows,

$$L_{ij}^w=\begin{cases}-w_{ij}, & \text{if } i\neq j \text{ and } w_{ij}>0,\\ \sum\limits_{s\in\mathcal{V}\setminus i} w_{is}, & \text{if } i=j,\\ 0, & \text{otherwise.}\end{cases} \quad (1)$$

Laplacian matrix L^w is positive semi-definite and its simple eigenvalue 0 is associated with the eigenvector $c\mathbf{1}_N,\forall c\in\mathbb{R}$.

B. Problem formulation

Consider a group of N sensors moving in a mission space $Q\in\mathbb{R}^2$. Assume that each sensor has first-order dynamics described by

$$\dot{p}_i(t)=u_i(t), \quad (2)$$

where $p_i(t)\in\mathbb{R}^2$ denotes the position of the sensor and u_i denotes its control input. The position of any point in Q is denoted by $q\in\mathbb{R}^2$. The mission space contains an event of interest, whose distribution can be modeled by a density function $\phi(q)$. Suppose that the MSN has no prior information of the density function. Let $K:Q\mapsto\mathbb{R}_+^m$ be a vector of bounded, continuous basis functions that are available to the MSN. A basis function approximation scheme is adopted by each sensor to estimate the density function, that is,

$$\hat{\phi}_i(q)=K(q)^{\mathrm{T}}\hat{a}_i(t), \quad (3)$$

where $\hat{a}_i(t)\in\mathbb{R}^m$ is the parameter vector that needs to be updated in an adaptive manner. To facilitate the analysis for density function estimation, the following standard assumption is introduced [6]–[9].

Assumption 1: There exists an ideal parameter vector $a\in\mathbb{R}^m$ such that

$$\phi(q)=K(q)^{\mathrm{T}}a, \quad (4)$$

where a is unknown to the MSN. Moreover,

$$a(j)\geq a_{\min},\forall j=1,\ldots,m, \quad (5)$$

where $a(j)$ denotes the j^{th} element of the vector a, and $a_{\min}\in\mathbb{R}_{>0}$ is a positive lower bound known to the MSN. Under this assumption, the estimation error of the event density function is defined as

$$\tilde{\phi}_i(q)=\hat{\phi}_i(q)-\phi(q)=K(q)^{\mathrm{T}}\hat{a}_i(t)-K(q)^{\mathrm{T}}a=K(q)^{\mathrm{T}}\tilde{a}_i(t), \quad (6)$$

with $\tilde{a}_i(t):=\hat{a}_i(t)-a$, which denotes the estimation error of the parameter vector. The choices of basis functions in Assumption 1 vary from Gaussian basis functions to other local basis functions, such as the wavelets, sigmoids, and splines. In the simulation part, we choose a set of Gaussian basis functions, which is a common choice for a coverage control problem [6]–[9].

The heterogeneous sensing abilities of the MSN are also considered and are handled by a Power diagram. Specifically, suppose that the sensing ability is described as follows,

$$\gamma_i(q,p_i(t))=-(||q-p_i(t)||^2-\alpha_i), \quad (7)$$

where $\alpha_i\in\mathbb{R}_+$ defines the sensing weight of sensor i. Note that Euclidean distance is a commonly used metric for coverage control problems under consideration of a convex environment. Intuitively, (7) reflects the phenomenon that as the distance of a point to a sensor increases and as the value of the sensing weight decreases, the value of γ_i decreases. The resulting Power diagram is now defined as follows,

$$V_i^\alpha=\{q\in Q:\gamma_i(q,p_i(t))\geq\gamma_j(q,p_j(t)),\forall j\in\mathcal{V}\setminus\{i\}\}, \quad (8)$$

which implies that environmental points will be partitioned to a sensor with a larger value of the sensing function γ_i.

