2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Quantum Optimal Control for the Shaping of Single Photons

Xue Dong[1] and Re-Bing Wu[1]

Abstract— The transmission of flying qubits carried by itinerant photons is fundamental in quantum communication networks. To physically match the receiver system, the single photons must be prepared in proper shapes, and this leads to a variety of flying-qubit control problems. In this paper, we introduce the optimal control theory to the shaping of single photons, where the control to be optimized are coherent driving fields. We design gradient-based algorithms to minimize the shape difference between the emitted and desired single photons. Simulation results show that the optimization can achieve high-fidelity in the generation of decaying photon shapes with a two-level atom. However, its performance is limited and hence has to be combined with incoherence controls when the target shape has a rising part.

I. INTRODUCTION

Towards networked quantum computation and communication, the high-fidelity transmission of qubit information is highly demanding between spatially separated nodes [1], [2]. In practice, the quantum channel can be established via the emission of photons from one standing qubit and the absorption by another remote standing qubit [3], [4]. The itinerant photon fields that transfer information between the standing qubits thus carry the so-called flying qubits.

Because realistic photons always contain a continuous band of electromagnetic modes, the mode distribution (or equivalently the temporal shape) must be controlled to match the remote receiver system. This leads to the shaping problem of single-photon wavepackets [5], [6], [7], [8]. As is shown in Fig. 1, the flying qubits are generated from a standing quantum system that is directly coupled to a waveguide or indirectly coupled to the waveguide through a cavity. The emission process can be manipulated by either coherent drivings on the generator or its tunable coupling to the waveguide.

*The author RBW acknowledges support from NSFC grants (Nos. 61833010 and 62173201).

[1]The authors are with Center for Intelligent and Networked Systems, Department of Automation, Tsinghua University, Beijing 100084, China rbwu@tsinghua.edu.cn

In the microwave domain, superconducting qubits are good candidates of single-photon sources owing to their tunable frequency, long coherence time, and the field confinement to one-dimensional transmission lines without additional spatial pattern matching [9]. Cavity-based photon-shaping schemes have been proposed and experimentally realized based on a tunable coupler that modulates the instantaneous emission rate of photons from the cavity into the transmission line [10], [11]. Additional tunable parameters can be introduced (e.g., the qubit-resonator detuning [12], [13] or microwave driving) to induce an effective tunable qubit-resonator coupling [6], [14], which may be applied to compensate the frequency shift effect.

It is relatively easy to design shaping control protocols with tunable couplings, because they directly adjust the rate of photon emission and preserve the number of excitation in the system. However, in practice the waveguide coupling strength is often fixed or only tunable within a limited range. In this regard, coherent drivings are much easier to implement, but they may introduce unwanted multi-photon emissions. Actually, coherent control have been proven to be effective in active suppression of decoherence by modulating the system's coupling to the environment [15], but to our knowledge, their application to flying-qubit contol has not been reported.

To maximally exploit coherent control resources, it is natural to treat the design using optimal control theory [16], which has stimulated a large amount of applications in the state or gate control of standing qubits. The shaping control to be explored here provides an intriguing new paradigm for quantum optimal control, and so far we have not yet seen any relevant investigations in the literature.

In this paper, we will introduce gradient-based optimal control algorithms to the shaping of single photons. The remainder of this paper is organized as follows. In Sec. II, we introduce the mathematical model of flying-qubit and the gradient-based

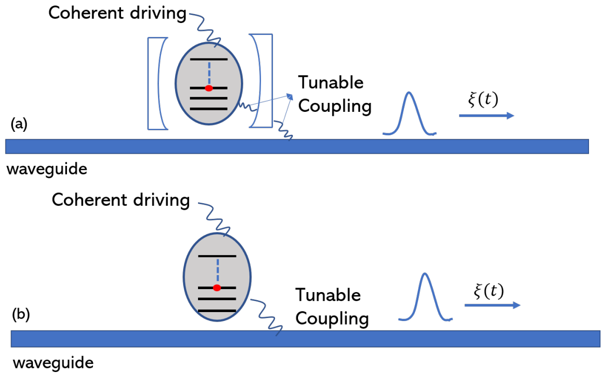

Fig. 1. Schematics of single-photon shaping systems: (a) the photons are generated from a standing quantum system in a cavity that is coupled to the waveguide; (b) the photons are generated from a standing quantum system that is directly coupled to the waveguide.

optimization algorithm for the shaping objective. In Sec. III, demonstrative simulations are done to achieve various single-photon shapes with ideal two-level quantum systems. In Sec. IV, conclusions are made.

