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Inverse Optimal Adaptive Prescribed Performance Control With Application to Compliant Actuator-Driven Robot Manipulators

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Abstract— **In this work, we formulate and solve the problem of inverse optimal adaptive prescribed performance control and consider its application to compliant actuator-driven robot manipulators. A definition and sufficient conditions for this problem are introduced and derived based on adaptive control Lyapunov function method. An auxiliary system is constructed and incorporated with prescribed performance bounds so as to design a new class of inverse optimal adaptive controllers for the control system. By exploring the links between inverse optimality and stability, it is proved that the proposed controller ensures both inverse optimality and prescribed transient performance of the control system. Above developments are illustrated via an application to robot manipulators driven by compliant actuators. The inverse optimal adaptive control problem for robot manipulators with guaranteed transient performance has not been addressed in the literature.**

I. INTRODUCTION

Inverse optimal control refers to the problem of searching a potential performance criterion for which a given control law is optimal, and it receives growing interests in many fields due to its wide applications from robotics to biomechanics (e.g., [1]–[4]). Inverse optimal control theory was motivated by the discovery found by Kalman that the optimal control laws with desirable properties are not rigidly tied to a single performance index and the most important aspects of optimality hold independently of the choice of the performance criterion [5]. In general, existing literatures on inverse optimal control can be classified into two major categories, one is known as *inverse reinforcement learning* [6], which aims for learning the cost function from the observed optimal behaviours or trajectories, the other is *inverse optimal gain assignment* [7], which solves the optimal control problem but avoids the need to solve the Hamilton-Jacobi-Bellman (HJB) equations. In this work, we focus on the latter category.

The problem of inverse optimal gain assignment was first proposed and solved by Krstic *et al* in [7]. Instead of finding the optimal control law based on a given cost function, the inverse approach proposed in [7] searches for not only the optimal control law but also the underlying cost functional simultaneously. With such an approach, the need of solving the HJB equation, which leads to a computational bottleneck, is avoided. In [8], Deng *et al* proposed a new criterion on inverse optimal stabilization and designed the inverse optimal controller directly without recourse to an auxiliary system as constructed in [7]. In regard to the systems with partial unknown dynamics, in [9], Li *et al* formulated the inverse optimal adaptive tracking problem and solved this problem by using adaptive control Lyapunov function method [10]. In [11], an inverse optimal adaptive tracking approach was proposed for spacecraft systems. Above pioneering works ensure global results and are effective for the systems whose dynamics are completely known or only some parameters are unknown. With respect to the systems with unknown functions that cannot be linearly parameterized, intelligent approximation tools, such as fuzzy logic systems (FLSs) and neural networks (NNs), are used to deal with the uncertain dynamics. But the inverse optimal designs developed in [7]–[9], [11] are not compatible with the fuzzy/neural control strategies. The reason is that solvability of the inverse optimal problem needs asymptotical or adaptive stabilization of the control system, which cannot be achieved by fuzzy/neural control. To address this issue, a criterion on inverse optimal *practical* stabilization was proposed in [12], which makes great senses to extend the results in [7]–[9], [11] to inverse optimal fuzzy/neural control.

Prescribed performance control enjoys many desirable properties beyond asymptotic tracking control since it guarantees a better transient response for a system with uncertain dynamics, such as a faster convergence rate and a smaller maximum overshoot of the regulation or tracking error [13], [14]. Prescribed performance control was originally presented in [15] and then extended in [16] based on prescribed performance bounds (PPB). The major idea of the PPB technique is to construct a new transformed system by incorporating the performance bounds into the original nonlinear system. Then by establishing the boundedness property of the transformed system, prescribed transient performance of the control system is then achieved.

With the growing role of robotics in practical implementations, the importance of improving the control performance of robots in regard to different performance indexes has also grown. These facts provide the main motivations for inverse optimal adaptive prescribed performance control considered in this paper. However, although the problems on inverse optimal control and prescribed performance control have been extensively investigated individually, very few results are reported on solving the inverse optimal control problem for uncertain nonlinear systems with guaranteed transient performance. The major difficulty lies in two aspects: i) most existing inverse optimal adaptive designs (e.g., [7], [9], [11]) require to construct an auxiliary system first and the control

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design is developed for the auxiliary system. It implies that if we combine the PPB technique with inverse optimal design in [7], [9], [11], the obtained prescribed performance results are only for the auxiliary system but not the control system; ii) the approach in [8] avoids the need to design an auxiliary system and the control design is for the control system, but it requires the controller to be of some specified forms. It is difficult to ensure existing prescribed performance controller to be of such forms.

In this paper, we consider the problem of inverse optimal adaptive prescribed performance control for strict-feedback systems and its application to robot manipulators driven by compliant actuators. Our contributions are listed as follows.

- A definition and sufficient conditions for inverse optimal adaptive prescribed performance control problem are introduced and derived based on adaptive control Lyapunov function method.
- An auxiliary system is constructed and the prescribed performance bounds are incorporated with the auxiliary system to design a non-adaptive controller. Then a new class of inverse optimal adaptive controllers is designed for the control system based on the proper design of the non-adaptive controller.
- Above developments are applied to design an inverse optimal adaptive prescribed performance controller for robot manipulators driven by compliant actuators.

The rest of the paper is organized as follows. Preliminaries and problem formulation are given in Section II. The main results are presented in Section III. Applications of the main results to robot manipulators driven by compliant actuators are shown in Section IV. Conclusions and future research are presented in Section V.

II. Preliminaries and Problem Formulation

Consider a class of strict-feedback systems as follows

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta^T\phi_i(\bar{x}_i),\ i = 1, ..., n-1 \\ x_n &= u + \theta^T\phi_n(\bar{x}_n) \\ y &= x_1, \end{aligned} \tag{1}$$

where $x_i \in R$ for $i = 1, ..., n$ is the system state, $\bar{x}_i = [x_1, ..., x_i]^T \in R^i$ is the state vector. $\phi_i(\bar{x}_i) \in R^m$ is a smooth function with $\phi_i(0) = 0$ and $\theta \in R^m$ is an unknown parameter. $y \in R$ and $u \in R$ are the system output and input, respectively.

