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Model Reference Adaptive Control for Time-Varying Command Following: A Gradient Descent Approach

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***Abstract*—Output regulation theory is a highly effective method for achieving accurate time-varying command following, but redesigning its control architecture for different time-varying command profiles can be necessary. Yet, this often results in control architectures with gain matrices of significantly increased dimensions. This paper focuses on model reference adaptive control of uncertain dynamical systems. Specifically, our contribution is a gradient descent-based term that alters both reference model trajectories and the control signal to enable time-varying command following. In contrast to output regulation theory, the proposed term can follow every potential time-varying command profile without specificity as the gain of this term increases. In addition to the presented theoretical results, illustrative numerical examples are included to show the efficacy of the proposed gradient descent-based term for time-varying command following.**

I. INTRODUCTION

The control design process requires an accurate mathematical model of a physical system. However, there are instances where an accurate mathematical model may not be possible to obtain, such as poor linearizations, changes in the equation of motion, degraded control output, simplified modeling assumptions, or environmental disturbances. When this occurs, problems arise in the control architecture implementation that may lead to poor closed-loop system performance or complete instability due to these uncertainties. Robust control (e.g., see [1] and [2]) and adaptive control (e.g., see [3] and [4]) provide effective solutions in addressing the negative effect from these system uncertainties. This paper focuses on the adaptive control approach due to its benefits over robust control, such as online learning and requiring less accurate system modeling.

Direct and indirect designs are two main classes of adaptive control architectures. While indirect designs first estimate the unknown parameters of the dynamical system and then utilize these estimates to tune the control parameters, direct designs tune the control parameters online without explicitly relying on unknown parameter estimation. In addition, direct methods do not require persistent excitation, which is required for accurate parameter estimation for indirect design methods. Because of these benefits, this paper contributes to an important class of direct adaptive control architectures called model reference adaptive control.

In control theory, output regulation is a highly effective method for achieving accurate time-varying command following [5]. Yet, it can demand a redesign of the control architecture to accommodate different time-varying command profiles. For example, suppose output regulation is utilized to design a controller that follows a sinusoidal command profile. In that case, a new controller may be redesigned to follow a sinusoidal command profile with a different frequency or simply another command profile such as a ramp or parabola. Because this process is not desired from a reconfigurability standpoint, a solution to address this problem is to choose a finite set of potential time-varying command profiles for the dynamical system to follow while using output regulation theory. Yet, this often results in control architectures with gain matrices of significantly increased dimensions. We refer to [6] and [7] for notable contributions using output regulation theory for model reference adaptive control.

The contribution of this paper is a new term for model-reference adaptive control architectures that alters both the reference model trajectories and the control signal. Specifically, this term is developed using the gradient descent approach, which minimizes an error criterion capturing the difference between the time rate of change of the reference model state and the time rate of change of a given command profile. The proposed term can follow every potential time-varying command profile without specificity as the gain of this term increases. In addition to the presented theoretical results, illustrative numerical examples are included to show the efficacy of the proposed gradient descent-based term for time-varying command following.

Finally, a standard mathematical notation is used throughout the paper. We denote $\mathbb{R}$ for the set of real numbers, $\mathbb{R}_+$ for the set of positive real numbers, $\mathbb{R}^n$ for the set of

$n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ for the set of $n \times m$ real matrices, $\mathbb{R}_+^{n\times n}$ for the set of positive-definite real matrices. In addition, "$\triangleq$" declares equality by definition, $(\cdot)^{-1}$ declares the inverse, $(\cdot)^{\mathrm{T}}$ declares the transpose, and $\mathrm{tr}(\cdot)$ declares the trace.

II. Model Reference Adaptive Control

This section presents the review of a standard model reference adaptive control architecture, where additional details can be found in [3] and [4].

