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Dynamic Curing and Network Design in SIS Epidemic Processes

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Abstract— **This paper studies efficient algorithms for dynamic curing policies and the corresponding network design problems to guarantee fast extinction of epidemic spread in a Markov process-based susceptible-infected-susceptible (SIS) model. We provide a computationally efficient curing algorithm based on the curing policy proposed by Drakopoulos, Ozdaglar, and Tsitsiklis (2014). Since the corresponding optimization problem is NP-hard, finding optimal policies is intractable for large graphs. We provide approximation guarantees on the curing budget of the proposed dynamic curing algorithm. To avoid the waiting period included in the original curing policy, we study network design problems to reduce the total infection rate by deleting edges or reducing the weight of edges. To this end, we provide algorithms with provable guarantees. In summary, the proposed curing and network design algorithms together provide an effective and computationally efficient approach that mitigates SIS epidemic spread in networks.**

I. Introduction

The modeling and control of contagious processes have received significant research interest due to the overwhelming societal cost of widespread epidemics. Extensive mathematical models have been proposed. The susceptible-infected-susceptible (SIS) model [1] and the susceptible-infected-recovered (SIR) model [2] are the simplest and most popular models. In an SIS model, individuals can be infected multiple times. Network-based compartmental models have been studied for SIS processes, including stochastic models [3], [4] and their mean-field approximations [5], [6].

In a wave of the epidemic, the demand for medical services could outrun local resources. Under such circumstances, policymakers sometimes need to coordinate regional resource allocation to improve medical services [7], [8]. Therefore, optimal resource allocation problems have been studied for various epidemic models. In practice, medical resource allocation strategies are complemented by non-pharmaceutical strategies [9] to reduce the number of infections and therefore the demand for medical resources. For example, travel restrictions, school closure, and stay-at-home policies. Simulation studies have shown the advantages of combined medical and non-pharmaceutical countermeasures in effectively mitigating epidemic spread [9]. However, the problem of allocating curing resources and designing the contact network optimally in an SIS model remains to be investigated from a theoretical viewpoint to attain better understanding of their interaction.

In this paper, we study a Markov process-based SIS model [4] on a weighted undirected graph. Each edge represents the contact between two individuals, and the weight of the edges is proportional to its infection rate. We provide algorithms for various interventions, including curing policies and contact restrictions. For curing policies, the resource constraint is the curing budget. To achieve fast extinction, an ordering of curing must be computed such that the total infection rate is maintained relatively small [10]. Thus, we aim to calculate a near-optimal ordering with an efficient algorithm. We also consider contact restrictions to mitigate the spread concurrently with curing policies. The contact restriction problems can be formulated as network design problems. The restriction of contact between two individuals is modeled as the reduction of the edge weight. The cost of contact restriction is the sum of edge weight reductions. For network design problems with the cost constraint, we present approximation algorithms to the optimal solutions. In the end, the algorithms for curing policies and network designs can be integrated to guarantee the fast extinction of the epidemic spread.

Both curing and network design algorithms for epidemic interventions have been proposed for various models. While the problems have been studied from various perspectives, most of the proposed policies are static, which means that the policy is fixed given the initial configuration.

For the curing policy, [11] studied a static resource allocation problem to minimize the cost to bound the spectral radius of a matrix in a mean-field approximation of an SIS model. The cost was defined as a convex function of the infection rate reductions. The authors formulated the problem as a semi-definite program that can be solved in polynomial time. Recently, the paper [12] presented an algorithm with lower time and space complexity than the SDP-based algorithm in [11]. The authors of the paper [13] proposed the optimal strategy to stabilize SIS processes by allocating curing resources to a single node. Spread minimization problems are also studied in SIS models without using mean-

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field approximation. Borgs et al. [14] proposed to allocate curing rates according to the degree of each node. Using this policy, logarithmic extinction time is achieved by allocating total curing rate proportional to the number of edges in the network. The authors of [15] improved the algorithm by using the personalized PageRank vectors, which exploit the local structure of networks.