Since the MSN has no prior information of the relative sensing weights in this work, it cannot partition the environment using the real sensing weights in (7). To this end, sensors are assigned some random initial weights and the corresponding Power diagram is defined as

$$V_i=\{q\in Q:||q-p_i(t)||^2-w_i(t)\leq||q-p_j(t)||^2-w_j(t),\ \forall j\in\mathcal{V}\setminus\{i\}\}, \quad (9)$$

where $w_i(t)$ and $w_j(t)$ represent the estimated sensing weights of sensor i and sensor j, respectively. The estimation errors of the sensing weights are defined by

$$e_i(t)=w_i(t)-\alpha_i,\forall i\in\mathcal{V}. \quad (10)$$

To facilitate the following analysis, some definitions are introduced as follows. The estimated mass, first moment, and centroid of each Power cell V_i are defined as

$$\hat{M}_{V_i}=\int_{V_i}\hat{\phi}(q)dq,\ \hat{L}_{V_i}=\int_{V_i}q\hat{\phi}(q)dq,\ \hat{C}_{V_i}=\frac{\hat{L}_{V_i}}{\hat{M}_{V_i}}, \quad (11)$$

respectively. Furthermore, the following definitions of a near-optimal coverage configuration and locally true density function approximation are adapted for this work from [6].

Definition 1 (Near-optimal coverage configuration):

An MSN is said to be in a near-optimal coverage configuration if each sensor is positioned at the estimated centroid of its Power cell, $p_i(t) = \hat{C}_{V_i}, \forall i \in \mathcal{V}$.

Definition 2 (Locally true density approximation):

A sensor i is said to have a locally true approximation of the density function over a subset $\Omega_i := \{p_i(\tau) \in Q : \tau \geq 0\}$, if its approximation is equal to the true density function at every point in the subset, $\hat{\phi}_i(p_i(\tau)) = \phi(p_i(\tau)), \forall p_i(\tau) \in \Omega_i$.

Now, the coverage control problem considered in this work can be formally stated as follows.

Problem 1: Consider an adaptive coverage control problem by a group of N heterogeneous sensors with unknown sensing abilities $[\gamma_1, \ldots, \gamma_N]$ in an environment $Q \in \mathbb{R}^2$, where an event density function is modeled by $\phi(q) : Q \mapsto \mathbb{R}_+$, design a weight learning law, a distributed control law and an adaptive law such that the following control objectives can be achieved.

i. The estimation errors of the sensing weights among the sensors can reach consensus, that is, $\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} e_j(t), \forall i, j \in \mathcal{V}$.
ii. The MSN converges to a near-optimal coverage configuration, that is, $\lim_{t\to\infty} ||p_i(t) - \hat{C}_{V_i}|| = 0, \forall i \in \mathcal{V}$.
iii. Each sensor converges to a locally true density approximation over the set $\Omega_i = \{p_i(\tau) \in Q : 0 \leq \tau \leq t\}$, made up of all points on the sensor's trajectory, that is, $\lim_{t\to\infty} ||\hat{\phi}_i(q, t) - \phi(q)||^2 = 0,\ \forall q \in \Omega_i,\ \forall i \in \mathcal{V}$.

III. Main Results

We begin this section by establishing an online weight learning law such that the estimation errors of the sensing weights among the sensors can reach consensus. Next, a distributed control law and an adaptive law are proposed so that each sensor in the MSN converges to a locally true density function approximation over its trajectory while reaching a near-optimal coverage configuration. Finally, we present an analysis of convergence properties for the concerned MSN. For the simplicity of notations, the time index t is omitted in the following analysis.

A. Online weight learning law

When the MSN has no prior information of the relative sensing weights, a weight learning law is proposed as follows,

$$\dot{w}_i = \frac{k_w}{\hat{M}_{V_i}} \sum_{j\in\mathcal{N}_i} \int_{q\in\partial V_{ij}} (\gamma_i(q, p_i) - \gamma_j(q, p_j))dq, \tag{12}$$

where $k_w > 0$ is the weight learning gain, $\mathcal{N}_i$ represents the neighbor set of sensor i, $q \in \partial V_{ij}$ represents all points located on the Power cell boundary between neighboring sensors i and j, and $\hat{M}_{V_i}$ is the estimated mass of Power cell V_i, defined in (11). An intuitive interpretation of this design in (12) is that the estimated weight of sensor i should be updated in a gradient-descent way such that the sensing differences at the Power cell boundaries among neighboring sensors are equal to zero.