Definition 1. The *inverse optimal adaptive prescribed performance control problem* of (1) is solvable if there exist positive functions $E(x, \hat{\theta})$, $M(x, \hat{\theta})$ and $R(x, \hat{\theta})$, a constant $\varepsilon > 0$, and a feedback control law $u = \omega(x, \hat{\theta})$ updated by $\dot{\hat{\theta}} = \vartheta(x, \hat{\theta})$, which solves the adaptive control problem of (1), minimizes the cost functional

$$\begin{aligned} \mathcal{J}(u) = \lim_{t\to\infty} \Big\{ & E(x(t), \hat{\theta}(t)) + \varepsilon\tilde{\theta}(t)^T\tilde{\theta}(t) \\ & + \int_0^t M(x(\upsilon), \hat{\theta}(\upsilon)) + u^T R(x(\upsilon), \hat{\theta}(\upsilon)) u d\upsilon \Big\} \end{aligned} \tag{2}$$

and the regulation error satisfies the prescribed performance bounds all the time.

Problem 1. In this work, the problem of interest is to solve the inverse optimal adaptive prescribed performance control problem of (1).

To address Problem 1, we design the following decreasing function $\eta(t) = (\eta_0 - \eta_\infty)e^{-at} + \eta_\infty$, where $\eta_0 = \eta(0)$, $\eta_\infty = \eta(\infty)$, $a > 0$ is a design parameter. The objective of Problem 1 is to ensure the inverse optimality and the regulation error $\xi(t) = y(t)$ remains in the prescribed performance bounds

$$-\sigma_m\eta(t) < \xi(t) < \sigma_M\eta(t), \quad t > 0 \tag{3}$$

all the time, where $\sigma_m, \sigma_M > 0$ are design parameters. To ensure (3), we design a smooth and increasing function

$$S(\gamma) = \frac{\sigma_M e^{(\gamma+w)} - \sigma_m e^{-(\gamma+w)}}{e^{(\gamma+w)} + e^{-(\gamma+w)}}, \tag{4}$$

where $w = \frac{\ln(\sigma_m/\sigma_M)}{2}$. The function $S(\gamma)$ has the following properties: i) $S(0) = 0$; ii) $-\sigma_m < S(\gamma) < \sigma_M$; and iii) $\lim\limits_{\gamma\to-\infty} S(\gamma) = -\sigma_m$ and $\lim\limits_{\gamma\to+\infty} S(\gamma) = \sigma_M$. With these properties, the performance condition (3) can be expressed as $\xi(t) = \eta(t)S(\gamma)$. Since $\eta(t) \neq 0$, and the inverse function S^{-1} exists and is

$$\begin{aligned} \gamma(t) &= S^{-1} \circ \rho(t) \\ &= \frac{1}{2}\ln\Big[\sigma_M\rho(t) + \sigma_m\sigma_M\Big] - \frac{1}{2}\ln\Big[\sigma_m\sigma_M - \sigma_m\rho(t)\Big]. \end{aligned} \tag{5}$$

We call $\gamma(t)$ as a transformation error and $\rho(t) = \xi(t)/\eta(t)$. Obviously, if the initialization $\xi(0)$ satisfies $-\sigma_m\eta(0) < \xi(0) < \sigma_M\eta(0)$ and $\gamma(t)$ remains bounded, the condition (3) holds and $\lim\limits_{t\to\infty} \xi(t) = 0$ is achieved if $\lim\limits_{t\to\infty} \gamma(t) = 0$ is followed.

III. Inverse Optimal Adaptive Prescribed Performance Control

Define the state error variables

$$\begin{aligned} \zeta_1 &= \gamma \\ \zeta_i &= x_i - \varrho_{i-1},\ i = 2, ..., n. \end{aligned} \tag{6}$$

Revisiting the definition of the transformation error γ in (5), we have

$$\dot{\gamma} = \frac{\partial S^{-1}}{\partial \rho}\dot{\rho} = \lambda\Big[\zeta_2 + \varrho_1 + \theta^T\phi_1 - \frac{y\dot{\eta}}{\eta}\Big], \tag{7}$$

where $\lambda = \frac{1}{2\eta}(\frac{1}{\sigma_m+\rho} + \frac{1}{\sigma_M-\rho})$. Then we construct an auxiliary system of the control system (1) as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_n^T \end{bmatrix} \Big[\theta + \Gamma\Big(\frac{\partial V}{\partial \theta}\Big)^T\Big] + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u, \tag{8}$$

where V is a Lyapunov function candidate chosen as

$$V(\zeta,\theta)=\sum_{i=1}^{n}\frac{1}{2}\zeta_i^2. \tag{9}$$

$\Gamma>0$ is a design parameter. Let $x=[x_1,x_2,...,x_n]^T$, $f(\bar{x}_n)=[x_2,x_3,...,x_n,0]^T$, $\Phi(\bar{x}_n)=[\phi_1,\phi_2,...,\phi_n]^T$, $g=[0,...0,1]^T$, $\mathcal{N}=[\eta,\dot{\eta},...,\eta^{(n-1)}]^T$, $\alpha_0=0$ and $x_{n+1}=0$. Then the Lie derivatives $L_fV=\frac{\partial V}{\partial x}f(\bar{x}_n)$ and $L_\phi V=\frac{\partial V}{\partial x}\Phi(\bar{x}_n)$. Specifically,

$$L_fV=\zeta_1\frac{\partial\gamma}{\partial x_1}x_2+\sum_{i=2}^{n}\zeta_i\Big(x_{i+1}-\sum_{j=1}^{i-1}\frac{\partial\varrho_{i-1}}{\partial x_j}x_{j+1}\Big) \tag{10}$$

$$L_\phi V=\zeta_1\frac{\partial\gamma}{\partial x_1}\phi_1+\sum_{i=2}^{n}\zeta_i\bar{\phi}_i \tag{11}$$

