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Symbiotic Control of Uncertain Dynamical Systems*

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Abstract— **In this paper, we consider both the fixed-gain control and adaptive learning architectures to suppress the effects of uncertainties. We note that fixed-gain control provides more predictable closed-loop system behavior, but it comes at the cost of knowing uncertainty bounds. On the other hand, adaptive learning removes the requirement of this knowledge at the expense of less predictable closed-loop system behavior compared to fixed-gain control. To this end, this paper presents a novel symbiotic control framework that integrates the advantages of both fixed-gain control and adaptive learning architectures. In particular, the proposed framework utilizes both control architectures to suppress the negative effects of uncertainties with more predictable closed-loop system behavior and without the knowledge of uncertainty bounds. Both parametric and nonparametric uncertainties are considered, where we use neural networks to approximate the unknown uncertainty basis for the latter case. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.**

I. Introduction

When the complexity of dynamical systems grows, the differences between mathematical models and actual physical systems widen due to idealized assumptions, simplifications, degraded modes of operation, and changes in equations of motion. These differences, known as uncertainties, can significantly degrade the closed-loop system performance and may even lead to instability. For the purpose of suppressing the negative effects of uncertainties, there are two fundamental approaches in the literature, namely, fixed-gain control and adaptive learning architectures. For example, robust control [1,2] and sliding mode control [3,4] are well-known fixed-gain control approaches, whereas adaptive control [5,6,7] and reinforcement learning [8,9] are well-known adaptive learning approaches.

In contrast to adaptive learning, fixed-gain control provides more predictable closed-loop system behavior since the gains of the resulting control algorithm are not a function of time or state. However, the fixed-gain control needs the knowledge of uncertainty bounds to ensure the stability of the closed-loop system (see [2, Chapter 2] and [10, Assumption 3.1] for examples). For dynamical systems of complex nature, obtaining such bounds can be challenging. In contrast, adaptive learning does not require such knowledge. Yet, because of their nonlinear parameter adjustment mechanism and the need for neural networks for approximating uncertainties of nonparametric nature, it often results in less predictable closed-loop system behavior compared to fixed-gain control especially during their transient period [11,12,13,14,15,16].

This paper contributes a novel control framework incorporating the advantages of fixed-gain control and adaptive learning architectures. Inspired by biology, we call this framework symbiotic control as symbiosis refers to the relationship or interaction between two dissimilar organisms [17] (i.e., two dissimilar organisms refer to the fixed-gain control and adaptive learning architectures). In particular, symbiotic control synergistically combines these architectures to suppress the negative effects of uncertainties in a more predictable manner as compared to adaptive learning alone and it does not require the bounds of such uncertainties. We also consider both parametric uncertainties and nonparametric uncertainties. For the latter, we resort to neural networks to approximate the unknown uncertainty basis function.

In the field of adaptive learning, it is known that an inadequate number of neurons can lead to a large approximation error in the neural network over a compact region. Since this often causes poor closed-loop system behavior, the studies [18,19] have recently investigated deep neural network methods to minimize this approximation error. Moreover, it is well-known that high leakage term parameters in the parameter adjustment mechanisms can degrade the closed-loop system performance by fighting against the learning process. Counterintuitively, the proposed symbiotic control framework can achieve a desired level of closed-loop system behavior even with an insufficient number of neurons, without a deep neural network method, or in the face of high leakage term parameters.

With regard to smoother response and better tracking performance, the combined/composite adaptive control approach and its variants use a model prediction error (see [20,21,22] and references therein). The prediction error is the difference between the output of a truth model, which can be computed at every instant, and its prediction. The truth model is obtained by low-pass filtering of the system dynamics and leaving the uncertain part alone. While the above studies add a term involving prediction error into the parameter adjustment mechanism, the proposed symbiotic framework incorporates the output of a truth model into both the fixed-gain control and adaptive learning architectures.

Finally, authors of [13,14,15] present symbiotic control frameworks that are relevant to the findings in this paper. However, we note that there are two key distinctions. First, these results do not focus on nonparametric uncertainties as

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opposed to the results documented in this paper. Second, while nonparametric uncertainties are not considered, the findings in [15] align more closely with the results of this paper. However, the authors of [15] make an assumption that requires some knowledge of uncertainty bounds for guaranteeing closed-loop system stability (i.e., [15, (34)]), where we here remove this assumption for not only parametric uncertainty but also nonparametric uncertainty cases.