Lemma 1: Under the proposed weight learning law (12), the estimation errors of the sensing weights among MSN can reach consensus, that is,

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} e_j, \forall i \neq j. \tag{13}$$

Proof: This lemma can be proved by using the properties of stable linear filter theory. First, observe that for any $q \in \partial V_{ij}$, the following relationship holds,

$$||q - p_j||^2 - ||q - p_i||^2 = w_j - w_i.$$

Then, substituting the sensing model (7) into the designed weight learning law (12) yields

$$\dot{w}_i = -\frac{k_w}{\hat{M}_{V_i}} \sum_{j\in\mathcal{N}_i} \int_{q\in\partial V_{ij}} [(w_i - \alpha_i) - (w_j - \alpha_j)]\, dq. \tag{14}$$

Rewrite (14) in terms of the estimation error e_i, one has

$$\dot{e}_i = -\frac{k_w}{\hat{M}_{V_i}} \sum_{j\in\mathcal{N}_i} l_{ij}(e_i - e_j), \tag{15}$$

where $l_{ij} = \int_{q\in\partial V_{ij}} dq$ denotes the length of the boundary ∂V_{ij}. Define $\boldsymbol{e} = [e_1, \ldots, e_N]^{\mathrm{T}}$, then the compact form of (15) can be obtained as

$$\dot{\boldsymbol{e}} = -ML^w\boldsymbol{e}, \tag{16}$$

where $M = \mathrm{Diag}\{\frac{k_w}{\hat{M}_{V_1}}, \ldots, \frac{k_w}{\hat{M}_{V_N}}\}$, and L^w is a positive semi-definite weighted Laplacian matrix whose element is defined by

$$L^w_{ij} = \begin{cases} -l_{ij}, & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i, \\ \sum_{s\in\mathcal{N}_i} l_{is}, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

It can be proved that the product ML^w has non-negative eigenvalues. Then, by properties of stable linear filter theory, $\lim_{t\to\infty} L^w\mathbf{e} = \mathbf{0}_N$. Furthermore, since L^w is a Laplacian matrix and zero is its simple eigenvalue, one has $\boldsymbol{e} = c_1\mathbf{1}_N, \forall c_1 \in \mathbb{R}$. Therefore, $\lim_{t\to\infty} e_i = \lim_{t\to\infty} e_j$. ■

B. Distributed control law and adaptive law

The control law is designed to drive each sensor to the estimated centroid of its Power cell as follows,

$$\dot{p}_i = k_c(\hat{C}_{V_i} - p_i), \tag{18}$$

where $k_c > 0$ is the control gain and $\hat{C}_{V_i}$ is the estimated centroid of Power cell V_i, defined in (11).

Before presenting the adaptive law, some quantities are introduced as follows,

$$\Lambda_i(t) = \int_{q\in\Omega_i(t)} K(q)K(q)^{\mathrm{T}} dq \ \in \mathbb{R}^{m\times m}, \tag{19a}$$

$$\lambda_i(t) = \int_{q\in\Omega_i(t)} K(q)\phi(q) dq \ \in \mathbb{R}^{m\times 1}, \tag{19b}$$

$$F_i = \frac{\int_{V_i} K(q)(q-p_i)^{\mathrm{T}} dq \int_{V_i}(q-p_i)K(q)^{\mathrm{T}} dq}{\hat{M}_{V_i}} \ \in \mathbb{R}^{m\times m}, \tag{19c}$$

$$S_i = \frac{\int_{V_i} K(q)dq}{\hat{M}_{V_i}} \ \in \mathbb{R}^{m\times 1}. \tag{19d}$$

It can be observed from (19) that each sensor is able to compute these quantities along its trajectory without knowing the true parameter a. Inspired by the approach in [6], we propose the following adaptive law to update the estimated parameter $\hat{a}_i$.

$$\begin{aligned}\dot{\hat{a}}_{\mathrm{pre}_i} = &- k_c F_i \hat{a}_i + \frac{k_w}{2} S_i \cdot \sum_{j\in\mathcal{N}_i} \int_{\partial V_{ij}} (\gamma_i(q,p_i) - \gamma_j(q,p_j))dq \\ &- \gamma(\Lambda_i \hat{a}_i - \lambda_i) - \zeta \sum_{j\in\mathcal{N}_i} l_{ij}(\hat{a}_i - \hat{a}_j),\end{aligned} \tag{20a}$$

$$\dot{\hat{a}}_i = \Gamma^{-1}(\dot{\hat{a}}_{\mathrm{pre}_i} - I_{\mathrm{proj}_i}\dot{\hat{a}}_{\mathrm{pre}_i}), \tag{20b}$$