All the approaches mentioned above use a static policy. In contrast, a dynamic curing policy can adapt to the observed history of epidemic spread. Bang-bang control and feedback control policies were studied in [16]. A feedback control policy [17] was proposed to suppress the SIS process. However, optimal curing resource allocation remains to be investigated rigorously. In this paper, we consider the same setting as in [10], where the curing budget is limited at any given moment, and a controller optimally distributes resources among all nodes. Drakopoulos et al. proposed a dynamic curing policy for the stochastic SIS model with budget constraint, namely the CURE policy [10]. The approach seeks to cure individuals in an optimal order. Following this order, the total infection rate is always bounded by the *impedance* of the initial infected set in the network. The impedance is the largest infection rate among any remaining subset of the initial infected set following the optimal order. Thus, the curing budget required by the policy depends on the *cutwidth* of the network, which is the impedance of the whole network.

For the network design, algorithms have been proposed for various epidemic models. For SI models, a $O(\log^2 n)$-approximation algorithm is proposed by [18] to find the minimum weight of edges such that a given number of vertices are separated from the initially infected nodes by removing these edges. Constant factor bi-criteria algorithms for this problem are later given by [19], [20]. Two modified problems considering demographic and individual fairness are investigated recently in [21]. For the independent cascade susceptible-infected-removed (IC-SIR) model, recent studies by [22], [23] show approximations to minimize the expected number of infections by deleting edges, under various assumptions about the underlying graph. For the SIS model, [24] provides approximation algorithms for spectral radius reduction problems by edge or node deletion.

Our paper is developed based on the dynamic curing policy in [10]. The CURE policy provides insights into curing rate allocation. However, it can be improved in various aspects. First of all, as mentioned by [10], computing the impedance is **NP**-hard. Therefore, efficient approximation algorithms are needed. Moreover, the impedance of an infected set can be greater than the curing capability. When the impedance is large, the policy sets a waiting period. We address the first issue by designing a computationally efficient curing algorithm under resources constraints, and the second issue by allowing a controller to modify network connections.

The main contribution of this paper is on providing efficient algorithms for curing policies and network design. We extend the $O(\log^2 n)$ approximation algorithm for cutwidth in [25] to the calculation of impedance, which was mentioned in [10] as an open problem. In addition, we study the problem of dynamically modifying the graph structure to ensure that the given curing budget is adequate to achieve fast extinction and avoid the waiting period. We propose algorithms with performance guarantees the SIS model with and without targeted curing rate allocation. By combining the proposed curing and network design algorithms, we provide a comprehensive solution to control SIS processes in networks.

Due to space constraints, proofs and simulation details are not included in the conference paper. For the full version of the papers, please see [26].

II. Preliminaries

In this section we first introduce some frequently used notations and definitions. Then we describe the considered SIS model.

A. Concepts and Notations

We consider an SIS epidemic process in an undirected weighted graph $G = (V, E, w)$, in which V is the set of nodes and E is the set of edges with a weight function $w : E \mapsto [0, 1]$. The degree d_u of a node u is defined as the sum of weights of all its incident edges. We denote by $d_{\max}$ the maximum degree of nodes in the graph. We adopt some of the terms used in [10], [27]. We refer to a subset of V as a *bag*. For any subset A of V, we use $A^c \stackrel{\text{def}}{=} V\backslash A$ to denote the complement of A. For any subset A and any node $u \in A^c$, let $A + u \stackrel{\text{def}}{=} A \cup \{u\}$ and $A - u \stackrel{\text{def}}{=} A\backslash\{u\}$. We then define the cut of the graph G with respect to a bag. We denote by $G[A]$ the subgraph of G supported on the bag A, i.e. $G[A] = (A, E', w')$ where $E' = \{(u, v) \mid u, v \in A\}$ and $w'(e) = w(e)$ for $e \in E'$.

