2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Simultaneous State and Unknown Input Interval Observer for Discrete-Time Linear Switched Systems Using Peak-to-Peak Analysis

Ghassen Marouani[1], Thach Ngoc Dinh[1], Zhenhua Wang[2], Xubin Ping[3] and Tarek Raïssi[1]

Abstract— **This paper investigates interval observer design for uncertain discrete-time linear switched systems under unknown inputs and a known switching signal. The approach introduces weighting matrices which allow one to relax design difficulties caused by classical coordinate transformation. To improve the accuracy, an L_∞ method minimizing the peak-to-peak gain is employed to reduce the influence of unknown uncertainties. The interval observer gains are computed by solving Linear Matrix Inequality (LMI) formulated based on multiple quadratic Lyapunov functions under average dwell time switching signals.**

I. Introduction

Estimating the current system state is crucial to obtain real-time information on a system for many purposes such as decision-making in model-based prognosis, fault detection, fault diagnosis and fault-tolerant control etc. A common way of addressing this problem is to place some sensors in the physical system and to design an algorithm, called an observer, whose role is to process the incomplete and imperfect information provided by the sensors and thereby to construct a reliable estimate of the whole system state [1]. Classical observers may have inherent weak points: first, they give only point estimate which is not useful for system supervising/monitoring; and second, they are generally sensitive to large deterministic uncertainties. On the other hand, modern industrial systems become more and more complex due to disturbances, measurement noise, unknown parameters, or unknown inputs which are unavoidable during the stage of system operation. Hence, dealing with uncertainties plays an important role in the safety and reliability assessment of dynamical systems.

In the above-mentioned context, interval observers appear to be an efficient tool to deal with systems affected by various types of uncertainties under the assumption that bounds of the initial state values as well as bounds of uncertainties are known. In return for these requirements, interval observers provide an upper and a lower bound of the actual state, which is helpful in monitoring and supervising systems state. This cutting-edge class of observers originating in [2] has been developed in many directions, e.g., interval observer design for linear and nonlinear systems ([3], [4]), and other concerns such as monitoring, fault detection and control purposes ([5], [6] etc).

Switched systems are an extremely momentous set of dynamic hybrid systems which consists of two parts, that is, a family of finite number of continuous or discrete-time subsystems and a switching signal that determines how to switch between these subsystems. The design of interval observers for switched systems has received a great interest ([7], [8], [9]). Most of the above works are based on a coordinate transformation which may cause conservatism. Indeed, it is often hard to design simultaneously observer gains and changes of coordinates ensuring at the same time the non-negativity property of the error dynamics and a good estimation accuracy. Besides noise and disturbances, real systems are often subject to unknown inputs. Such a case has been already investigated for non switched systems (the readers can for instance refer to [10], [11], [12]). Furthermore, some works have considered the case of continuous-time switched systems with unknown inputs ([13], [14]). However, to the best of the authors' knowledge, the problem of simultaneous state and unknown input interval observer for discrete-time linear switched systems has not yet been fully studied in the literature.

The present work continues the previous works [15], [16]. In [15], the design approach is based on the use of two state transformations. The first one is a nonsingular "disturbance-decoupling" state transformation that allows one to decouple the unknown input from the state. The second transformation is performed in order to ensure the non-negativity property of the observation error. Such approach is limited by the fact that coordinate transformation matrices, interval observer gain matrices and the bounds enclosing the unknown input cannot be simultaneously synthesized to fulfill framer property, stability property and other performances such as robust constraints. To handle design difficulty and computational complexity caused by coordinate transformation in unknown input interval observer design, a new structure inspired by [17] which provides more design degrees of freedom is presented in [16]. [16] incorporates an H_∞ technique to attenuate uncertainties in order to obtain tighter intervals. However, the H_∞ norm is a measurement of energy to energy bounds and many signals have only bounded peak values. Thus, the goal of this paper is to complement [16] by proposing the L_∞ norm analysis which describes the peak-to-peak performance index.

The remainder of this paper is organized as follows.

[1]Ghassen Marouani, Thach Ngoc Dinh and Tarek Raïssi are with Conservatoire National des Arts et Métiers (CNAM), Cedric - Laetitia, Rue St-Martin, 75141 Paris Cedex 03, France {ghassen.marouani, ngoc-thach.dinh}@lecnam.net, tarek.raissi@cnam.fr

[2]Zhenhua Wang is with School of Astronautics, Harbin Institute of Technology, West Da-Zhi Street, 150001 Harbin, PR China zhenhua.wang@hit.edu.cn

[3]Xubin Ping is with Department of Automation, School of Electro-Mechanical Engineering, Xidian University, 710075 Xi'an, PR China pingxubin@126.com

Section II gives general prerequisites, formulates the addressed problem, and introduces the so-called TNL framer design (where T, N and L denote the weighting matrices and gain used in this strategy). Peak-to-peak analysis is presented in Section III. Section IV draws comparison studies to assess the efficiency of the proposed simultaneous state and unknown input estimation. Section V gives concluding remarks and perspectives.