In the remainder of this paper, we first outline the problem formulation and provide the relevant preliminaries on fixed-gain control and adaptive learning architectures in Section II. We then introduce the symbiotic control framework in Section III (respectively, Section IV) for dynamical systems with parametric uncertainty (respectively, nonparametric uncertainty). We finally present illustrative numerical examples in Section V to demonstrate the efficacy of the contributions of this paper and summarize our conclusions in Section VI.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ respectively are the sets of real numbers, real vectors, and real matrices; $\mathbb{R}_+$, $\overline{\mathbb{R}}_+$, $\mathbb{R}_+^{n\times n}$, and $\overline{\mathbb{R}}_+^{n\times n}$ respectively are the sets of positive real numbers, nonnegative real numbers, symmetric positive-definite matrices, and symmetric nonnegative-definite matrices; and "$\triangleq$" is the equality by definition. Moreover, $(\cdot)^{-1}$ is used for the inverse, $(\cdot)^{\mathrm{T}}$ is used for the transpose, and $\mathrm{diag}(a)$ is used for the diagonal matrix with the real vector $a \in \mathbb{R}^n$ on its diagonal.

II. Problem Formulation and Preliminaries

Consider the following uncertain dynamical system

$$\dot{x}(t) = Ax(t) + B\Lambda\big(u(t) + \delta(x(t))\big), \quad x(0) = x_0. \quad (1)$$

Here, $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$ denotes the measurable state and $u(t) \in \mathbb{R}^m$ denotes the control signal. In (1), $A \in \mathbb{R}^{n\times n}$ denotes the known system matrix and $B \in \mathbb{R}^{n\times m}$ denotes the known full column rank control matrix. Note that the pair (A, B) is stabilizable. Moreover, $\Lambda = \mathrm{diag}(\lambda) \in \mathbb{R}_+^{m\times m}$, $\lambda = [\lambda_1, \ldots, \lambda_m]^{\mathrm{T}}$, denotes an unknown control effectiveness matrix and $\delta(x(t)) : \mathcal{D} \to \mathbb{R}^m$ denotes a parametric ($\mathcal{D} = \mathbb{R}^n$ in this case) or nonparametric ($\mathcal{D} \subset \mathbb{R}^n$ in this case with $\mathcal{D}$ being a compact set) uncertainty whose components are real-valued locally Lipschitz functions of the state.

We define the nominal control signal, denoted by $u_{\mathrm{n}}(t) \in \mathbb{R}^m$, as follows

$$u_{\mathrm{n}}(t) = -K_1 x(t) + K_2 r(t). \quad (2)$$

Here, $K_1 \in \mathbb{R}^{m\times n}$ denotes a feedback gain matrix such that $A - BK_1$ is Hurwitz, $K_2 \in \mathbb{R}^{m\times p}$ denotes a feedforward gain matrix, and $r(t) \in \mathbb{R}^p$ denotes a reference signal, which is uniformly continuous and bounded on $[0, \infty)$. The goal of (2) is to define the nominal (i.e., ideal) closed-loop system behavior

$$\dot{x}_{\mathrm{n}}(t) = A_{\mathrm{n}} x_{\mathrm{n}}(t) + B_{\mathrm{n}} r(t), \quad x_{\mathrm{n}}(0) = x_{\mathrm{n}0}, \quad (3)$$

when there are no uncertainties (i.e., $\delta(x(t)) \equiv 0$ and $\Lambda = I$) and $u(t) \equiv u_{\mathrm{n}}(t)$, where $A_{\mathrm{n}} \triangleq A - BK_1$ and $B_{\mathrm{n}} \triangleq BK_2$.

Next, we can denote the fixed-gain control signal and adaptive learning signal as $u_{\mathrm{f}}(t) \in \mathbb{R}^m$ and $u_{\mathrm{a}}(t) \in \mathbb{R}^m$, respectively. The control signal satisfies

$$u(t) = u_{\mathrm{n}}(t) + u_{\mathrm{f}}(t) + u_{\mathrm{a}}(t). \quad (4)$$

We present the definitions of $u_{\mathrm{f}}(t)$ and $u_{\mathrm{a}}(t)$ in the following sections. Note that we can obtain

$$\begin{aligned} \dot{x}(t) \;&=\; A_{\mathrm{n}} x(t) + B_{\mathrm{n}} r(t) + B\Lambda\big(u_{\mathrm{f}}(t) + u_{\mathrm{a}}(t) \\ &\quad + \pi(x(t), u_{\mathrm{n}}(t))\big), \end{aligned} \quad (5)$$

with (1), (2), and (4). In (5),

$$\pi(x(t), u_{\mathrm{n}}(t)) \triangleq \delta(x(t)) + (I - \Lambda^{-1}) u_{\mathrm{n}}(t) \quad (6)$$

denotes the total uncertainty.

