2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Optimal orientation for automated vehicles on large lane-free roundabouts *

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***Abstract*— Path planning for vehicles on large, complex, lane-free roundabouts is challenging due to the geometrical features and frequent conflicts among entering, navigating, and exiting vehicles. A key difficulty is to properly determine the desired vehicle orientations on the roundabout so that vehicles enter the roundabout and move towards their corresponding exits smoothly and safely. Specification of vehicle orientations should consider the resulting trip distance, the angle difference from other vehicles, and the exploitation of the available roundabout surface for efficient traffic flow. This paper proposes an optimal control approach to determine optimal vehicle orientations at each point on the roundabout, in dependence of the exit branch, so as to minimize a weighted sum of the trip distance and the deviation from the circular motion. Analytical solutions for two extreme cases, addressing only the shortest path or only the minimum deviation from the circular angle, respectively, are derived. For the general weighted problem solution, a Dynamic Programming-based (backward Dijkstra) algorithm is employed to deliver the optimal orientations in a 2-D space-discretized grid of the roundabout surface. In the light of the optimal solution, a computationally light near-optimal approach is also proposed. As a challenging case study, the methods are applied to the famous roundabout of Place Charles de Gaulle in Paris, which features a road width of 38 m and comprises 12 bidirectional radial streets, hence a total of 144 origin-destination movements for the vehicles.**

I. Introduction

To tackle traffic congestion and its consequences, like excessive delays, environmental pollution, and reduced traffic safety, traffic control of various kinds [1, 2] has been developed for decades. More recently, the development of a variety of Vehicle Automation and Communication Systems (VACS) that significantly improve vehicles' individual capabilities, have been considered in a new generation of traffic management tools [3, 4]. This trend continues with the development of high-automation or virtually driverless vehicles that are tested in real traffic conditions, see e.g. [5]. In the not-too-far future, vehicles may communicate with each other and with the infrastructure; and drive automatically, based on own sensors, communications, and appropriate movement control strategies.

Recently, the TrafficFluid concept was proposed [6], which is a novel paradigm for vehicular traffic, applicable at high levels of vehicle automation and communication. The TrafficFluid concept is based on two combined principles: (a) Lane-free traffic, whereby vehicles are not bound to fixed traffic lanes, as in conventional traffic, but may drive anywhere on the 2-D surface of the road; and (b) Vehicle nudging, whereby vehicles communicate their presence to other vehicles in front of them (or are sensed by them), and this may influence the movement of vehicles in front. Over the last couple of years, a number of movement strategies for automated vehicles on lane-free highways were developed, in accordance with the TrafficFluid paradigm, using different methodologies, such as: ad-hoc strategies [6], optimal model predictive control [9], reinforcement learning [10], nonlinear feedback control [11]; and a generic simulation environment for lane-free traffic has also been developed [12]; see [13] for a brief review. Most of these strategies require availability of a desired vehicle orientation that determines the local vehicle movement direction if no collision-avoidance maneuver is required.

Roundabout is a key element in urban traffic which improves traffic efficiency in light traffic conditions [14]; but may become a bottleneck point in higher demands. Hence, efficient operation of roundabouts, which is indeed considered challenging because of the geometric complexities, can enhance traffic in its surrounding area. Several works in the literature focus on controlling automated vehicles on roundabouts, most of which addressing simple infrastructures. Specifically, a noticeable number of works consider single-lane roundabouts using various control approaches. Some research [15-17] suggests priority management approaches, whereby a suitable policy, like "First-Come-First-Served", is utilized to assign the priorities to vehicles. If two vehicles have

* The research leading to these results has received funding from the European Research Council under the European Union's Horizon 2020 Research and Innovation Programme / ERC Grant Agreement no. 833915, project TrafficFluid, see: https://www.trafficfluid.tuc.gr

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979-8-3503-0123-6/23/$31.00 ©2023 IEEE

a conflict in the roundabout, the vehicle with a lower priority should stop or decrease its speed to let the higher-priority vehicle pass. Also, several presented methodologies [18-23] formulate the vehicles' movements, either for complete navigation or only for the merging part, on roundabouts as an optimal model predictive control problem aiming to minimize different criteria, including travel time, fuel consumption, and distance from the destination. Furthermore, a hierarchical structure is proposed in [24], to determine the optimum roundabout inflow and guarantee vehicles' safety.

Other works propose control approaches for two-lane roundabouts. In [25], two fuzzy controllers, designed based on real data, drivers' knowledge, and common reasoning, are employed to control steering angle and angular speed for a two-lane roundabout. Also, [26 - 28] combine optimal control and game theory to make decisions at the merging points or change lanes on the roundabout. An optimization embedded reinforcement learning method is suggested in [29] to make lane changing decisions at a four-lane roundabout.