$$\frac{\partial V}{\partial\theta}=\frac{\partial\sum_{i=1}^{n}\frac{1}{2}\zeta_i^2}{\partial\theta}=-\sum_{i=1}^{n}\zeta_i\frac{\partial\varrho_{i-1}}{\partial\theta} \tag{12}$$

$$\frac{\partial V}{\partial\mathcal{N}}\dot{\mathcal{N}}=-\lambda\zeta_1\frac{x_1}{\eta}\dot{\eta}-\sum_{i=2}^{n}\zeta_i\sum_{j=1}^{i}\frac{\partial\varrho_{i-1}}{\partial\eta^{(j-1)}}\eta^{(j)}, \tag{13}$$

where $\bar{\phi}_i=\phi_i-\sum_{j=1}^{i-1}\frac{\partial\varrho_{i-1}}{\partial x_j}\phi_j$ for $i=1,...,n$. From (5), one has $\frac{\partial\gamma}{\partial x_1}=\frac{\partial\gamma}{\partial\rho}\frac{\partial\rho}{\partial x_1}=\lambda$. Based on (10)-(13), we have

$$\begin{aligned}\dot{V}=&\lambda\zeta_1x_2+\lambda\zeta_1\theta^T\phi_1+\sum_{i=2}^{n}\zeta_i\Big(\zeta_{i+1}+\varrho_i-\sum_{j=1}^{i-1}\frac{\partial\varrho_{i-1}}{\partial x_j}x_{j+1}\Big)\\&+\sum_{i=2}^{n}\zeta_i\theta^T\bar{\phi}_i\underbrace{-\Gamma\Big(\lambda\zeta_1\phi_1^T+\sum_{i=2}^{n}\zeta_i\bar{\phi}_i^T\Big)\sum_{j=1}^{n}\zeta_j\frac{\partial\varrho_{j-1}}{\partial\theta}}_{\tau}\\&-\sum_{i=1}^{n}\zeta_i\sum_{j=1}^{i}\frac{\partial\varrho_{i-1}}{\partial\eta^{(j-1)}}\eta^{(j)}+\zeta_nu.\end{aligned} \tag{14}$$

Before designing the virtual controller ϱ_i, we rearrange the under-braced term τ as

$$\begin{aligned}\tau=&-\Gamma\sum_{i=2}^{n}\zeta_i\Big(\sum_{j=1}^{i}\zeta_j\bar{\phi}_i^T\frac{\partial\varrho_{j-1}}{\partial\theta}+\sum_{j=2}^{i-1}\zeta_j\bar{\phi}_j^T\frac{\partial\varrho_{i-1}}{\partial\theta}\Big)\\&-\Gamma\lambda\zeta_1\phi_1^T\sum_{i=1}^{n}\zeta_i\frac{\partial\varrho_{i-1}}{\partial\theta}=-\Gamma\sum_{i=1}^{n}\zeta_i\tau_i,\end{aligned} \tag{15}$$

where $\tau_i=\sum_{j=1}^{i}\zeta_j\bar{\phi}_i^T\frac{\partial\varrho_{j-1}}{\partial\theta}+(\lambda\zeta_1\phi_1^T+\sum_{j=2}^{i-1}\zeta_j\bar{\phi}_j^T)\frac{\partial\varrho_{i-1}}{\partial\theta}$. For the definition of τ_i, we have $\tau_1=0$ and $\tau_2=(\lambda\zeta_1\phi_1+\zeta_2\bar{\phi}_2)^T\frac{\partial\varrho_1}{\partial\theta}$ since $\varrho_0=0$ and $\sum_{j=2}^{i-1}\zeta_j\bar{\phi}_j^T\frac{\partial\varrho_{i-1}}{\partial\theta}=0$ for $i=2$. Now we design the virtual controller ϱ_i as follows

$$\varrho_1=-\frac{c_1\zeta_1}{\lambda}-\theta^T\phi_1+\frac{x_1}{\eta}\dot{\eta} \tag{16}$$

$$\varrho_2=-c_2\zeta_2-\lambda\zeta_1-\theta^T\bar{\phi}_2+\frac{\partial\varrho_1}{\partial x_1}x_2+\Gamma\tau_2+\sum_{j=1}^{2}\frac{\partial\varrho_1}{\partial\eta^{(j-1)}}\eta^{(j)} \tag{17}$$

$$\begin{aligned}\varrho_i=&-c_i\zeta_i-\zeta_{i-1}-\theta^T\bar{\phi}_i+\sum_{j=1}^{i-1}\frac{\partial\varrho_{i-1}}{\partial x_j}x_{j+1}+\Gamma\tau_i\\&+\sum_{j=1}^{i}\frac{\partial\varrho_{i-1}}{\partial\eta^{(j-1)}}\eta^{(j)},\ i=3,...,n.\end{aligned} \tag{18}$$

Substituting ϱ_i into $\dot{V}$ (14), we have

$$\dot{V}=-\sum_{i=1}^{n-1}c_i\zeta_i^2+\zeta_n(u+\Delta_\zeta+\Delta_x+\Delta_\eta), \tag{19}$$