II. Quantum Optimal Control Theory for Single-Photon Pulse Shaping

In this section, we introduce the model of single-photon shaping sysytems and present optimal control protocols using gradient-based algorithms for the shaping of single photon pulses.

A. The modeling of single-photon shaping systems

Consider a single-photon field that travels in a unidirectional continuous-model waveguide. Let $|\mathrm{vac}\rangle$ be its vacuum state and $b(t)$ be the temporal annihilation operator that satisfies the singular commutation relation $[b(t), b^\dagger(t')] = \delta(t-t')$ [17], [18]. Throughout the paper, we assume that the waveguide is empty when $t<0$, and hence a single-photon state can be defined as:

$$|1_\xi\rangle = \int_0^\infty \xi(t) b^\dagger(t) dt |\mathrm{vac}\rangle, \qquad (1)$$

in which the complex-number valued normalized function $\xi(t)$ represents the temporal pulse shape of the single photon.

Consider the case that the waveguide is directly coupled to a standing quantum system. As is shown in Fig. 1(b), the joint system may be manipulated by either coherent control fields on the standing system or its tunable coupling with the waveguide. Under most general circumstances, the emitted photon field can be expressed in the following form of superposition state:

$$|\Xi\rangle = \xi^{(0)}|\mathrm{vac}\rangle + \int_0^\infty \xi^{(1)}(t) b^\dagger(t) dt |\mathrm{vac}\rangle + |\Xi^{'}\rangle, \qquad (2)$$

where $\xi^{(0)}$ is the probability amplitude of the vaccum state, $\xi^{(1)}(t)$ is the shape of the single-photon component, and $|\Xi^{'}\rangle$ represents the rest multi-photon components that we are not concerned with in this paper.

In our previous studies [8], it is shown that the photon fields emitted from the standing system can be calculated from the following non-unitary differential equation

$$\dot{V}(t) = -iH_{\mathrm{eff}}(t)V(t), \quad V(0) = \mathbb{I}, \qquad (3)$$

where the effective non-Hermitian Hamiltonian

$$H_{\mathrm{eff}}(t) = H_0 + \sum_{k=1}^{m} u_k(t) H_k - \frac{i}{2} L^\dagger(t) L(t) \qquad (4)$$

includes the system's free Hamiltonian H_0, the coherent controls $u_1(t)\cdots, u_m(t)$ associated with Hamiltonians $H_1, \cdots, H_m$, respectively, and the incoherent control through the tunable coupling operator $L(t)$. Here, Markovianity is implicitly assumed in the sense that all modes of the field are uniformed coupled to the system throught $L(t)$.

Suppose that the standing system is initially prepared at $|\psi_0\rangle$, and the controls are imposed during a finite time interval $[0, T]$, after which the system eventually decays to its ground state $|g\rangle$. Then the probablity amplitude of the output field being in vacuum state is [8]

$$\xi^{(0)} = \langle g|G(\infty, 0)|\psi_0\rangle, \qquad (5)$$

and the single-photon shape function is:

$$\xi^{(1)}(t) = \langle g|G(\infty, t)L(t)G(t, 0)|\psi_0\rangle, \qquad (6)$$

where $G(t, t') = V(t)V^{-1}(t')$ is the propagator from time t' to t. The expression (6) indicates that the single-photon shape can be extracted from the non-unitary evolution of the standing system intervened by quantum jumps at t.