A lane-free roundabout was first addressed in [30], where we presented a comprehensive control strategy for vehicles on the basis of the bicycle model for vehicle dynamics. A nonlinear controller, which had been developed in [11] and guarantees several features for straight lane-free roads, including collision and boundary-violation avoidance, desired speed tracking, and convergence of acceleration and orientation to zero, was modified to appropriately control vehicles in the roundabout. Since the modified controller requires a desired vehicle orientation, we proposed in [30] a heuristic approach to determine desired orientations to be fed to the nonlinear controller. The approach was applied to the overly complex roundabout of Place Charles de Gaulle (Paris), which, due to its high complexity, is anyhow a lane-free road infrastructure even for today's conventional traffic.

In this paper, we present a more transparent, systematic, and potentially more efficient way to determine the desired orientation of vehicles moving on lane-free roundabouts, based on the current vehicle location and its destination. For this goal, we formulate and solve an optimal control problem that minimizes a weighted sum of two criteria: (a) the trip distance to the destination, and (b) deviation from the circular angle. Regarding (b), it should be noted that, if all vehicle orientations are close to the circular angle, then they will be close to each other, something that mitigates the strength of any required collision-avoidance maneuvers. The defined problem is solved analytically for two extreme cases, namely the shortest path and the minimum deviation from the circular angle, respectively. For intermediate cases, it is difficult to find the solution analytically. Therefore, a backward Dijkstra algorithm is suggested to determine the optimal orientation in a discretized grid of the roundabout surface. The resulting desired orientations can be stored as an offline database, such that the vehicles can extract their current desired orientation based on their position and exit, while moving on the roundabout. Alternatively, an approximation of the optimal orientation is also suggested, which can be implemented online with negligible computational requirements. The approximation combines locally the orientations related to the shortest path and the minimum angle deviation, which can be simply calculated. The methodologies are applied to a specific, overly complex case study, the roundabout of Place Charles de Gaulle.

The structure of the paper is as follows. The optimal control problem, analytical solutions, and the backward Dijkstra algorithm are presented in Section II. Section III describes the sub-optimal approach. Demonstration results are presented in Section IV. Finally, concluding remarks are given in Section V.

II. Optimal Desired Orientations

In this section, a systematic approach is proposed to determine the vehicle's desired orientation at any location within a circular roundabout, separately for each exit branch. We formulate the problem of specifying desired orientations for the vehicle advancement as an optimal control problem that minimizes a cost function consisting of two weighted terms: (a) the trip distance to the destination; and (b) the deviation from circular angle. Note that, by connecting the desired orientations, a complete path from any point in the roundabout to any exit may be obtained.

It is important to highlight that the considered problem does not concern the actual vehicle movement, but merely the desired orientations to be fed to the vehicle's movement strategy. In other words, the specified orientations would coincide with the actual vehicle path, only if there are no other vehicles around that might call for deviations from the desired path to avoid collisions. For the same reason, the addressed problem does not involve vehicle acceleration or speed, as it aims at merely specifying optimal orientations and resulting paths, whereon the vehicle may drive at acceleration and speed specified by its movement strategy. Therefore, our approach does not need a vehicle model to be explicitly considered.

A. Continuous optimal control problem

The optimal control problem may first be presented in a continuous framework. The vehicle position on the roundabout is represented by a radius r and an angle φ in polar coordinates. Since our goal is to determine the desired orientations without referring to the vehicle dynamics, φ (rather than the time t) is considered as the independent variable, as the vehicle advances forward (increasing φ) within the roundabout. Thus, we have the state equation

$$\dot{r} := dr/d\varphi = u \tag{1}$$

where u is the radius change rate, which may be considered as the control signal. For example, if $u = 0$, then (1) states that the radius does not change, hence the vehicle orientation θ equals the circular angle, i.e., it coincides with the tangent of the circle with the current radius r; while the deviation s

from the circular angle is zero. More generally, we have the relationship $s = \theta - (\varphi + \pi/2)$ where $(\varphi + \pi/2)$ is the circular angle. Based on elementary geometric considerations, we may also derive the relationship between the deviation s and the control signal u as

$$s = \tan^{-1}(-u / r) . \tag{2}$$

The admissible state region for a circular roundabout is obviously $r \in [R_{\text{in}}, R_{\text{out}}]$, where R_{in} and R_{out} are the inner and outer roundabout radiuses, respectively.