where $\Delta_\zeta=\zeta_{n-1}-\Gamma\tau_n$, $\Delta_x=\theta^T\bar{\phi}_n-\sum_{i=1}^{n-1}\frac{\partial\varrho_{n-1}}{\partial x_i}x_{i+1}$ and $\Delta_\eta=-\sum_{i=1}^{n}\frac{\partial\varrho_{n-1}}{\partial\eta^{(i-1)}}\eta^{(i)}$. With the definition of τ_i, we see that Δ_ζ can be rewritten as $\sum_{i=1}^{n}\alpha_i\zeta_i$. Now we analyze Δ_x and show that Δ_x vanishes at $\bar{\zeta}_n=0$. If $\zeta_1=0$, then $\gamma=\zeta_1=0$, $x_1=0$, $\bar{\phi}_1=\phi_1=0$ and $\varrho_1=0$. If $\zeta_2=\zeta_1=0$, then $x_1=0$, $x_2=\zeta_2+\varrho_1=0$ and $\bar{\phi}_2=\phi_2-\frac{\partial\varrho_1}{\partial x_1}\phi_1=0$. Let $\eta_1=\eta$ and $\eta_2=\dot{\eta}$ and we have $\frac{\partial\zeta_1}{\partial\eta}=\frac{\partial\gamma}{\partial\rho}\frac{\partial\rho}{\partial\eta}=-\lambda\frac{x_1}{\eta}=0$, $\frac{\partial\varrho_1}{\partial\eta}=\frac{\partial\varrho_1}{\partial\eta_1}=\frac{\partial}{\partial\eta_1}(-\frac{c_1\zeta_1}{\lambda}+\frac{x_1}{\eta_1}\eta_2)=-\frac{c1}{\lambda^2}(\frac{\partial\zeta_1}{\partial\eta_1}\lambda-\zeta_1\frac{\partial\lambda}{\partial\eta_1})-\frac{x_1}{\eta_1^2}\eta_2=0$, $\frac{\partial\varrho_1}{\partial\dot{\eta}}=\frac{\partial\varrho_1}{\partial\eta_2}=\frac{x_1}{\eta_1}=0$ and $\varrho_2=-c_2\zeta_2-\lambda\zeta_1-\theta^T\bar{\phi}_2+\frac{\partial\varrho_1}{\partial x_1}x_2+\Gamma\tau_2+\sum_{j=1}^{2}\frac{\partial\varrho_1}{\partial\eta^{(j-1)}}\eta^{(j)}=0$. For the similar analysis, we see that if $\bar{\zeta}_n=0$, then $\bar{\phi}_n=0$ and $x_1=x_2=\cdots x_n=0$. Thus, Δ_x can be also rewritten as $\Delta_x=\sum_{i=1}^{n}\beta_i\zeta_i$. From the analysis for Δ_x, we see that Δ_η also vanishes at $\bar{\zeta}_n=0$ and thus it can be rewritten as $\Delta_\eta=\sum_{i=1}^{n}\varphi_i\zeta_i$. Now we design the auxiliary controller u as

$$u=-R(\zeta,\theta)^{-1}\zeta_n, \tag{20}$$

where

$$R(\zeta,\theta)=\Big[c_n+\sum_{i=1}^{n}\frac{(\alpha_i+\beta_i+\varphi_i)^2}{2c_i}\Big]^{-1}. \tag{21}$$

The main results are summarized in the following Theorem.

Theorem 1. *Consider the control system (1). If the controller u (35) asymptotically stabilizes the auxiliary system (8), then the adaptive feedback control*

$$u^*=-\kappa R(\zeta,\hat{\theta})^{-1}\zeta_n,\quad \kappa\geq 2 \tag{22}$$

$$\dot{\hat{\theta}}=\Gamma L_\phi V \tag{23}$$

solves the inverse optimal adaptive prescribed performance control problem of (1) by minimizing the cost functional

$$\begin{aligned}\mathcal{J}(u)=\lim_{t\to\infty}\Big\{&2\kappa V(\zeta,\hat{\theta})+\frac{\kappa}{\Gamma}\tilde{\theta}^T\tilde{\theta}\\&+\int_0^t\Big[M(\zeta,\hat{\theta})+u^TR(\zeta,\hat{\theta})u\Big]d\upsilon\Big\},\end{aligned} \tag{24}$$

where

$$\begin{aligned}M(\zeta,\hat{\theta})=&-2\kappa\Big[L_fV+L_\phi V\Big(\hat{\theta}+\Gamma\Big(\frac{\partial V}{\partial\hat{\theta}}\Big)^T\Big)\\&-L_gVR^{-1}L_gV\Big]+\kappa(\kappa-2)L_gVR^{-1}L_gV.\end{aligned} \tag{25}$$

Proof. Substituting u (35) into the auxiliary system (8), we have

$$\dot{V}=-\sum_{i=1}^{n}\frac{1}{2}c_i\zeta_i^2-\sum_{i=1}^{n}\frac{c_i}{2}\Big(\zeta_i-\frac{\alpha_i+\beta_i+\varphi_i}{c_i}\zeta_n\Big)^2 \tag{26}$$

for all $\theta \in R^m$. From (9) and recalling that $g = [0, ..., 0, 1]^T$, we have $L_g V = \zeta_n$. Then by denoting $W(\zeta, \theta) = -\sum_{i=1}^{n} \frac{1}{2} c_i \zeta_i^2 - \sum_{i=1}^{n} \frac{c_i}{2}(\zeta_i - \frac{\alpha_i+\beta_i+\varphi_i}{c_i}\zeta_n)^2$, we have

$$L_f V + L_\phi V\Big[\hat{\theta} + \Gamma\Big(\frac{\partial V}{\partial \hat{\theta}}\Big)^T\Big] - L_g V R^{-1} L_g V \leq -W(\zeta, \hat{\theta}).$$

Thus, $M(\zeta, \hat{\theta}) \geq 2\kappa W(\zeta, \hat{\theta})$ is positive-definite and $\mathcal{J}(u)$ is meaningful. Substituting (22)-(23) into (24), it yields that

$$\begin{aligned}
&\mathcal{J}(u) \\
&= \lim_{t\to\infty} \Bigg\{ 2\kappa V(\zeta, \hat{\theta}) + \frac{\kappa}{\Gamma}\tilde{\theta}^T\tilde{\theta} - 2\kappa \int_0^t \Bigg(L_f V + L_\phi V \theta \\
&\quad + L_g V u + \frac{\partial V}{\partial \hat{\theta}}\dot{\hat{\theta}}\Bigg) d\upsilon + 2\kappa \int_0^t \Bigg[L_\phi V \tilde{\theta} - L_\phi V \Gamma \Bigg(\frac{\partial V}{\partial \hat{\theta}}\Bigg)^T \\
&\quad + \frac{\partial V}{\partial \hat{\theta}}\dot{\hat{\theta}}\Bigg] d\upsilon + \int_0^t \Bigg(2\kappa L_g V u + \kappa^2 L_g V R^{-1} L_g V \\
&\quad + u^T R u \Bigg) d\upsilon \Bigg\} \\
&= 2\kappa V(\zeta(\infty), \tilde{\theta}(\infty)) + \frac{\kappa}{\Gamma}\tilde{\theta}(\infty)^T\tilde{\theta}(\infty) - 2\kappa \int_0^\infty dV \\
&\quad - 2\kappa \int_0^\infty \frac{1}{\Gamma}\tilde{\theta}^T\dot{\tilde{\theta}} dt + \int_0^\infty (u - u^*) R (u - u^*) dt. \qquad (27)
\end{aligned}$$

