2024 IEEE 63rd Conference on Decision and Control (CDC)
December 16-19, 2024. MiCo, Milan, Italy

Multiagent Placement to Spatiotemporal Points: A Finite-Time Distributed Control Protocol over Directed Acyclic Graphs*

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Abstract— **Many multiagent system applications necessitate each agent to separately visit a specific location in space at a particular moment in time (i.e., spatiotemporal points). To this end, we propose a finite-time distributed control protocol over directed acyclic graphs that addresses the problem of multiagent placement at spatiotemporal points. In particular, we demonstrate that the proposed protocol can drive the trajectories of agents to their spatiotemporal points at different user-defined times by employing methods ranging from time transformation to input-to-state stability and Lyapunov stability. To show the efficacy of the proposed protocol, we also give two illustrative numerical examples.**

I. Introduction

Multiagent systems are essential in scientific, civilian, and military applications such as collaborative surveillance, reconnaissance, and control of underwater, ground, aerial, and space teams. With growing interest, significant research has advanced distributed control protocols that are based on local, agent-to-agent information exchange (e.g., see [1]–[3] for the seminal books on the topic). Specifically, consensus [4]–[6] and bipartite consensus [7]–[9] are two widely studied "leaderless" problems (i.e., all agents perform a task without an external command), in which the trajectories of agents converge to an agreement point or disagreement points. On the other hand, pinning [10]–[12] and containment [13]–[15] are two widely studied "leader-follower" problems, in which the trajectories of agents synchronize with an external command for the former problem and the trajectories of agents converge to the convex hull spanned by external commands for the latter problem (i.e., agents that know the external commands are referred to as leaders or root agents). The current state-of-the-art literature also extends leaderless and leader-follower problems to formation control, coverage control, flocking, axial alignment, and distributed optimization, to name but a few examples.

The shared feature of all the above problems is that agents achieve a task at the same time, either asymptotically or in finite time. Yet, many multiagent system applications such as environmental monitoring, information gathering and resource extraction, transportation and delivery, and task assignment, necessitate each agent to separately visit a specific location in space at a particular moment in time (i.e., spatiotemporal points). As an example, consider a scenario with two agents, where the first agent is required to visit a spatial point p_1 at a time T_1 and the second agent is required to visit another spatial point p_2 at a time T_2 (within this setting, both these spatial points and their corresponding times are available to one of these agents that is the leader or the root agent). To address this problem, a distributed control protocol is needed since the root agent possesses the information on the multiagent task. However, none of the existing distributed control protocols given in the above paragraph can be used since T_1 does not necessarily equal T_2, which means that these agents do not perform this task simultaneously (e.g., $T_1 < T_2$).

To this end, this paper introduces a new and novel finite-time distributed control protocol over directed acyclic graphs to address the problem of multiagent placement to spatiotemporal points. First, the conditions at the user-defined times are defined to make the problem feasible. Next, it is demonstrated that the proposed protocol can drive the trajectories of agents to their spatiotemporal points at different used-defined times by employing methods ranging from time transformation to input-to-state stability and Lyapunov stability. Then, it is shown that the control signals of each agent are bounded and continuous. In what follows, Section II rigorously define the considered problem, Section III presents our main system-theoretical results, and Section IV presents two illustrative numerical examples. Conclusions are then summarized in Section V.

II. Problem Formulation

In this section, we first state the fairly standard mathematical and graph-theoretical notation used in this paper and we then rigorously define the problem of multiagent placement to spatiotemporal points.

*This research was supported by the Air Force Research Laboratory Aerospace Systems Directorate under the Grant FA8650-21-D-2602.

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A. Notation

Throughout this paper, $\mathbb{R}$ represents the set of real numbers, $\mathbb{R}_+$ represents the set of positive real numbers, $\overline{\mathbb{R}}_+$ represents the set of nonnegative real numbers, and "$\triangleq$" represents the equality by definition. Furthermore, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents a fixed, connected, and directed graph with $\mathcal{V} = \{v_1, \ldots, v_p\}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ respectively being a nonempty finite set of n nodes and a set of edges. We also say that an edge is rooted at node v_j and is ended at node v_i when v_i gets information from v_j ($i \sim j$ is used to represent this neighboring relation). In addition, a directed path from node v_i to node v_j is a sequence of successive edges in the form $(v_i, v_p), (v_p, v_q), \ldots, (v_r, v_j)$. A directed graph $\mathcal{G}$ has a spanning tree when there is a directed path to every other node from the root nodes in the graph $\mathcal{G}$. For the main results of this paper, we consider a fixed, connected, and directed acyclic graph $\mathcal{G}$ that models agent-to-agent information exchange, where we make the standard spanning tree assumption.

