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Event-triggered Pricing-based Frequency Control in Power System via Passivity Analysis

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Abstract— **This study addresses the problem of frequency control in a power system with the participation of demand response (DR). The pricing-based controller is applied to the power system so that DR participants and suppliers can contribute to the frequency regulation via electricity price. However, the high frequency of communication between frequency dynamics and pricing-based controllers is thought to be a problem. To address this problem, we derive event-triggered conditions that reduce the communication frequency while maintaining stability based on the passivity property of the system, and the asymptotical stability around the equilibrium point of the overall system is derived under the proposed event-triggered conditions. Furthermore, numerical simulation with simplified power network system is conducted so the effectiveness of the proposed system design and system stability is confirmed.**

I. INTRODUCTION

Renewable energy resources have been introduced into intermittent power systems and their generations are difficult to predict, which can result in severe power fluctuations. Therefore, load frequency control (LFC), the method of adjusting the energy balance via frequency regulation, is essential to suppress the fluctuations. Meanwhile, demand response (DR) where consumers control their electricity consumption and adjust the balance between supply and demand attracts attention to the development of distributed energy resources. DR aims to make effective use of distributed energy resources.

These situations lead to numerous studies on the optimal operation of power systems with controllable loads such as DR and generators. For example, in [1], frequency control has been proposed for power systems, including electric vehicles and DRs. In addition, studies on optimal frequency control based on generation costs also take into account consumer utility [2].

However, in the literature, the electricity consumption of DR participants is determined by the controller of the power system. In the case of actual operation, consumers should make their own decisions. One method to solve this issue is to use the pricing-based control method [3]. The concern of pricing-based control for LFC systems is that the power systems will become complex and heterogeneous, integrating physical systems and human systems. This means that stability analysis is necessary for such a heterogeneous system. Additionally, differences in timescales between physical systems and human systems also need to be addressed.

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One of the methods to analyze the stability of such complex systems is to use passivity and passivity-short, which is a relaxation of passivity [4]. Passivity is deeply related to the system stability and its modularity makes it possible to analyze the passivity and stability of the whole system by analyzing its subsystem. There have been many studies on the control of power systems using passivity [5], [6]. In [7], the passivity framework is performed for an LFC system that considers the profit of the consumer and supplier, the difference in time scale is not discussed. For practicality, it is desirable to reduce the frequency of demand/supply updates.

The objective of this paper is to reduce the frequency of decision updating for DR participants' and suppliers' model while ensuring system stability. To achieve this objective, we design a pricing-based controller using event-based triggers to control DR participants and suppliers.

The main contributions of this paper are as follows: i) Design the LFC system considering the DR so that the system stability and the optimality of the profit maximization problem of each area are guaranteed. ii) Derive the event-triggered conditions that guarantee the asymptotical stability of the equilibrium for the proposed load-frequency control system through the passivity property, thereby reducing the decision-making frequency in the pricing-based controller.

II. PROBLEM FORMULATION

Fig. 1 shows a model of a power network system including DR participants in this study. Suppose that multiple suppliers, who have generators, and DR participants, who behave as energy consumers, exist in each area connected by transmission lines. Those players determine their electricity operations via electricity price provided by an Independent system operator (ISO) serving as a power grid manager in each area. LFC is done through these electricity operations by suppliers and DR participants. Note that, in the rest of this paper, $\mathcal{A}$ denotes a set of areas in this power network system, $\mathcal{E}$ denotes a set of links between each area, $\mathcal{G}_i$ and $\mathcal{L}_i$ denote a set of suppliers and DR participants in area $i \in \mathcal{A}$ respectively. Also, $\|\cdot\|$ denotes the number of elements of an arbitrary set.

In this study, the following assumptions are made for the power network systems:

Assumption 1: The power grid is assumed to satisfy the following properties: i) The transmission lines are lossless. ii) The voltage of each node is approximately equal to 1 [p.u.]. iii) Reactive power flows do not affect bus voltage phase angles and frequencies. iv) The voltage phase difference between each node is sufficiently small.

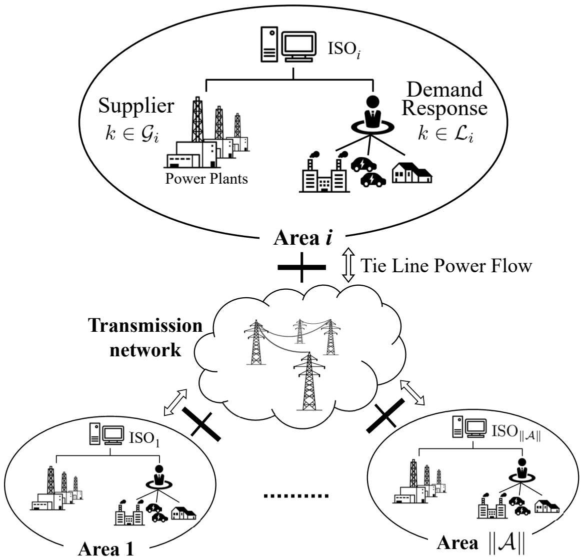


Fig. 1: Structure of Power System with DR

These assumptions are widely used in studies in the field of frequency regulation such as [8].