II. PRELIMINARIES

A. Notation, definitions, basic result

The set of natural numbers, integers and real numbers are denoted by $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$, respectively. The set of nonnegative real numbers and nonnegative integers are denoted by $\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$ and $\mathbb{Z}_+ = \mathbb{Z} \cap \mathbb{R}_+$, respectively. The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $|x|$, and for a measurable and locally essentially bounded input $u : \mathbb{Z} \to \mathbb{R}$, the symbol $\|u\|_{[t_0,t_1]}$ denotes its L_∞ norm,

$$\|u\|_{[t_0,t_1]} = \sup\{|u|, t \in [t_0, t_1]\}.$$

If $t_1 = \infty$ then we will simply write $\|u\|$. We denote $\mathcal{L}_\infty$ as the set of all inputs u with the property $\|u\| < \infty$. We denote the sequence of integers $1, \ldots, N$ as $\overline{1, N}$. Inequalities must be understood *component-wise*, i.e., for $x_a = [x_{a,1}, ..., x_{a,n}]^\top \in \mathbb{R}^n$ and $x_b = [x_{b,1}, ..., x_{b,n}]^\top \in \mathbb{R}^n$, $x_a \leq x_b$ if and only if, for all $i \in \overline{1, N}$, $x_{a,i} \leq x_{b,i}$. For a square matrix $Q \in \mathbb{R}^{n\times n}$, let the matrix $Q^+ \in \mathbb{R}^{n\times n}$ denote $Q^+ = (\max\{q_{i,j}, 0\})_{i,j=1,1}^{n,n}$, where the notation $Q = (q_{i,j})_{i,j=1,1}^{n,n}$ is used. Let $Q^- \in \mathbb{R}^{n\times n}$ be defined by $Q^- = Q^+ - Q$ and the matrix of absolute values of all elements be defined by $|Q| = Q^+ + Q^-$, the superscripts $+$ and $-$ for other purposes are defined appropriately when they appear. The asterisk $\star$ denotes the symmetric term in a symmetric matrix. A square matrix $Q \in \mathbb{R}^{n\times n}$ is said to be nonnegative if all its entries are nonnegative. I is the identity matrix of appropriate dimension. A positive (res. negative) (semi) definite matrix $P \in \mathbb{R}^{n\times n}$ is denoted as $P \succ (\succcurlyeq)\ 0$ (resp. $P \prec (\preccurlyeq)\ 0$). For a non-square matrix B, the left pseudo-inverse of matrix B is $B^\dagger = (B^T B)^{-1} B^T$.

B. Average dwell time

Definition *1:* [18] For a switching signal σ and any $0 \leq k_l \leq k_s$, let $N_\sigma(k_l, k_s)$ denote the number of discontinuities of σ on the interval $[k_l, k_s)$. If there exist a scalar $\tau_a > 0$ and an integer $N_0 \geq 0$, such that

$$N_\sigma(k_l, k_s) \leq N_0 + \frac{k_s - k_l}{\tau_a} \tag{1}$$

holds for all k_l and k_s, then the scalar $\tau_a > 0$ is called an average dwell time (ADT) and N_0 the chatter bound. In this paper, we assume that $N_0 = 0$ for simplicity as commonly used in the literature.

C. Input to state stability

Input-to-State Stability (ISS) is an approach to analyse the effect of external disturbance on the stability of systems. Lemma 1 gives sufficient conditions on Input-to- State Stability for discrete-time switched systems using multiple Lyapunov function.

Lemma 1: [19] Consider the discrete-time switched system $x(k+1) = f_q(\xi(k), \eta(k))$, $q \in \overline{1, N}$. Suppose that there exists $\mathcal{C}^1$ functions $V_q : \mathbb{R}^n \longrightarrow \mathbb{R}_+$, class $\mathcal{K}_\infty$ functions α_1, α_2, γ and constants $0 < \alpha < 1$, $\mu \geq 1$ such that $\forall \xi \in \mathbb{R}^n$, $\eta \in \mathbb{R}^l$ we have

$$\alpha_1(\|\xi\|) \leq V_q(\xi) \leq \alpha_2(\|\xi\|), \tag{2}$$

$$V_q(\xi(k+1)) - V_q(\xi(k)) \leq -\alpha V_q(\xi(k)) + \varrho(\|\eta\|), \tag{3}$$

and for each switching instant k_l, $l = 0, 1\ 2, 3, \ldots$, for $k = k_l$ the active subsystem is the i^{th} and for $k = k_l^-$ the active subsystem is the j^{th}, then $\forall\ (i,\ j) \in \left(\overline{1, N},\ \overline{1, N}\right)$, $i \neq j$:

$$V_i(\xi) \leq \mu V_j(\xi). \tag{4}$$

Then the system $x(k+1) = f_q(\xi(k), \eta(k))$, $q \in \overline{1, N}$ is Input-to-State Stable for any switching signal satisfying the average dwell time

$$\tau_a \geq \tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)}. \tag{5}$$

Lemma 2: [20] Given matrices $\Xi \in \mathbb{R}^{a\times b}$, $\Psi \in \mathbb{R}^{b\times c}$ and $\Upsilon \in \mathbb{R}^{a\times c}$ with $rank(\Psi) = c$. The general solution Ξ of the equation $\Xi\Psi = \Upsilon$ is

$$\Xi = \Upsilon\Psi^\dagger + S\left(I - \Psi\Psi^\dagger\right) \tag{6}$$

where $S \in \mathbb{R}^{a\times b}$ is an arbitrary matrix.