Definition 1: A cut of the graph G is defined for a bag A as the vertex partition (A, A^c). The size of the cut is defined as $c(A) \stackrel{\text{def}}{=} \sum_{(u,v)} w_{uv}$, where $(u, v) \in E$, $u \in A$, $v \in A^c$, and w_{uv} is the weight of the edge (u, v).

We also use the standard definition for the balanced cut in [28], where a cut is α-balanced if $\min\{|A|, |A^c|\} \geq \alpha |V|$. We introduce the following definition of maximum restricted cut (MRC).

Definition 2: Given a graph $\mathcal{G}$ and a bag A, the MRC of the bag A is defined by

$$\phi(A) \stackrel{\text{def}}{=} \max_{Q \subseteq A} c(Q)\,. \tag{1}$$

We further recall some concepts defined in [10], [27].

Definition 3: For any two bags A and B satisfying $B \subseteq A$, a monotone crusade from A to B is a sequence of bags $p(A, B) \stackrel{\text{def}}{=} (p_0, p_1, \ldots, p_k)$ where $p_0 = A$, $p_k = B$; $p_i \subseteq p_{i-1}$ and $|p_{i-1}\backslash p_i| = 1$ for any $i \in [k]$. We denote by $\mathcal{C}(A, B)$ the set of all crusades from A to B.

Definition 4: The *width* of a crusade $p = (p_0, p_1, \ldots, p_k)$ is defined by $z(p) \stackrel{\text{def}}{=} \max_{0 \leq i \leq k}\{c(p_i)\}$.

Definition 5: Given a bag A, its *impedance* $\delta(A)$ is defined as $\delta(A) \stackrel{\text{def}}{=} \min_{p \in \mathcal{C}(A, \varnothing)} z(p)$, namely the minimum width of a crusade from A to $\varnothing$.

The impedance $\delta(V)$ is referred to as the *cutwidth* of a graph, denoted by $\mathcal{W}$.

B. Epidemic Model

We consider a networked SIS model in which each node can be in one of two states: Susceptible or Infected. The infection spreads according to a continuous time Markov process $\{I(t)\}_{t\geq 0}$ on the state space $\{0,1\}^V$ where $I_u(t) = 0$ if node u is susceptible and $I_u(t) = 1$ if it is infected. Without ambiguity, we also use $I(t)$ to denote the set of all infected nodes at time t.

The process starts with a given initial state $I(0)$. Let w_{uv} be the infection rate for each edge $(u, v) \in E$. At any time $t \geq 0$, the transition rate of a susceptible node v to the infected state is defined by $\sum_{(u,v)\in E} w_{uv} I_u(t)$, which is the sum of infection rates w_{uv} over all infected neighbors of v.

On the other hand, the curing rate of an infected node u is denoted by $\rho_u(t) \geq 0$. We consider two typical scenarios: (a) $\rho_u(t)$ is decided by a network controller with the budget constraint $\sum_{u\in V} \rho_u(t) \leq r$ where r is the curing budget; (b) the allocation of $\rho_u(t)$ for each u is decided by the environment and can be adversarial. We define the total curing capacity be $r(t) \stackrel{\text{def}}{=} \sum_{u\in V} \rho_u(t)$.

III. Problem Formulations

In this section we present formal definitions for the optimization problems that arise in curing ordering computation and targeted contact restrictions.

A. Curing Policies

The CURE policy proposed in [10] seeks to find paths along which the cuts are maintained as small as possible such that the spread stops in sublinear time with high probability. We briefly recall the CURE policy as follows:

- Wait until $c(I(t)) \leq r/8$. Let A be the set of infected nodes right after the waiting period.
- Start a segment. Calculate the optimal crusade and obtain an ordering $\{v_1, \ldots, v_{|A|}\}$ in the beginning of a segment. Let $C \stackrel{\text{def}}{=} \{v_2, \ldots, v_{|A|}\}$. At any t before the segment ends, allocate the entire curing budget to cure an arbitrary node in the set $D(t) \stackrel{\text{def}}{=} I(t)\backslash C$. A segment ends when $I(t) = C$ or $|D(t)| \geq r/(8d_{\max})$. If $I(t) = C$, a new segment will be started. If $|D(t)| \geq r/(8d_{max})$, a waiting period will be started.