Under the above assumptions, the power network system is formulated as follows:

$$M_i\dot{\omega}_i = -D_i\omega_i + \sum_{k\in\mathcal{G}_i} P^k_{M,i} - \sum_{k\in\mathcal{L}_i} P^k_{DR,i} + P_{tie,i} + \Delta P_{L,i}, \tag{1}$$

$$\dot{\eta}_{ij} = \omega_i - \omega_j, \ \forall (i,j) \in \mathcal{E}, \tag{2}$$

$$P_{tie,i} = -\sum\nolimits_{j\in\mathcal{N}_i} T_{ij}\eta_{ij}, \ \forall i \in \mathcal{A}. \tag{3}$$

Equations (1)–(3) represent the swing dynamics where ω_i is the deviation from the nominal value of the frequency in area i. The time-dependent variable $P^k_{M,i}$ and $P^k_{DR,i}$ represent the mechanical power input of k–th supplier in area i and the electricity consumption of k–th DR participant in area i respectively. $P_{tie,i}$ represents the tie-line power from area i's neighbor, where the set of neighbors being defined as $\mathcal{N}_i$, and $\Delta P_{L,i}$ indicates load change in area i. M_i and D_i are inertia constant and damping constant of area i's generator respectively. η_{ij} represents the power angle difference and T_{ij} is transmission coefficient.

Each generator is supposed to be equipped with a governor controller, hence we use first-order turbine model for each supplier such that:

$$\tau^k_i \dot{P}^k_{M,i} = -P^k_{M,i} + u^k_{G,i} - \frac{1}{R^k_i}\omega_i, \ \forall k \in \mathcal{G}_i \forall i \in \mathcal{A} \tag{4}$$

where τ^k_i and R^k_i represent time constant and droop gain of the governor operation respectively. $u^k_{G,i}$ is the variable of k–th supplier's input command to the turbine.

III. PRICING-BASED CONTROLLER DESIGN

In the study of optimal frequency control, generator operation cost for suppliers should be minimized and the profit of consumers should be maximized [9]. Therefore, the following optimization problem is considered while letting $c^k_i(\cdot)$ be the cost function of k–th supplier in area i and $v^k_i(\cdot)$ be the utility function of k–th DR participant in area i.

$$\max_{P^k_{DR,i}, u^k_{G,i}} \quad \sum_{k\in\mathcal{L}_i} v^k_i(P^k_{DR,i}) - \sum_{k\in\mathcal{G}_i} c^k_i(u^k_{G,i}), \ \forall i \in \mathcal{A}, \tag{5}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{G}_i} u^k_{G,i} - \sum_{k\in\mathcal{L}_i} P^k_{DR,i} = -\Delta P_{L,i}, \ \forall i \in \mathcal{A}, \tag{6}$$

$$P^{k,min}_{DR,i} \le P^k_{DR,i} \le P^{k,max}_{DR,i}, \quad \forall k \in \mathcal{L}_i \ \forall i \in \mathcal{A}, \tag{7}$$

$$u^{k,min}_{G,i} \le u^k_{G,i} \le u^{k,max}_{G,i}, \quad \forall k \in \mathcal{G}_i \ \forall i \in \mathcal{A}. \tag{8}$$

The constraint given by (6) indicates energy balance constraints. Observing that, load change $\Delta P_{L,i}$ is supplemented by both suppliers and DR participants. The inequality constraints (7) and (8) indicate the lower/upper bounds of supplier's generation and DR electricity consumption. For the cost function and utility function, we assume:

Assumption 2: Utility function $v^k_i(\cdot)$ of k–th consumer in area i is in $C^2[0, P^{k,max}_{L,i}]$ and is $\alpha^k_{v,i}$–strongly concave

Assumption 3: Cost function $c^k_i(\cdot)$ of k–th supplier in area i is in $C^2[0,\infty]$ and is $\alpha^k_{c,i}$–strongly convex.

These assumptions make (5) a convex optimization problem.

Generally, by using the dual decomposition method, optimization problem (5) can be solved separately as a consumer's problem, a supplier's problem, and Lagrange multipliers update. Especially, in the study of the power market design, it is well known that Lagrange multiplier for the energy balance is equivalent to electricity price [6], [8]. This study also utilizes this pricing-based method as the LFC to consider both DR participants' and suppliers' decision-making problems as shown in many kind of literatures [10]. The following Fig. 2 shows the structure of one area LFC system implementing pricing-based controller. As shown in