D. Problem formulation

Consider the following discrete-time linear switched system

$$\begin{cases} x(k+1) = A_q x(k) + B_q u(k) + D_q d(k) + \omega(k), \\ y(k) = Cx(k) + v(k), q \in \overline{1, N},\ N \in \mathbb{N}, \end{cases} \tag{7}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output, $\omega \in \mathbb{R}^n$ and $v \in \mathbb{R}^p$ are respectively the disturbances and the measurement noise. $d \in \mathbb{R}^l$ is the unknown input. A_q, B_q and D_q and C are time-invariant matrices with suitable dimensions. q is the index of the active subsystem. N is the number of subsystems. Some assumptions are introduced.

Assumption 1: The switching signal is assumed to be known.

Assumption 2: The initial condition, the disturbances and the measurement noise are assumed to be bounded such that

$$\underline{x}_0 \leq x(0) \leq \overline{x}_0,\ -\overline{\omega} \leq \omega(k) \leq \overline{\omega},\ -\overline{v} \leq v(k) \leq \overline{v}$$

where $\underline{x}_0$, $\overline{x}_0$, $\overline{\omega} \in \mathbb{R}^n$ and $\overline{v} \in \mathbb{R}^p$ are known vectors.

Assumption 3: Matrices C and D_q for each $q \in \overline{1, N}$, are of full row rank and of full column rank respectively. As a further matter, the dimension of mesurable output y is always greater than or equal to the dimension of the unknown input d.

Most of works in the literature employ change of coordinates to guarantee the non-negativity of error dynamics. However, this relaxing technique can cause some limitations in the choices of optimal gain to ensure at the same time the non-negativity, the stability analysis as well as the accuracy improvement of the proposed bounds. Motivated by [17], the previous work [16] achieves the non-negativity by introducing new matrices, called T, N and L to relax design conditions after defining an auxiliary system which its state vector incorporates the unknown input d and it is given as:

$$\tilde{x}(k+1) = \begin{bmatrix} x(k+1) \\ d(k) \end{bmatrix}, \text{ where } \tilde{x}(0) = \begin{bmatrix} x(0) \\ 0 \end{bmatrix} \quad (8)$$

In fact, (7) can be rewritten as

$$\begin{cases} E_q\tilde{x}(k+1) & = \tilde{A}_q\tilde{x}(k) + \tilde{B}_q u(k) + \tilde{I}\omega(k), \\ y(k) & = \tilde{C}\tilde{x}(k) + v(k), \end{cases} \quad (9)$$

where

$$\tilde{A}_q = \begin{bmatrix} A_q & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B}_q = \begin{bmatrix} B_q \\ 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$
$$E_q = \begin{bmatrix} I & -D_q \\ 0 & 0 \end{bmatrix}, \; \tilde{I} = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

By designing the interval observer of the augmented state $\tilde{x}(k+1)$, i.e., two bounds $\underline{\tilde{x}}(k)$, $\overline{\tilde{x}}(k)$ such that

$$\underline{\tilde{x}}(k) \leq \tilde{x}(k) \leq \overline{\tilde{x}}(k), \forall k \in \mathbb{Z}_+, \quad (10)$$

one can immediately deduce the bounds that enclose the state and the unknown input.

Remark 1: By augmenting the unknown input $d(k)$ as a part of the state vector $\tilde{x}(k+1)$, the structural conditions for decoupling unknown inputs in ([13], [11]) are relaxed. Subsequently, the proposed method possesses a wider application scope than the above-mentioned works.

In the sequel, we reuse the framer candidate given in [16] for the augmented state (9) and we introduce a new criterion of stability such that the framer becomes an interval observer. Usually, it is not very difficult to achieve the framer property, which is the notion of providing intervals in which state variable stay, if one does not care the length of estimated intervals. Thus, sharing similar framers is quite frequent in solving interval estimation design problems.