B. Single-photon shaping as an optimal control problem

Under most circumstances, the coupling operator can be written as $L(t) = \sqrt{\gamma_c} L_0$, where γ_c is the fixed coupling strength and L_0 is a constant operator. Through the coherent driving controls $u_k(t)$, we can formulate the shaping problem as an optimization problem subject to some distance function between the real and desired single-photon wavepackets. Let complex-number $\xi_0(t)$ be the desired single-photon shape function. We can define the distance function as follows:

$$J[u_k(t)] = \int_0^\infty |\xi^{(1)}(t) - \xi_0(t)|^2 dt, \tag{7}$$

where the actual controlled single-photon shape $\xi^{(1)}(t)$ is calculated from Eq. (6).

To numerically evaluate the above objective functions, we first choose a sufficiently large time interval $[0, T]$ so that the field emitted after $t = T$ is negligible. We split this time interval evenly into M pieces, and denote $t_k = k\Delta t$ with $\Delta t = T/M$ and $k = 1, \cdots, M$. The objective functions can then be approximated as

$$J \approx \sum_{n=1}^{M} |\xi^{(1)}(t_n) - \xi_0(t_n)|^2 \Delta t \tag{8}$$

Under piecewise constant controls, the transition operator from t_j to t_n can be expressed as

$$G_{n,j} \triangleq G(t_n, t_j) = V_n \cdots V_{j+2} V_{j+1}, \tag{9}$$

where $V_k \approx e^{-iH_{\mathrm{eff}}(t_k)\Delta t}, k = 1 \cdots M$.

C. The Gradient Formula

In the following, we will apply gradient-descent algorithms to optimize the control functions $u_k(t)$ for single-photon shaping. According to the derived objective functions (8), we can see that

$$\begin{aligned} \frac{\partial J}{\partial u_k(t_j)} &= 2\sum_{n=1}^{M} \mathrm{Re}\left\{ \frac{\partial \xi^{(1)*}(t_n)}{\partial u_k(t_j)} \right. \\ &\quad \left. \cdot \left[\xi^{(1)}(t_n) - \xi_0(t_n)\right] \right\} \Delta t, \end{aligned}$$

where $k = 1, \cdots, m$ and $j = 1. \cdots, M$. Therefore, the gradient evalulation attributes to the calculation of $\frac{\partial \xi^{(1)}(t_n)}{\partial u_k(t_j)}$, which are complicated because they depend on the ordinal relation of t_n and t_j. Concretely, we have

$$\frac{\partial \xi^{(1)}(t_n)}{\partial u_k(t_j)} \approx -i\sqrt{\gamma_c}\langle g|G_{M,n} L_0 G_{n,j} H_k G_{j,0}|\psi_0\rangle \Delta t, \tag{10}$$

for $t_j \leqslant t_n$ and

$$\frac{\partial \xi^{(1)}(t_n)}{\partial u_k(t_j)} \approx -i\sqrt{\gamma_c}\langle g|G_{M,j} H_k G_{j,n} L_0 G_{n,0}|\psi_0\rangle \Delta t, \tag{11}$$

for $t_j > t_n$.

Based on the above formulas, we can apply gradient-descent algorithms to the design of single-photon shaping protocols. The basic procedure is, for any coherent control $u(t)$ that is tunable in the system, to choose some initial guess and gradually update it along the steepest descent direction:

$$u_k^{(i+1)}(t_j) = u_k^{(i)}(t_j) - \epsilon_i \frac{\partial J}{\partial u_k^{(i)}(t_j)}, \tag{12}$$

where $u_k^{(i)}(t)$ is the value of coherent control function $u_k(t)$ in the ith iteration and ϵ_i is the stepsize that can be adaptively adjusted during the optimization process. The optimal control is obtained when, ideally, the objective function J is decreased to zero. Howerver, as will be seen below, this is usually not achievable when the system is under coherent control and fixed incoherent coupling, because the underlying Markovian dynamics is fully uncontrollable [19].