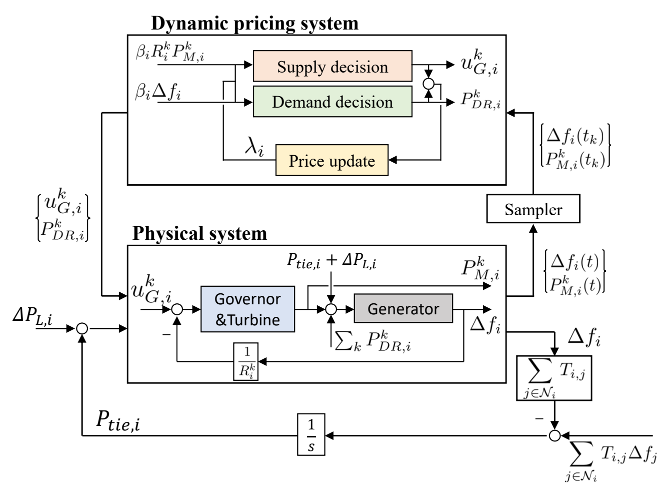


Fig. 2: One area model description

Fig. 2, the plant, called "physical system", includes the governor-turbine model and the generator model described in (1),(4). Tie-line power flow $P_{tie,i}$ is also modeled according to (2),(3). The controller, called "dynamic pricing system", consists of an update of the electricity price λ_i by ISO, the demand decision systems by DR participants and the supply decision systems by suppliers. Each DR participant and supplier determines its electricity consumption and generation depending on the electricity price and state of the physical system. Additionally, the input to the dynamic pricing system is updated according to the event-triggered condition as shown in Fig. 2, where t indicates the current time, and t_k is the event time, when the trigger condition is satisfied.

We propose the following dynamic pricing system to ensure that its equilibrium point satisfies the optimal solution of (5) and to guarantee the stability of the system under the physical dynamics (1)–(4).

A. DR participants' demand decision system

The behavior of k–th DR participant is formulated as follows:

$$\dot{P}^k_{DR,i} = \alpha^k_{DR,i}\Big(\nabla v^k_i(P^k_{DR,i}) - \lambda_i + \mu^{k,-}_{DR,i} - \mu^{k,+}_{DR,i} + \beta_i\Delta f_i\Big), \quad k\in\mathcal{L}_i\ i\in\mathcal{A}, \tag{9}$$

$$\dot{\mu}^{k,-}_{DR,i} = k_{\mu^{k,-}_{DR,i}}\left(P^{k,\min}_{DR,i} - P^k_{DR,i}\right)^+_{\mu^{k,-}_{DR,i}}, k\in\mathcal{L}_i\ i\in\mathcal{A}, \tag{10}$$

$$\dot{\mu}^{k,+}_{DR,i} = k_{\mu^{k,+}_{DR,i}}\left(P^k_{DR,i} - P^{k,\max}_{DR,i}\right)^+_{\mu^{k,+}_{DR,i}}, k\in\mathcal{L}_i\ i\in\mathcal{A} \tag{11}$$

where $\alpha^k_{DR,i}$, β_i, $k_{\mu^{k,-}_{DR,i}}$ and $k_{\mu^{k,+}_{DR,i}}$ are positive gains. $\mu^{k,-}_{DR,i}$ and $\mu^{k,+}_{DR,i}$ are the Lagrange multipliers, respectively, for the lower and upper bounds on the electricity consumption. In equation (10) and (11), the operator $(\cdot)^+_x$ is defined, with arbitrary functions $f(x), x\geq 0$, as follows:

$$(f(x))^+_x := \begin{cases} f(x), & \text{if } x>0, \text{ or } x=0 \text{ and } f(x)\geq 0,\\ 0, & \text{if } x=0 \text{ and } f(x)<0.\end{cases} \tag{12}$$

B. Supplier's power generation decision system

The behavior of k–th supplier is formulated as follows:

$$\dot{u}^k_{G,i} = -\alpha^k_{G,i}\Big(\nabla c^k_i(u^k_{G,i}) - \lambda_i - \mu^{k,-}_{G,i} + \mu^{k,+}_{G,i} - \beta_i R^k_i u^k_{G,i} + \beta_i R^k_i P^k_{M,i}\Big), \quad k\in\mathcal{G}_i\ i\in\mathcal{A}, \tag{13}$$

$$\dot{\mu}^{k,-}_{G,i} = k_{\mu^{k,-}_{G,i}}\left(u^{k,\min}_{G,i} - u^k_{G,i}\right)^+_{\mu^-_{G,i}}, k\in\mathcal{G}_i\ i\in\mathcal{A}, \tag{14}$$

$$\dot{\mu}^{k,+}_{G,i} = k_{\mu^{k,+}_{G,i}}\left(u^k_{G,i} - u^{k,\max}_{G,i}\right)^+_{\mu^{k,+}_{G,i}}, k\in\mathcal{G}_i\ i\in\mathcal{A} \tag{15}$$

where $\alpha^k_{G,i}$, $k_{\mu^{k,-}_{G,i}}$ and $k_{\mu^{k,+}_{G,i}}$ are positive gains. $\mu^{k,-}_{G,i}$ and $\mu^{k,+}_{G,i}$ are the Lagrange multipliers, respectively, for the lower and upper bounds on the power generation.

C. ISO electricity price updating system

The electricity price updating algorithm by ISO is described as follows:

$$\dot{\lambda}_i = -\Big(\sum\nolimits_{k\in\mathcal{G}_i} u^k_{G,i} - \sum\nolimits_{k\in\mathcal{L}_i} P^k_{DR,i} + \Delta P_{L,i}\Big). \tag{16}$$

Remark 1: In equation (9) and (13), the terms of $\beta_i\Delta f_i$ and $\beta_i R^k_i P^k_{M,i}$ are added to general gradient method dynamics so that a closed-loop is formed between the physical system and the dynamic pricing system. An advantage of this design is that it does not disturb the convergence of the equilibrium point, which is the same as the optimal solution guaranteed by the gradient method.