For every initial angle and admissible state (r_0, φ_0), the final angle φ_{e} is determined by the angle of the specific destination branch considered; while the final state, at the exit angle φ_{e}, is, for all branches, $r(\varphi_{\text{e}}) = R_{\text{out}}$, since all branches are located at the outer radius of the roundabout. The control objective to be minimized along the corresponding path is specified as

$$J = \int_{\varphi_0}^{\varphi_{\text{e}}} \left(\sqrt{u^2 + r^2} + w(u / r)^2 \right) d\varphi \tag{3}$$

where the first term reflects the trip distance from the origin to the exit point; the second term penalizes quadratically the deviation from the circular angle (see (2)); and w is a weight determining the relative importance of the two terms. Some control constraints may be added to the problem to suppress strong deviations from the circular angle. In conclusion, the optimal control problem (per destination) reads:

$$\begin{aligned} &\text{Minimize } J \\ &\text{subject to: } \dot{r} = u \\ &\qquad \left| \tan^{-1}(u / r) \right| < \bar{s} \\ &\qquad R_{\text{in}} \le r \le R_{\text{out}} \\ &\qquad r(\varphi_e) = R_{\text{out}} \end{aligned} \tag{4}$$

where $\bar{s}$ is the maximum admissible deviation.

If the maximum-deviation constraint is disregarded, problem (4) can be analytically solved for two extreme cases: (a) the shortest path problem, i.e., for $w = 0$; and (b) the minimum deviation problem, i.e., for $w \to \infty$.

B. Extreme Case 1: The shortest path problem

The shortest path has a clear physical meaning and can be readily derived. For better comprehension, we distinguish among two distinct cases:

Visible destination: If the straight line connecting a roundabout point (origin) (r, φ) with the destination lies completely within the roundabout, then we call the destination "visible"; and the shortest path obviously coincides with that straight line; while the slope of the line is the desired orientation at (r, φ). The visible area for an exit branch, grey-shaded in Fig. 1, is described by

$$V = \{(r, \varphi); R_{\text{in}} \le r \le R_{\text{out}}, 0 \le \Delta\varphi \le \Delta\varphi_{\text{vis}}(r)\} \tag{5}$$

where $\Delta\varphi \in [0, 2\pi)$ is the vehicle's angular distance from the exit point, and $\Delta\varphi_{\text{vis}}(r)$ is a radius-dependent visibility threshold. The visible area is delineated upstream by the inner-circle tangent connected to the exit point, which is displayed light blue in Fig. 1. Using trigonometric relationships, $\Delta\varphi_{\text{vis}}(r)$ can be derived as below:

$$\Delta\varphi_{\text{vis}}(r) = \cos^{-1}(R_{\text{in}}/r) + \Delta\varphi_{\text{vis}}(R_{\text{in}}) \tag{6}$$

where

$$\Delta\varphi_{\text{vis}}(R_{\text{in}}) = \cos^{-1}(R_{\text{in}}/R_{\text{out}}) . \tag{7}$$

Invisible destination: The shortest path from a roundabout point (origin) (r, φ) to an invisible destination consists of three parts (Fig. 2). The first part is on the tangent of the inner circle that is connected to the origin, with touch point $(R_{\text{in}}, \varphi + \Delta\varphi_{\tan}(r))$, where $\Delta\varphi_{\tan}(r)$ satisfies

$$\Delta\varphi_{\tan}(r) = \cos^{-1}(R_{\text{in}}/r) . \tag{8}$$

The desired orientation in this part is the slope of the tangent. In the second part, the path follows the inner boundary, i.e., the desired orientation is the circular angle, until the destination gets visible; after which we have again the case of visible destination, and the desired orientation is the slope of a line connected to the exit point, see Fig.2.

In conclusion, the desired orientation at every point on the roundabout (r, φ), with either visible or invisible destination $(R_{\text{out}}, \varphi_{\text{e}})$, is:

$$\theta_{\text{d,SP}}(r, \varphi) = \begin{cases} \tan^{-1}\left(\dfrac{R_{\text{out}} \sin\varphi_{\text{e}} - r\sin\varphi}{R_{\text{out}} \cos\varphi_{\text{e}} - r\cos\varphi}\right) & 0 \le \Delta\varphi \le \Delta\varphi_{\text{vis}}(r) \\ \varphi + \dfrac{\pi}{2} & \Delta\varphi > \Delta\varphi_{\text{vis}}(r) \ \&\ r = R_{\text{in}} \\ \tan^{-1}\left(\dfrac{R_{\text{in}} \sin(\varphi + \Delta\varphi_{\tan}(r)) - r\sin\varphi}{R_{\text{in}} \cos(\varphi + \Delta\varphi_{\tan}(r)) - r\cos\varphi}\right) & \text{otherwise} \end{cases} \tag{9}$$

where the first condition reflects the points in the visible area of an exit branch; while the second and third conditions apply when the destination is invisible. Note that the respective tangent slopes, leading to the desired orientations for the first and third conditions, are calculated, after transforming the respective two points' positions to Cartesian coordinates, as the ratio $\Delta y / \Delta x$ of their difference in y coordinate (Δy) over their difference in x coordinate (Δx).