The paper [10] does not specify the algorithm to calculate the optimal crusade. A dynamic programming algorithm with exponential time and space complexity is obtained by directly applying the Bellman equation given in the paper. In [10], improving the computational complexity of the algorithm was listed as an open problem. In this paper we study the following optimization problem.

Problem 1: Given a network G and a bag A, find a crusade p from A to $\varnothing$ such that

$$\underset{p\in C(A,\varnothing)}{\text{minimize}} \quad z(p)\,. \tag{2}$$

It has been shown that the problem of calculating the cutwidth (and hence the impedance) of a graph is **NP**-hard [29]. Therefore, we resort to approximation algorithms which compute a crusade whose width is bounded within a certain factor compared to the minimum width.

B. Network Design

A drawback of the CURE policy is that when $c(I(t)) > r/8$, the policy starts a waiting period. This is clearly undesired since the epidemic spreads to more nodes while no measures are taken. For similar reasons, the CURE policy only works when the budget is $\Omega(\mathcal{W})$, where $\mathcal{W}$ is the cutwidth of the network. Moreover, the authors have shown in subsequent work [30], [27] that there exists a phase transition such that when $r = o(\mathcal{W})$, the extinction time is exponential regardless of the curing policy.

Real-world epidemic response is often a composition of medical resource allocation and contact restrictions. Previous work has shown that, by applying along with preferentially vaccinating urban locations, travel restrictions can postpone the arrival of peak in the spread of influenza [31]. School closure and social distancing are combined with antiviral treatment or vaccination to reduce the cost of an influenza pandemic [32]. A simulation of smallpox shows the effectiveness of combining vaccination with school closure [33]. Motivated by these simulation studies, we investigate the problem of reducing network connections to achieve fast extinction with insufficient medical resources.

Given a graph G and a budget r, if r is insufficient for any policy to suppress the spread, we ask the question of how to modify the network to effectively reduce its impedance and cutwidth. Since modifying the network results in the change of optimal curing order, and changing the curing order affects the optimal weight reduction of the network, optimizing the two policies simultaneously is challenging. We consider a simplified problem with any fixed curing order[1].

Given the CURE policy, we study an alternative problem, in which we modify the graph to effectively reduce the width of a *fixed* crusade computed in the beginning of each curing segment. Specifically, we modify the graph by reducing the weight of each edge $(u, v) \in E$ to $w'_{uv} = w_{uv} - \Delta_{uv}$. Our goal is to minimize the total weight reduction such that the width of the current crusade is no more than a threshold b.

Problem 2: Given a graph G with the edge weight function w, a bag A, a crusade p from A to $\varnothing$, and a threshold $b \in \mathbb{R}_+$, find the weight reduction Δ_{uv} of each edge $(u, v) \in E$ for the following optimization program:

$$\begin{aligned} \underset{\Delta}{\text{minimize}} \quad & \sum_{(u,v)\in E} \Delta_{uv}, \\ \text{subject to} \quad & 0 \leq \Delta_{uv} \leq w_{uv}, \forall (u, v) \in E\,, \\ & z_{G'}(p) \leq b\,, \end{aligned} \tag{3}$$

where $G' = (V, E, w')$ is the modified graph with weight $w'_{uv} = w_{uv} - \Delta_{uv}$ for $(u, v) \in E$.

We also consider this problem under the constraint that weight reductions $\Delta_{uv} \in \{0, w_{uv}\}$ for all edges, which means each edge is either left intact or removed completely.

Problem 3: Given a graph G with the edge weight function w, a bag A and a crusade p from A to $\varnothing$, a threshold

[1] In practice we use the ordering attained by applying curing policies to the original contact network.