IV. PASSIVITY ANALYSIS

In this section, passivity analysis is performed for the LFC system considering DR and the pricing-based control scheme described in the previous sections. Firstly, we establish the passivity of the physical system through the following lemma. Note that, in the following the optimal solution for each variable is represented using a respective superscript $(\cdot)^*$. Additionally, $\tilde{(\cdot)}$ indicates difference between an arbitrary variable from its optimal solution such as $\tilde{x} := x - x^*$.

Lemma 1: Physical system in area $i\in\mathcal{A}$ described by (1) and (4) is output strictly passive under the inequality (18) with positive definite storage function $V_{phy,i}$, input $\boldsymbol{u}_{phy,i}\in\mathbb{R}^{\|\mathcal{G}_i\|+1}$ and output $\boldsymbol{y}_{phy,i}\in\mathbb{R}^{\|\mathcal{G}_i\|+1}$, defined as follows:

$$\boldsymbol{u}_{phy,i}=\begin{bmatrix} -\sum_{k\in\mathcal{L}_i}\tilde{P}^k_{DR,i}+\tilde{P}_{tie,i}\\ \tilde{u}^1_{G,i}\\ \vdots\\ \tilde{u}^{\|\mathcal{G}_i\|}_{G,i}\end{bmatrix}, \boldsymbol{y}_{phy,i}=\begin{bmatrix}\beta_i\Delta\tilde{f}_i\\ \beta_i R^1_i\tilde{P}^1_{M,i}\\ \vdots\\ \beta_i R^{\|\mathcal{G}_i\|}_i\tilde{P}^{\|\mathcal{G}_i\|}_{M,i}\end{bmatrix}. \tag{17}$$

$$\dot{V}_{phy,i} \leq \boldsymbol{u}^\top_{phy,i}\boldsymbol{y}_{phy,i} - \frac{D_i}{\beta_i}(\beta_i\Delta\tilde{f}_i)^2 - \sum_{k\in\mathcal{G}_i}\frac{1}{\beta_i R^k_i}(\beta_i R^k_i\tilde{P}^k_{M,i})^2. \tag{18}$$

Proof: Consider the storage function $V_{phy,i}$ as

$$V_{phy,i} = \frac{\beta_i}{2}\left(M_i\Delta\tilde{f}^2_i + \sum\nolimits_{k\in\mathcal{G}_i} R^k_i\tau^k_i\tilde{P}^{k^2}_{M,i}\right). \tag{19}$$

Calculating its derivative along the trajectory yields (18). ■

Lemma 2: Suppose that Assumption 2 holds. The k–th DR participant's demand decision system (9)–(11) is output strictly passive under the equation (20) with positive definite storage function $V^k_{DR,i}$, input $-\tilde{\lambda}_i + \Delta\tilde{f}_i$ and output $\tilde{P}^k_{DR,i}$.

$$\dot{V}^k_{DR,i} \leq \tilde{P}^k_{DR,i}\left(-\tilde{\lambda}_i + \beta_i\Delta\tilde{f}_i\right) - \alpha_v\tilde{P}^2_{DR,i}. \tag{20}$$

Proof: Consider the storage function as

$$V_{P^k_{DR,i}} = \frac{1}{2\alpha^k_{DR,i}}\tilde{P}^{k^2}_{DR,i}. \tag{21}$$

Its derivative is calculated as follows taking into account Assumption 2:

$$\dot{V}_{P^k_{DR,i}} \leq \tilde{P}^k_{DR,i}\left(-\alpha^k_{v,i}\tilde{P}^k_{DR,i} - \tilde{\lambda}_i + \tilde{\mu}^{k,-}_{DR,i} - \tilde{\mu}^{k,+}_{DR,i} + \beta_i\Delta\tilde{f}_i\right). \tag{22}$$

In addition, it is also known that the inputs and outputs $\tilde{P}^k_{DR,i}$ and $-\tilde{\mu}^{k,-}_{DR,i}$ are passive [6]. Similarly for inputs and outputs $\tilde{P}^k_{DR,i}$ and $\tilde{\mu}^{k,+}_{DR,i}$. Therefore, inequality (20) is proven to hold under the positive definite storage function $V^k_{DR,i}$. ■

Lemma 3: Suppose that Assumption 3 holds. The k–th supplier's power generation decision system (13)–(15) is output strictly passive or output passivity-short under the equation (23) with positive definite function $V_{G,i}^k$, input $-\tilde{\lambda}_i + \beta_i R_i^k \tilde{P}_{M,i}^k$ and output $-\tilde{u}_{G,i}^k$.

$$\dot{V}_{G,i}^k \leq -\tilde{u}_{G,i}^k \left(-\tilde{\lambda}_i + \beta_i R_i^k \tilde{P}_{M,i}^k\right) - (\alpha_{c,i}^k - \beta_i R_i^k)\tilde{u}_{G,i}^2. \tag{23}$$

This lemma can be proven similar to Lemma 2.