Thus, $\mathcal{J}(u)$ achieves its minimum at $u = u^*$ and $\mathcal{J}(u)_{\min} = 2\kappa V(\zeta(0), \tilde{\theta}(0)) + \frac{\kappa}{\Gamma}\tilde{\theta}(0)^T\tilde{\theta}(0)$. Choose a Lyapunov function candidate for (1)

$$\bar{V} = V(\zeta, \hat{\theta}) + \frac{1}{2\Gamma}\tilde{\theta}^T\tilde{\theta}. \qquad (28)$$

The time derivative of $\bar{V}$ is

$$\begin{aligned}
\dot{\bar{V}} = &-\frac{1}{2\kappa}M(\zeta, \hat{\theta}) + L_\phi V \tilde{\theta} - L_\phi V \Gamma \Bigg(\frac{\partial V}{\partial \hat{\theta}}\Bigg)^T \\
&- \frac{\kappa}{2} L_g V R^{-1} L_g V + \frac{\partial V}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{1}{\Gamma}\tilde{\theta}^T\dot{\tilde{\theta}} \leq -\frac{1}{2\kappa}M(\zeta, \hat{\theta}).
\end{aligned} \qquad (29)$$

From (29), we see that all the closed-loop signals ζ_1, ..., ζ_n and $\tilde{\theta}$ remain bounded. Since $\gamma = \zeta_1$ is bounded, the regulation error ξ satisfies the performance condition (3). Since ϱ_1 is a function of the bounded signals ζ_1 and $\hat{\theta}$, $x_2 = \zeta_2 - \varrho_1$ is bounded. By the similar analysis, all the states $x_1, ..., x_n$ are bounded. Thus, the inverse optimal adaptive prescribed performance control problem of (1) is solved. The proof is completed. ■

IV. Application to Compliant Actuator-Driven Robot Manipulators

In this section, we use Theorem 1 to design an inverse optimal adaptive control approach for compliant actuator-driven robot manipulators with prescribed performance.

Consider an $n-$link robot manipulator driven by a compliant actuator modeling as follows

$$\begin{aligned}
B(q)\ddot{q} + K(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) &= C(\alpha - q) \\
J\ddot{\alpha} + C(\alpha - q) &= u
\end{aligned} \qquad (30)$$

where $q \in R^n$ and $\alpha \in R^n$ denote the joint and actuator angular position, respectively. $u \in R^n$ is the control input. $B(q) \in R^{n\times n}$ and $K(q, \dot{q}) \in R^{n\times n}$ are the inertia matrix and centripetal-coriolis matrix, respectively. $G(q) \in R^n$ and $F(\dot{q}) \in R^n$ are the gravitational torque and friction torque, respectively. $J \in R^{n\times n}$ and $C \in R^{n\times n}$ are the inertia and stiffness matrix, respectively.

Property 1. $B(q)$, $K(q, \dot{q})$ and $G(q)$ are first-order differentiable and besides, $B(q)$ is symmetric and positive definite. J and C are constant, diagonal and positive definite.

Property 2. The functions $Cq + K(q, \dot{q})\dot{q} + G(q) + F(\dot{q})$ can be linearly parameterized such that $-[Cq + K(q, \dot{q})\dot{q} + G(q) + F(\dot{q})] = [\vartheta_{21}^T\varphi_{21}, \vartheta_{22}^T\varphi_{22}, ..., \vartheta_{2n}^T\varphi_{2n}]^T \in R^n$, where for $i = 1, ..., n$, $\varphi_{2i} \in R^{m_i}$ is a known function and $\vartheta_{2i} \in R^{m_i}$ is an unknown parameter.

Assumption 1. The parameters C and J, and the function $B(q)$ are known for control design.

By denoting $x_1 = q$, $x_2 = \dot{q}$, $x_3 = \alpha$ and $x_4 = \dot{\alpha}$, the plant (30) can be written as a strict-feedback system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= B^{-1}(x_1)\Big[Cx_3 - Cx_1 - K(\bar{x}_2)x_2 - G(x_1) - F(x_2)\Big] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= J^{-1}\Big[u - C(x_3 - x_1)\Big].
\end{aligned} \qquad (31)$$

By denoting $\phi_1 = \phi_3 = \phi_4 = 0$, $\phi_2 = \varphi_2$, where $\varphi_2 = [\varphi_{21}, ..., \varphi_{2n}]^T \in R^{mn}$, $\theta =$diag$(\vartheta_{21}, \vartheta_{22}, ..., \vartheta_{2n}) \in R^{mn\times n}$ and defining the state error variables as in (6), we construct the auxiliary system (8) for (30) and design the virtual controllers $\varrho_i(\zeta, \theta)$ for $i = 1, 2, 3$ as follows

$$\varrho_1 = -\frac{c_1\zeta_1}{\lambda} + \frac{x_1}{\eta}\dot{\eta} \qquad (32)$$

$$\begin{aligned}
\varrho_2 = B(x_1)C^{-1}\Bigg[&-c_2\zeta_2 - \lambda\zeta_1 - \theta^T\bar{\phi}_2 + \frac{\partial\varrho_1}{\partial x_1}x_2 \\
&+ \Gamma\tau_2 + \sum_{j=1}^{2}\frac{\partial\varrho_1}{\partial\eta^{(j-1)}}\eta^{(j)}\Bigg]
\end{aligned} \qquad (33)$$