where $k_c > 0$ and $k_w > 0$ are the control gain and the weight learning gain, respectively; $\gamma > 0$ and $\zeta > 0$ represent the adaptive gain and the consensus gain of the estimated parameter $\hat{a}_i$, respectively, $\Gamma \in \mathbb{R}^{m\times m}$ is a diagonal positive-definite matrix, and $I_{\mathrm{proj}_i} \in \mathbb{R}^{m\times m}$ denotes a diagonal matrix defined as in [6].The adaptive law in (20) contains two parts. The first part (20a) is often referred to as the main adaptive law. Specifically, the first two terms in (20a) compensate for uncertainties in the centroid estimation and the Power cell weight learning, respectively; the third term carries out a gradient descent for density function approximation error $\tilde{\phi}_i$ accumulated along the trajectory of each sensor; and the last consensus term enables neighboring sensors to exchange their estimated parameters $\hat{a}_i$ to one another. The second part (20b) is often called the projection law, which eliminates the singularity point of $\hat{C}_{V_i}$ at $\hat{a}_i = 0$ by preventing it from dropping below $a_{\min}$.

C. Convergence analysis of the MSN

Consider a candidate Lyapunov function as follows,

$$\begin{aligned}V &= H_1(w_i,p_i) + H_2(\tilde{a}_i) \\ &= \sum_{i=1}^{N} \int_{V_i} \frac{1}{2}(\|q-p_i\|^2 - w_i)\phi_i(q)dq + \sum_{i=1}^{N} \frac{1}{2}\tilde{a}_i^{\mathrm{T}}\Gamma\tilde{a}_i,\end{aligned} \tag{21}$$

where $H_1(w_i,p_i)$ is a standard locational cost function in the coverage control problem with heterogeneous MSNs, and $H_2(\tilde{a}_i)$ is added to analyze the adaptive law for density function estimation.

Now, we are ready to give the main result of this work.

Theorem 1: Under Assumption 1, the MSN (2) under the weight learning law (12), the distributed controller (18), and the adaptive law (20) will converge to a near-optimal coverage configuration, that is,

$$\lim_{t\to\infty} ||p_i(t) - \hat{C}_{V_i}|| = 0, \forall i \in \mathcal{V}. \tag{22}$$

Meanwhile, each sensor converges to a locally true density approximation over the set $\Omega_i = \{p_i(\tau) \in Q : 0 \le \tau \le t\}$, made up of all points on the sensor's trajectory, that is,

$$\lim_{t\to\infty} ||\hat{\phi}_i(q,t) - \phi(q)||^2 = 0, \ \forall q \in \Omega_i, \ \forall i \in \mathcal{V}. \tag{23}$$

Furthermore, the estimated parameter vector $\hat{a}_i$ for all $i \in \mathcal{V}$ by the MSN reaches consensus, that is, $\lim_{t\to\infty} \hat{a}_i(t) = \lim_{t\to\infty} \hat{a}_j(t), \forall i,j \in \mathcal{V}$.

Proof: First, we show that the time derivative of the candidate Lyapunov function (21) satisfies that $\dot{V} \le 0$. Taking the time derivative of V yields

$$\begin{aligned}\dot{V} &= \dot{H}_1(w_i,p_i) + \dot{H}_2(\tilde{a}_i) \\ &= \sum_{i=1}^{N}\left(\frac{\partial H_1}{\partial w_i}\dot{w}_i + \frac{\partial H_1}{\partial p_i}\dot{p}_i + \frac{dH_2}{d\tilde{a}_i}\dot{\tilde{a}}_i\right).\end{aligned} \tag{24}$$

The terms $\frac{\partial H_1}{\partial w_i}\dot{w}_i$ and $\frac{\partial H_1}{\partial p_i}\dot{p}_i$ can be computed using the proposed weight learning law (12) and the control law (18) as follows,