B. Multiagent Placement to Spatiotemporal Points

For the purpose of defining the problem of multiagent placement to spatiotemporal points, consider a multiagent system with n agents over a fixed, connected, and directed acyclic graph $\mathcal{G}$. Consider also n spatiotemporal points (p_i, T_i) that each agent must visit, where T_i represents a nonidentical user-defined time for each agent. In addition, consider that the root agent(s) generates a command that is bounded and the time rate change of this command is also bounded (see Figure 1 for such an exemplary command generated through spline interpolation).

Note that the spatiotemporal points are available only to the root agent(s) because of the operation's security since root agent(s) generally constitute a sufficiently small portion of the entire multiagent system. Note also that the user-defined time T_i is available to the corresponding agent i before executing the proposed protocol below. If this is not desired, one can use the theory of multiplex networks [12] to add another graph layer to locally distribute the user-defined times T_i. In this case, the added graph layer needs to have the leader-follower nullspace control structure [16] blended with the finite-time approach [17] so that the local distribution of these user-defined times can be completed before the minimum user-defined time $T_{\min} \triangleq \min\{T_1, \ldots, T_n\}$. Finally, one needs to complete generating the command signal $c(t)$ before executing the proposed protocol below.

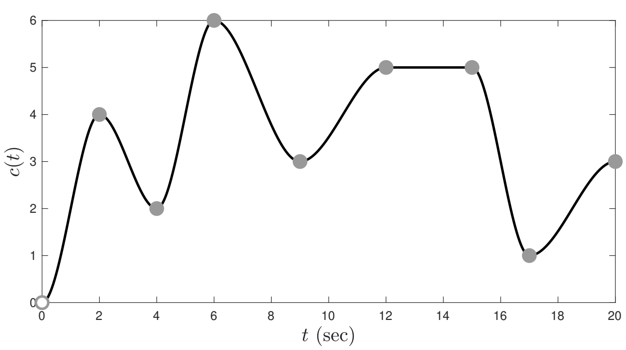

Fig. 1. Through spline interpolation, we generate a command signal $c(t)$ (black line) for the purpose of connecting the initial command signal $c(0)$ (gray dot with white fill) with given spatiotemporal points (p_i, T_i) (gray dots with gray fills) such that $c(T_i) = (p_i, T_i)$, where $i = 1, \ldots, 8$. Compared with a single high-degree polynomial fit to all spatiotemporal points at once, spline interpolation does not result in over-fitting.

Next, the results of this paper apply to the scalar dynamics of each agent given by

$$\dot{x}_i(t) \quad = \quad u_i(t), \quad x_i(0) = x_{i0}. \tag{1}$$

In (1), the state and the control signal of an agent are respectively represented by $x_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$, $i = 1, \ldots, n$. Furthermore, the proposed multiagent placement control protocol has the form given by

$$u_i(t) \;=\; \begin{cases} -\frac{\alpha}{T_i - t} f_i(x_i(t), \hat{x}_j(t), c(t)), & t < T_i, \\ 0, & t \geq T_i, \end{cases} \tag{2}$$

$$f_i(\cdot) \;=\; \sum_{i \sim j} (x_i(t) - \hat{x}_j(t)) + k_i(x_i(t) - c(t)), \tag{3}$$

with $\alpha \in \mathbb{R}_+$ being a gain chosen before this protocol's execution and $k_i = 1$ for the root agent(s) and otherwise $k_i = 0$. Moreover, $\hat{x}_i(t) \in \mathbb{R}$ represents the virtual state of an agent satisfying

$$\dot{\hat{x}}_i(t) = \begin{cases} -\frac{\alpha}{T_i - t} \hat{f}_i(\hat{x}_i(t), \hat{x}_j(t), c(t)), & t < T_i, \\ -\beta_1 \mathrm{sgn}\left(\hat{f}_i(\cdot)\right) - \beta_2 \hat{f}_i(\cdot), & t \geq T_i, \end{cases} \tag{4}$$

