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Bearing-Based Formation Maneuver Control of Leader-Follower Multi-Agent Systems

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Abstract— **In this paper, we study the bearing-based formation maneuver control problem of the leader-follower multi-agent system. The objectives are achieving the rotation, translation, and scaling maneuvers with a transformable formation shape. Unlike existing works where the target formation is defined by displacements, distances, or constant bearings, we propose a novel target formation with time-varying bearings. The feasibility and uniqueness of the target formation are analyzed by extending the properties of bearing rigidity to time-varying cases. Compared to the existing methods where the positions and velocities of all the agents are required, an estimation-based control method is proposed to achieve the target formation using relative bearings and only the leaders' positions and velocities. Both the estimation error and tracking error converge to zero under the extended properties of bearing rigidity and cascade system theories. A sufficient condition for collision avoidance among the agents is also proposed. A numerical example illustrates the effectiveness of the proposed method.**

I. INTRODUCTION

Formation control, which is one of the most attractive tasks in cooperative control, has received increasing attention. To achieve the desired formation, two subtasks are needed, i.e., formation shape control and maneuver control, which aim to steer agents to form a target geometric shape and maneuver as a whole such that the translation, rotation, scaling, and other motions can be changed, respectively [1].

Based on the invariance to the translation, the displacement-based method can be applied to achieve translation formation maneuver [2]. The work [3] proposes a distance-based approach with desired motion parameters to achieve translation and rotation formation maneuver control based on the distances' invariance to the maneuvers. In addition, the complex-valued Laplacian method [4], [5], can achieve translation, rotation, and scaling formation maneuver control in the 2D complex field. The stress matrices, which are invariant to any transformation of the formation, can be used in the 3D cases [1], [6]. However, the weights of the inter-agent edges are difficult to be determined since the equilibrium stress of the nominal formation is hard to predefine.

In addition to the above approaches, the bearing-based methods, where the target formation is defined by the relative bearings, have attracted a growing research interest [7]–[9]. Because of the invariance to the scalings and translations, it is easier to achieve translation and scaling formation maneuver control in 3D cases using bearing-based methods than the displacement-based, distance-based, and complex-Laplacian-based ones [10], [11]. The weights of the inter-agent edges can be easily determined by the desired bearings, which is more convenient than the stress-matrix-based methods. However, it is still challenging to achieve rotation maneuver control and formation transformation, which lead to time-varying desired bearings and result in complex relations among the agents' desired positions and velocities. The relations can not be described by the bearing rigidity theories [7]–[11], where the desired bearings must be constant. Recently, a relaxed bearing rigidity-based method [12] can achieve rotation maneuver control. However, it requires the bearings to be persistently exciting, which means the target formation can not be static.

Moreover, the aforementioned approaches [1]–[6], [10], [11] need real-time position or displacement measurements of all the agents. It requires some GPS devices or visual cameras, which may hardly be used in severe environments [13]. In comparison, relative bearings, which impose the minimum requirements on the agents' sensing abilities [14], can be easily obtained by passive sonars or radars in many environments [15]. However, since the bearings only describe the inverse tangent relations among the displacements of the agents, it is usually difficult to design a control method using bearings, especially when the desired bearings are time-varying [14]. The varying rates of the desired bearings lead to complicated desired velocities of the agents, which can not be assumed to know priorly [12] or estimated by the first-order consensus algorithm [8]. That even increases the difficulties of bearing-based control method design.

Based on the observation, in this paper, we aim to solve the bearing-based formation maneuver control problem with rotation, translation, and scaling maneuvers with transformable formation shapes. The contributions of this paper are three-fold. **First**, we propose a target formation defined by time-varying bearings that contain the desired rotation, translation and scaling maneuvers with transformable formation shapes. The target formation can be either static or time-varying, which contain those in [7]–[12] as special cases. The feasibility and uniqueness of the target formation are analyzed by extending some of the bearing rigidity properties to the time-varying cases. **Second**, an estimation-based control method

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is designed to achieve the target formation. Compared with [12], the desired positions and velocities of the followers are not required priorly. Different from [1]–[6], [10], [11], only the leaders' position and velocity information is required. **Third**, the convergence of the estimation and tracking errors is analyzed by employing extended bearing rigidity properties and cascade system theories. A sufficient condition that guarantees inter-agent collision avoidance is also proposed.

The rest of this paper is organized as follows. In Section II, preliminaries and problem statements are presented. In Section III, the bearing-based control method is presented. The convergence analysis of the errors is also shown. Section IV shows the numerical example. Section V draws the conclusions.

Notations: Given a real vector $x = [x_1, ..., x_n]^T \in \mathbb{R}^n$, define the norms for x as $\|x\| = \sqrt{x^T x}$ and $\|x\|_1 = \sum_{j=1}^{n} |x_j|$, where $|\cdot|$ denotes the absolute value. The signum function $\text{sgn}(x) = [\text{sgn}(x_1), ..., \text{sgn}(x_n)]^T$. Let I_d and $\mathbf{0}_d$ be the identity matrix and vectors of all zeros with d dimensions, respectively. $\otimes$ denotes the Kronecker product. $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximal eigenvalues of the matrix, respectively.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Preliminaries

Consider n mobile agents, where the first $n_l (n_l \geq 2)$ agents are the leaders and the others labeled by $n_l + 1, n_l + 2, \ldots, n$ are the followers. Let $p_i, u_i \in \mathbb{R}^d (d \geq 2)$ denote the position and velocity control input of agent i in the global reference frame, respectively. The dynamic model of each agent i is

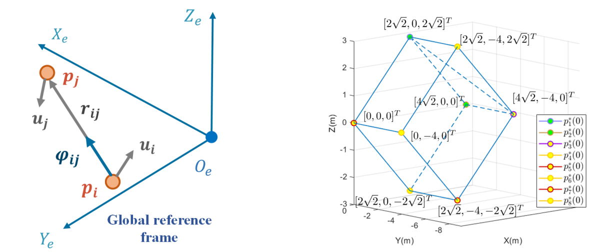
$$\dot{p}_i = u_i. \tag{1}$$

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote an undirected graph with a vertex set $\mathcal{V} = \{1, \ldots, n\}$ and an edge set $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ composed of m edges. For each edge $(i, j) \in \mathcal{E}$, j is called the neighbor of i. Agent i can sense the relative measurements and receive the information from j. $\mathcal{N}_i$ denotes the neighbor set of agent i. In the set $\mathcal{V}$, the vertexes in $\mathcal{V}_l := \{1, \ldots, n_l\}$ and $\mathcal{V}_f := \{n_l+1, \ldots, n\}$ denote the leaders and followers, respectively.