As a solution to (9), the following framer candidate is considered

$$\begin{cases} \overline{\xi}(k+1) & = T_q\tilde{A}_q\overline{\tilde{x}}(k) + T_q\tilde{B}_q u(k) + L_q(y(k) - \tilde{C}\overline{\tilde{x}}(k)) \\ & +\Delta \\ \overline{\tilde{x}}(k) & = \overline{\xi}(k) + N_q y(k) \\ \underline{\xi}(k+1) & = T_q\tilde{A}_q\underline{\tilde{x}}(k) + T_q\tilde{B}_q u(k) + L_q(y(k) - \tilde{C}\underline{\tilde{x}}(k)) \\ & -\Delta \\ \underline{\tilde{x}}(k) & = \underline{\xi}(k) + N_q y(k) \end{cases} \quad (11)$$

with

$$\Delta = |T_q\tilde{I}|\overline{\omega} + |L_q|\overline{v} + |N_q|\overline{v}, \quad (12)$$

where L_q is an appropriate observer gain associated to the q-subsystem with $q \in \overline{1,N}$ to be computed later.

The matrices T_q, N_q, with $q \in \overline{1,N}$, are computed satisfying the following condition

$$T_q E_q + N_q\tilde{C} \quad = I. \quad (13)$$

Equation (13) can be rewritten as

$$\begin{bmatrix} T_q & N_q \end{bmatrix} \begin{bmatrix} E_q \\ \tilde{C} \end{bmatrix} = I \quad (14)$$

The matrices T_q, N_q are given based on Lemma 2, then the general solution is

$$\begin{bmatrix} T_q \; N_q \end{bmatrix} \quad = I \begin{bmatrix} E_q \\ \tilde{C} \end{bmatrix}^{\dagger} + S_q \left(I - \begin{bmatrix} E_q \\ \tilde{C} \end{bmatrix} \begin{bmatrix} E_q \\ \tilde{C} \end{bmatrix}^{\dagger} \right) \quad (15)$$

with S_q is an arbitrary matrix.

Lemma 3: Let Assumptions 1-3 hold, the lower bound $\underline{\tilde{x}}(k)$ and upper bound $\overline{\tilde{x}}(k)$ for the state $\tilde{x}(k)$ given by (11) satisfy (10), if (13) hold and $(T_q\tilde{A}_q - L_q\tilde{C}) \geq 0$, $\forall\, q \in \overline{1,N}$ provided that $\underline{\tilde{x}}_0 := \begin{bmatrix} \underline{x}(0) \\ 0 \end{bmatrix} \leq \tilde{x}(0) \leq \overline{\tilde{x}}_0 := \begin{bmatrix} \overline{x}(0) \\ 0 \end{bmatrix}$.

Proof 1: The proof is similar to the one in [16].

Remark 2: The main difference between the approach used in literature and the one presented in (11) is the introduction of additional parameters T_q, N_q in the framer structure. If we choose $T_q = I$ and $N_q = 0$ for all $q \in \overline{1,N}$, (10) reduces to the interval observer presented in [9].

III. Interval observer design using peak-to-peak analysis

The novelty of this paper is to introduce stability criteria based on peak-to-peak analysis which are different from those presented in [16]. This section is focused on L_q gain computation to ensure both the ISS of the estimation errors and the non-negativity of the matrices $T_q\tilde{A}_q - L_q\tilde{C}$.

Let define the estimation error as follows

$$e(k) = \overline{e}(k) - \underline{e}(k) \quad (16)$$

where $\overline{e}(k) = \overline{\tilde{x}}(k) - \tilde{x}(k)$, $\underline{e}(k) = \underline{\tilde{x}}(k) - \tilde{x}(k)$ are , respectively, the upper and the lower observation errors. Thus,

$$e(k+1) = (T_q\tilde{A}_q - L_q\tilde{C})e(k) + \Phi_q\delta(k) \quad (17)$$

with

$$\delta(k) = \begin{bmatrix} -T_q\tilde{I}\omega(k) \\ v(k) \\ v(k+1) \end{bmatrix} \quad (18)$$

and

$$\Phi_q = 2\begin{bmatrix} I & L_q & N_q \end{bmatrix} \quad (19)$$

Next, the conditions ensuring the ISS of the error system (16) in sense of Lemma 1 under an L_∞ criterion are presented. Sufficient design conditions are given in the following theorem.

Theorem 1: Let Assumptions 1-3 hold. For a given $0 < \alpha < 1$, if there exist positive scalars $\alpha_2 > \alpha_1 > 0$, $\gamma > 0$, diagonal $n \times n$ matrices $P_q \succ 0$, and matrices $W_l \in \mathbb{R}^{n\times n}$, $G_q \in \mathbb{R}^{n\times p}$, $H_q \in \mathbb{R}^{n\times(n+p)}$ such that

$$P_q\ \Theta_q^{\dagger}\lambda_1\tilde{A}_q + H_q\psi_q\lambda_1\tilde{A}_q - G_q\tilde{C} \geq 0,\ \forall\ q \in \overline{1,N}, \quad (20)$$

$$\alpha_1 I_n \preceq P_q \preceq \alpha_2 I_n,\ \forall\ q \in \overline{1,N}\ , \quad (21)$$