C. Extreme Case 2: The minimum deviation problem

If we drop the first term of the cost function, the solution minimizes the deviation from the circular angle. We rewrite (4) as below:

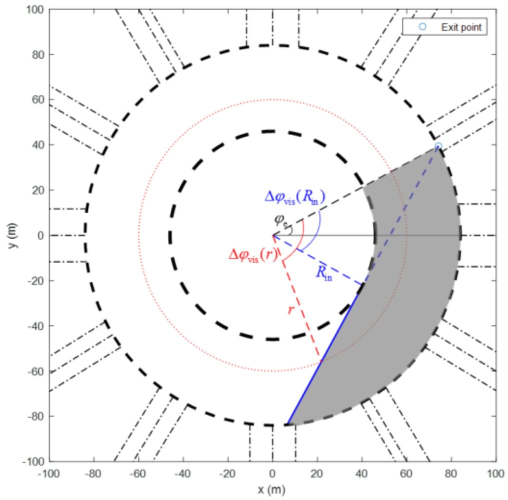

Figure 1. The visible area (grey-shaded) for an exit point

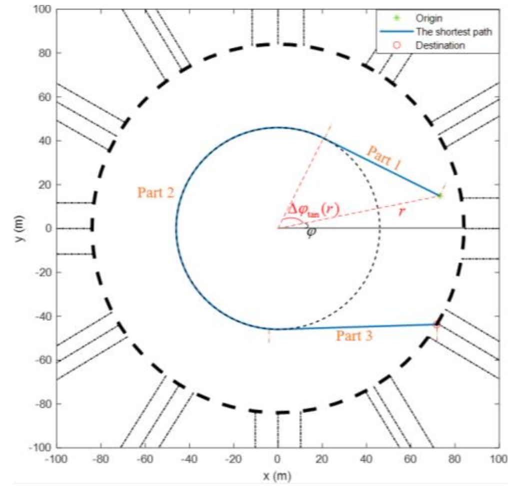

Figure 2. The shortest path for an invisible destination

$$J = \frac{1}{2}\int_{\varphi_0}^{\varphi_e} z^2 d\varphi \quad \text{subject to: } \dot{r} = -rz \tag{10}$$

where, for convenience, z is defined as $z := \tan(s)$ and treated as the control input. The Hamiltonian of this problem reads

$$H = \frac{z^2}{2} - \lambda r z \,. \tag{11}$$

Considering Pontryagin's principle, the optimal solution should satisfy the following conditions:

$$\begin{cases} \dfrac{\partial H}{\partial z} = z - \lambda r = 0 \\ \dfrac{d\lambda}{d\varphi} = -\dfrac{\partial H}{\partial r} = \lambda^2 r \\ \dfrac{dr}{d\varphi} = \dfrac{\partial H}{\partial \lambda} = -\lambda r^2 \end{cases} \tag{12}$$

An intuitive solution is to have a constant deviation for the whole path, from origin to destination, i.e., $z(\varphi) = c; \varphi_0 \le \varphi \le \varphi_e$, which indeed satisfies the mentioned conditions in (12) and is the optimal solution of (10). To calculate the constant value, we solve the state equation:

$$\dot{r} = -cr \Rightarrow r(\varphi) = r_0 \exp\left(-c(\varphi - \varphi_0)\right) \tag{13}$$

Then, by substituting the final condition, the constant value can be found as below:

$$\begin{aligned} R_{\text{out}} &= r_0 \exp\left(-c(\varphi_e - \varphi_0)\right) \\ \Rightarrow c &= -\frac{\ln(R_{\text{out}}) - \ln(r_0)}{\varphi_e - \varphi_0} \end{aligned} \tag{14}$$

Finally, the desired orientation for the minimum-deviation problem is

$$\theta_{\text{d,MD}}(r,\varphi) = \tan^{-1}\left(-\frac{\ln(R_{\text{out}}) - \ln(r)}{\varphi_e - \varphi}\right) + \varphi + \frac{\pi}{2} \,. \tag{15}$$

D. Backward Dijkstra algorithm

The solutions of the mentioned extreme cases may not be desirable due to sharp vehicle movements or uneven exploitation of the roundabout area. In fact, for the shortest path case, many vehicles may tend to move along the inner circle; while for the minimum deviation case, the outer boundary would be more crowded. Therefore, it is interesting to have a combination of these two cases by choosing a finite and non-zero weight w value in (3). However, finding the analytical solution in this general case is not straightforward. To overcome this issue, a Dynamic Programming-based approach, called backward Dijkstra algorithm, is proposed to compute a numerical closed-loop solution for the space-discretized problem.