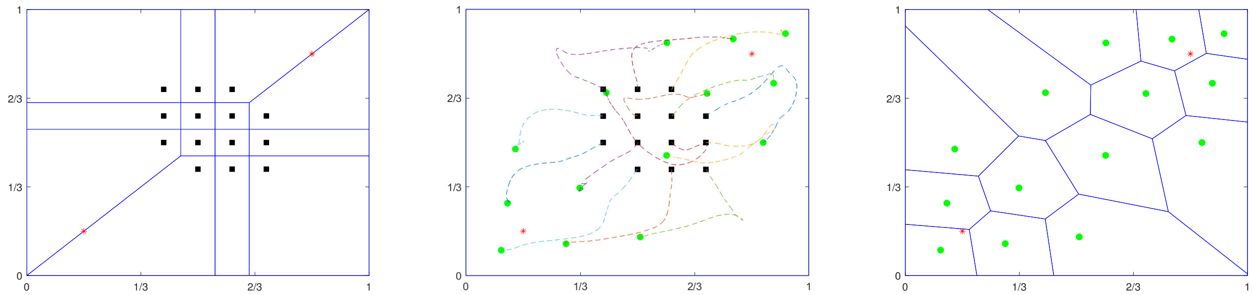
$$\begin{aligned}\frac{\partial H_1}{\partial w_i}\dot{w}_i &= -\frac{1}{2}\int_{V_i}\hat{\phi}_i(q)dq\dot{w}_i + \frac{1}{2}\int_{V_i}\tilde{\phi}_i(q)dq\dot{w}_i \\ &= \frac{k_w}{2}\sum_{j\in\mathcal{N}_i} l_{ij}(e_i - e_j) - \frac{k_w}{2}\tilde{a}_i^{\mathrm{T}} S_i \sum_{j\in\mathcal{N}_i} l_{ij}(e_i - e_j),\end{aligned} \tag{25a}$$

$$\begin{aligned}\frac{\partial H_1}{\partial p_i}\dot{p}_i &= -\int_{V_i}(q-p_i)^{\mathrm{T}}\hat{\phi}_i(q)dq\dot{p}_i + \int_{V_i}(q-p_i)^{\mathrm{T}}\tilde{\phi}_i(q)dq\dot{p}_i \\ &= -k_c\hat{M}_{V_i}||\hat{C}_{V_i} - p_i||^2 + k_c\tilde{a}_i^{\mathrm{T}}F_i\hat{a}_i.\end{aligned} \tag{25b}$$

Substituting the partial derivatives obtained in (25) and the adaptive law (20) into (24) results in

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{N}\left[\tfrac{k_w}{2}\sum_{j\in\mathcal{N}_i}(e_i - e_j)l_{ij} - \tfrac{k_w}{2}\tilde{a}_i^{\mathrm{T}}S_i\sum_{j\in\mathcal{N}_i}(e_i-e_j)l_{ij}\right] \\ &+ \sum_{i=1}^{N}(-k_c\hat{M}_{V_i}||\hat{C}_{V_i} - p_i||^2) + \sum_{i=1}^{N}\left[k_c\tilde{a}_i^{\mathrm{T}}F_i\hat{a}_i + \tilde{a}_i^{\mathrm{T}}\Gamma\dot{\hat{a}}_i\right] \\ =& \sum_{i=1}^{N}\left[-\frac{k_w}{2}\tilde{a}_i^{\mathrm{T}}S_i\sum_{j\in\mathcal{N}_i}(e_i-e_j)l_{ij} - k_c\hat{M}_{V_i}||\hat{C}_{V_i} - p_i||^2\right] \\ &+ \sum_{i=1}^{N}\left[k_c\tilde{a}_i^{\mathrm{T}}F_i\hat{a}_i + \tilde{a}_i^{\mathrm{T}}\Gamma\dot{\hat{a}}_i\right] \\ =& \sum_{i=1}^{N}\Bigg[-k_c\hat{M}_{V_i}||\hat{C}_{V_i} - p_i||^2 - \gamma\int_{q\in\Omega_i}||K(q)^{\mathrm{T}}\tilde{a}_i||^2 dq \\ &- \zeta\tilde{a}_i^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} l_{ij}(\tilde{a}_i - \tilde{a}_j) - \tilde{a}_i^{\mathrm{T}}I_{\mathrm{proj}_i}\dot{\hat{a}}_{\mathrm{pre}_i}\Bigg].\end{aligned} \tag{26}$$

where the second equality holds because the underlying communication topology is undirected weight-balanced.



(a) Initial configuration of the MSN.

(b) Trajectories of the MSN.

(c) Final configuration of the MSN.

Fig. 1: Evolution of the MSN at initial and final time instant.