For each edge $(i, j) \in \mathcal{E}$, define $r_{ij} := p_j - p_i$ as the displacement from agent i to j, and $\|r_{ij}\|$ as the distance between i and j. $\varphi_{ij} := \frac{r_{ij}}{\|r_{ij}\|}$ represents the relative bearing from i to j. For convenience, define $P_{\varphi_{ij}} := I_d - \varphi_{ij}\varphi_{ij}^T$. Moreover, define $\boldsymbol{p} := [p_1^T, \ldots, p_n^T]^T$, $\boldsymbol{r} := [r_1^T, \ldots, r_m^T]^T$ and $\boldsymbol{u} := [u_1^T, \ldots, u_n^T]^T$ as the positions, displacements, and velocities of the n agents. Then one has $\boldsymbol{r} = (H \otimes I_d)\boldsymbol{p} := \bar{H}\boldsymbol{p}$, where $H \in \mathbb{R}^{m \times n}$ is the incidence matrix related to graph $\mathcal{G}$ [7]. The notations shown in Fig.1(a) are all defined in the global reference frame. An example of the topology with 2 leaders and 6 followers is shown in Fig.1(b).

B. Problem Statement

To achieve formation maneuver control using bearing measurements, we first define the target formation to be achieved.



(a) (b)

Fig. 1. The notations and an example of the topology.

Definition 1 (Target Formation): A formation $\mathcal{G}(\boldsymbol{p}(t))$ is a bearing-based target formation with desired rotation, scaling, and translation maneuvers with transformable formation shape if it satisfies the following conditions,

1) $p_k(t) = p_k^*(t), \forall k \in \mathcal{V}_l$,
2) $\frac{p_j(t) - p_i(t)}{\|p_j(t) - p_i(t)\|} = \varphi_{ij}^*(t), \forall (i, j) \in \mathcal{E}$,
3) $\dot{p}_i(t) = u_i^*(t), \forall i \in \mathcal{V}$,

where $p_i^*(t)$ and $u_i^*(t)$ denote the desired position and velocity of each agent i, respectively. $\varphi_{ij}^*(t)$ denotes the desired bearing of each pair of neighboring agents.

The desired position p_k^* and velocity u_k^* of each leader $\forall k \in \mathcal{V}_l$ in condition 1) and part of condition 3) determine the translation and scaling maneuvers. The time-varying desired bearings $\varphi_{ij}^*, \forall (i, j) \in \mathcal{E}$ in condition 2) determines the rotation maneuver. The formation transformation is determined by $p_k^*, u_k^*, \forall k \in \mathcal{V}_l$ and $\varphi_{ij}^*, \forall (i, j) \in \mathcal{E}$ together. The proposed target formation is a general form containing those defined in the existing bearing-based works [7]–[12].

In this paper, each leader $k \in \mathcal{V}_l$ knows its desired states p_k^*, u_k^* as prior information and it can sense its position measurement p_k to move with the desired dynamics. For each follower $i \in \mathcal{V}_f$, the desired bearings and varying rates $\varphi_{ij}^*, \dot{\varphi}_{ij}^*$ with respect to the neighbors $j \in \mathcal{N}_i$ is known as prior information. During the maneuvers, follower i can measure the bearings φ_{ij} and obtain the estimations of the desired states p_j^*, u_j^* from its neighbors $j \in \mathcal{N}_i$ via communication.

Remark 1: A position-based controller can be easily designed for each leader to track the desired dynamics very fast. Hence, the assumption that each leader moves in its desired dynamics does not simplify the controller design and theoretical analysis of the whole closed-loop system greatly.

We aim to study the following two problems in this paper:

Problem 1: What is the sufficient condition such that the target formation in *Definition 1* is feasible and unique?

Problem 2: How to design a control method for each follower agent $i \in \mathcal{V}_f$ using the bearing-based information such that the target formation in *Definition 1* can be achieved?

III. BEARING-BASED FORMATION MANEUVER CONTROL

In this section, the feasibility and uniqueness of the time-varying target formation are first analyzed. Then, a distributed estimator is designed for each follower to estimate the desired states. After that, a bearing-based controller is designed for each follower to achieve the target formation.

Finally, the convergence of the estimation and tracking error is analyzed.

A. Feasibility and Uniqueness of the Target Formation

To analyze the feasibility and uniqueness of the target formation in *Definition 1*, a time-varying bearing Laplacian matrix is defined as follows:

$$[B(t)]_{ij}=\begin{cases}\mathbf{0}_{d\times d}, & i\neq j,(i,j)\notin\mathcal{E},\\ -(I_d-\varphi_{ij}^*(t)\varphi_{ij}^{*T}(t)), & i\neq j,(i,j)\in\mathcal{E},\\ \sum_{j\in\mathcal{N}_i}(I_d-\varphi_{ij}^*(t)\varphi_{ij}^{*T}(t)), & i=j,i\in\mathcal{V}.\end{cases}\tag{2}$$

$B(t)$ is partitioned to

$$B(t)=\begin{bmatrix}B_{ll}(t) & B_{lf}(t)\\ B_{fl}(t) & B_{ff}(t)\end{bmatrix},$$

where $B_{ff}(t)\in\mathbb{R}^{dn_f\times dn_f}$ only contains the desired bearings among the followers.