We study how to synergistically blend fixed-gain control signal $u_{\mathrm{f}}(t)$ and adaptive learning signal $u_{\mathrm{a}}(t)$ (i.e., symbiotic control) in order to suppress the total uncertainty $\pi(x(t), u_{\mathrm{n}}(t))$ in a more predictable manner as compared to adaptive learning alone and without requiring any knowledge on the bound of this uncertainty. Note that we address the case when $\delta(x(t))$ in $\pi(x(t), u_{\mathrm{n}}(t))$ is parametric (respectively, nonparametric) in Section III (respectively, Section IV). Before presenting our main results, we now introduce important preliminaries in the following subsections.

A. Preliminaries on Fixed-Gain Control

In this subsection, we consider that adaptive learning signal satisfies $u_{\mathrm{a}}(t) \equiv 0$. Moreover, let the fixed-gain control signal satisfy

$$u_{\mathrm{f}}(t) = -\alpha B_{\mathrm{i}}(x(t) - x_0) + \alpha B_{\mathrm{i}} \int_0^t \big(A_{\mathrm{n}} x(s) + B_{\mathrm{n}} r(s)\big) \mathrm{d}s. \quad (7)$$

Here, $\alpha \in \mathbb{R}_+$ denotes the fixed-gain control parameter and $B_{\mathrm{i}} \triangleq (B^{\mathrm{T}} B)^{-1} B^{\mathrm{T}}$. We note that $B^{\mathrm{T}} B$ is always invertible due to B having full column rank. Now, we are ready to present an important result.

Proposition 1. If α in (7) is sufficiently large, then the solution to (5) approximately behaves as the solution to the nominal (i.e., ideal) closed-loop system given by (3).

Due to the page limitations, the proofs of Proposition 1 as well as other results presented in this paper are omitted and they will be presented elsewhere.

Remark 1. Let the fixed-gain control parameter α be sufficiently large. Then, one can conclude from Proposition 1 that the solution to the uncertain dynamical system given in (5) approaches the solution to the nominal (i.e., ideal) closed-loop system given in (3). We note that this result is highly valuable since it applies to both parametric and nonparametric uncertainty cases, even in the absence of an adaptive learning signal. We also note that it is hard to determine how large α we need to select in practice. Moreover, analyzing closed-loop system stability on how α needs to be properly chosen without an adaptive learning signal requires a specific uncertainty structure and an upper bound on this uncertainty, where such analysis can be conservative as well (see [10] for an exemplary study involving small gain theorem). In order to get rid of these constraints, we utilize a form of the fixed-gain control signal given by (7) in the following sections with an adaptive learning signal (i.e., symbiotic control). Notably, we utilize this fixed-gain control signal for achieving a more predictable closed-loop system behavior as α increases.

B. Preliminaries on Adaptive Learning

In this subsection, we now consider that the fixed-gain control signal satisfies $u_{\mathrm{f}}(t) \equiv 0$. Moreover, let $\delta(x(t))$ be a parametric uncertainty, which satisfies

$$\delta(x(t)) = W_\delta^{\mathrm{T}} \sigma_\delta(x(t)), \quad x(t) \in \mathbb{R}^n. \tag{8}$$

Here, $W_\delta \in \mathbb{R}^{s\times m}$ denotes an unknown weight and $\sigma_\delta(x(t)) : \mathbb{R}^n \to \mathbb{R}^s$ denotes a known basis function. Then, one can present the total uncertainty (6) in the following form

$$\pi(x(t), u_{\mathrm{n}}(t)) \triangleq W^{\mathrm{T}} \sigma(x(t), u_{\mathrm{n}}(t)). \tag{9}$$

Here, $W \triangleq [W_\delta^{\mathrm{T}}, (I - \Lambda^{-1})]^{\mathrm{T}} \in \mathbb{R}^{(s+m)\times m}$ is unknown and $\sigma(x(t), u_{\mathrm{n}}(t)) \triangleq [\sigma_\delta^{\mathrm{T}}(x(t)), u_{\mathrm{n}}^{\mathrm{T}}(t)]^{\mathrm{T}} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{s+m}$ is known by definition. Moreover, define the adaptive learning signal as

$$u_{\mathrm{a}}(t) = -\hat{W}^{\mathrm{T}}(t) \sigma(x(t), u_{\mathrm{n}}(t)). \tag{10}$$

Here, $\hat{W}(t) \in \mathbb{R}^{(s+m)\times m}$ denotes the estimate of W obtained through the parameter adjustment mechanism given by

$$\dot{\hat{W}}(t) = \beta \sigma(x(t), u_{\mathrm{n}}(t)) \big(x(t) - x_{\mathrm{n}}(t)\big)^{\mathrm{T}} PB, \ \hat{W}(0) = \hat{W}_0. \tag{11}$$

Here, $\beta \in \mathbb{R}_+$ denotes the adaptive learning parameter, $x_{\mathrm{n}}(t)$ satisfying (3), and $P \in \mathbb{R}_+^{n\times n}$ denotes the unique solution to the Lyapunov equation

$$0 = A_{\mathrm{n}}^{\mathrm{T}} P + P A_{\mathrm{n}} + R \tag{12}$$

for a given $R \in \mathbb{R}_+^{n\times n}$. We now state the next proposition.