$$\begin{aligned}
\varrho_3 = &-c_3\zeta_3 - B(x_1)C^{-1}\zeta_2 - \theta^T\bar{\phi}_3 + \sum_{j=1}^{2}\frac{\partial\varrho_2}{\partial x_j}x_{j+1} \\
&+ \Gamma\tau_3 + \sum_{j=1}^{3}\frac{\partial\varrho_2}{\partial\eta^{(j-1)}}\eta^{(j)},
\end{aligned} \qquad (34)$$

where $c_i \in R$ for $i = 1, 2, 3, 4$ is a positive design parameter. Then the auxiliary controller $u(x, \theta)$ is designed as

$$u = -JR(\zeta, \theta)^{-1}\zeta_4, \qquad (35)$$

where

$$R(\zeta, \theta) = \Bigg[c_4 + \sum_{i=4}^{n}\frac{\Phi_i^T\Phi_i}{2c_i}\Bigg]^{-1} \qquad (36)$$

and the function Φ_i satisfies the equation $\sum_{i=1}^{4}\Phi_i\zeta_i = -C(x_1 - x_3) + \zeta_3 - \Gamma\tau_4 + \theta^T\bar{\phi}_4 - \sum_{i=1}^{3}\frac{\partial\varrho_3}{\partial x_i}x_{i+1} -$

$\sum_{i=1}^{4} \frac{\partial \varrho_{n-1}}{\partial \eta^{(i-1)}} \eta^{(i)}$. Choosing a Lyapunov function candidate $V(\zeta,\theta) = \sum_{i=1}^{4} \frac{1}{2}\zeta_i^T \zeta_i$ and applying the controller u, we have

$$\dot{V} = -\sum_{i=1}^{4} \frac{1}{2} c_i \zeta_i^T \zeta_i - \sum_{i=1}^{4} \frac{c_i}{2} \left\| \zeta_i - \frac{\Phi_i}{c_i} \zeta_n \right\|^2. \tag{37}$$

By Theorem 1, we see that the adaptive feedback control

$$u^* = -\kappa J R(\zeta, \hat{\theta})^{-1} \zeta_n, \quad \kappa \geq 2 \tag{38}$$

$$\dot{\hat{\theta}} = \Gamma \sum_{i=2}^{n} \zeta_i \bar{\phi}_i \tag{39}$$

solve the inverse optimal adaptive prescribed performance control problem of (30).

In the following, we use an example to demonstrate the established results.

Example 1. Consider a double-link robot manipulator driven by series elastic actuator (SEA), in which $F(\dot{q}) = 0$ (zero damping). The dynamics of

$$B(q) = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \; K(q,\dot{q}) = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}, \; G(\dot{q}) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

are $b_{11} = \varsigma_{11} + 2\varsigma_{22}\cos q_2$, $b_{12} = b_{21} = \varsigma_{21} + \varsigma_{22}\cos q_2$, $b_{22} = \varsigma_{21}$, $k_{11} = -\varsigma_{22}(2q_1q_2 + q_2^2)\sin q_2$, $k_{12} = k_{21} = 0$, $k_{22} = -\varsigma_{22} q_1^2 \sin q_2$, $G_1 = \cos(q_1+q_2) m_2 g L_{k2} + \varsigma_{12}\cos q_1$ and $G_2 = m_2 g L_{k2}\cos(q_1+q_2)$, where $q = [q_1, q_2]^T$ are the joint position of the links, $g = 10\text{m/s}^2$ is the gravitational acceleration and ς_{ij} for $i,j = 1,2$ are parameters given as $\varsigma_{11} = m_1 L_{k2}^2 + m_2(L_1^2 + L_{k2}^2) + \ell_1 + \ell_2$, $\varsigma_{21} = m_2 L_{k2}^2 + \ell_2$, $\varsigma_{12} = g(m_1 L_{k1} + m_2 \ell_1)$ and $\varsigma_{22} = m_2 \ell_1 L_{k2}$, where $m_1 = 1.1$kg, $m_2 = 0.4$kg are the link mass, $L_1 = 0.37$m and $L_2 = 0.285$m are the link length, $L_{k1} = 0.185$m and $L_{k2} = 0.1425$m are the length to the mass center. $\ell_1 = 1\text{kg}\cdot\text{m}^2$ and $\ell_2 = 0.8\text{kg}\cdot\text{m}^2$ are the inertial tensor. The inertia and stiffness matrices are $J = \text{diag}(J_1, J_2)$ with $J_1 = J_2 = 0.1562\text{kg}\cdot\text{m}^2$ and $C = \text{diag}(C_1, C_2)$ with $C_1 = C_2 = 29.4$Nm/rad. The nonlinear functions in Property 2 are $\varphi_{21}(q,\dot{q}) = [q_1, \dot{q}_1(2q_1q_2 + q_2^2)\sin q_2, \cos(q_1 + q_2), \cos q_1]^T \in R^4$ and $\varphi_{22}(q,\dot{q}) = [q_2, \dot{q}_2 q_1^2 \sin q_2, \cos(q_1 + q_2)]^T \in R^3$, which are known for control design. The parameter vectors $\vartheta_{21} = [C_1, -\varsigma_{22}, m_2 g L_{k2}, \varsigma_{12}]^T \in R^4$ and $\vartheta_{22} = [C_2, -\varsigma_{22}, m_2 g L_{k2}]^T \in R^3$ are unknown for control design.

Control objectives and parameter selections. Our goal is designing an inverse optimal adaptive control law so that: i) all the signals in (30) are bounded; ii) inverse optimality is achieved; iii) the joint position $q = [q_1, q_2]^T$ converges to zero and, q_1 and q_2 satisfy the performance bound: $\sigma_m \eta_1(t) < q_1(t) < \sigma_M \eta_1(t)$ and $\sigma_m \eta_2(t) < q_2(t) < \sigma_M \eta_2(t)$ all the time, where $\eta_1(t) = 1.5e^{-t} + 0.05$, $\eta_2(t) = 1.5e^{-t} + 0.1$, $\sigma_m = 0.5$ and $\sigma_M = 1$. Recalling the performance function, we can choose a smaller η_0 to acquire a smaller overshoot, and choose a larger a to acquire a faster convergence, and choose a smaller η_∞, σ_m and σ_M to acquire a smaller tracking error. To achieve this goal, we choose the parameters $c_1 = c_2 = 3$, $c_3 = c_4 = 1$, $\Gamma = \text{diag}(0.1, 0.3)$ and $\kappa = 2$.