Lemma 4: Suppose that Assumptions 2,3 hold. The dynamic pricing system in area $i \in \mathcal{A}$ described in (9)–(16) is passive or output passivity-short under the equation (25) with positive definite storage function $V_{dp,i}$, input $\boldsymbol{u}_{dp,i}$ and output $\boldsymbol{y}_{dp,i}$, described in (24).

$$\boldsymbol{u}_{dp,i} = \begin{bmatrix} \beta_i \Delta \tilde{f}_i \\ \beta_i R_i^1 \tilde{P}_{M,i}^1 \\ \vdots \\ \beta_i R_i^{\|\mathcal{G}_i\|} \tilde{P}_{M,i}^{\|\mathcal{G}_i\|} \end{bmatrix}, \boldsymbol{y}_{dp,i} = \begin{bmatrix} \sum_{k\in\mathcal{L}_i} \tilde{P}_{DR,i}^k \\ -\tilde{u}_{G,i}^1 \\ \vdots \\ -\tilde{u}_{G,i}^{\|\mathcal{G}_i\|} \end{bmatrix}. \tag{24}$$

$$\dot{V}_{dp,i} = \boldsymbol{u}_{dp,i}^\top \boldsymbol{y}_{dp,i} - \sum_{k\in\mathcal{G}_i} (\alpha_{c,i}^k - \beta_i R_i^k)\tilde{u}_{G,i}^{k^2} - \sum_{k\in\mathcal{L}_i} \alpha_{v,i}^k \tilde{P}_{DR,i}^{k^2}. \tag{25}$$

Proof: Consider the storage function $V_{dp,i}$ as

$$V_{dp,i} = \frac{1}{2}\tilde{\lambda}_i^2 + \sum\nolimits_{k\in\mathcal{L}_i} V_{DR,i}^k + \sum\nolimits_{k\in\mathcal{G}_i} V_{sup,i}^k. \tag{26}$$

Calculating the derivative of $V_{dp,i}$ yields (25) with (24). ■

We can get the following lemma for passivity of the power flow model indicated in (2) and (3) as shown in [11]

Lemma 5: Power flow model described in (2) and (3) is passive under the equation (27) with positive definite storage function V_{net}, input $\boldsymbol{\beta}\Delta\tilde{\boldsymbol{f}}$ and output $-\tilde{\boldsymbol{P}}_{tie}$, where $\Delta\tilde{\boldsymbol{f}}, \tilde{\boldsymbol{P}}_{tie} \in \mathbb{R}^{\|\mathcal{A}\|}$, $\boldsymbol{\beta} = \mathrm{diag}(\beta_i, \cdots, \beta_{\|\mathcal{A}\|})$.

$$\dot{V}_{net} \leq \left(-\tilde{\boldsymbol{P}}_{tie}\right)^\top \boldsymbol{\beta}\boldsymbol{\Delta}\tilde{\boldsymbol{f}}. \tag{27}$$

The results of the above passivity analysis indicate that the physical system is output strictly passive, while the dynamic pricing system is passive or output passivity-short, depending on the strong concavity and strong convexity of the utility and cost functions of each of DR participants and suppliers. In the next section, we use the results of this passivity analysis to design event-triggering conditions.

V. EVENT-TRIGGERED CONDITION

In general, frequency control is performed on a shorter time scale than consumer or supplier behavior models. Therefore, in this study, we derive a trigger condition that reduces the number of information updates while maintaining the stability of the system. Specifically, we design a system in which the input to the dynamic pricing system from the physical system is updated according to its triggering conditions as shown in Fig. 2.

To design the trigger conditions, strictly passivity is required for both the physical system and the dynamic pricing system. Therefore, the following assumption is considered.

Assumption 4: LFC system considering DR described by (1),(4), (9)–(16) satisfies the following inequalities for each area $i \in \mathcal{A}$.

$$\alpha_{c,i}^k - \beta_i R_i^k > 0, \ i \in \mathcal{A} \tag{28}$$

where the gain β_i is a hyperparameter, indicating that β_i must be designed to satisfy Assumption 4.

The input $\boldsymbol{u}_{dp,i}$ to the dynamic pricing system is updated only at the time t_k when the trigger condition is satisfied. Accordingly, the input to the dynamic pricing system is given using zero-order hold as follows:

$$\boldsymbol{u}_{dp,i}(t) = \boldsymbol{y}_{phy,i}(t_k), \quad \forall t \in [t_k, t_k + 1). \tag{29}$$

In the rest of this paper, a variable without an argument is indicated to be a value at time t, and the argument is written as $\boldsymbol{y}_{phy,i}(t_k)$ when it is a value at event time t_k. The trigger condition for the LFC system with DR is given by the following lemma [12].