Proposition 2[1]**.** Consider the dynamical system given by (5) with the parametric uncertainty given by (9), where $u_{\mathrm{f}}(t) \equiv 0$. Consider also the adaptive learning signal given by (10), (11), and (3). The trajectories of the closed-loop system are then bounded and $\lim_{t\to\infty} \big(x(t) - x_{\mathrm{n}}(t)\big) = 0$.

We now consider that $\delta(x(t))$ is a nonparametric uncertainty, which satisfies

$$\delta(x(t)) = W_\delta^{\mathrm{T}} \sigma_\delta(x(t)) + \epsilon(x(t)), \quad x(t) \in \mathcal{D}. \tag{13}$$

Here, $W_\delta \in \mathbb{R}^{s\times m}$ denotes an unknown weight, $\sigma_\delta(x(t)) : \mathcal{D} \to \mathbb{R}^s$ denotes a selected basis function containing a unity bias and $s_{\mathrm{f}} \triangleq s - 1$ radial basis functions, and $\epsilon(x(t))$ denotes a bounded approximation error[2]. In this scenario, one can present the total uncertainty (6) in the following form

$$\pi(x(t), u_{\mathrm{n}}(t)) \triangleq W^{\mathrm{T}} \sigma(x(t), u_{\mathrm{n}}(t)) + \epsilon(x(t)). \tag{14}$$

Here, W and $\sigma(x(t), u_{\mathrm{n}}(t))$ have the same forms given after (9). Moreover, consider the adaptive learning signal given by (10) with $\hat{W}(t)$ being an estimate of W, which is obtained through the parameter adjustment mechanism given by

$$\dot{\hat{W}}(t) = \beta_1 \sigma(x(t), u_{\mathrm{n}}(t)) \big(x(t) - x_{\mathrm{n}}(t)\big)^{\mathrm{T}} PB - \beta_2 \hat{W}(t), \quad \hat{W}(0) = \hat{W}_0. \tag{15}$$

Here, $\beta_1 \in \mathbb{R}_+$ denotes the adaptive learning parameter, $\beta_2 \in \mathbb{R}_+$ denotes the leakage parameter, $x_{\mathrm{n}}(t)$ satisfying (3), and $P \in \mathbb{R}_+^{n\times n}$ denotes the unique solution to the Lyapunov equation (12). We are now ready for the next proposition.

Proposition 3[3]**.** Consider the dynamical system given by (5) with the nonparametric uncertainty given by (14), where $u_{\mathrm{f}}(t) \equiv 0$. Consider also the adaptive learning signal given by (10), (15), and (3). The trajectories of the closed-loop system are then bounded.

Note that we use a form of direct adaptive control method given in Propositions 2 and 3 in the next sections for constructing the adaptive learning signal $u_{\mathrm{a}}(t)$. One can also utilize other adaptive learning methods to use together with fixed-gain control. However, it is worth noting that direct adaptive control, like other adaptive learning methods, can exhibit less predictable, poor closed-loop system behavior due to their nonlinear parameter adjustment mechanism and in the presence of high neural network approximation errors (i.e., $\epsilon(x(t))$ in (13)) and high leakage parameter (i.e., β_2 in (15)). We address this issue by synergistically integrating the fixed-gain control signal with the adaptive learning signal (i.e., symbiotic control) in the next sections.

Now, we present two observations about Proposition 3. First, it holds when $x(t)$ stays in $\mathcal{D}$. To enforce $x(t)$ to stay in $\mathcal{D}$ without necessarily making $\mathcal{D}$ arbitrarily large, one can use set-theoretic direct adaptive control method [23]. Second, Proposition 3 holds when one uses a projection operator in (15) instead of the leakage term (i.e., $-\beta_2 \hat{W}(t)$). We prefer not to use a projection operator in order not to make any assumptions on the bounds of W.

III. Symbiotic Control of Dynamical Systems with Parametric Uncertainty

In this section, we consider the uncertain dynamical system (5), which is subject to (6). Moreover, we investigate the parametric uncertainty case such that (8) holds over $x(t) \in \mathbb{R}^n$. Note that one can consider the total uncertainty as given in (9).