$$\begin{bmatrix} W_l & P_q \\ P_q & P_q \end{bmatrix} \succeq 0,\ \forall q, l \in \overline{1,N}, \quad (22)$$

$$\begin{bmatrix} (\alpha-1)P_q & 0 & 0 & 0 & \star \\ \star & -\rho I & 0 & 0 & \star \\ \star & \star & -\rho I & 0 & \star \\ \star & \star & \star & -\rho I & \star \\ \kappa_{1q} & 2P_q & 2G_q & 2\kappa_{2q} & -P_q \end{bmatrix} \prec 0, \quad (23)$$

$$\begin{bmatrix} \alpha P_q & 0 & I \\ \star & (\gamma-\rho)I & 0 \\ \star & \star & \gamma I \end{bmatrix} \succ 0, \quad (24)$$

with

$$W_l = \mu P_l,\ G_q = P_q L_q,\ H_q = P_q S_q,$$

$$\kappa_{1q} = P_q\ \Theta_q^{\dagger}\lambda_1\tilde{A}_q + H_q\psi_q\lambda_1\tilde{A}_q - G_q\tilde{C},$$
$$\kappa_{2q} = P_q\ \Theta_q^{\dagger}\lambda_2 + H_q\psi_q\lambda_2,\ \forall q \in \overline{1,N},$$

where

$$\Theta_q = \begin{bmatrix} E_q \\ \tilde{C} \end{bmatrix}, \lambda_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}, \lambda_2 = \begin{bmatrix} 0 \\ I \end{bmatrix},$$
$$\psi_q = I - \Theta_q\Theta_q^{\dagger},\ \forall q \in \overline{1,N},$$

then, the gains of the interval observer can be computed by

$$L_q = P_q^{-1}G_q,\ \forall q \in \overline{1,N}, \quad (25)$$

via the solution of the following constrained minimization problem

$$\begin{aligned} &\underset{P_q, G_q, H_q}{\text{minimize}} && \beta\mu + (1-\beta)\rho,\quad \forall q \in \overline{1,N}, \beta \in [0\ 1] \\ &\text{subject to} && (20), (21), (22), (23), (24). \end{aligned} \quad (26)$$

Additionally, the framer (11) is an interval observer for (7). The estimation error (16) have the following L_∞ performances

$$\|e(k)\| \leq \sqrt{\gamma\left(\alpha\,(1-\alpha)^k V_q(e(0)) + \gamma\|\delta\|_\infty^2\right)}. \quad (27)$$

Proof 2: Consider the multiple Lyapunov function

$$V_q(k) = e^T(k)P_q e(k) \quad (28)$$

where P_q are diagonal positive definite matrices for all $q \in \overline{1,N}$. The increment of the Lyapunov function (28) is

$$\Delta V_q(e) = V_q(e(k+1)) - V_q(e(k)) \quad (29)$$

Thus,

$$\Delta V_q(e(k)) = \begin{bmatrix} e(k) & \delta(k) \end{bmatrix}^T \Upsilon_q \begin{bmatrix} e(k) & \delta(k) \end{bmatrix} \quad (30)$$

where

$$\Upsilon_q = \begin{bmatrix} \Pi_q^T P_q \Pi_q - P_q & \Pi_q^T P_q \Phi_q \\ \Phi_q^T P_q \Pi_q & \Phi_q^T P_q \Phi_q \end{bmatrix}. \quad (31)$$

and

$$\Pi_q = T_q\tilde{A}_q - L_q\tilde{C}.$$

Note that inequality (23) can be rewritten as

$$\begin{bmatrix} (\alpha-1)P_q & 0 & \Pi_q^T P_q \\ 0 & -\rho I & \Phi_q^T P_q \\ P_q\Pi_q & P_q\Phi_q & -P_q \end{bmatrix} \prec 0. \quad (32)$$

By pre- and post- multiplying (32) with

$$\begin{bmatrix} I & 0 & \Pi_q^T \\ 0 & I & \Phi_q^T \end{bmatrix} \quad (33)$$

and its transpose respectively, we obtain

$$\Upsilon_q + \begin{bmatrix} \alpha P_q & 0 \\ 0 & -\rho I \end{bmatrix} \prec 0. \quad (34)$$

Let post- and pre- multiply equation (34) by $\begin{bmatrix} e^T(k) & \delta^T(k) \end{bmatrix}$ and its transpose, we have

$$\Delta V_q(e(k)) \leq -\alpha V_q(e(k)) + \rho\ \delta^T(k)\delta(k). \quad (35)$$

Then, the condition (3) presented in Lemma 1 is ensured.