Stack the desired positions and velocities of the agents into vectors as $\boldsymbol{p^*}:=[\boldsymbol{p_l^{*T}},\boldsymbol{p_f^{*T}}]^T$ and $\boldsymbol{u^*}:=[\boldsymbol{u_l^{*T}},\boldsymbol{u_f^{*T}}]^T$, respectively, where $\boldsymbol{p_l^*}:=[p_1^{*T},...,p_{n_l}^{*T}]^T$, $\boldsymbol{u_l^*}:=[u_1^{*T},...,u_{n_l}^{*T}]^T$, $\boldsymbol{p_f^*}:=[p_{n_l+1}^{*T},...,p_n^{*T}]^T$, and $\boldsymbol{u_f^*}:=[u_{n_l+1}^{*T},...,u_n^{*T}]^T$.

To guarantee the feasibility of the target formation, it is essential to require that if $\lim_{t\to\infty}p_i=p_i^*$, then $\lim_{t\to\infty}\dot{p}_i=u_i^*$. This can be satisfied by setting $\dot{p}_i^*=u_i^*,\forall i\in\mathcal{V}$, which requires p_i^* to be differential. For each leader $k\in\mathcal{V}_l$, the differentiability can be satisfied by designing a smooth $p_k^*(t)$, which yields $\boldsymbol{\dot{p}_l^*}=\boldsymbol{u_l^*}$. For each follower $i\in\mathcal{V}_f$, however, p_i^* is determined by the leaders' desired positions and velocities in $\boldsymbol{p_l^*}=\mathrm{col}(p_k^*)_{k\in\mathcal{V}_l}$ and the desired bearing Laplacian B. To be specific, the feasibility and uniqueness of the followers' desired positions and velocities in the target formation is shown as follows.

Proposition 1: The desired position $\boldsymbol{p_f^*}$ and velocity $\boldsymbol{u_f^*}$ of the followers are feasible and uniquely determined by the leaders' desired positions $\boldsymbol{p_l^*}$ and velocities $\boldsymbol{u_l^*}$, and the bearing Laplacian matrix B, if B_{ff} is positive definite, $B_{fl}\neq 0$, and B and $\boldsymbol{p_l^*}$ are differential for all $t\geq 0$. Specifically, one has

$$\boldsymbol{p_f^*}=-B_{ff}^{-1}B_{fl}\boldsymbol{p_l^*},\tag{3}$$

$$\boldsymbol{u_f^*}=B_{ff}^{-1}\left[(\dot{B}_{ff}B_{ff}^{-1}B_{fl}-\dot{B}_{fl})\boldsymbol{p_l^*}-B_{fl}\boldsymbol{u_l^*}\right].\tag{4}$$

Proof: Similar to the bearing-based network localizability properties [16], we can obtain that the desired positions of the followers can be uniquely determined by $B_{ff}\boldsymbol{p_f^*}=-B_{fl}\boldsymbol{p_l^*}$. Since B_{ff} is positive definite for all $t\geq 0$, we have $\boldsymbol{p_f^*}=-B_{ff}^{-1}B_{fl}\boldsymbol{p_l^*}$. Since B and $\boldsymbol{p_l^*}$ are differential, we take the time derivative of both sides of $B_{ff}\boldsymbol{p_f^*}=-B_{fl}\boldsymbol{p_l^*}$ and obtain that $\dot{B}_{ff}\boldsymbol{p_f^*}+B_{ff}\boldsymbol{\dot{p}_f^*}=-\dot{B}_{fl}\boldsymbol{p_l^*}-B_{fl}\boldsymbol{u_l^*}$. Then the feasibility and uniqueness can be guaranteed by taking $\boldsymbol{u_f^*}=\boldsymbol{\dot{p}_f^*}$, which yields (4). The proof is completed. ∎

Proposition 1 is obtained by extending some of the bearing rigidity properties in [16], [17] to the cases with time-varying desired bearings. The positive definiteness of B_{ff} guarantees that (3) and (4) are well-defined. Moreover, the topology needs to be connected, i.e. $B_{fl}\neq 0$ so that the leaders' desired positions and velocities influence those of the followers. Hence, we adopt the following assumptions.

Assumption 1: The target formation defined in *Definition 1* is feasible and unique, i.e., $B_{ff}(t)$ is positive definite, $B_{fl}(t)\neq 0$, $B(t)$ and $\boldsymbol{p_l^*}(t)$ are second-order differential for all $t\geq 0$.

Assumption 2: $B(t)$, $\dot{B}(t)$, $\ddot{B}(t)$ are uniformly bounded for all $t\geq 0$.

Remark 2: Bearing-based localizability is different from bearing rigidity. A target formation is localizable if $\boldsymbol{p^*}$ is uniquely determined by $\varphi_{ij}=\varphi_{ij}^*,\forall(i,j)\in\mathcal{E}$, and $p_k=p_k^*,\forall k\in\mathcal{V}_l$. In this regard, there may not exist a constant $c_p>0$ such that any $\|\boldsymbol{p}-\boldsymbol{p^*}\|<c_p$ satisfying $P_{\varphi_{ij}}(p_i^*-p_j^*)=0,\forall(i,j)\in\mathcal{E}$ also yields $P_{\varphi_{ij}}(p_i^*-p_j^*)=0,\forall i,j\in\mathcal{V}$. More detailed discussions can be referred to [16].

B. A bearing-based formation maneuver control method

According to *Proposition 1*, the desired positions and velocities of the followers are both determined by the leaders' positions and velocities as (3)-(4). However, (3)-(4) can not be directly used since it is hard to know the complete knowledge of the whole network topology. Hence, it is required to design a distributed estimator based on the local information with respect to the neighbors for each follower to estimate its desired states. Based on the estimations, a distributed controller is required to achieve the time-varying target formation.