A. Control Architecture

We first present a form of the fixed-gain (like) control signal satisfying

$$u_{\mathrm{f}}(t) = -\alpha B_{\mathrm{i}}(x(t) - x_0) + \alpha B_{\mathrm{i}} \int_0^t \big(A_{\mathrm{n}} x(s) + B_{\mathrm{n}} r(s)\big) \mathrm{d}s + \int_0^t u_{\mathrm{g}}(s) \mathrm{d}s. \tag{16}$$

Here, $\alpha \in \mathbb{R}_+$ denotes the fixed-gain control parameter and $B_{\mathrm{i}} \triangleq (B^{\mathrm{T}} B)^{-1} B^{\mathrm{T}}$. Moreover, $u_{\mathrm{g}}(t) \in \mathbb{R}^m$ satisfies

[1]See [6, Section 4.3] for the proof of this proposition.

[2]Radial basis function neural networks are universal approximators (see [5, Section 12.3]).

[3]From [6, Section 4.5], the proof of this proposition follows.

$$u_{\rm g}(t) = -\beta_1\beta_2^{-1}\hat{\Lambda}(t)B^{\rm T}Pe(t). \quad (17)$$

Here, $e(t) \triangleq x(t) - x_{\rm n}(t) \in \mathbb{R}^n$ denotes the error signal and $P \in \mathbb{R}_+^{n\times n}$ denotes the unique solution to the Lyapunov equation (12) for a given $R \in \mathbb{R}_+^{n\times n}$ ($\beta_1 \in \mathbb{R}_+$ and $\beta_2 \in \mathbb{R}_+$ are defined below). Moreover, $\hat{\Lambda}(t) \in \mathbb{R}^{m\times m}$ denotes an estimate of Λ obtained through the parameter adjustment mechanism

$$\dot{\hat{\Lambda}}(t) = \gamma B^{\rm T}Pe(t)u_{\rm f}^{\rm T}(t),\ \hat{\Lambda}(0) = \hat{\Lambda}_0. \quad (18)$$

Here, $\gamma \in \mathbb{R}_+$ denotes the adaptive learning parameter. We next consider the adaptive learning signal (10), where $\hat{W}(t) \in \mathbb{R}^{(s+m)\times m}$ denotes an estimate of W obtained through the parameter adjustment mechanism[4]

$$\begin{aligned}\dot{\hat{W}}(t) \ &= \ \beta_1\sigma(x(t),u_{\rm n}(t))e^{\rm T}(t)PB \\ &\quad -\beta_2\alpha\sigma(x(t),u_{\rm n}(t))u_{\rm f}^{\rm T}(t),\ \hat{W}(0) = \hat{W}_0. \end{aligned} \quad (19)$$

Here, $\beta_1 \in \mathbb{R}_+$ and $\beta_2 \in \mathbb{R}_+$ denote the adaptive learning parameters.

Remark 2. The proposed symbiotic control architecture for dynamical systems with parametric uncertainty is given above. Observe that the proposed fixed-gain control signal (16) is a version of the original fixed-gain control signal (7) with an added integral of (17) containing the adaptive parameter $\hat{\Lambda}(t)$. Likewise, (19) is a version of (11) with an added term containing the fixed-gain control signal $u_{\rm f}(t)$. In other words, the fixed-gain control and adaptive learning architectures interact with each other to mitigate the effects of uncertainties in a more predictable manner without requiring any knowledge on the bounds of such uncertainties.

B. System-Theoretical Analysis

We begin with a key lemma.

Lemma 1. The fixed-gain control signal (16) satisfies

$$\begin{aligned}\dot{u}_{\rm f}(t) \ &= \ -\alpha\Lambda\big(u_{\rm f}(t)+u_{\rm a}(t)+\pi(x(t),u_{\rm n}(t))\big)+u_{\rm g}(t), \\ &\qquad u_{\rm f}(0) = 0. \end{aligned} \quad (20)$$

While the fixed-gain control signal (16) is implementable, its equivalent representation (20) is not. The main reason is that the matrix Λ and the term $\pi(x(t),u_{\rm n}(t))$ are unknown. We note that this equivalent representation is only needed for the system-theoretical analysis given in this subsection. Now, we can demonstrate an important theorem.

Theorem 1. If α in (16) is sufficiently large, then the solution to (5) approximately behaves as the solution to the nominal (i.e., ideal) closed-loop system given by (3).

One can deduce from Theorem 1 that the closed-loop system behavior becomes more predictable if we increase α. Note that one can also apply the similar discussion given in Remark 1 to this theorem. Now, we use (9) and (10) in (20) to obtain

$$\dot{u}_{\rm f}(t) = -\alpha\Lambda\big(u_{\rm f}(t) - \tilde{W}^{\rm T}(t)\sigma(x(t),u_{\rm n}(t))\big)+u_{\rm g}(t). \quad (21)$$