III. Simulation Examples

In this section, we will demonstrate the proposed optimal control algorithm with the BFGS method to the shaping of single-photon pulses with coherent controls in a two-level atom, where the coupling strength is fixed.

To fully test the performance, we selected three typical target pulse shapes in the simulation, the real part of the shape is exponential decay, exponential rise and exponential symmetric function respectively, and the imaginary part is zero. The first type corresponds to the naturally emitted single photon via spontaneous decay of an excited two-level atom, while the second and third types can be perfectly caught by a two-level atom and thus are favored for quantum information transmission to remote receivers [20]. These three types of shapes are uniformly described by:

$$\xi_0(t) = \begin{cases} Ae^{\alpha_1(t-T_0)/2} & t \leqslant T_0 \\ Ae^{-\alpha_2(t-T_0)/2} & t > T_0 \end{cases}, \tag{13}$$

which consists of an exponentially rising part from $t = 0$ to $t = T_0$ and an exponentially decaying part from $t = T_0$ to $t = T$. The exponentially decaying shape corresponds $\alpha_1 \gg \alpha_2$ and $T \gg T_0$, while the exponentially rising shape corresponds to $\alpha_2 \gg \alpha_1$ and $T \approx T_0$. The symmetrical shape corresponds to $\alpha_1 = \alpha_2$ and $T = 2T_0$.

We start the numerical test with an ideal two-level quantum system, whose coupling strength to the waveguide is fixed at $\gamma(t) \equiv \gamma_c$. Under this circumstance, $L(t) = \sqrt{\gamma_c}\sigma_-$ and the effective Hamiltonian can be written as

$$H_{\text{eff}}(t) = u(t)\sigma_+ + u^*(t)\sigma_- - \frac{i\gamma_c\sigma_+\sigma_-}{2}, \qquad (14)$$

where the control $u(t) = u_x(t) + iu_y(t)$ includes both in-phase and phase-quadrature components $u_x(t)$ and $u_y(t)$, and $\sigma_\pm$ are the standard Pauli raising and lowering operators.

In our simulations, we always assume that the standing system is initially prepared at its ground state. The coupling strength is set to $\gamma_c/2\pi = 5$ MHz. The choice of duration time T is related to the coupling strength γ. The probability of generating a complete single photon is $1 - e^{-\gamma T}$. Therefore, the duration time is chosen as $T = 1000\text{ns} \gg \gamma_c^{-1}$, The sampling period of control pulses is estimated based on the signal frequency of the microwave source in the experiment. The frequency unit of the transmitted signal is GHz. The sampling interval is chosen as $\Delta t = 1$ ns.

We first apply the algorithm to the optimal generation of single photons using the objective function J. As are shown in Figs. 2 and 3, the emitted single-photon component $\xi^{(1)}(t)$ can be optimized to be close to the target shapes, but the deviation cannot be completely eliminated due to the unavoidable vacuum and multi-photon output in (2). The comparison between different target shape functions indicate that the optimization is less effective when the targe shape has a rising part or a sharp edge. The best performance is achieved when the target shape is exponentially decaying with $\alpha = \gamma_c$, while the poorest performance is obtained when the target shape is exponentially rising with $\alpha = 2\gamma_c$.

To better understand the shaping ability under fixed coupling, we performed more extensive simulations on shaping single photons with different target shapes and available fixed coupling strengths. The simulation results are shown in Fig. 4. For

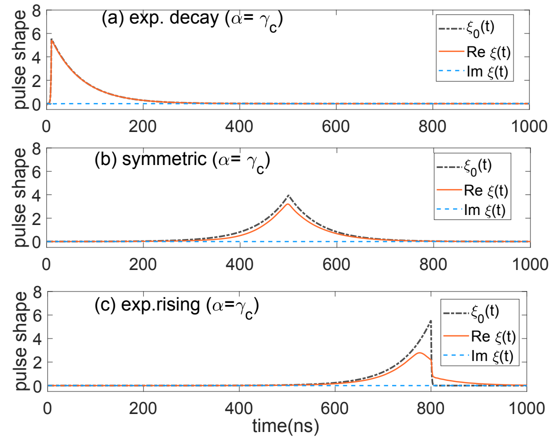

Fig. 2. The optimized single-photon pulse shapes under coherent driving controls and fixed coupling strength $\gamma_c/2\pi =$ 5MHz. Three types of single-photon pulses are simulated, including the exponentially decaying, symmetric and exponentially rising shapes with different decaying/rising rates $\alpha = \gamma_c$.