From (35), we deduce that

$$V_q(e(k+1)) \leq (1-\alpha)V_q(e(k)) + \rho\|\delta\|_\infty^2. \quad (36)$$

Consequently,

$$\begin{aligned} V_q(e(k)) &\leq (1-\alpha)^k V_q(e(0)) + \rho \sum_{i=0}^{i=k-1} (1-\alpha)^i \|\delta\|_\infty^2 \\ &\leq (1-\alpha)^k V_q(e(0)) + \rho\frac{(1-\alpha^k)}{\alpha}\|\delta\|_\infty^2 \\ &\leq (1-\alpha)^k V_q(e(0)) + \frac{\rho}{\alpha}\|\delta\|_\infty^2. \end{aligned} \quad (37)$$

From Schur complement, the inequality (24) can take the following form

$$\begin{bmatrix} \alpha P_q & 0 \\ 0 & (\gamma-\rho)I \end{bmatrix} - \frac{1}{\gamma}\begin{bmatrix} I \\ 0 \end{bmatrix}\begin{bmatrix} I & 0 \end{bmatrix} \succ 0. \quad (38)$$

By pre-multiplying and post-multiplying inequality (38) with $\begin{bmatrix} e^T(k) & \delta^T(k) \end{bmatrix}$ and its transpose respectively, we arrive at

$$e^T(k)e(k) \leq \gamma\left(\alpha V_q(e(k)) + (\gamma-\rho)\|\delta\|_\infty^2\right). \quad (39)$$

Taking in mind (37), then the inequality (39) can be alternatively written

$$e^T(k)e(k) \leq \gamma \left(\alpha (1-\alpha)^k V_q(e(0)) + \gamma \|\delta\|_\infty^2 \right). \quad (40)$$

Hence,

$$\|e\| \leq \sqrt{\gamma \left(\alpha (1-\alpha)^k V_q(e(0)) + \gamma \|\delta\|_\infty^2 \right)} \quad (41)$$

which implies that $e(k)$ satisfies the L_∞ performance.

In addition, let inequality (2) hold, then

$$\alpha_1(\|e(k)\|) \leq V_q(e(k)). \quad (42)$$

From (37) and (42), the following inequality is satisfied

$$\|e(k)\| \leq \frac{1}{\alpha_1} \left[(1-\alpha)^k V_q(e(0)) + \frac{\rho}{\alpha} \|\delta\|_\infty^2 \right]. \quad (43)$$

Thus,

$$\lim_{k \longrightarrow \infty} \|e(k)\| < \frac{\rho}{\alpha \alpha_1} \|\delta\|_\infty^2. \quad (44)$$

Through the inequality (44), we can deduce that the interval error width is bounded by $\dfrac{\rho}{\alpha \ \alpha_1}\|\delta\|_\infty^2$. Note that for given α_1 and α_2, (44) depends only on ρ. Accordingly, the minimization of ρ allows us to improve the estimation accuracy. Furthermore, we aim to minimize μ in order to obtain an optimum dwell time. For these purposes, we add the following objective function

$$\beta\mu + (1-\beta)\rho, \text{ with } \beta \in [0 \ 1]. \quad (45)$$

Moreover, by using LMI conditions (20), (21) and (22), one can conclude that the ISS conditions presented in Lemma 1 are verified for $e = \overline{e} - \underline{e}$ (see [16, Theorem 2]). Therefore the framer (11) is an interval observer for (7) with L_∞ criterion.

IV. Comparison Studies

Given the system (7) with three modes ($N = 3$) where

$$A_1 = \begin{bmatrix} 0.55 & 0.5 & 0.7 \\ 0 & 0.8 & 0.5 \\ 0 & 0 & 0.4 \end{bmatrix}, A_2 = \begin{bmatrix} -0.44 & -0.4 & -0.56 \\ 0 & -0.64 & -0.4 \\ 0 & 0 & -0.32 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.1 & 1 & 1 \\ 0 & 0.2 & -0.5 \\ 0 & 0 & 0.2 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 0.5 \\ 0.7 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.4 \\ 0.6 \\ 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.1 \\ 0.0 \\ 0.1 \end{bmatrix}, \ D_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \ D_2 = \begin{bmatrix} 1 \\ 0 \\ 4.73 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

$w(k)$ and $v(k)$ are respectively the disturbances and the measurement noises which are uniformly bounded such that $-\overline{w} \leq w(k) \leq \overline{w}$ with $\overline{w} = \begin{bmatrix} 0.6 & 0.6 & 0.6 \end{bmatrix}$, and $-\overline{v} \leq v(k) \leq \overline{v}$ with $\overline{v} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}$. The unknown input is given as $d(k) = 0.5\sin(0.5k)$.