Here, $\tilde{W}(t) \triangleq \hat{W}(t) - W \in \mathbb{R}^{(s+m)\times m}$. Moreover, one can obtain the time derivative of the error signal as

$$\begin{aligned}\dot{e}(t) \ &= \ A_{\rm n}e(t) + B\Lambda\big(u_{\rm f}(t)+u_{\rm a}(t)+\pi(x(t),u_{\rm n}(t))\big), \\ &\qquad e(0) = 0, \end{aligned} \quad (22)$$

by using (5) and (3). Next, we obtain

$$\dot{e}(t) = A_{\rm n}e(t) + B\Lambda\big(u_{\rm f}(t) - \tilde{W}^{\rm T}(t)\sigma(x(t),u_{\rm n}(t))\big), \quad (23)$$

by using (9) and (10) in (22). Now, we can present our first main result. To this end, let $\tilde{\Lambda}(t) \triangleq \hat{\Lambda}(t) - \Lambda \in \mathbb{R}^{m\times m}$.

Theorem 2. Consider the dynamical system given by (5) with the parametric uncertainty given by (9). In addition, consider the form of the fixed-gain control signal given by (16) with (17) and (18). Consider also the form of the adaptive learning signal given by (10), (19), and (3). The trajectories $\big(e(t), u_{\rm f}(t), \tilde{W}(t), \tilde{\Lambda}(t)\big)$ of the closed-loop system are then bounded and

$$\lim_{t\to\infty}\Big(e(t),\, u_{\rm f}(t)\Big) = \big(0,0\big). \quad (24)$$

IV. Symbiotic Control of Dynamical Systems with Nonparametric Uncertainty

In this section, we consider the uncertain dynamical system (5), which is subject to (6). Here, we investigate the nonparametric uncertainty case such that (13) holds over $x(t) \in \mathcal{D}$, where the total uncertainty can be given in (14).

A. Control Architecture

We here consider the fixed-gain control signal (16), where $\alpha \in \mathbb{R}_+$ denotes the fixed-gain control parameter and $B_{\rm i} \triangleq (B^{\rm T}B)^{-1}B^{\rm T}$. In (16), $u_{\rm g}(t) \in \mathbb{R}^m$ satisfies (17), where $e(t) \triangleq x(t) - x_{\rm n}(t) \in \mathbb{R}^n$ denotes the error signal, and $P \in \mathbb{R}_+^{n\times n}$ denotes the unique solution to the Lyapunov equation (12) for a given $R \in \mathbb{R}_+^{n\times n}$ ($\beta_1 \in \mathbb{R}_+$ and $\beta_2 \in \mathbb{R}_+$ are defined below). In (17), $\hat{\Lambda}(t) \in \mathbb{R}^{m\times m}$ denotes an estimate of Λ obtained by the parameter adjustment mechanism

$$\dot{\hat{\Lambda}}(t) = \gamma_1 B^{\rm T}Pe(t)u_{\rm f}^{\rm T}(t) - \gamma_2\hat{\Lambda}(t),\ \hat{\Lambda}(0) = \hat{\Lambda}_0. \quad (25)$$

Here, $\gamma_1 \in \mathbb{R}_+$ denotes the adaptive learning parameter and $\gamma_2 \in \mathbb{R}_+$ denotes the leakage parameter. We next consider the adaptive learning signal (10), where $\hat{W}(t) \in \mathbb{R}^{(s+m)\times m}$ denotes an estimate of W obtained through the parameter adjustment mechanism

$$\begin{aligned}\dot{\hat{W}}(t) \ &= \ \beta_1\sigma(x(t),u_{\rm n}(t))e^{\rm T}(t)PB - \beta_3\hat{W}(t) \\ &\quad -\beta_2\alpha\sigma(x(t),u_{\rm n}(t))u_{\rm f}^{\rm T}(t),\ \hat{W}(0) = \hat{W}_0. \end{aligned} \quad (26)$$

Here, $\beta_1 \in \mathbb{R}_+$ and $\beta_2 \in \mathbb{R}_+$ denote the adaptive learning parameters, and $\beta_3 \in \mathbb{R}_+$ denotes the leakage parameter. We note that one can apply a similar version of Remark 2 for the proposed symbiotic control architecture given above for dynamical systems with nonparametric uncertainty.

[4]The results documented in [21] also consider a parameter adjustment mechanism with a second term added to its right-hand side that is predicated on a filtering technique. Yet, that result does not have a fixed-gain control law as opposed to the findings reported in this paper.