the symmetry or decaying shape target field, the emmited single photon is closest to the target by optimizing coherent control when α is around γ_c. While for the rising shape, the error between the generated photon and the target is rising when α/γ_c increases.

IV. CONCLUSIONS

To conclude, we introduced quantum optimal control theory to the generation of arbitrary-shapes single photons based gradient-descent algorithms. Simulation results show that single photons can be effectively shaped by coherent controls when the waveguide coupling is fixed and when the target pulse shape is decaying. However, the shaping precision is not ideal when the desired single photon wavepacket has rising slopes.

These results show that the ability of coherent control for shaping single photons is very limited because the underlying control system is essentially a Markovian quantum system that is inherently uncontrollable. To improve the shaping performance, a straightforward manner is to combine the coherent control pulse with incoherent control resources (such as tunable waveguide coupler). This topic will be explored in our future studies.

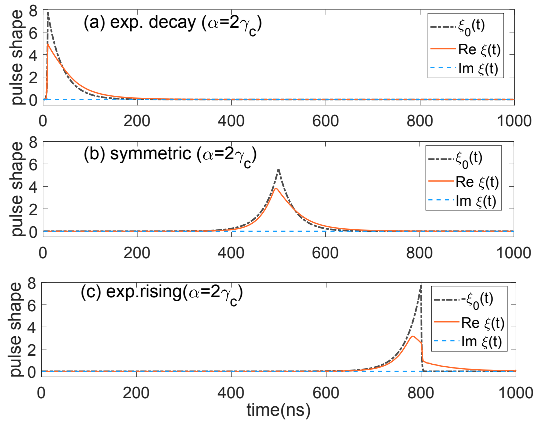

Fig. 3. The optimized single-photon pulse shapes under coherent driving controls and fixed coupling strength $\gamma_c/2\pi =$ 5MHz. Three types of single-photon pulses are simulated, including the exponentially decaying, symmetric and exponentially rising shapes with different decaying/rising rates $\alpha = 2\gamma_c$.

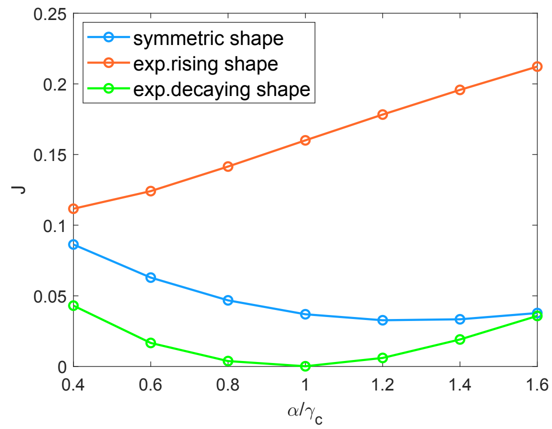

Fig. 4. The dependence of the shaping error on the slope α/γ_c of the target shape functions for exponentially decaying, exponentially rising and exponentially symmetric shaped photons. Here α is the decay/rising rate of the desired photon shapes and γ_c is the coupling constant.

References

[1] D. P. DiVincenzo, "Quantum computation," Science, vol. 270, no. 5234, pp. 255–261, 1995.

[2] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, "Quantum state transfer and entanglement distribution among distant nodes in a quantum network," Physical Review Letters, vol. 78, no. 16, p. 3221, 1997.

[3] A. A. Houck, D. Schuster, J. Gambetta, J. Schreier, B. Johnson, J. Chow, L. Frunzio, J. Majer, M. Devoret, S. Girvin, et al., "Generating single microwave photons in a circuit," Nature, vol. 449, no. 7160, pp. 328–331, 2007.