We verify that Assumptions 1-2 hold. The matrices T_q and N_q are determined as follows

$$T_1 = \begin{bmatrix} 0.7293 & -0.2293 & -0.2707 & 0 \\ -0.4227 & 0.4227 & -0.4227 & 0 \\ -1.0717 & 0.5717 & -0.0717 & 0 \\ -0.0087 & -0.4913 & -0.0087 & 0 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 0.8253 & -0.0932 & -0.1745 & 0 \\ -0.0001 & 0.0675 & 0 & 0 \\ -0.8256 & 0.1249 & 0.1746 & 0 \\ -0.1747 & 0.1965 & -0.1744 & 0 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} 0.3926 & 0.4295 & -0.6074 & 0 \\ -0.0083 & 0.0331 & -0.0083 & 0 \\ -0.3944 & -0.4223 & 0.6056 & 0 \\ -0.3299 & -0.6804 & -0.3229 & 0 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.2293 & 0.2707 \\ 0.5773 & 0.4227 \\ -0.5717 & 1.0717 \\ 0.4913 & 0.0087 \end{bmatrix}, N_2 = \begin{bmatrix} 0.0931 & 0.1745 \\ 0.9322 & 0 \\ -0.1248 & 0.8253 \\ -0.1964 & 0.1744 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} -0.4295 & 0.6074 \\ 0.9669 & 0.0083 \\ 0.4223 & 0.3944 \\ 0.6804 & 0.3299 \end{bmatrix}.$$

Let Theorem 1 hold for $\alpha = 0.9$ and $\alpha_1 = 0.1$, then the interval observer gains are computed by $L_q = P_q^{-1}G_q$ with

$$L_1 = \begin{bmatrix} 0.1812 & 0.2876 \\ 0.1268 & -0.2536 \\ -0.1128 & -0.5922 \\ -0.3974 & -0.2552 \end{bmatrix}, L_2 = \begin{bmatrix} -0.3304 & -0.4271 \\ -0.0984 & -0.0722 \\ 0.1934 & 0.3024 \\ -0.1231 & 0.0077 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 0.3864 & -0.0422 \\ -0.0303 & -0.0498 \\ -0.5092 & -0.0879 \\ -0.4935 & -0.0788 \end{bmatrix}.$$

We get $\mu = 1.3446$ which leads to an average dwell time $\tau_a > 0.1286$ and $\rho = 1.2615$.

The switched signal verifying the average dwell time is plotted in Figure 1. The intervals that enclose the state components and the unknown input by using the technique in [15] and in the present paper are depicted respectively in Figures 2 and 3. Under the same simulation conditions, the interval observer using peak-to-peak analysis provides more accurate interval estimation than the approach presented in [15]. This result is explained by the fact that the coordinate transformation used in [15] to design the unknown input interval observer leads to more conservatism so the choice of gains is more limited. Furthermore, the use of peak-to-peak analysis allows to attenuate the effect of disturbances and noise measurement. As a consequence, it makes the frame tighter.

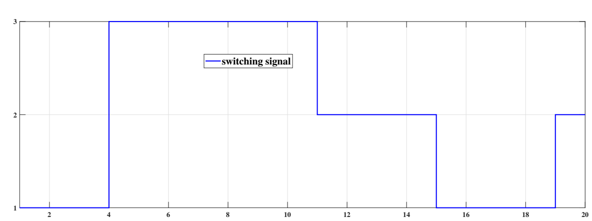

Fig. 1. The switching signal

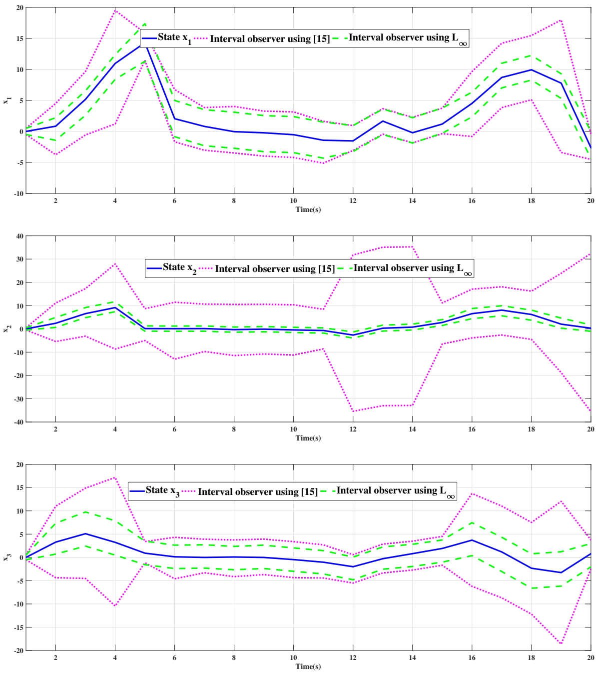

Fig. 2. State and estimated bounds

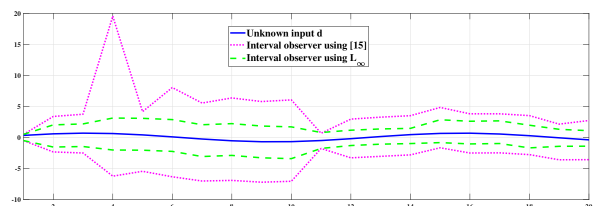

Fig. 3. The unknown input

V. Conclusion

A simultaneous state and unknown input interval observer for discrete-time linear switched systems is addressed in this paper. We incorporate TNL design introducing flexible weighting matrices and gain with peak-to-peak analysis to provide not only the non-negativity property and the stability of estimation errors but also the accuracy improvement. The design conditions are formulated in terms of LMIs. Simulation results illustrate the effectiveness of the proposed methods. The study of switched systems under an unknown switching rule or the extension of those results to Linear-Parameter Varying switched systems are promising directions for future works.