1) A distributed bearing-based state estimator: From (3) and (4), the desired positions and velocities satisfy

$$\sum_{j\in\mathcal{V}_f\cap\mathcal{N}_i}B_{ij}(p_i^*-p_j^*)+\sum_{k\in\mathcal{V}_l\cap\mathcal{N}_i}B_{ik}(p_i^*-p_k^*)=0,\tag{5}$$

$$\begin{aligned}&\sum_{j\in\mathcal{V}_f\cap\mathcal{N}_i}\dot{B}_{ij}(p_i^*-p_j^*)+\sum_{k\in\mathcal{V}_l\cap\mathcal{N}_i}\dot{B}_{ik}(p_i^*-p_k^*)\\ =&-\sum_{j\in\mathcal{V}_f\cap\mathcal{N}_i}B_{ij}(u_i^*-u_j^*)-\sum_{k\in\mathcal{V}_l\cap\mathcal{N}_i}B_{ik}(u_i^*-u_k^*),\end{aligned}\tag{6}$$

for all $i\in\mathcal{V}$, respectively. Based on (5) and (6), we adopt the following estimator for each follower $i\in\mathcal{V}_f$:

$$\begin{cases}\dot{\hat{p}}_i^*=\hat{u}_i^*\\ \dot{\hat{u}}_i^*=-k_1\Big\{\sum_{j\in\mathcal{V}_f\cap\mathcal{N}_i}[B_{ij}(\hat{u}_i^*-\hat{u}_j^*)+(k_2B_{ij}+\dot{B}_{ij})(\hat{p}_i^*-\hat{p}_j^*)]\\ \quad+\sum_{k\in\mathcal{V}_l\cap\mathcal{N}_i}[B_{ik}(\hat{u}_i^*-u_k^*)+(k_2B_{ik}+\dot{B}_{ik})(\hat{p}_i^*-p_k^*)]\Big\}\\ \quad-k_3\mathrm{sgn}\Big\{\sum_{j\in\mathcal{V}_f\cap\mathcal{N}_i}[B_{ij}(\hat{u}_i^*-\hat{u}_j^*)+(k_2B_{ij}+\dot{B}_{ij})(\hat{p}_i^*-\hat{p}_j^*)]\\ \quad+\sum_{k\in\mathcal{V}_l\cap\mathcal{N}_i}[B_{ik}(\hat{u}_i^*-u_k^*)+(k_2B_{ik}+\dot{B}_{ik})(\hat{p}_i^*-p_k^*)]\Big\},\end{cases}\tag{7}$$

where $\hat{p}_i^*$ and $\hat{u}_i^*$ are the estimations of agent i's desired position p_i^* and velocity u_i^*, respectively. B is defined in (2). k_1,k_2 and k_3 are all positive and constant gains. Different from (3) and (4), the estimator (7) is based on (5) and (6), which only require the local information φ_{ij}^*, $\dot{\varphi}_{ij}^*$, φ_{ij}, $\hat{u}_j^*$, $\hat{p}_j^*$ from the follower neighbors $j\in\mathcal{N}_i\cup\mathcal{V}_f$ and φ_{ik}^*, $\dot{\varphi}_{ik}^*$, φ_{ik}, u_k, p_k from their leader neighbors $k\in\mathcal{N}_i\cup\mathcal{V}_l$.

Define $P(t)=\begin{bmatrix}P_{11}(t) & P_{12}(t)\\ P_{21}(t) & P_{22}\end{bmatrix},Q(t)=\begin{bmatrix}Q_{11}(t) & Q_{12}(t)\\ Q_{21}(t) & Q_{22}(t)\end{bmatrix}$, where $P_{11}(t)=B_{ff}(t)$, $P_{12}(t)=P_{21}(t)=k_2B_{ff}(t)+\dot{B}_{ff}(t)$, $P_{22}=k_pI_{dn_f}$, $Q_{11}(t)=k_1B_{ff}(t)^2-M_2(t)$, $Q_{12}(t)=Q_{21}(t)^T=k_1B_{ff}(t)P_{12}(t)-M_1(t)$, $Q_{22}(t)=k_1P_{12}(t)P_{21}(t)$, $M_1(t)=M_3(t)+\frac{kpI_{dn_f}}{2}$, $M_2(t)=$

$k_2B_{ff}(t) + \frac{3\dot{B}_{ff}(t)}{2}$, $M_3(t) = \frac{k_2\dot{B}_{ff}(t)+\ddot{B}_{ff}(t)}{2}$, and k_p is a constant to be determined. Let $\boldsymbol{r}^* = \bar{H}\boldsymbol{p}^*$ contain all the desired relative positions and $\dot{\boldsymbol{u}}_l^*$ be the time derivative of $\boldsymbol{u}_l^*$. Define the spectral radius of a uniformly bounded matrix $A(t) : R \to R^{n\times n}$ at the time instant t as $\rho(t) = \max\{|\lambda_1(t)|, ..., |\lambda_n(t)|\}$, where $\lambda_1(t)$, $\lambda_2(t)$,..., and $\lambda_n(t)$ are the eigenvalues of $A(t)$. Then the values of the constants k_1, k_2, k_3, k_p are determined in the following steps:

Step 1: Choose k_3 such that

$$k_3 > \sup_{t\geq 0}\{\rho(B_{ff}^{-1}(t))\left[\rho(\ddot{B}(t))\|\boldsymbol{r}^*(t)\|+ 2\rho(\dot{B}(t))\|\boldsymbol{u}^*(t)\| + \rho(B(t))\|\dot{\boldsymbol{u}}_l^*(t)\|\right]\}, \tag{8}$$

Step 2: Choose k_2 such that

$$k_2 > \sup_{t\geq 0}\left[1/2 + \rho(B_{ff}^{-\frac{1}{2}}(t)\dot{B}_{ff}(t)B_{ff}^{-\frac{1}{2}}(t)), \sqrt{\rho(B_{ff}(t)^{-1}\dot{B}_{ff}^2(t)B_{ff}^{-1}(t))}\right]. \tag{9}$$

Step 3: Choose k_p such that

$$k_p > \sup_{t\geq 0}\{\rho(P_{12}(t)B_{ff}^{-1}(t)P_{12}(t)), 4k_2[4\rho(M_3(t))+\rho(M_2(t))]\}. \tag{10}$$

Step 4: Choose k_1 such that

$$k_1 > \sup_{t\geq 0}\left[(4k_2/k_p)\rho(M_1(t)P_{12}^{-2}(t)M_1(t))\right]. \tag{11}$$

For convenience, we omit (t) in the following. The feasibility of the above steps is shown in the following lemma.