Lemma 6: Suppose that Assumptions 2–4 hold. The event time t_k is determined when the event-triggered condition (30) is satisfied, then the LFC system of the area given by (1),(4), (9)–(16) is passive with input $\tilde{P}_{tie,i}$ and output $\beta_i \Delta \tilde{f}_i$.

$$\boldsymbol{y}_{phy,i}^\top(t)\boldsymbol{Q}_i \boldsymbol{y}_{phy,i}(t) < \boldsymbol{e}_{phy,i}^\top(t)\boldsymbol{S}_i \boldsymbol{e}_{phy,i}(t) \tag{30}$$

where,

$$\boldsymbol{e}_{phy,i} = \boldsymbol{y}_{phy,i}(t) - \boldsymbol{y}_{phy,i}(t_k), \tag{31}$$

$$\boldsymbol{Q}_i = \mathrm{diag}\Big(\frac{D_i}{\beta_i}, \frac{1}{\beta_i R_i^1}, \cdots, \frac{1}{\beta_i R_i^{\|\mathcal{G}_i\|}}\Big), \tag{32}$$

$$\boldsymbol{S}_i = \mathrm{diag}\Big(\sum_{k\in\mathcal{L}_i} \frac{1}{4\alpha_{v,i}^k}, \frac{1}{4(\alpha_{c,i}^1 - \beta_i R_i^1)}, \cdots, \frac{1}{4(\alpha_{c,i}^{\|\mathcal{G}_i\|} - \beta_i R_i^{\|\mathcal{G}_i\|})}\Big). \tag{33}$$

Proof: First, consider the storage function as:

$$V_i = V_{phy,i} + V_{dp,i} \tag{34}$$

and its time-derivative along the trajectories is given as follows:

$$\begin{aligned}
\dot{V}_i \leq\ & \boldsymbol{u}_{dp,i}^\top \boldsymbol{y}_{dp,i} + \boldsymbol{u}_{phy,i}^\top \boldsymbol{y}_{phy,i} - \sum_{k\in\mathcal{G}_i} (\alpha_{c,i}^k - \beta_i R_i^k)\tilde{u}_{G,i}^{k^2} \\
& - \sum_{k\in\mathcal{L}_i} \alpha_{v,i}^k \tilde{P}_{DR,i}^{k^2} - \frac{D_i}{\beta_i}(\beta_i \Delta \tilde{f}_i)^2 - \sum_{k\in\mathcal{G}_i} \frac{1}{\beta_i R_i^k}(\beta_i R_i^k \tilde{P}_{M,i}^k)^2 \\
=& - \boldsymbol{e}_{phy,i}^\top \boldsymbol{y}_{dp,i} + \tilde{P}_{tie,i}\beta_i \Delta \tilde{f}_i - \sum_{k\in\mathcal{G}_i} (\alpha_{c,i}^k - \beta_i R_i^k)\tilde{u}_{G,i}^{k^2} \\
& - \sum_{k\in\mathcal{L}_i} \alpha_{v,i}^k \tilde{P}_{DR,i}^{k^2} - \boldsymbol{y}_{phy,i}^\top \boldsymbol{Q}_i \boldsymbol{y}_{phy,i} \\
\leq\ & \tilde{P}_{tie,i}\beta_i \Delta \tilde{f}_i + \boldsymbol{e}_{phy,i}^\top \boldsymbol{S}_i \boldsymbol{e}_{phy,i} - \boldsymbol{y}_{phy,i}^\top \boldsymbol{Q}_i \boldsymbol{y}_{phy,i}.
\end{aligned} \tag{35}$$

Therefore, while the inequality,

$$\boldsymbol{y}_{phy,i}^\top \boldsymbol{Q}_i \boldsymbol{y}_{phy,i} \geq \boldsymbol{e}_{phy,i}^\top \boldsymbol{S}_i \boldsymbol{e}_{phy,i} \tag{36}$$

holds, the system can maintain its passivity with input $\tilde{P}_{tie,i}$ and output $\beta_i \Delta \tilde{f}_i$. The trigger condition is thus derived as in (30). ■

The stability of an overall event-triggered LFC system is guaranteed by the following theorem, using Lemma 5 and 6.

Theorem 1: Suppose that Assumptions 2–4 hold. The LFC system considering DR described by (1)– (4), (9)–(16) is asymptotically stable with respect to the equilibrium point when the dynamic pricing system in each area $i \in \mathcal{A}$ updates its inputs according to the event trigger conditions in (30).

Proof: Let the Lyapunov function for the overall LFC system be:

$$V_{LFC} := V_{net} + \sum\nolimits_{i\in\mathcal{A}} V_i. \tag{37}$$

Then, asymptotical stability is proven by calculating the time derivative of V_{LFC} along the trajectory with event-triggered condition (30). ∎

Given the above results, we design the following event-triggered condition for the numerical simulation.

$$\sigma \boldsymbol{y}_{phy,i}^{\top} \boldsymbol{Q}_i \boldsymbol{y}_{phy,i}(t) < \boldsymbol{e}_{phy,i}^{\top} \boldsymbol{S}_i \boldsymbol{e}_{phy,i} \tag{38}$$

where $\sigma \in (0,1]$ is a coefficient indicating the strictness of the inequality condition. The closer sigma is to 0, the more frequently the dynamic pricing system is updated, and also the gradient of the Lyapunov function can be kept with a negative larger value, which is supposed to improve convergence to the equilibrium point.

VI. Simulation Verification

This section shows the numerical simulation results to verify the effectiveness of the method proposed in this paper.