$$\hat{f}_i(\cdot) = \sum_{i \sim j} (\hat{x}_i(t) - \hat{x}_j(t)) + k_i(\hat{x}_i(t) - c(t)), \tag{5}$$

with $\beta_1 \in \mathbb{R}_+$ and $\beta_2 \in \mathbb{R}_+$ being the gains that are also chosen before this protocol's execution. Considering the proposed multiagent placement control protocol given above, one may use heterogeneous gains $\alpha_i \in \mathbb{R}_+$, $\beta_{1i} \in \mathbb{R}_+$, and $\beta_{2i} \in \mathbb{R}_+$ as desired, where the main results of this paper only experience minor modifications in this case.

At this point, we can proceed to define the multiagent placement problem:

i) The state $x_i(t)$ of agent i approaches its spatiotemporal point (p_i, T_i) at the user-defined time T_i; that is,

$$\lim_{t \to T_i} \big(x_i(t) - (p_i, T_i)\big) = 0, \quad i = 1, \ldots, n. \tag{6}$$

ii) The state $x_i(t)$ of agent i stays at its spatial point (p_i, T_i) over $t \in [T_i, \infty)$; that is,

$$x_i(t) - (p_i, T_i) = 0, \quad t \geq T_i, \quad i = 1, \ldots, n. \tag{7}$$

iii) All closed-loop signals including the control signals of agents remain bounded for all time.

iv) The control signals of agents remain continuous at their switching instants.

Although ***i)*** and ***ii)*** provide a mathematical definition of the multiagent placement problem, ***iii)*** and ***iv)*** are also necessary to ensure the well-posedness of the problem. The next section demonstrates that the proposed protocol can solve the problem when the following assumptions are met.

Assumption 1. The user-defined time T_i of node v_i is greater than the user-defined times of its neighbors in which this node receives information.

Assumption 2. The condition given by

$$\alpha(d_i + k_i) - 1 \tag{8}$$

is positive for each agent with d_i being the degree of node i and $\beta_1 > \bar{c}$, $|\dot{c}(t)| \leq \bar{c}$.

Assumption 3. The condition given by

$$\lim_{t \to T_i} \dot{c}(t) = 0 \tag{9}$$

holds at each switching instant.

Assumption 1 can be easily satisfied for a given fixed, connected, and directed acyclic graph $\mathcal{G}$. To clarify this, consider the graph $\mathcal{G}$ in Figure 2, where nodes 1, 2, and 3 are root agents, and the graph clearly meets the spanning tree assumption. Now, examine three specific cases a), b), and c) as highlighted in the figure. For the agents in case a), Assumption 1 dictates that their user-defined times should be chosen as $T_6 > T_4 > T_1$. For the agents in case b), the assumption requires $T_7 > T_5 > T_2$ and $T_7 > T_3$, with the selection of T_3 being independent of T_2 and T_5. In case c), Assumption 1 further necessitates that $T_{12} > T_{10} > T_9$ and $T_{12} > T_{11} > T_9$ with the selection of T_{11} not affecting T_{10} as long as both exceed T_9. In addition, T_9 must be greater than both T_7 and T_8. A similar reasoning applies to all other agents in this graph $\mathcal{G}$. Assumption 2 can also be easily satisfied not only for α but also for β_1. This is because the root node(s) generate the bounded command signal $c(t)$ with a bounded time rate of change before executing the proposed protocol. As a result, they aware of the upper bound $\bar{c}$ or a conservative estimate of it, which can be used to determine β_1. Note that Assumptions 1 and 2 are needed to address ***i)***, ***ii)***, and ***iii)*** in the next section. Finally, Assumption 3 is necessary to address ***iv)*** as jumps in the control signals of agents occur when it is not satisfied. The constraint

$$\dot{c}(T_i) = 0 \tag{10}$$

must be considered when applying the spline interpolation approach to ensure this assumption is met. For instance, this constraint is incorporated in Figure 1, where Assumption 3 is satisfied for all points $i = 1, \ldots, 8$.