Lemma 1: Under *Assumptions 1* and *2*, there exist bounded constants k_3, k_2, k_p and k_1 such that (8)-(11) are satisfied, if the leaders' desired velocities $\|\boldsymbol{u}_l^*\|$, $\|\dot{\boldsymbol{u}}_l^*\|$ are uniformly bounded for all $t \geq 0$.

Proof: According to *Assumption 1*, B_{ff} is positive definite so that B_{ff}^{-1} exists for all $t \geq 0$. Hence, there exist feasible k_3, k_2, and k_p to satisfy (8)-(10) respectively. From (9), it leads to $k_2I_{dn_f} > -B_{ff}^{-\frac{1}{2}}\dot{B}_{ff}B_{ff}^{-\frac{1}{2}}$, which yields $P_{12} = k_2B_{ff} + \dot{B}_{ff} > 0$ for all $t \geq 0$. Hence, P_{12}^{-1} exists for all $t \geq 0$ and there exists k_1 such that (11) is satisfied.

Moreover, according to *Assumption 2*, $\rho(B_{ff}^{-1})$, $\rho(B)$, $\rho(\dot{B})$, and $\rho(\ddot{B})$ are all uniformly bounded. $\boldsymbol{r}^*$ contains the desired displacements which are also uniformly bounded. According to *Proposition 1*, $\boldsymbol{u}_f^*$ is uniformly bounded if B, $\dot{B}$, $\boldsymbol{r}^*$ and $\boldsymbol{u}_l^*$ are uniformly bounded. Therefore, $\boldsymbol{u}^*$ is uniformly bounded. Moreover, $\dot{\boldsymbol{u}}_l^*$ is considered to be uniformly bounded as we have stated, and hence k_3 can be chosen as a bounded constant to satisfy (8). By following a similar analysis, k_2 can also be set with a bounded value. With a bounded k_2, k_p can be chosen as a bounded constant to satisfy (10), and then k_1 can also be bounded to satisfy (11). The proof is completed. ■

Lemma 2: Under *Assumptions 1* and *2*, P and Q are both positive definite and $k_3 > \|\dot{\boldsymbol{u}}_f^*\|$ for all $t \geq 0$ if k_3, k_2, k_p and k_1 satisfy (8)-(11), respectively.

Proof: According to *Lemma 1*, $P_{11} = B_{ff} > 0$ for all $t \geq 0$. Then from (10) one has $P_{22} = k_pI_{dn_f} > \rho(P_{12}B_{ff}^{-1}P_{12})I_{dn_f} \geq P_{12}P_{11}^{-1}P_{12}$. Then, according to Schur Complement [18], P is positive definite for all $t \geq 0$. By following a similar analysis, one can also get that $k_3 > \|\dot{\boldsymbol{u}}_f^*\|$ if (8) is satisfied and Q is positive definite if (9)-(11) are satisfied for all $t \geq 0$. The proof is completed. ■

The effectiveness of the estimator (7) is shown as follows.

Theorem 1: Under *Assumptions 1* and *2*, the estimator (7) guarantees that $\hat{p}_i^* \to p_i^*$ and $\hat{u}_i^* \to u_i^*$ exponentially fast as $t \to \infty$ for all $i \in \mathcal{V}_f$ if (8)-(11) are satisfied.

Proof: Denote the position and velocity estimation errors of each follower i as $\tilde{p}_i := \hat{p}_i^* - p_i^*$ and $\tilde{u}_i := \hat{u}_i^* - u_i^*$, respectively. $\tilde{\boldsymbol{p}}_f := [\tilde{p}_{n_l+1}^T, ..., \tilde{p}_n^T]^T$ and $\tilde{\boldsymbol{u}}_f := [\tilde{u}_{n_l+1}^T, ..., \tilde{u}_n^T]^T$. Consider the positive definite Lyapunov function $V_e = \frac{1}{2}[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]P[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]^T$. Taking the time derivative of V_e gives

$$\begin{aligned}\dot{V}_e =&[\tilde{\boldsymbol{u}}_f^TB_{ff} + \tilde{\boldsymbol{p}}_f^T(k_2B_{ff} + \dot{B}_{ff})]\dot{\tilde{\boldsymbol{u}}}_f + \tilde{\boldsymbol{u}}_f^T(\frac{3\dot{B}_{ff}}{2}+ \\ &k_2B_{ff})\tilde{\boldsymbol{u}}_f + \tilde{\boldsymbol{p}}_f^T(k_2\dot{B}_{ff} + \ddot{B}_{ff} + kpI_{dn_f})\tilde{\boldsymbol{u}}_f \\ \leq& -(k_3 - \|\dot{\boldsymbol{u}}_f^*\|)\|\tilde{\boldsymbol{u}}_f^TB_{ff} + \tilde{\boldsymbol{p}}_f^T(k_2B_{ff} + \dot{B}_{ff})\|_1 \\ &- \left[\tilde{\boldsymbol{u}}_f^T \quad \tilde{\boldsymbol{p}}_f^T\right] Q \left[\tilde{\boldsymbol{u}}_f^T \quad \tilde{\boldsymbol{p}}_f^T\right]^T \\ \leq& -\left[\tilde{\boldsymbol{u}}_f^T \quad \tilde{\boldsymbol{p}}_f^T\right] Q \left[\tilde{\boldsymbol{u}}_f^T \quad \tilde{\boldsymbol{p}}_f^T\right]^T,\end{aligned} \tag{12}$$

where the last inequality is due to $k_3 > \|\dot{\boldsymbol{u}}_f^*\|$ from *Lemma 2*. According to *Lemma 2*, one has $P > 0$ and $Q > 0$ for all $t \geq 0$, which leads to $\max_{t\geq 0}[\lambda_M(P)] > \min_{t\geq 0}[\lambda_m(P)] > 0$ and $\min_{t\geq 0}[\lambda_m(Q)] > 0$. For convenience, define

$$\lambda_1 := \min_{t\geq 0}[\lambda_m(P)], \lambda_2 := \max_{t\geq 0}[\lambda_M(P)], \lambda_3 := \min_{t\geq 0}[\lambda_m(Q)]. \tag{13}$$