$b \in \mathbb{R}_+$, find the weight reduction Δ_{uv} of each edge $(u,v) \in E$ for the following optimization program:

$$\begin{aligned} \underset{\Delta}{\text{minimize}} \quad & \sum_{(u,v)\in E} \Delta_{uv}, \\ \text{subject to} \quad & \Delta_{uv} \in \{0, w_{uv}\}, \forall (u,v) \in E\,, \\ & z_{G'}(p) \le b\,, \end{aligned} \tag{4}$$

where $G' = (V, E, w')$ is the modified graph with weight $w'_{uv} = w_{uv} - \Delta_{uv}$ for $(u,v) \in E$.

When the curing policy is decided by the environment, the ordering of curing is arbitrary. Therefore, a variant of the maximum cut of the network decides the extinction time of the SIS process. We also investigate the problem of minimizing the maximum cut of a given bag.

Problem 4: Given a graph G with the edge weight function w, a bag A, and a threshold $b \in \mathbb{R}_+$, find the weight reduction Δ_{uv} of each edge $(u,v) \in E$ for the following optimization program:

$$\begin{aligned} \underset{\Delta}{\text{minimize}} \quad & \sum_{(u,v)\in E} \Delta_{uv}, \\ \text{subject to} \quad & 0 \le \Delta_{uv} \le w_{uv}, \forall (u,v) \in E\,, \\ & \phi_{G'}(A) \le b\,. \end{aligned} \tag{5}$$

where $G' = (V, E, w')$ is the modified graph with weight $w'_{uv} = w_{uv} - \Delta_{uv}$ for $(u,v) \in E$.

IV. Curing Algorithms

In this section we provide algorithms for approximating impedance of a bag. The definition of these problems are given in Section III-A.

A. Algorithm for CURE Policy

In this section, we provide a polynomial-time curing algorithm for the networked SIS model. Specifically, we combine a polynomial-time approximation algorithm for computing the crusade with the CURE policy proposed in [10]. We present the following theorem.

Theorem 1: Given a graph G, suppose the curing budget $r \ge \max\{\alpha \mathcal{W} \log^2 n, 8d_{max} \log n\}$, where $\mathcal{W}$ is the cutwidth of graph G and α is a fixed constant. Then, there exists a polynomial-time curing algorithm such that the expected extinction time is at most $O(n \log^2 n / r)$.

To prove this theorem, we use a polynomial-time algorithm that, given any bag A, finds a crusade of A with width at most $O(\log^2 k)$ times the impedance of A. This algorithm is shown in Algorithm 1. Our algorithm follows the approaches of approximation algorithms for multiway cut [34] and cutwidth [25]. The crux of the algorithm is an approximation algorithm for the balanced cut problem [25].

Algorithm 1: ApprImpe(G, A)

```
Input  : a graph G = (V, E, w), a bag A with |A| = k;
Output: a crusade p ∈ C(A, ∅);
if |A| = 1 then
  | return p ← ({u}, ∅);
else
  | (V_1, V_2) ← BalancedCut(G[A], A);
  | (q_0, ..., q_{|V_1|}) ← ApprImpe(G[V_1], V_1);
  | (p_{|V_1|}, ..., p_{|A|}) ← ApprImpe(G[V_2], V_2);
  | for i ← 0 to |V_1| − 1 do
  |   | p_i ← q_i ∪ V_2;
  | end
  | return p ← (p_0, ..., p_{|A|});
end
```

Algorithm 2: WidthOpt(G, A, p)

```
Input  : a graph G = (V, E, w), a bag A with |A| = k, a
         crusade p from A to ∅
Output: Δ_uv ∈ {0, w_uv} for (u, v) ∈ E
Let {((u, v), Δ*_uv) : (u, v) ∈ E} be an optimal solution for
  the LP in Problem 2;
v_i ← p_{i−1} \ p_i for i ∈ [k];
Set an arbitrary ordering {v_{k+1}, v_{k+2}, ..., v_n} for nodes
  in A^c;
for i ← 1 to k do
  | Let E⃗_i be the sorted list of {(v_i, v_j) : j > i} in
  |   non-increasing order of j;
  | x ← 0;
  | while x < Σ_{(u,v)∈E⃗_i} Δ*_uv do
  |   | Let the edge (u, v) be the first edge in E⃗_i;
  |   | x ← x + w_uv;
  |   | Δ_uv ← w_uv and remove (u, v) from E⃗_i;
  | end
  | Δ_uv ← 0 for the remaining edges (u, v) in E⃗_i;
end
```

V. Network Design

In this section we present algorithms for the network design problems posed in Section III-B.