B. System-Theoretical Analysis

First, note that Lemma 1 and Theorem 1 are also applicable for the nonparametric uncertainty case. Then, we obtain

$$\dot{u}_{\mathrm{f}}(t) = -\alpha\Lambda\big(u_{\mathrm{f}}(t) - \tilde{W}^{\mathrm{T}}(t)\sigma(x(t), u_{\mathrm{n}}(t)) + \epsilon(x(t))\big) + u_{\mathrm{g}}(t), \quad u_{\mathrm{f}}(0) = 0, \tag{27}$$

by using (14) and (10) in (20). One can now obtain the time derivative of the error signal given in (22) once again by using (5) and (3). Similarly, we have

$$\dot{e}(t) = A_{\mathrm{n}}e(t) + B\Lambda\big(u_{\mathrm{f}}(t) - \tilde{W}^{\mathrm{T}}(t)\sigma(x(t), u_{\mathrm{n}}(t)) + \epsilon(x(t))\big), \quad e(0) = 0, \tag{28}$$

by using (14) and (10) in (22). Now, we can present our second main result. To this end, recall that $\tilde{W}(t) \triangleq \hat{W}(t) - W$ and $\tilde{\Lambda}(t) \triangleq \hat{\Lambda}(t) - \Lambda$.

Theorem 3. Consider the dynamical system given by (5) with the nonparametric uncertainty given by (14). In addition, consider the form of the fixed-gain control signal given by (16) with (17) and (25). Consider also the form of the adaptive learning signal given by (10), (26), and (3). Then, the trajectories $\big(e(t), u_{\mathrm{f}}(t), \tilde{W}(t), \tilde{\Lambda}(t)\big)$ of the closed-loop system are then bounded.

V. Illustrative Numerical Examples

In this section, we show the efficacy of the contributions given in Sections III and IV, where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are considered for (1); $K_1 = [0.16 \quad 0.57]$, $K_2 = 0.16$, and a filtered square-wave reference signal are considered for (2); and $R = I$ is considered for (12). Moreover, the initial conditions are set zero.

A. Parametric Uncertainty Example

In parametric uncertainty example, we set the unknown term $\Lambda = 0.9$, which represents a 10% reduction in control effectiveness and the uncertainty $\delta(x(t)) = 0.2x_1(t) + 0.2x_2(t) + 0.8x_1(t)x_2(t) + 0.1x_1^3(t) + 0.1x_2^2(t)$ for (1), where this uncertainty is regarded as parametric (i.e., $\sigma_\delta(x(t))$ in (8) is known). Figure 1 illustrates the results, where the thick (yellow) line shows the nominal (i.e., ideal) closed-loop system behavior and the thin (green, red, and black) lines show the actual closed-loop system behavior for three different cases.

Specifically, the green line shows the closed-loop system behavior with standard adaptive learning signal and without fixed-gain control signal (i.e., Proposition 2 with $\beta = 1$). It is clear that state and control responses exhibit oscillations. The red line shows the closed-loop system behavior with symbiotic control signal (i.e., Theorem 2 with $\alpha = 1$, $\beta_1 = 1$, $\beta_2 = 1$, and $\gamma = 1$). In this case, state and control responses have less oscillations compared to the former case. The black line shows the closed-loop system behavior again with symbiotic control signal, but with the increased fixed-gain control parameter $\alpha = 3$ motivated by Theorem 1 (i.e., Theorem 2 with $\alpha = 3$, $\beta_1 = 1$, $\beta_2 = 1$, and $\gamma = 1$). This adjustment intends to achieve a smoother closed-loop system behavior that stays close to its nominal performance.

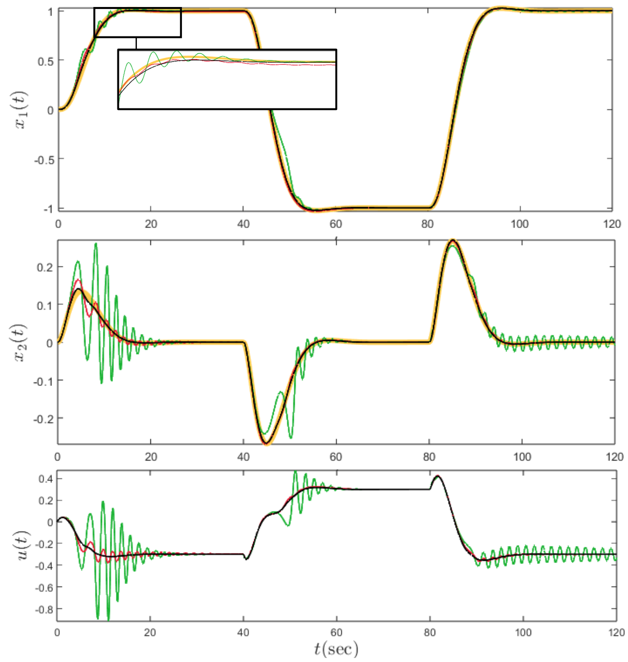

Fig. 1. Closed-loop system behavior under parametric uncertainty.