III. System-Theoretical Results

This section presents several theorems to demonstrate that the proposed protocol given in the previous section can drive the trajectories of agents to their spatiotemporal points at different user-defined times. While the proofs of these theorems will be reported elsewhere, it is worth mentioning that the time transformation method [17] is used in the proofs

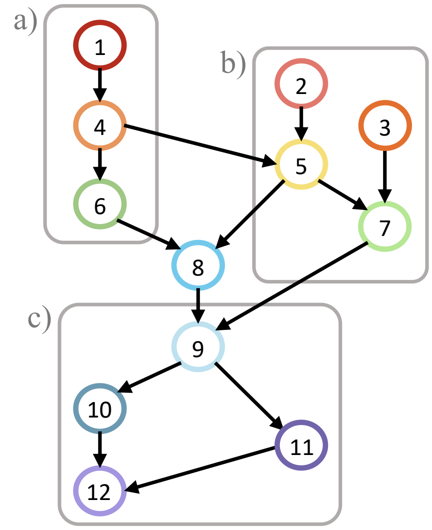

Fig. 2. A fixed, connected, and directed acyclic graph with nodes 1, 2, and 3 being root agents, where this graph satisfies the spanning tree assumption.

of Theorems 1 and 4; input-to-state stability is used in the proofs of Theorems 1, 4, and 5; Lyapunov stability is used in the proof of Theorem 3; and an analysis predicated on the initial conditions of each agent at their switching instants is used in the proof of Theorem 2.

The first main result of this paper is now introduced, which shows how the proposed control protocol for multiagent placement solves problem ***i)***.

Theorem 1. Consider a multiagent system over a fixed, connected, and directed acyclic graph $\mathcal{G}$, where the dynamics of each agent satisfies (1). Consider also the multiagent placement control protocol given by (2), (3), (4), and (5). If the user-defined times satisfy Assumption 1, then ***i)*** holds.

The second main result of this paper is now introduced, which shows how the proposed control protocol for multiagent placement solves problem ***ii)***.

Theorem 2. Consider a multiagent system over a fixed, connected, and directed graph $\mathcal{G}$, where the dynamics of each agent satisfies (1). Consider also the multiagent placement control protocol given by (2), (3), (4), and (5). If the user-defined times satisfy Assumption 1, then ***ii)*** holds.

The third main result of this paper is now introduced, which shows how the proposed control protocol for multiagent placement solves problem ***iii)***. To this end, the boundedness of all state and virtual state signals is first shown in Theorem 3, and the boundedness of the control and virtual control signals (i.e., the right side of (4)) is then shown in Theorem 4.

Theorem 3. Consider a multiagent system over a fixed, connected, and directed graph $\mathcal{G}$, where the dynamics of each agent satisfies (1). Consider also the multiagent placement control protocol given by (2), (3), (4), and (5). If the user-defined times satisfies Assumption 1 and the gain β_1 satisfies the condition in Assumption 2, then the state $x_i(t)$ and the virtual state $\hat{x}_i(t)$ of agent i are bounded, and

$$\hat{x}_i(t) - c(t) = 0, \quad t \geq T_i, \quad i = 1, \ldots, n. \tag{11}$$

Theorem 4. Consider a multiagent system over a fixed, connected, and directed graph $\mathcal{G}$, where the dynamics of each agent satisfies (1). Consider also the multiagent placement control protocol given by (2), (3), (4), and (5). If the user-defined times satisfy Assumption 1 and the gain α satisfies the condition in Assumption 2, then the control signal $u_i(t)$ and the virtual control signal $\hat{u}_i(t)$ (i.e., the right side of (4)) are bounded.

The boundedness of the control signal $u_i(t)$ and the virtual control signal $\hat{u}_i(t)$ for agent i, $i = 1, \ldots, n$, is demonstrated in Theorem 4. If these signals are to be kept from taking large values as t approaches T_i due to the term "$\frac{1}{T_i - t}$," one can saturate this term over $t \in [T_{si}, T_i)$ as "$\frac{1}{T_i - T_{si}}$" for some $T_{si} < T_i$. We refer to [18, Section 4] for details.

Finally, the fourth main result of this paper is now introduced, which shows how the proposed control protocol for multiagent placement solves problem ***iv)***.

Theorem 5. Consider a multiagent system over a fixed, connected, and directed graph $\mathcal{G}$, where the dynamics of each agent satisfies (1). Consider also the multiagent placement control protocol given by (2), (3), (4), and (5). If Assumptions 1, 2, and 3 hold, then ***iv)*** holds.