A. Network Design for Curing Policies

In this section we consider the problems of designing the structure of the network to minimize the width of a fixed crusade, namely Problems 2 and 3.

For a fixed crusade, Problem 2 is a linear program with a cost function that is linear in the edge weights and k linear constraints for each cut to be no more than the curing budget. Therefore we obtain the optimal solution in polynomial time.

Theorem 2: There exists a polynomial-time algorithm which finds the optimal solution for Problem 2.

Next we consider Problem 3, in which Δ_{uv} takes the value of either w_{uv} or 0. We provide an algorithm with the following guarantee:

Theorem 3: Given a graph G, a bag A with $|A| = k$, and a crusade p from A to $\varnothing$, Algorithm 2 provides a solution of Problem 3 with the total weight reduction $\mathrm{OPT}(G, A, p) + k$, where $\mathrm{OPT}(G, A, p)$ is the optimal solution of Problem 3.

We then consider an unweighted version of Problem 3 where $w_{uv} = 1$ and $\Delta_{uv} \in \{0, 1\}$ for all $(u,v) \in E$, which we refer to as the Unweighted Crusade Width Cost Minimization Problem (UWCMP). We show an algorithm which finds the optimal solution of UWCMP in polynomial time. Details are shown in the techical report [26].

Remark 1: By applying the proposed network design algorithms, we can discard the waiting period in the CURE policy. Instead we modify the network such that $z_{G'}(p) \leq r/4$ and start a segment. The target path is given by p. Then we start a new segment. When $|D(t)| \geq r/(4d_{\max})$, we calculate a new nearly optimal crusade p and a modified graph G' with $z_{G'}(p) \leq r/4$ and start a new segment.

B. Network Design for Minimizing the MRC

In this section, we consider the problem of designing the structure of the network to minimize the MRC of a given bag, namely Problem 4. Given an algorithm for Problem 4, we propose a policy to guarantee sublinear expected extinction time, with arbitrary ordering of curing. Let $r(t)$ be the sum of curing rates of all nodes in $I(t)$. Suppose $r(t)$ is always greater than a fixed constant r' for all t, the policy is:

- Network design: modify the graph such that $\phi_{G'}(I(t)) \leq r'/4$ and start a segment. Let A be the set of infected nodes.
- Segment: Define $D(t) = I(t) \setminus A$. If $|D(t)| \geq r'/(4d_{\max}) - 1$ then start a new network design period.

Theorem 4: If $r' \geq 2 \log n$ always holds, by modifying G' such that $\phi_{G'}(I(t)) \leq r'/4$, the policy achieves sublinear expected extinction time.

Now we present the algorithm for Problem 4. We consider the following minimax program for the modified graph $G' = (V, E, w')$ where $w'_{uv} = w_{uv} - \Delta_{uv}$:

$$\begin{aligned} \underset{G'}{\text{minimize}} \quad & \phi_{G'}(A)\,, \qquad (6)\\ \text{subject to} \quad & 0 \leq \Delta_{uv} \leq w_{uv}, \forall (u,v) \in E\,,\\ & \sum_{(u,v)\in E} \Delta_{uv} \leq b'\,, \end{aligned}$$

where b' is a given edge reduction budget. We note that by running a binary search on b' we solve Problem 4.

Theorem 5: There exists a polynomial time algorithm which computes a graph G' with weight reductions Δ_{uv}, in which $\phi_{G'}(A) \leq 1.14 \cdot \phi_{\tilde{G}}(A)$ where $\tilde{G}$ is any modified graph with the same reduction budget b.