References

[1] P. Bernard, *Observer design for nonlinear systems*. Springer, 2019, vol. 479.

[2] J.-L. Gouze, A. Rapaport, and M. Z. Hadj-Sadok, "Interval observers for uncertain biological systems," *Ecological modelling*, vol. 133, no. 1-2, pp. 45–56, 2000.

[3] F. Mazenc, T. N. Dinh, and S. I. Niculescu, "Interval observers for discrete-time systems," *International journal of robust and nonlinear control*, vol. 24, no. 17, pp. 2867–2890, 2014.

[4] T. Raïssi, D. Efimov, and A. Zolghadri, "Interval state estimation for a class of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 1, pp. 260–265, 2012.

[5] F. Xu, V. Puig, C. Ocampo-Martinez, F. Stoican, and S. Olaru, "Actuator-fault detection and isolation based on set-theoretic approaches," *Journal of Process Control*, vol. 24, no. 6, pp. 947–956, 2014.

[6] D. Efimov, T. Raïssi, and A. Zolghadri, "Control of nonlinear and lpv systems: interval observer-based framework," *IEEE Transactions on Automatic Control*, vol. 58, no. 3, pp. 773–778, 2013.

[7] N. Otsuka and D. Kakehi, "Interval switched positive observers for continuous-time switched positive systems under arbitrary switching," in *5th IFAC Conference on Intelligent Control and Automation Sciences*, vol. 52. IFAC, 2019, pp. 250–255.

[8] Y.-E. Wang, H. R. Karimi, and D. Wu, "Construction of hybrid interval observers for switched linear systems," *Information Sciences*, vol. 454, pp. 242–254, 2018.

[9] T. N. Dinh, G. Marouani, T. Raïssi, Z. Wang, and H. Messaoud, "Optimal interval observers for discrete-time linear switched systems," *International Journal of Control*, vol. 93, no. 11, pp. 2613–2621, 2020.

[10] N. Ellero, D. Gucik-Derigny, and D. Henry, "An unknown input interval observer for lpv systems under l-2-gain and l-infinity-gain criteria," *Automatica*, vol. 103, pp. 294–301, 2019.

[11] E. I. Robinson, J. Marzat, and T. Raïssi, "Interval observer design for unknown input estimation of linear time-invariant discrete-time systems," in *20th IFAC World Congress, Toulouse, France*, 2017, pp. 4021–4026.

[12] J. Li, T. Raïssi, Z. Wang, X. Wang, and Y. Shen, "Interval estimation of state and unknown input for linear discrete-time systems," *Journal of the Franklin Institute*, vol. 357, no. 13, pp. 9045–9062, 2020.

[13] D. Gucik-Derigny, T. Raïssi, and A. Zolghadri, "A note on interval observer design for unknown input estimation," *International Journal of Control*, vol. 89, no. 1, pp. 25–37, 2016.

[14] S. Ifqir, D. Ichalal, N. A. Oufroukh, and S. Mammar, "Robust interval observer for switched systems with unknown inputs: Application to vehicle dynamics estimation," *European Journal of Control*, vol. 44, pp. 3–14, 2018.

[15] G. Marouani, T. N. Dinh, T. Raïssi, X. Wang, and H. Messaoud, "Unknown input interval observers for discrete-time linear switched systems," *European Journal of Control*, vol. 59, pp. 165–174, 2021.

[16] G. Marouani, T. N. Dinh, N. Sehli, T. Raïssi, Z. Wang, and H. Messaoud, "H_∞ interval observer design for uncertain discrete-time linear switched systems with unknown inputs," in *5th International Conf. on Control, Automation and Diagnosis, Grenoble, France*, 2021, pp. 1–6.

[17] Z. Wang, C.-C. Lim, and Y. Shen, "Interval observer design for uncertain discrete-time linear systems," *Systems & Control Letters*, vol. 116, pp. 41–46, 2018.

[18] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *38th IEEE Conf. on Decision and Control, AZ, USA*, 1999, pp. 2655–2660.

[19] K. Zhu, Y. Song, D. Ding, G. Wei, and H. Liu, "Robust mpc under event-triggered mechanism and round-robin protocol: An average dwell-time approach," *Information Sciences*, vol. 457, pp. 126–140, 2018.

[20] C. R. Rao, S. K. Mitra, *et al.*, "Generalized inverse of a matrix and its applications," in *Proceedings of the sixth Berkeley symposium on mathematical statistics and probability*, vol. 1. University of California Press Oakland, CA, USA, 1972, pp. 601–620.