To this end, we discretize the roundabout surface to form a grid of nodes, with resolution defined by a selectable radius step size Δr and angle step size $\Delta\varphi$. The angle step size should be sufficiently small so that at least one node is located at each (entrance and exit) radial branch. For the edges of the grid, it is reasonable to assume that vehicle paths on the roundabout point only forward, in discrete $\Delta\varphi$ steps. The next vehicle path radius at each forward step depends on vehicle orientation, i.e., we have $r(k+1) = r(k) + q\Delta r$, where $k = 1, 2, \cdots$ is the discrete angle step forward; $q \in \left\{\underline{q}, \cdots, -1, 0, 1, \cdots, \overline{q}\right\}$ reflects corresponding edges (transitions) to next-step nodes in the grid, with corresponding orientations; and $\underline{q}$ and $\overline{q}$ reflect lower and upper limits, respectively, for the admissible orientations, similarly to $\overline{s}$ in (4). Clearly, the allowable range of radiuses is $r \in [R_{\text{in}}, R_{\text{out}}]$, and any transitions leading out of the roundabout are suppressed while constructing the grid. For instance, for nodes on the outer boundary, it is not allowed to select a bigger radius, else outer-boundary violation would occur.

The employed cost criterion for transitions between two nodes, where the radius changes by q steps, is defined as

$$J(k) = d_{r,q}(k) + w \tan^2(s_{r,q}(k)) \tag{16}$$

where, as in (3), $d_{r,q}(k)$ and $s_{r,q}(k)$ are the transition distance and deviation from the circular angle, respectively, if the vehicle decides to change its current radius r by q steps. These terms can be obtained by following equations (see Fig. 3):

$$d_{r,q} = \sqrt{2(r^2 + r \cdot q\Delta r) \cdot (1 - \cos(\Delta\varphi)) + q^2 \Delta r^2} \tag{17}$$

$$s_{r,q} = \tan^{-1}(-q\Delta r / d_{r,0}) . \tag{18}$$

This way, we have a discretized grid of the roundabout including all admissible transitions, along with their costs, which may be used for numerical optimal control problem solutions, separate for each exit point.

The classical Dijkstra algorithm [31] finds the shortest (least-cost) paths between an origin node and all other nodes in a graph. However, what we need here is to find the optimal orientation (transition) at each discrete point (node) of the roundabout grid towards a specific destination point. To this end, we modify the Dijkstra algorithm in a Dynamic Programming-like way, whereby we begin from the destination point and move backward iteratively to determine the optimal transition for all nodes of the roundabout grid. The algorithm determines the optimal orientations at each node, such that the summation of the defined cost criterion from any origin to the exit point is minimized, i.e., the algorithm delivers a (discrete) closed-loop solution.

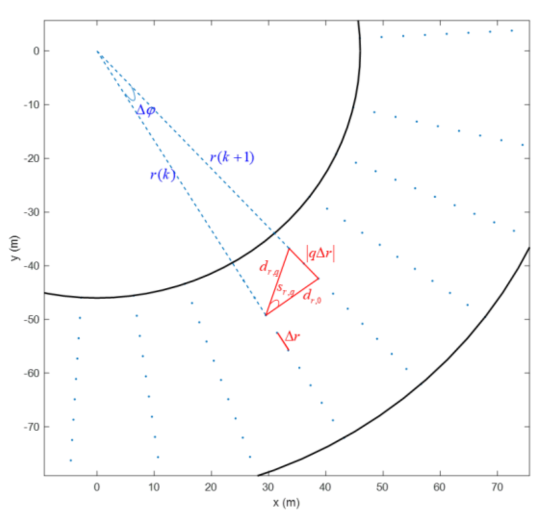

Figure 3. Construction of the roundabout grid

The backward Dijkstra algorithm executes the following steps:

1. Mark all nodes unvisited. Create a set of all the unvisited nodes, called the unvisited set.
2. Assign to every node a tentative optimal cost (TOC) value; set it to zero for the final node and to infinity for all other nodes. [The TOC of a node v is the cost summation on the best path discovered so far between the node v and the final node. Since initially no path is known from any other node than the destination itself (which is a path of cost zero), all other tentative OCs are initially set to infinity.] Set the final node as current.
3. For the current node, consider all its unvisited neighbors and calculate their TOC values through the current node. Compare the newly calculated TOC to the one currently assigned to that neighbor and assign it the smaller one, along with the corresponding tentatively optimal transition. [For example, if the current node A is marked with a TOC of 6, and the edge connecting it with a neighbor B has cost of 2, then the total cost to B through A is 6 + 2 = 8. If B was previously marked with a TOC greater than 8, then change it to 8; and change the optimal transition to point towards A; otherwise, the current TOC value and optimal transition are kept.]
4. When you are done considering all the unvisited neighbors of the current node, mark the current node as visited and remove it from the unvisited set. [A visited node will never be checked again.]
5. If the smallest TOC among the nodes in the unvisited set is infinity [occurs when there is no connection between the remaining unvisited nodes and the final node] or if all nodes have been marked visited, then stop. [The algorithm has finished.]
6. Otherwise, select the unvisited node that is marked with the smallest TOC, set it as the new current node, and go back to step 3.

At the end of iterations, the algorithm delivers, besides the final optimal cost, also the optimal transition (orientation) at each grid node towards the destination. Starting from any discrete point (r, φ) in the grid and following the optimal transitions at the encountered nodes, an optimal path connecting (r, φ) with the destination may be obtained.

E. Real-time implementation

Due to the high computational effort of the numerical solution via the backward Dijkstra algorithm, specifically for a dense grid, it is not possible to run it online. Hence, we have to implement a real-time scheme to determine the optimal orientation for each vehicle based on its current position and exit branch.

This may be achieved by storing the desired orientations for each roundabout location and exit branch, calculated offline by the backward Dijkstra algorithm, as a database which is accessible for the vehicles moving on the roundabout. At each location in the roundabout, a vehicle, depending on its exit branch, recalls the optimal desired orientation for its current location from the corresponding table.

III. A Sub-Optimal Online Approach

In the light of the optimal results of the previous section, an alternative, sub-optimal method with negligible online computational effort may be proposed, which uses the optimal orientations related to the mentioned extreme cases that can be computed online very fast. Specifically, we suggest finding the desired orientation at any location by calculating a weighted average of the orientations resulted from the shortest path and the minimum deviation cases as below:

$$\hat{\theta}_d(r,\varphi) = \alpha\theta_{\mathrm{d,SP}}(r,\varphi) + (1-\alpha)\theta_{\mathrm{d,MD}}(r,\varphi) \tag{19}$$

where $\theta_{\mathrm{d,SP}}(r,\varphi)$ and $\theta_{\mathrm{d,MD}}(r,\varphi)$ are the respective desired orientations corresponding to the shortest path (i.e. (9)) and the minimum deviation (i.e. (15)) problems. Moreover, $0 \le \alpha \le 1$ is a selectable parameter. Note that the orientations derived from (19) combine the outcome of the two respective extreme problems, which is different than combining the two criteria as in the numerically solved general problem.

If desired, one may offline optimize α such that (19) yields an orientation close, as much as possible, to the result of the backward Dijkstra algorithm for a specific weight w. Specifically, we can find α by employing a Least Square (LS) approach, where (19) is employed to construct a regression equation as below:

$$\boldsymbol{\theta}_{\mathrm{d}} - \boldsymbol{\theta}_{\mathrm{d,MD}} = \alpha(\boldsymbol{\theta}_{\mathrm{d,SP}} - \boldsymbol{\theta}_{\mathrm{d,MD}}) \tag{20}$$

where $\boldsymbol{\theta}_{\mathrm{d}}$ is a vector containing the desired orientation, determined by the backward Dijkstra algorithm, at all points of the roundabout grid; while $\boldsymbol{\theta}_{\mathrm{d,SP}}$ and $\boldsymbol{\theta}_{\mathrm{d,MD}}$ contain the shortest path and the minimum deviation orientations, respectively, at those points. Then, α can be calculated by the LS solution:

$$\alpha = \left((\boldsymbol{\theta}_{\mathrm{d,SP}} - \boldsymbol{\theta}_{\mathrm{d,MD}})^T(\boldsymbol{\theta}_{\mathrm{d,SP}} - \boldsymbol{\theta}_{\mathrm{d,MD}})\right)^{-1}(\boldsymbol{\theta}_{\mathrm{d,SP}} - \boldsymbol{\theta}_{\mathrm{d,MD}})^T(\boldsymbol{\theta}_{\mathrm{d}} - \boldsymbol{\theta}_{\mathrm{d,MD}}) \tag{21}$$

The easiness of producing desired orientations with this approach offers an additional advantage, namely the possibility to modify in real time the value of α, and hence the produced orientations, in dependence of the current traffic. Specifically, if the traffic density in the roundabout is low, vehicle conflicts are accordingly few, hence it may be preferable to tend towards shortest paths (α small) to save trip time and fuel consumption. In contrast, if the traffic density in the roundabout is high, vehicle conflicts are accordingly frequent, hence it may be preferable to tend towards minimum-deviation paths (α big) to mitigate the required vehicle maneuver intensity.