To prove Theorem 5, we transform the problem into a Semidefinite Programming (SDP) relaxation [35] solvable in polynomial time. For details, refer to techincal report [26].

VI. Numerical Simulations

In this section, we examine the effectiveness of the proposed curing policies as well as the network design algorithms through numerical simulations. The model for the SIS process is described in Section II-B. We only present an overview of the experiment. For detailed parameters and configurations, please refer to technical report [26].

A. Simulations for Curing Policies

We compare Algorithm 1 against the following four baseline curing policies, including two static policies and their dynamic variations.

1) Uniform (static): the curing budget is uniformly allocated to all nodes, $\rho_u(t) = 1/n, \forall u \in V$.
2) Degree (static) [14]: the curing rate of a node $u \in V$ is set to $r \cdot d_u / (\sum_{u\in V} d_u), \forall u \in V$.
3) Uniform (dynamic): the curing budget is uniformly allocated to all *currently infected* nodes, i.e. $\rho_u(t) = 1/|I(t)|, \forall u \in I(t)$, and otherwise set to 0.
4) Degree (dynamic): the curing rate of a node $u \in I(t)$ is set to $\rho_u(t) = r \cdot d_u I_u(t) / (\sum_{u\in V} d_u I_u(t))$, and otherwise set to 0.

To test the effectiveness of the CURE policy using Algorithm 1, we simulate the SIS process on two typical networks: the locally connected network and the binary tree network. For information on how to build these two networks, please refer to technical report [26].

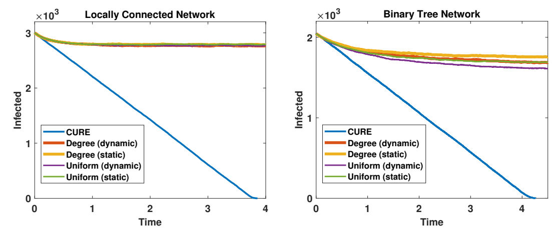

Fig. 1. Comparison of CURE (using Algorithm 1) with baseline policies

Figure 1 shows the performance of the CURE policy and the baselines. In both networks, the CURE policy is the only one that succeeds in curing all nodes in all 10 runs.

B. Simulations for Network Design Algorithms

For the network design algorithms, we simulate the process on a human contact network and an email network. The human contact network is constructed by collecting data using mobile technologies [36], [37]. The email network was generated using email data from a research institute [38].

For both networks, we compare the CURE policy with the CURE policy augmented by linear programming-based network design, as well as random curing budget allocation complemented by semi-definite programming-based network design. Figure 2 shows the average trajectories of 10 runs for each method for both networks. Simulations on both networks show consistent results.

For the human contact network, the trajectory of the CURE policy clearly shows a waiting period at the beginning of the process. After the curing rates are dynamically allocated, the number of infections plateaus subsequent to a transient declination. With network design, the curing processes are continued and quickly stop the SIS processes. We note that the SDP approach considers the worst curing rate allocation strategy and hence is more conservative. The SDP network algorithm achieves shorter extinction time with random curing targets at any given time but with more edge weight reduction cost. Details are shown in the technical report [22].

VII. Conclusion

In this paper, we have studied efficient algorithms for dynamic curing policies of SIS epidemic models. We have proposed a computationally efficient approximation algorithm to the impedance of an infected set in a given graph. We

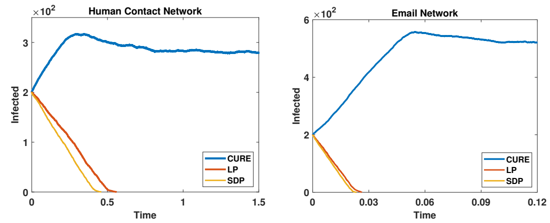

Fig. 2. Comparison of network design algorithms: 1) CURE without network design; 2) CURE with linear programming network design; 3) Random curing rate allocation with SDP.

have also proposed algorithms for network design problems to help cure the network. Provable guarantees have been provided. Additionally, We have shown the effectiveness of the policy and network co-design approach by running simulations on real contact networks.

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