Then one has $\frac{1}{2}\lambda_1\|[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]\|^2 \leq V_e \leq \frac{1}{2}\lambda_2\|[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]\|^2$ and $\dot{V}_e \leq -\lambda_3\|[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]\|^2 \leq -\frac{2\lambda_3}{\lambda_2}V_e$. Hence, one has

$$\|[\tilde{\boldsymbol{u}}_f^T(t), \tilde{\boldsymbol{p}}_f^T(t)]\| \leq \sqrt{\frac{2V_e(t)}{\lambda_1}} \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|[\tilde{\boldsymbol{u}}_f^T(0), \tilde{\boldsymbol{p}}_f^T(0)]\|e^{-\frac{\lambda_3 t}{\lambda_2}}. \tag{14}$$

Then according to [19, Th 4.10], the error $[\tilde{\boldsymbol{u}}_f^T, \tilde{\boldsymbol{p}}_f^T]^T$ converges to zero exponentially fast as $t \to \infty$. The proof is completed. ■

Remark 3: To avoid using the global information in designing the gains as in (8)-(11), one idea is to use the local information B_{ij}, $\dot{B}_{ij}$, $\hat{p}_j^*$, $\hat{u}_j^*$ for all $j \in \mathcal{N}_i$ as feedback terms to adaptively change k_3, k_p, k_2, and k_1 until the convergence of the estimation errors is guaranteed [20]. However, it is hard to guarantee the exponential convergence. Hence, the integration of the estimation errors may be unbounded. That yields the controller design and convergence analysis more difficulties. We leave this topic to our future work.

2) A distributed bearing-based controller: According to *Proposition 1*, the target formation in *Definition 1* can be realized if $p_i \to p_i^*$ and $u_i \to u_i^*$ as $t \to \infty$ for all $i \in \mathcal{V}_f$.

We propose the following bearing-based controller for each follower agent $i \in \mathcal{V}_f$:

$$\dot{p}_i = k_\varphi \sum_{j\in\mathcal{N}_i}(\varphi_{ij} - \varphi_{ij}^*) + \hat{u}_i^*, \tag{15}$$

where $k_\varphi > 0$ is a constant gain. The controller (15) is inspired by the bearing-based method in [7]. For each follower $i \in \mathcal{V}_f$, the used information in (15) is $\hat{u}_i^*$, φ_{ij}, and $\varphi_{ij}^*, \forall j \in \mathcal{N}_i$, which are also local information with follower i's neighbors. Hence, (15) is a distributed controller without requiring global network-wide information.

Define the tracking error as $\delta_{p_i} := p_i - p_i^*$ for all $i \in \mathcal{V}_f$ and stack all the errors into $\boldsymbol{\delta_p} = [\mathbf{0}_{dn_l}^T, \delta_{p_{n_l+1}}^T, ..., \delta_{p_n}^T]^T$. Let $\boldsymbol{\varphi} = [\varphi_1^T, ..., \varphi_m^T]^T$ and $\boldsymbol{\varphi^*} = [\varphi_1^{*T}, ..., \varphi_m^{*T}]^T$ contain the bearing measurements and desired bearings corresponding to H. Define $M_I = \begin{bmatrix} O_{dn_l} & O_{dn_l \times dn_f} \\ O_{dn_f \times dn_l} & I_{dn_f} \end{bmatrix}$. With the controller (15), the closed-loop dynamic of $\boldsymbol{\delta_p}$ is

$$\dot{\boldsymbol{\delta}}_{\boldsymbol{p}} = -k_\varphi M_I \bar{H}^T(\boldsymbol{\varphi} - \boldsymbol{\varphi^*}) + \tilde{\boldsymbol{u}} := f(\boldsymbol{\delta_p}) + \tilde{\boldsymbol{u}}, \qquad (16)$$

which can be seen as a perturbed system with $\tilde{\boldsymbol{u}} = [\mathbf{0}_{dn_l}^T, \tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T]^T$ being the external input of the nominal system $\dot{\boldsymbol{\delta}}_{\boldsymbol{p}} = f(\boldsymbol{\delta_p})$.

The convergence of $\boldsymbol{\delta_p}$ in the closed-loop system (16) can be shown in the following.

Theorem 2: Under the state estimator (7) and the controller (15), the tracking error $\boldsymbol{\delta_p}$ in the closed-loop system (16) converges to zero asymptotically as $t \to \infty$ if *Assumptions 1* and *2* are satisfied and no inter-agent collision happens.

Proof: According to *Theorem 1* and [7], $\boldsymbol{\delta_p}$ of the nominal system $\dot{\boldsymbol{\delta}}_{\boldsymbol{p}} = f(\boldsymbol{\delta_p})$ and the external input $\tilde{\boldsymbol{u}}$ converge to zero exponentially fast. According to the cascade system theories [21], the rest of the proof is to show that $\tilde{\boldsymbol{u}}$ will not lead to the unboundedness of the state $\boldsymbol{\delta_p}$ in (16).