B. Nonparametric Uncertainty Example

In this example, we set the unknown term $\Lambda = 0.8$, which represents a 20% reduction in control effectiveness and the uncertainty $\delta(x(t)) = 0.4x_1(t) + 0.4x_2(t) + 1.6x_1(t)x_2(t) + 0.2x_1^3(t) + 0.2x_2^2(t)$ for (1), where this uncertainty is regarded as nonparametric (i.e., $\sigma_\delta(x(t))$ in (13) is constructed with a unity bias and 4 radial basis functions over $\mathcal{D} = [-4, 4] \times [-4, 4]$ according to $\sigma_\delta(x(t)) = [1, \mathrm{e}^{-0.5(x_1(t)-1)^2}, \mathrm{e}^{-0.5(x_1(t)+1)^2}, \mathrm{e}^{-0.5(x_2(t)-1)^2}, \mathrm{e}^{-0.5(x_2(t)+1)^2}]^{\mathrm{T}}$). Figure 2 shows the results, where the thick (yellow) line shows the nominal (i.e., ideal) closed-loop system behavior and the thin (green, red, blue, and black) lines show the actual closed-loop system behavior for four different cases.

Specifically, the green line shows the closed-loop system behavior with standard adaptive learning signal and without fixed-gain control signal (i.e., Proposition 3 with $\beta_1 = 1$ and $\beta_2 = 1$). Note that state and control responses show oscillations. The red line shows the closed-loop system behavior also with standard adaptive learning signal and without fixed-gain control signal (i.e., Proposition 3 with $\beta_1 = 1$ and $\beta_2 = 2$). Comparing to the previous example, we have increased the leakage term parameter β_2 to achieve a smoother closed-loop system behavior. However, this leads the resulting behavior to deviate more from the nominal closed-loop system behavior due to the neural network approximation error and high leakage term parameter. The blue line shows the closed-loop system behavior with symbiotic control signal (i.e., Theorem 3 with $\alpha = 3$, $\beta_1 = 1$, $\beta_2 = 1$, $\beta_3 = 2$, $\gamma_1 = 1$, and $\gamma_2 = 2$). In this case, state responses stay close to their ideal ones. Hence, one can conclude that the proposed symbiotic framework has the ability to attain the desired closed-loop system performance, despite the presence of neural network approximation errors and high leakage term parameters. The black line shows the

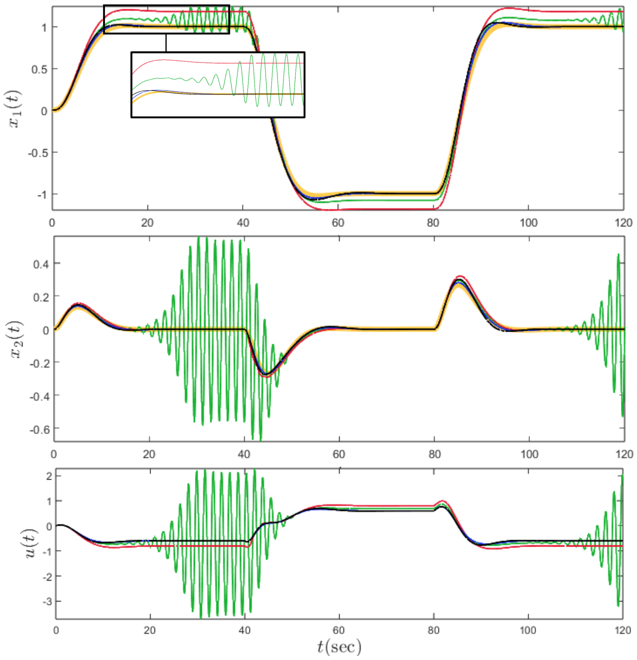

Fig. 2. Closed-loop system behavior under nonparametric uncertainty.

closed-loop system behavior with fixed-gain control signal and without any adaptive learning signal (i.e., Proposition 1 with $\alpha = 9$). Note that we increase α to obtain a similar response to the previous case[5].

VI. Conclusion

In this paper, we present the symbiotic control framework, which integrates the advantages of both fixed-gain control and adaptive learning architectures. In particular, we show how to achieve a more predictable closed-loop system behavior with the fixed-gain control signal (see Theorem 1). Moreover, we consider the adaptive learning signal, which avoids a certain uncertainty structure and an upper bound on this uncertainty. We investigate both parametric (see Theorem 2) and nonparametric (see Theorem 3) uncertainty cases. Alongside the system-theoretical results given in this paper, we also present illustrative numerical examples to show that the proposed symbiotic control framework is capable to obtain the desired closed-loop performance even with an insufficient number of neurons and without a deep neural network method, or in the presence of high leakage term parameters. Future research can focus on implementing the symbiotic control framework in a real-world system.

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[5] While this closed-loop system behavior is remarkable without an adaptive learning signal, we may not know in practice how large α needs to be as noted in Remark 1, and therefore, we include this result here only to support Proposition 1.