Define $\tilde{A} = \left[\tilde{a}_1^{\mathrm{T}}, \ldots, \tilde{a}_N^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{mN}$, then one can rewrite the term $\sum_{i=1}^{N} \tilde{a}_i^{\mathrm{T}} \sum_{j\in\mathcal{N}_i} l_{ij}(\tilde{a}_i - \tilde{a}_j)$ in (26) as $\tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A}$. Furthermore, it can be verified that each element in the last term of (26), that is, $\tilde{a}_i^{\mathrm{T}}(s) I_{\mathrm{proj}_i}(s) \dot{\hat{a}}_{\mathrm{pre}_i}(s)$, is non-negative under the projection law (20b). Therefore, the following inequalities hold,

$$\dot{V} \leq \sum_{i=1}^{N} \left[-k_c \hat{M}_{V_i} ||\hat{C}_{V_i} - p_i||^2 - \gamma \int_{q\in\Omega_i} ||K(q)^{\mathrm{T}} \tilde{a}_i||^2 dq \right] - \tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A} \leq 0. \tag{27}$$

Note that $\tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A} \geq 0$ due to that matrix L^w is positive semi-definite. Then, by La Salle's Invariant Set theory, one can conclude that $\lim_{t\to\infty} \dot{V} = 0$, which further implies that (22), (23) hold, and $\lim_{t\to\infty} \tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A} = 0$. It is known that 0 is a simple eigenvalue of matrix L^w and the corresponding eigenvector is $c_2 \mathbf{1}_N, \forall c_2 \in \mathbb{R}$. Then, one can obtain that $\lim_{t\to\infty} \hat{a}_i(t) = \lim_{t\to\infty} \hat{a}_j(t),\ i, j \in \mathcal{V}$. ■

IV. Simulation Results

In this section, simulations are conducted to illustrate that the proposed approach can solve the coverage control problem for a group of mobile sensors with heterogeneous sensing abilities in an unknown environment.

Consider a group of $N = 14$ robots with single integrator dynamics deployed in a unit square mission space, which is divided into 3×3 grids. The coordinate origin is located at the bottom left corner of the unit square. The unknown event density function is parameterized according to (4), where the true parameter vector is chosen to be $a = [100, a_{\min}, \ldots, a_{\min}, 100]^{\mathrm{T}}$ with $a_{\min} = 0.1$. In this work, the basis functions $K(q) = [K_1(q), \cdots, K_9(q)]^{\mathrm{T}}$ are chosen to be nine radially symmetric Gaussian functions, whose element is defined as $K_j(q) = \frac{1}{2\pi\sigma^2} \exp\{-\frac{\|q-\mu_j\|^2}{2\sigma^2}\}$, with a fixed width $\sigma = 0.18$ and a fixed center μ_j being the center position of each grid. The estimated parameter vectors $\hat{a}_i$ are initialized as $\hat{a}_i = a_{\min} \mathbf{1}_9$ for all 14 sensors. In this work, it is assumed that each sensor has no prior knowledge of other sensors' sensing abilities. The sensing weights α_i that represent the heterogeneity among the sensors are given by a set of random numbers drawn from the uniform distribution over $[0, 0.01]$. The estimated sensing weights w_i used to generate the Power cells are initialized as 0.1 for all sensors.

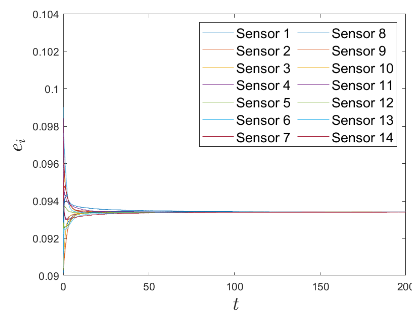
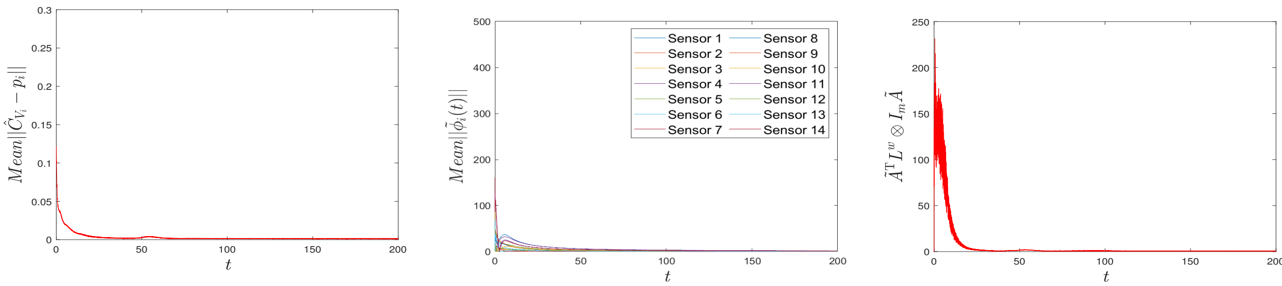


Fig. 2: Time responses of the learning weight errors.