Consider the following Lyapunov function $V_u = \frac{1}{2}\boldsymbol{\delta_p}^T\boldsymbol{\delta_p}$. Similar to [7, Th.1], it yields $\boldsymbol{\delta_p}^T f(\boldsymbol{\delta_p}) \leq 0$. Hence, taking the time derivative of V_u gives $\dot{V}_u = \boldsymbol{\delta_p}^T[f(\boldsymbol{\delta_p}) + \tilde{\boldsymbol{u}}] \leq \|\boldsymbol{\delta_p}\|\|\tilde{\boldsymbol{u}}\|$. Note that $\|\boldsymbol{\delta_p}\| \leq \frac{1}{2}\|\boldsymbol{\delta_p}\|^2 = V_u$ if $\|\boldsymbol{\delta_p}\| \geq 2$. Then one has $\|\boldsymbol{\delta_p}\| \leq 2 + V_u$. Moreover, according to (14) one has

$$\dot{V}_u/(2+V_u) \leq \sqrt{\lambda_2/\lambda_1}\|[\tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T(0), \tilde{\boldsymbol{p}}_{\boldsymbol{f}}^T(0)]\|e^{-\frac{\lambda_3 t}{\lambda_2}}. \qquad (17)$$

Integrating both sides of (17) from 0 to t gives $\log\frac{2+V_u(t)}{2+V_u(0)} \leq \frac{\lambda_2}{\lambda_3}\sqrt{\frac{\lambda_2}{\lambda_1}}\|[\tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T(0), \tilde{\boldsymbol{p}}_{\boldsymbol{f}}^T(0)]\|$, which leads to $V_u(t) \leq (2 + V_u(0))e^{\frac{\lambda_2}{\lambda_3}\sqrt{\frac{\lambda_2}{\lambda_1}}\|[\tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T(0), \tilde{\boldsymbol{p}}_{\boldsymbol{f}}^T(0)]\|} - 2$. Since the initial value $\|[\tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T(0), \tilde{\boldsymbol{p}}_{\boldsymbol{f}}^T(0)]\|$ is bounded, i.e., $\|\tilde{\boldsymbol{u}}(0)\|$ is bounded, $V_u(t)$ is bounded and so is $\|\boldsymbol{\delta_p}(t)\| \leq 2 + V_u(t)$.

As a result, before $\|\tilde{\boldsymbol{u}}\|$ converges to zero, the tracking error state $\|\boldsymbol{\delta_p}\|$ is always bounded. Recall that $\boldsymbol{\delta_p}$ of the nominal system $\dot{\boldsymbol{\delta}}_{\boldsymbol{p}} = f(\boldsymbol{\delta_p})$ converges to zero asymptotically and the external input $\tilde{\boldsymbol{u}}$ converges to zero exponentially. According to the cascade system theories [21], $\boldsymbol{\delta_p}$ converges to zero as $t \to \infty$. The proof is completed. ■

Theorem 2 indicates that all the agents achieve their desired positions. This also means $\dot{\boldsymbol{\delta}}_{\boldsymbol{p}} \to 0$ as $t \to \infty$. Hence, the target formation is achieved, and *Problem 2* is solved.

To drop the assumption of collision avoidance, a sufficient condition on the initial tracking error $\boldsymbol{\delta_p}(0)$ and estimation error $[\tilde{\boldsymbol{p}}_{\boldsymbol{f}}(0)^T, \tilde{\boldsymbol{u}}_{\boldsymbol{f}}(0)^T]^T$ is given as follows.

Proposition 2: Under *Assumptions 1* and *2*, if the initial estimation error $[\tilde{\boldsymbol{p}}_{\boldsymbol{f}}(0)^T, \tilde{\boldsymbol{u}}_{\boldsymbol{f}}(0)^T]^T$ and the tracking error $\boldsymbol{\delta_p}(0)$ are sufficiently small so that

$$\begin{aligned}&(2 + \tfrac{1}{2}\|\boldsymbol{\delta_p}(0)\|^2)e^{(\lambda_2/\lambda_3)\sqrt{(\lambda_2/\lambda_1)}\|[\tilde{\boldsymbol{u}}_{\boldsymbol{f}}^T(0), \tilde{\boldsymbol{p}}_{\boldsymbol{f}}^T(0)]\|} \\ &\leq \big(\min{}_{t\geq 0, i\in\mathcal{V}, j\in\mathcal{V}}\|p_i^*(t) - p_j^*(t)\| - \gamma\big)/\sqrt{n},\end{aligned} \qquad (18)$$

then $\|p_i(t) - p_j(t)\| \geq \gamma$ for all $i, j \in \mathcal{V}, t \geq 0$ and $\boldsymbol{\delta_p}$ converges to zero asymptotically, where γ satisfies $\min_{t\geq 0, i\in\mathcal{V}, j\in\mathcal{V}}\|p_i^*(t) - p_j^*(t)\| > \gamma > 0$.

The proof can be completed by a similar way as [7, Corollary 1] and hence omitted here.

IV. NUMERICAL EXAMPLE

In this section, a general scenario is considered to validate the effectiveness of our method. The topology of the multi-agent system is shown in Fig.1(b), where $1, 2$ are the leaders and the others are the followers. The bearing Laplacian matrix B corresponding to the topology is composed of

$$\varphi_{21}^* = -\varphi_{43}^* = R_2^T R_1^T \varphi_{21}^*(0), \varphi_{65}^* = R_1^T\varphi_{65}^*(0),$$

$$\varphi_{31}^* = \varphi_{75}^* = R_1^T\varphi_{31}^*(0), \varphi_{32}^* = \varphi_{41}^* = \varphi_{76}^* = \varphi_{85}^* = R_1^T\varphi_{32}^*(0),$$

$$\varphi_{51}^* = \varphi_{62}^* = \varphi_{73}^* = \varphi_{84}^* = R_1^T\varphi_{51}^*(0),$$

where the initial values $\varphi_{21}^*(0)$, $\varphi_{65}^*(0)$, $\varphi_{31}^*(0)$, $\varphi_{32}^*(0)$, $\varphi_{51}^*(0)$ can be calculated from the initial desired positions in Fig.1(b). R_1 and R_2 are two orthogonal rotation matrices with $R_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 \\ 0 & 1 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 \end{bmatrix}$ and $R_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$. θ_1 and θ_2 are two rotation angles. The matrices $\dot{B}$ and $\ddot{B}$ are the first and second time derivatives of B. $\dot{R}_1 = R_1 S_1$ and $\dot{R}_2 = R_2 S_2$, where S_1 and S_2 contains the angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$. S_1 and S_2 are continuous and uniformly bounded which satisfy $\|S_1\| \leq 0.03$, $\|S_2\| \leq 0.03$, $\|\dot{S}_1\| \leq 0.1$, $\|\dot{S}_2\| \leq 0.1$. The gains k_3, k_2, k_p, and k_1 are chosen as *Step 1-Step 4* in Section III.B to satisfy (8)-(11), which yields $k_3 = 23$, $k_2 = 0.8$, $k_p = 13$, and $k_1 = 55$.