IV. Results

In this section, results of applying both above approaches to the Place Charles de Gaulle roundabout (Paris, France) are presented. Our case study has outer and inner radiuses of 84 m and 46 m, respectively, i.e., its width is 38 m; and comprises 12 bi-directional radial branches, which results in 144 possible Origin- Destination (OD) vehicle trips.

Despite the infrastructure complexity, it is easy to generate the desired orientation for the two extreme cases in the continuous framework. For the general case, we may generate the results in a discretized grid using the backward Dijkstra algorithm. These different solutions enable demonstration and comparison of the orientations and paths resulting from different approaches. The roundabout surface is discretized with $\Delta r = 0.38m$ and $\Delta\varphi = 3^\circ$, which leads to a total of 10797 grid nodes. Also, the maximum admissible deviation is set to 40°, which corresponds roughly to a maximum of 11 possible transitions at each node, i.e. we set $\overline{q} = -\underline{q} = 5$. Note that, for a given maximum admissible deviation, the corresponding $\underline{q}$ and $\overline{q}$ depend on the radius and are not the same for different radiuses; but we run the code with a constant number of possible transitions for simplicity. To have visually understandable results, the obtained desired orientations will be displayed for only 20% of the nodes on the roundabout surface. All presented results concern one specific exit branch. The computation time to run the backward Dijkstra algorithm in Matlab code for this big roundabout and dense discretization (for one destination) amounts to 2557 seconds on an Intel(R) Core™i5-10500 CPU @ 3.10GHz with 8.0 GB of installed RAM.

Fig. 4 shows the results of the backward Dijkstra algorithm for the shortest path case, i.e., for $w = 0$, albeit, in distinction to Section II.B., subject to the maximum admissible deviation mentioned above. It can be observed that at points from which the destination is not visible, vehicles tend to reach the inner circle as straight as possible, provided that their deviation from the circular angle does not violate the defined limit. Furthermore, where the destination is visible, orientations indicate an almost straight path towards it, if following the direct line does not cause maximum deviation violation.

Fig. 5, on the other hand, illustrates the desired orientations for the case of minimum deviation, i.e., for $w \to \infty$, again subject to the maximum admissible deviation. In this case, vehicles have mostly quasi-circular paths, approaching the exit branch gradually. For nodes located on the outer boundary, e.g., when a vehicle enters the roundabout, the orientations and induced paths follow the outer boundary. Note that the blank spaces in both (and other) figures comprise nodes that cannot be feasibly connected to the destination, as this would violate the maximum-deviation limit. Since this area is mainly determined by the deviation limit, it is similar for all values of the weight. If a vehicle is located in that blank space, the slope of the line connecting its current position to the exit point is considered as the desired orientation.

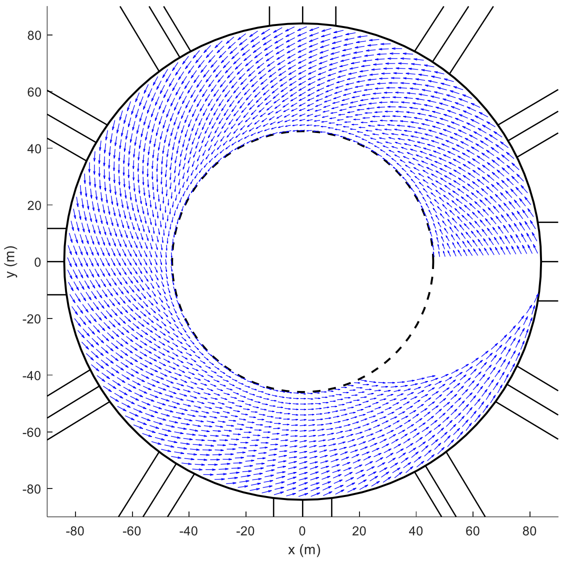

Figure 4. Desired orientations for shortest path ($w = 0$) subject to the maximum deviation constraint