[4] A. N. Korotkov, "Flying microwave qubits with nearly perfect transfer efficiency," Physical Review B, vol. 84, no. 1, p. 014510, 2011.

[5] M. Keller, B. Lange, K. Hayasaka, W. Lange, and H. Walther, "Continuous generation of single photons with controlled waveform in an ion-trap cavity system," Nature, vol. 431, no. 7012, pp. 1075–1078, 2004.

[6] M. Pechal, L. Huthmacher, C. Eichler, S. Zeytinoğlu, A. Abdumalikov Jr, S. Berger, A. Wallraff, and S. Filipp, "Microwave-controlled generation of shaped single photons in circuit quantum electrodynamics," Physical Review X, vol. 4, no. 4, p. 041010, 2014.

[7] P. Forn-Diaz, C. Warren, C. Chang, A. Vadiraj, and C. Wilson, "On-demand microwave generator of shaped single photons," Physical Review Applied, vol. 8, no. 5, p. 054015, 2017.

[8] W.-L. Li, G. Zhang, and R.-B. Wu, "On the control of flying qubits," Automatica, vol. 143, p. 110338, 2022.

[9] X. Gu, A. F. Kockum, A. Miranowicz, Y.-x. Liu, and F. Nori, "Microwave photonics with superconducting quantum circuits," Physics Reports, vol. 718, pp. 1–102, 2017.

[10] M. Pierre, I.-M. Svensson, S. Raman Sathyamoorthy, G. Johansson, and P. Delsing, "Storage and on-demand release of microwaves using superconducting resonators with tunable coupling," Applied Physics Letters, vol. 104, no. 23, p. 232604, 2014.

[11] Y. Yin, Y. Chen, D. Sank, P. OMalley, T. White, R. Barends, J. Kelly, E. Lucero, M. Mariantoni, A. Megrant, et al., "Catch and release of microwave photon states," Physical review letters, vol. 110, no. 10, p. 107001, 2013.

[12] S. J. Srinivasan, N. M. Sundaresan, D. Sadri, Y. Liu, J. M. Gambetta, T. Yu, S. Girvin, and A. A. Houck, "Time-reversal symmetrization of spontaneous emission for quantum state transfer," Physical Review A, vol. 89, no. 3, p. 033857, 2014.

[13] G. Hornecker, E. Peinke, J. Claudon, A. Auffèves, and J.-M. Gérard, "Temporal shaping of single-photon pulses," in Quantum Optics and Quantum Information Transfer and Processing 2015, vol. 9505. SPIE, 2015, pp. 66–72.

[14] S. Zeytinoğlu, M. Pechal, S. Berger, A. Abdumalikov Jr, A. Wallraff, and S. Filipp, "Microwave-induced amplitude-and phase-tunable qubit-resonator coupling in circuit quantum electrodynamics," Physical Review A, vol. 91, no. 4, p. 043846, 2015.

[15] L. Viola, E. Knill, and S. Lloyd, "Dynamical decoupling of open quantum systems," Physical Review Letters, vol. 82, no. 12, p. 2417, 1999.

[16] F. L. Lewis, D. Vrabie, and V. L. Syrmos, Optimal control. John Wiley & Sons, 2012.

[17] R. Loudon, The quantum theory of light. OUP Oxford, 2000.

[18] G. Zhang and M. R. James, "On the response of quantum linear systems to single photon input fields," IEEE Transactions on Automatic Control, vol. 58, no. 5, pp. 1221–1235, 2012.

[19] C. Altafini, "Coherent control of open quantum dynami-

cal systems," Physical Review A, vol. 70, no. 6, p. 062321, 2004.

[20] Y. Wang, J. Minář, L. Sheridan, and V. Scarani, "Efficient excitation of a two-level atom by a single photon in a propagating mode," Physical Review A, vol. 83, no. 6, p. 063842, 2011.