A. Simulation conditions

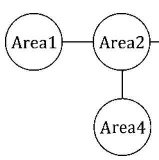


Fig. 3: Network Structure

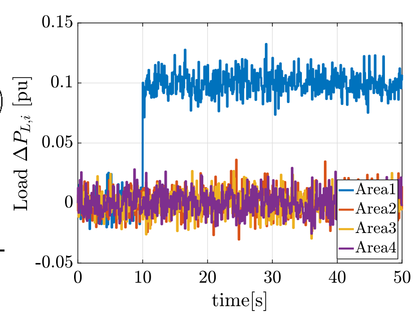


Fig. 4: Load Change

As a simplified model, we assume the power network structure shown in Fig. 3, where the number of areas $\|\mathcal{A}\| = 4$, links $\|\mathcal{E}\| = 3$, suppliers in each area $\|\mathcal{G}_i\| = 1$, and DR participants in each area $\|\mathcal{L}_i\| = 3$.

The system-rated capacity is 1[pu] = 1000[MW], and the utility function of each DR participant and the cost function of each supplier in each area are considered quadratic functions as shown below.

$$v_i^k(P_{DR,i}^k) = a_{DR,i}^k P_{DR,i}^{k^2} + b_{DR,i}^k P_{DR,i}^k,\ \forall i \in \mathcal{A}, \forall k \in \mathcal{L}_i, \tag{39}$$

$$c_i^k(u_{G,i}^k) = a_{G,i}^k u_{G,i}^{k^2} + b_{G,i}^k u_{G,i}^k + c_{G,i}^k,\ \forall i \in \mathcal{A}, \forall k \in \mathcal{G}_i. \tag{40}$$

Each coefficient is given so that Assumptions 2–4 are satisfied.

$$\begin{aligned}
[a_{DR,1}^1\ a_{DR,1}^2\ a_{DR,1}^3] &= [-6.06\ \ -6.03\ \ -6.08],\\
[b_{DR,1}^1\ b_{DR,1}^2\ b_{DR,1}^3] &= [7.85\ 7.60\ 8.10],\\
[a_{DR,2}^1\ a_{DR,2}^2\ a_{DR,2}^3] &= [-6.54 - 6.52 - 6.56],\\
[b_{DR,2}^1\ b_{DR,2}^2\ b_{DR,2}^3] &= [5.655.405.90],\\
[a_{DR,3}^1\ a_{DR,3}^2\ a_{DR,3}^3] &= [-17.3 - 20.3 - 16.9],\\
[b_{DR,3}^1\ b_{DR,3}^2\ b_{DR,3}^3] &= [5.60\ 5.85\ 6.10],\\
[a_{DR,4}^1\ a_{DR,4}^2\ a_{DR,4}^3] &= [-5.71 - 5.67 - 5.74],\\
[b_{DR,4}^1\ b_{DR,4}^2\ b_{DR,4}^3] &= [7.40\ 7.15\ 7.65]\\
[a_{G,1}^1\ a_{G,2}^1\ a_{G,3}^1\ a_{G,4}^1] &= [0.127\ 0.400\ 0.830\ 0.700],\\
[b_{G,1}^1\ b_{G,2}^1\ b_{G,3}^1\ b_{G,4}^1] &= [5.37\ 2.85\ 2.00\ 0.996],\\
[c_{G,1}^1\ c_{G,2}^1\ c_{G,3}^1\ c_{G,4}^1] &= [0.780\ 0.117\ 0.200\ 0.550].
\end{aligned}$$

For simplicity, the upper and lower bound constraints are not considered in this simulation. Other parameters are decided as:

$$\begin{aligned}
&M_i = 7.0, \tau_i^k = 9.0, R_i = 0.05, D_i = 4.0, \quad \forall i \in \mathcal{A},\\
&T_{i,j} = 1 \ \ \forall (i,j) \in \mathcal{E}.
\end{aligned}$$

The hyper parameters are set as follows:

$$\begin{aligned}
&\beta_i = 3,\ \ \alpha_{DR,i}^k = \alpha_{G,i}^k = 2,\\
&k_{\mu_{DR,i}^{k,-}} = k_{\mu_{DR,i}^{k,+}} = k_{\mu_{G,i}^{k,-}} = k_{\mu_{G,i}^{k,-}} = 10, \quad \forall i \in \mathcal{A}.
\end{aligned}$$