The above-mentioned values and coordinate frame are illustrated in Fig. 1(a). The two Gaussian peaks of the density function are shown in red stars. The sensors are initially positioned at the middle part of the mission space, represented by the black squares. The initial Power diagram that is generated using the positions of the sensors and the weights at $t = 0$ is illustrated by the straight lines in Fig. 1(a). The weight learning gain k_w is selected to be a large value that equals 850000 due to that the denominator term $\hat{M}_{V_i}$ in (12) tends to be a large value during the evolution of the MSN, which results in a very slow convergence speed for the weight learning process. The control gain k_c, the adaptive gain γ, the consensus gain ζ, and matrix Γ are chosen to be 1, 10, and 18, and I_9, respectively. The adaptive gain and the consensus gain are selected by trial and error in this simulation example.

Figs. 1(b) and 1(c) shows the trajectories and the final configuration of the MSN, respectively. In Fig. 1(b), the black squares and green circles correspond to the initial positions and the final positions of the MSN, respectively. It can be observed that more sensors are driven to the two Gaussian peaks under the proposed move-to-centroid controller in Fig. 1(c).

The time responses of the sensing weights errors for all sensors are shown in Fig. 2. It can be seen that the errors



(a) Time response of the distance of each sensor's current position to its estimated centroid averaged over all sensors.

(b) Time responses of local event density function approximation errors.

(c) Consensus of estimated parameters $\hat{a}_i$ among sensors.

Fig. 3: Time responses of the MSN to assert Theorem 1.

$e_i = w_i - \alpha_i$ of all sensors converge to the same value of 0.0934 as time goes to infinity, which implies that the weight learning errors among the MSN achieve consensus under the proposed weight learning law (12).

The effectiveness of the decentralized control law (18) and the adaptive law (20) are also verified. Fig. 3(a) illustrates the time response of the distance of each sensor's current position p_i to its estimated centroid $\hat{C}_{V_i}$ averaged over all sensors, that is, $Mean\|\hat{C}_{V_i} - p_i\| = \frac{1}{14}\sum_{i=1}^{14}\|\hat{C}_{V_i} - p_i\|$. It can be observed that as time goes to infinity, this value converges to zero, which implies that each sensor arrives at the estimated centroid of its Power cell. Therefore, the MSN achieves a near-optimal coverage configuration as time goes to infinity, as asserted in Theorem 1. Fig. 3(b) shows that the density function estimation error averaged along the trajectory Ω_i of each sensor converges to zero, which implies that $\lim_{t\to\infty} ||\hat{\phi}_i(q,t) - \phi(q)||^2 = 0,\ \forall q \in \Omega_i,\ \forall i \in \mathcal{V}$. Finally, the consensus of the estimated parameter $\hat{a}_i$ among the sensors are evaluated by the time response of the value $\tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A}$, shown in Fig. 3(c). It can be observed that $\lim_{t\to\infty} \tilde{A}^{\mathrm{T}} L^w \otimes I_m \tilde{A} = 0$, which implies that the estimated parameter $\hat{a}_i$ by the MSN achieves consensus, that is, $\lim_{t\to\infty} \hat{a}_i(t) = \lim_{t\to\infty} \hat{a}_j(t), \forall i,j \in \mathcal{V}$.

V. Conclusions

In this article, the coverage control problem for a group of mobile sensors subject to heterogeneous sensing abilities in an unknown environment has been investigated. A new framework that can handle heterogeneous sensing weight learning and unknown event density function estimation simultaneously has been designed. It has been shown that the MSN will achieve a near-optimal coverage configuration and that each sensor converges to a locally true density function approximation along its trajectory as time goes to infinity under the proposed approach. In the future, it would be interesting to extend current results to achieve optimal coverage configurations with an exponential rate.

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