IV. Illustrative Numerical Examples

In this section, two illustrative numerical examples are presented to demonstrate the efficacy of the multiagent placement control protocol introduced in Section II.

A. One Dimensional Numerical Example

In the first example, eight agents (i.e., $n = 8$) are considered over a fixed, connected, and directed path graph $\mathcal{G}$, where the first agent is the root agent. The spatiotemporal points (p_i, T_i) are selected as shown in Figure 1 with the root agent utilizing the generated command depicted in the same figure. We choose the gains in (2) and (4) as $\alpha = 2$, $\beta_1 = 4$, and $\beta_2 = 4$, and we randomly select the initial conditions of all agents from the interval $(-1, 1)$. It should be noted that Assumptions 1, 2, and 3 hold, and to prevent the chattering phenomenon in the control signals of agents, $\text{sgn}(x)$ is approximated as $\tanh(\rho x)$ with $\rho = 50$. Figures 3 and 4 respectively show the state and control histories of the multiagent system. Observe that the proposed protocol achieves ***i)***, ***ii)***, ***iii)***, and ***iv)*** as we expect from the system-theoretical results given in Section III.

B. Two Dimensional Numerical Example

In the second example, it is shown that the proposed multiagent placement control protocol can be applied to a multiple-dimensional problem without modification. In particular, twelve agents (i.e., $n = 12$) are considered over

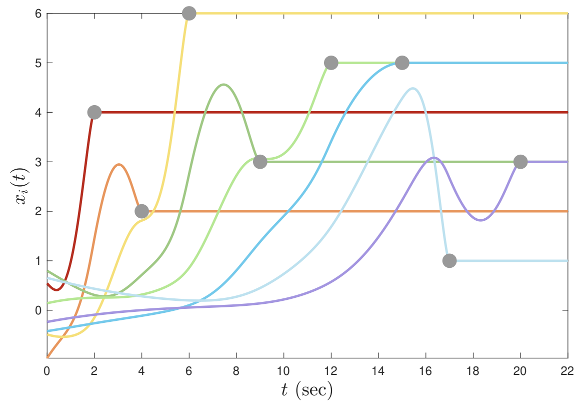

Fig. 3. State histories of the multiagent system.

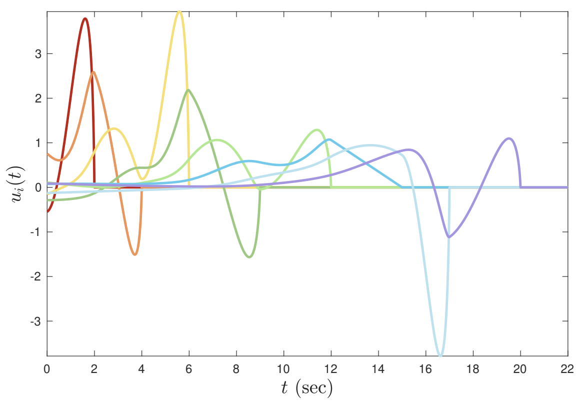

Fig. 4. Control histories of the multiagent system.

the fixed, connected, and directed graph $\mathcal{G}$ shown in Figure 2, where the first three agents are as the root agents. We select the spatiotemporal points (p_i, T_i) to be uniformly distributed along a circle with a 4-unit radius. In addition, we choose the gains as $\alpha = 2$, $\beta_1 = 5$, and $\beta_2 = 5$, and we randomly select the initial conditions of all agents from the interval $(-1, 1)$. It should be noted that Assumptions 1, 2, and 3 hold, and to prevent the chattering phenomenon in the control signals of agents, $\text{sgn}(x)$ is approximated as $\tanh(\rho x)$ with $\rho = 50$. Figure 5 shows the state histories of the multiagent system, where Figures 6 and 7 respectively show the control histories of the multiagent system along x-axis and y-axis. Once again, observe that the proposed protocol achieves ***i)***, ***ii)***, ***iii)***, and ***iv)*** as we expect from the system-theoretical results given in Section III.