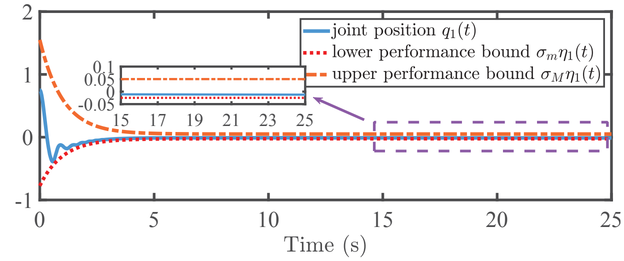

Fig. 1. Evolutions of the joint position $q_1(t)$.

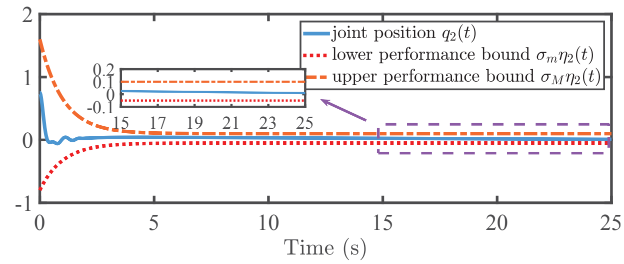

Fig. 2. Evolutions of the joint position $q_2(t)$.

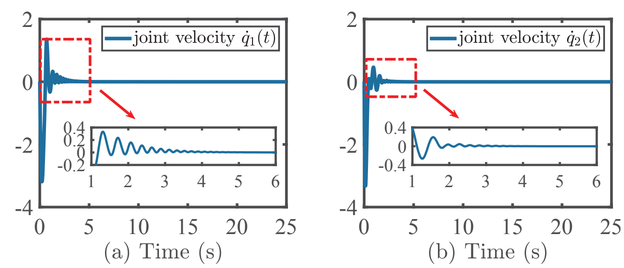

Fig. 3. Evolutions of the joint velocity $\dot{q}(t) = [\dot{q}_1(t), \dot{q}_2(t)]^T$. (a) evolutions of $\dot{q}_1(t)$; (b) evolutions of $\dot{q}_2(t)$.

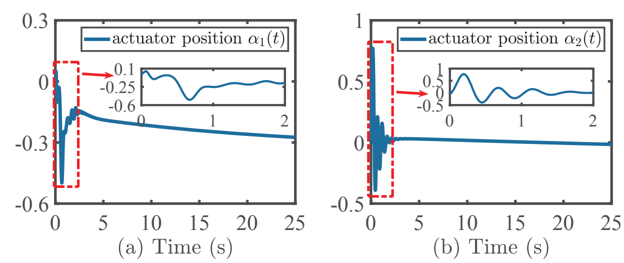

Fig. 4. Evolutions of the actuator position $\alpha(t) = [\alpha_1(t), \alpha_2(t)]^T$. (a) evolutions of $\alpha_1(t)$; (b) evolutions of $\alpha_2(t)$.

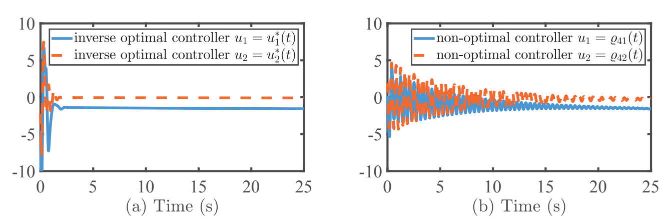

Fig. 5. Comparison of the inverse optimal controller and typical controller. (a) inverse optimal controller $u = u^*$ with $u = [u_1, u_2]^T$ and $u^* = [u_1^*, u_2^*]^T$; (b) typical controller $u = \varrho_4$ with $u = [u_1, u_2]^T$ and $\varrho_4 = [\varrho_{41}, \varrho_{42}]^T$.

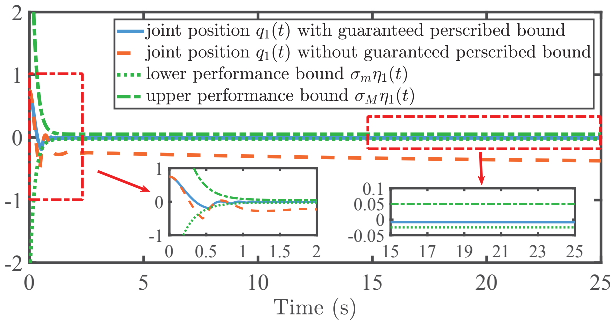

Fig. 6. Inverse optimal controller of the first robot link with and without the performance bounds.

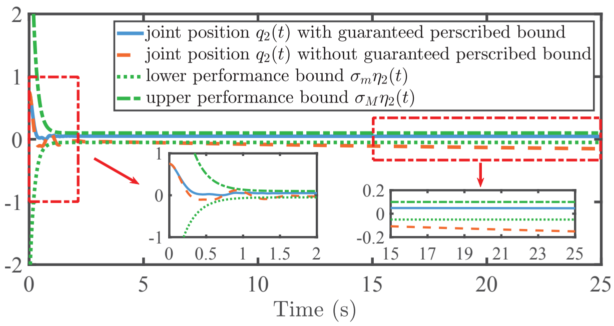

Fig. 7. Inverse optimal controller of the second robot link with and without the performance bounds.