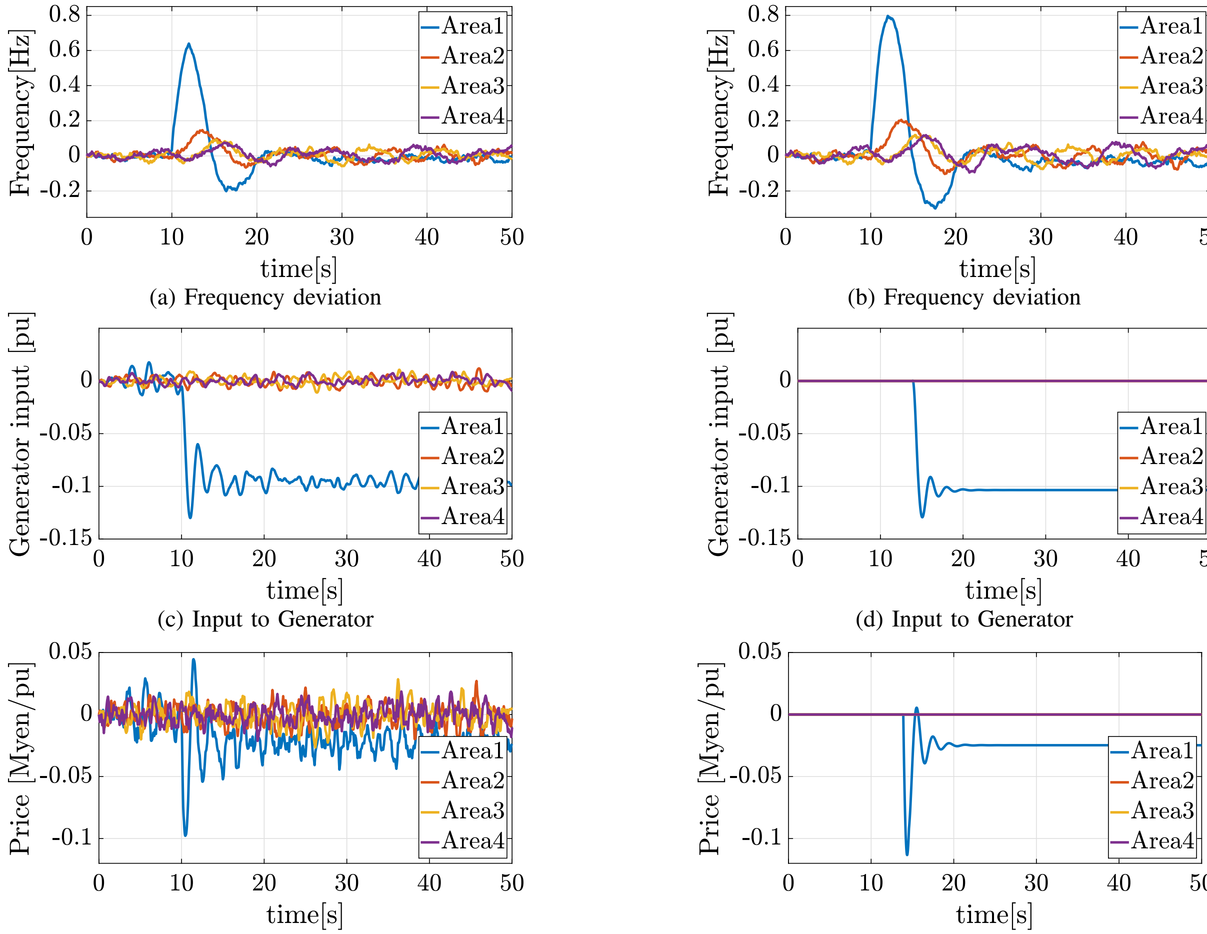
In order to observe the system stability from the equilibrium, we assume the situation where the load changes shown by Fig. 4 occur in each area.

B. Simulation results

A comparison of the simulation results with and without event-triggered condition is shown in Fig. 5. The results of frequency deviation, input to the generator and the electricity price are described. Note that, all these results represent the state around the equilibrium point. Fig. 6 shows the time when the event-triggered condition is satisfied. When the event-triggered condition is not designed, the frequency oscillates due to the load change, and DR participants and suppliers behave with extremely high frequency. In contrast, when the event-triggered condition is designed, the output of the physical system is not communicated with the dynamic pricing system at high frequency, while the behavior of DR participants, suppliers, and electricity price updates are stable. Discussing control performance, the system with event-trigger is worse than that without event-trigger as shown in Fig. 5d and 5c. The numerical simulations show that system states are not diverging even when the load fluctuations change the system equilibrium, also these are converging to constant values, thus asymptotical stability of the system equilibrium is satisfied and the effectiveness of the proposed design is demonstrated.

VII. Conclusion

This study deals with an LFC system with DR participants using a pricing-based controller. The objective of this study is to reduce the communication frequency between the physical system and the pricing-based controller. Specifically, the passivity of the physical system and the dynamic pricing system are analyzed to derive the passivity and output strict passivity, and the triggered condition is derived by using this passivity index. Simulation verification shows that the proposed triggered condition reduces the communication frequency and is asymptotically stable around the equilibrium point. Future



(a) Frequency deviation (b) Frequency deviation

(c) Input to Generator (d) Input to Generator

(e) Price (f) Price

Fig. 5: Simulation results where the left and right columns show the results without and with event-triggering, respectively.

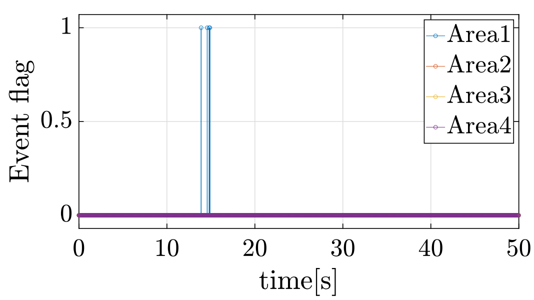


Fig. 6: Triggering time

work is to derive a lower bound for the update frequency of the triggered condition, which will guarantee the reduction of the update frequency of the proposed method.

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