The leaders' initial velocities are set as $u_1(0) = [0.2 + 0.02\sqrt{2}\pi, 0, 0]m/s$ and $u_2(0) = [0.2, 0, -0.02\sqrt{2}\pi]m/s$, respectively. The initial desired positions are shown in Fig.1(b). Define $h_a = [\cos\theta_a, 0, \sin\theta_a]^T$ and $h_a^\perp = [-\sin\theta_a, 0, \cos\theta_a]^T$ for $a = 1, 2, 3$. Fig.2 illustrates that the target formation is achieved and Fig.3 shows the details of the agents' velocities. When $0 \leq t \leq 200$, the formation rotates and translates with a constant angular velocity $\dot{\theta}_1 = -\frac{\pi}{100}$ and a constant translation velocity, i.e., $u_1 = 2\sqrt{2}|\dot{\theta}_1|h_1 + [0.2, 0, 0]^T$ and $u_2 = -2\sqrt{2}|\dot{\theta}_1|h_1^\perp + [0.2, 0, 0]^T$. $\dot{\theta}_2 = \dot{\theta}_3 = 0$. Then the translation velocity is $0.2h_3$ when $200 \leq t \leq 300$, where $\dot{\theta}_3 = -\frac{\pi}{100}$. When $328.2 \leq t \leq 352.8$, the formation shape transforms into a right-angled trapezoid by changing the velocities of Leader 2 and Follower 2. The other agents only move with a constant translation velocity. Then one has $u_1 = -[0.2, 0, 0]^T$ and $u_2 = -4\dot{\theta}_2 h_2 - [0.2, 0, 0]^T$, where $\dot{\theta}_2 = \frac{1}{32}(1 - \cos(t - 328.2))$. $\dot{\theta}_1 = \dot{\theta}_3 = 0$. When $433.2 \leq t \leq 458.4$, the formation scale shrinks to half of the original scale, and $u_1 = \frac{\cos t - 1}{4\pi}(\varphi_{21}^* + \varphi_{51}^* + \varphi_{41}^*) - [0.2, 0, 0]^T$, $u_2 = \frac{\cos t - 1}{4\pi}(\varphi_{21}^* + \varphi_{51}^* - \varphi_{41}^*) - [0.2, 0, 0]^T$. When $518.4 \leq t \leq 543.4$, the scale restores. The angular velocities are $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ from $t = 353.2$ to $t = 543.4$ since there is no rotation maneuver during the time phase. Different from [12], the formation is not required to keep rotating all the time. Fig.4(a) and Fig.4(b) of the revised manuscript show that the velocity estimation errors and the formation tracking errors converge to zero within 5 seconds, and they remain null during the different maneuvers and formation transformations.

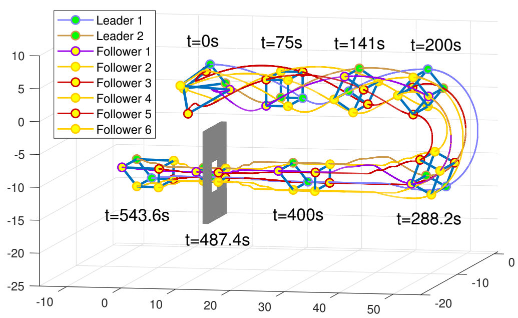


Fig. 2. Trajectories of all the agents.

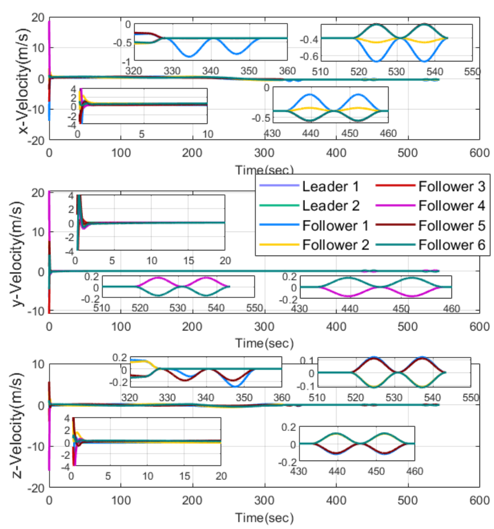
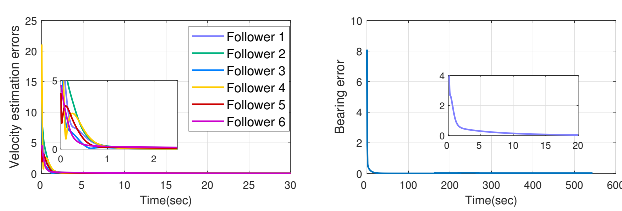


Fig. 3. Velocities of all the agents.



(a) (b)

Fig. 4. Errors of the agents. (a) Velocity estimation error of each follower $\|\tilde{u}_i\|$. Bearing error of all the agents $\sum_{(i,j)\in\mathcal{E}} \|\varphi_{ij} - \varphi_{ij}^*\|$.

V. Conclusion

This paper considers the bearing-based problem of achieving rotation, translation, and scaling formation maneuver control with transformable formation shapes. The target formation corresponding to the control objectives is defined by the time-varying desired bearings. By extending the bearing rigidity properties, the time-varying target formation is analyzed to be feasible and unique. An estimation-based control method is proposed using the relative bearings and the leaders' positions and velocities. Both the estimation error and tracking error converge to zero as time goes to infinity. A sufficient condition for guaranteeing collision avoidance between each pair of agents is proposed. In the future, the proposed method will be extended to cases with agents of more complex models.

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