V. Conclusion

We proposed a finite-time distributed control protocol over directed acyclic graphs to solve the problem of multiagent placement to spatiotemporal points. To this end, we first mathematically defined the problem in ***i)***, ***ii)***, ***iii)***, and ***iv)***, and then provided the conditions in Assumptions 1, 2, and 3 to ensure the feasibility of the problem. Section III presented Theorem 1 to solve ***i)***, Theorem 2 to solve ***ii)***, Theorems 3 and 4 to solve ***iii)***, and Theorem 5 to solve ***iv)*** using methods

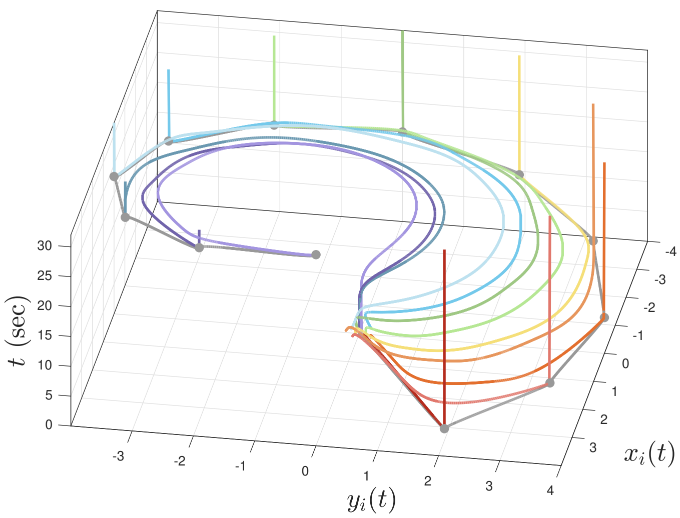

Fig. 5. State histories of the multiagent system.

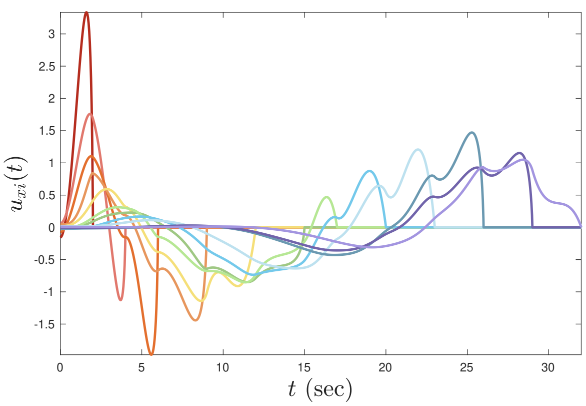

Fig. 6. Control histories of the multiagent system along x-axis.

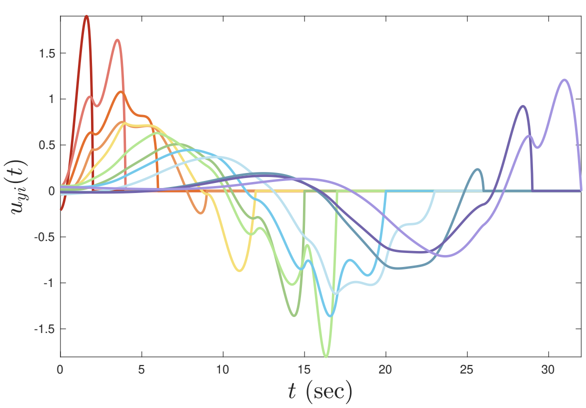

Fig. 7. Control histories of the multiagent system along y-axis.

ranging from time transformation to input-to-state stability and Lyapunov stability. Finally, to demonstrate the efficacy of the proposed finite-time distributed control protocol in driving the trajectories of agents to the given spatiotemporal points at different user-defined times, we provided two illustrative numerical examples in Section IV.

While this study introduces a novel finite-time distributed control protocol for multiagent placement to spatiotemporal points, future research could focus on several essential directions. One critical area is generalizing the proposed protocol from acyclic directed graphs to general directed graphs. To this end, the first future research direction may involve this extension as many real-world networks are represented by general directed graphs that may contain cycles and do not necessarily have a clear hierarchical structure. The second future research direction may involve experimentally validating the proposed protocol in real-world scenarios including exogenous disturbances and system uncertainties as well as communication delays and measurement noise. Finally, the third future research direction may involve the extension of our results to second or higher-order agent dynamics.

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