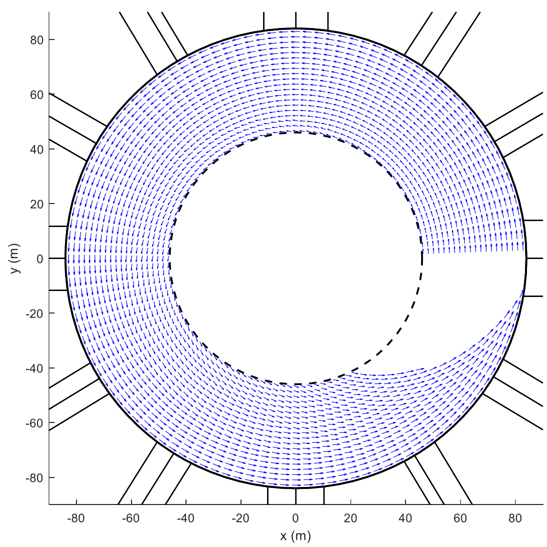

Figure 5. Desired orientations for minimum deviation ($w \to \infty$)

To reach a better exploitation level of the roundabout area and have smoother paths, it is preferable to use a finite and non-zero weight value. The result of the backward Dijkstra algorithm for is depicted in Fig. 6. Paths from very far origins have a tendency to reach first the inner boundary; while for closer origins, paths do not approach the inner boundary. Consequently, the traffic will be more evenly distributed in the whole roundabout area.

Finally, to have a comparison for a certain OD, and also to evaluate the sub-optimal approach, the optimal paths for a vehicle starting from a point in the middle of the roundabout with different weight values are shown in Fig. 7. It can be seen that for the shortest path case, the created path reaches the inner boundary, roughly along a tangent line, and stays thereon till the destination gets visible, after which it tends quasi-linearly to the exit. Small deviations from the continuous solution are due to discretization or due to the imposed maximum-deviation limit. For the minimum deviation case, in contrast, the path approaches gradually the outer boundary, reaching it at the exit angle. When $w = 10$, the path lies between those corresponding to the mentioned extreme cases.

Lastly, the path determined based on the sub-optimal approach (16), with $\alpha = 0.29$ obtained from (21), is also displayed in Fig. 7. It may be seen that this path is acceptably close to the optimal solution.

V. Conclusion

This paper proposes a transparent and systematic way to determine the desired orientation of vehicles moving on large lane-free roundabouts, which is considered challenging because of complex geometrical features and numerous potential vehicle conflicts. The presented approach determines the desired orientation through solving an optimal control problem minimizing a weighted sum of the trip distance and deviation from the circular angle. An analytical solution is derived for two extreme cases, the shortest path and the minimum deviation from the circular angle, respectively. Furthermore, the backward Dijkstra algorithm is employed to find the optimal path, for general weighted situations. Finally, a sub-optimal scheme is suggested in the light of the optimal derivations. All approaches are illustrated and compared for an overly complex case study, the Charles de Gaulle roundabout in Paris, France.

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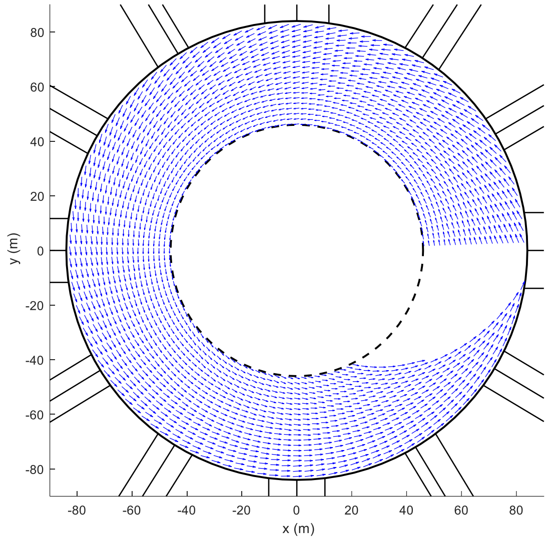

Figure 6. Desired orientations for the general case ($w = 10$)

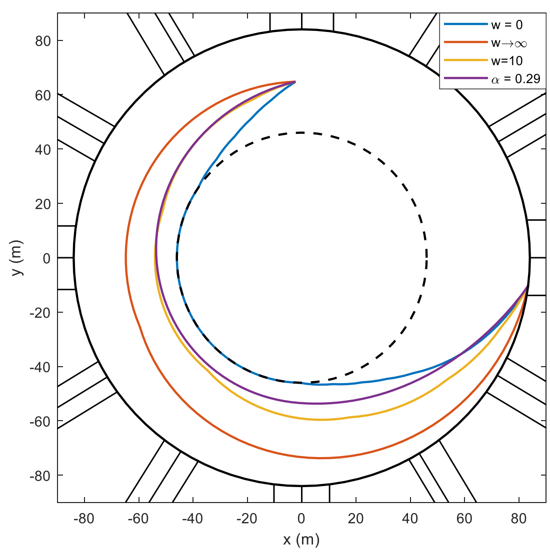

Figure 7. The paths for a certain OD with different weights

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