Simulation results. Simulation results are shown in Fig. 1-Fig. 5. The joint positions of the robot manipulator are shown in Fig. 1 and Fig. 2. We see that both the outputs $q_1(t)$ and $q_2(t)$ converge to the origin ultimately and remain in the prescribed bounds all the time. The joint velocity $\dot{q}$ and actuator position α are shown in Fig. 3 and Fig. 4, respectively. The inverse optimal controller u^* is shown in Fig. 5 (a).

Comparisons. To further illustrate the effectiveness of the proposed approach, two comparisons are carried out. We apply the typical adaptive backstepping design in [20] to the system (30), in which the controller is not an inverse optimal one. For a fair comparison, all the parameters are the same as given above. Comparative results are shown in Fig. 5 (b). Obviously, a larger control effort is needed than the inverse optimal controller shown in Fig. 5 (a). Moreover, we also apply the inverse optimal controller proposed in [7] to (30). For a more apparent result, parameters of the performance bound are modified as $\eta_1(t) = 5e^{-5t} + 0.05$, $\eta_2(t) = 5e^{-5t} + 0.1$, $\sigma_m = 0.5$ and $\sigma_M = 1$. Simulation results are shown in Fig. 6 and Fig. 7. It is obviously that by incorporating the performance bound into the inverse optimal design, a better transient performance is achieved.

V. CONCLUSIONS AND FUTURE WORKS

In this work, we solve the inverse optimal adaptive prescribed performance control problem. An auxiliary system is constructed and incorporated with prescribed performance bounds to design a non-adaptive controller. Then a new class of inverse optimal adaptive controllers are proposed based on the non-adaptive controller.

Extension of our results to tracking control will be considered in future. Moreover, safety is important for robot control. Inverse optimal safety control motivated by pioneering works [21] and [22], [23] is a future research line.

REFERENCES

[1] W. Jin, D. Kuli, J. F. S. Lin, S. Mou and S. Hirche. Inverse optimal control for multiphase cost functions. *IEEE Transactions on Robotics*, vol. 35, no. 6, pp. 1387-1398, Dec. 2019, doi: 10.1109/TRO.2019.2926388.

[2] Li Y, Yao Y, Hu X. Continuous-time inverse quadratic optimal control problem. *Automatica*, 2020, 117: 108977.

[3] F. Jean and S. Maslovskaya. Inverse optimal control problem: the linear-quadratic case. *2018 IEEE Conference on Decision and Control (CDC)*, Miami, FL, USA, 2018, pp. 888-893, doi: 10.1109/CDC.2018.8619204.

[4] J. Mainprice, R. Hayne and D. Berenson. Goal set inverse optimal control and iterative replanning for predicting human reaching motions in shared workspaces. *IEEE Transactions on Robotics*, vol. 32, no. 4, pp. 897-908, Aug. 2016, doi: 10.1109/TRO.2016.2581216.

[5] Kalman, R. E. When is a linear control system optimal? *ASME. J. Basic Eng*, March 1964; 86(1): 5160.

[6] Ng, Andrew Y., and Stuart Russell. Algorithms for inverse reinforcement learning. Icml. Vol. 1. 2000.

[7] Krstic M, Li Z H. Inverse optimal design of input-to-state stabilizing nonlinear controllers. *IEEE Transactions on Automatic Control*, 1998, 43(3): 336-350.

[8] Deng H, Krstic M. Stochastic nonlinear stabilizationII: Inverse optimality. *Systems & Control Letters*, 1997, 32(3): 151-159.

[9] Li Z H, Krstic M. Optimal design of adaptive tracking controllers for non-linear systems. *Automatica*, 1997, 33(8): 1459-1473.

[10] Krstic M, Kokotovic P V. Control Lyapunov functions for adaptive nonlinear stabilization. *Systems & Control Letters*, 1995, 26(1): 17-23.

[11] Wencheng Luo, Yun-Chung Chu and Keck-Voon Ling. Inverse optimal adaptive control for attitude tracking of spacecraft. *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1639-1654, Nov. 2005, doi: 10.1109/TAC.2005.858694.

[12] Do K D. Inverse optimal gain assignment control of evolution systems and its application to boundary control of marine risers. *Automatica*, 2019, 106: 242-256.

[13] Zhao K, Song Y, Ma T, et al. Prescribed performance control of uncertain EulerLagrange systems subject to full-state constraints. *IEEE Transactions on Neural Networks and Learning Systems*, 2017, 29(8): 3478-3489.

[14] Huang J, Wen C, Wang W, et al. Design of adaptive finite-time controllers for nonlinear uncertain systems based on given transient specifications. *Automatica*, 2016, 69: 395-404.

[15] Bechlioulis C P, Rovithakis G A. Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems. *Automatica*, 2009, 45(2): 532-538.

[16] Wang W, Wen C. Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance, *Automatica*, 2010, 46(12): 2082-2091.

[17] Pan Y, Wang H, Li X, et al. Adaptive command-filtered backstepping control of robot arms with compliant actuators. *IEEE Transactions on Control Systems Technology*, 2017, 26(3): 1149-1156.

[18] Pan Y, Li X, Yu H. Efficient PID tracking control of robotic manipulators driven by compliant actuators. *IEEE Transactions on Control Systems Technology*, 2018, 27(2): 915-922.

[19] Lv M, Chen Z, De Schutter B, et al. Prescribed-performance tracking for high-power nonlinear dynamics with time-varying unknown control coefficients. *Automatica*, 2022, 146: 110584.

[20] Krstic, Miroslav and Kokotovic, Petar V and Kanellakopoulos, Ioannis. Nonlinear and adaptive control design. *John Wiley & Sons, Inc*, 1995.

[21] Krstic, Miroslav. Inverse Optimal Safety Filters. *IEEE Transactions on Automatic Control*, doi: 10.1109/TAC.2023.3278788.

[22] Lyu Z, Xu X, Hong Y. Small-gain theorem for safety verification of interconnected systems. *Automatica*, 2022, 139: 110178.

[23] Lyu Z, Xu X, Hong Y. Small-gain theorem for safety verification under high-relative-degree constraints. arXiv preprint arXiv:2204.04376, 2022.