A. Uncertain Dynamical System

We consider an uncertain dynamical system in the form given by

$$\dot{x}(t) = Ax(t) + B\big(u(t) + \theta_{\mathrm{p}}(x(t))\big), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the measurable state and $u(t) \in \mathbb{R}^m$ is the control signal. The respective state and control matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are known and controllable. In addition, $\theta_{\mathrm{p}}(x(t)) : \mathbb{R}^n \to \mathbb{R}^m$ is a system uncertainty that is composed of locally Lipschitz functions. A parametric system uncertainty for $\theta_{\mathrm{p}}(x(t))$ is considered that takes the form given by

$$\theta_{\mathrm{p}}(x(t)) = W_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}(x(t)). \tag{2}$$

In (2), $W_{\mathrm{p}} \in \mathbb{R}^{s\times m}$ is an unknown weight and $\sigma_{\mathrm{p}}(x(t)) : \mathbb{R}^n \to \mathbb{R}^s$ is a known basis function.

We now consider the control signal in the form

$$u(t) = u_{\mathrm{n}}(t) + u_{\mathrm{a}}(t), \tag{3}$$

where $u_{\mathrm{n}}(t) \in \mathbb{R}^m$ and $u_{\mathrm{a}}(t) \in \mathbb{R}^m$ respectively denote the nominal and adaptive control signals. The nominal control signal is given by

$$u_{\mathrm{n}}(t) = -K_1 x(t) + K_2 c(t), \tag{4}$$

where $K_1 \in \mathbb{R}^{m\times n}$ and $K_2 \in \mathbb{R}^{m\times p}$ respectively denote the feedback and feedforward gain matrices. The gain matrix K_1 needs to be chosen such that $A - BK_1$ is Hurwitz and the gain matrix K_2 needs to be chosen such that

$$-E(A - BK_1)^{-1}(BK_2) = I \tag{5}$$

holds, where $E \in \mathbb{R}^{p\times n}$ denotes a subset of $x(t)$ to be followed for command tracking. In addition, the bounded command profile is denoted by $c(t) \in \mathbb{R}^p$ and is assumed to be continuously differentiable such that $\dot{c}(t) \in \mathbb{R}^p$ is bounded and measurable.

Note that the results of this paper require that $\dot{c}(t)$ is measurable and this is not restrictive. To elucidate this point, let $c_{\mathrm{f}}(t) \in \mathbb{R}^p$ and consider the low-pass filter given by

$$\dot{c}_{\mathrm{f}}(t) = -\zeta(c_{\mathrm{f}}(t) - c(t)), \quad c_{\mathrm{f}}(0) = c_{\mathrm{f}0}, \tag{6}$$

where $\zeta \in \mathbb{R}_+$. Because $c(t)$ is bounded, it follows from the input-to-state stability that $c_{\mathrm{f}}(t)$ is bounded [8]. In addition, $\dot{c}_{\mathrm{f}}(t)$ is bounded as well due to the right side of (6) containing only bounded terms. Therefore, when $c(t)$ is not continuously differentiable and $\dot{c}(t)$ is not measurable, $c_{\mathrm{f}}(t)$ and $\dot{c}_{\mathrm{f}}(t)$ can respectively be used instead of $c(t)$ and $\dot{c}(t)$ in the results presented in this paper. When in use, the low-pass filter gain ζ should be adjusted judiciously to allow $c_{\mathrm{f}}(t)$ and $\dot{c}_{\mathrm{f}}(t)$ to respectively remain sufficiently close to $c(t)$ and $\dot{c}(t)$.

B. Reference Model

A reference model is created to capture the desired closed-loop system performance. It is constructed by focusing on the behavior of (1) in the absence of system uncertainties (i.e., $\theta_{\mathrm{p}}(x(t)) \equiv 0$) and without an adaptive control signal (i.e., $u_{\mathrm{a}}(t) \equiv 0$). The reference model dynamics can now take the form given by

$$\dot{x}_{\mathrm{r}}(t) = Ax_{\mathrm{r}}(t) + Bu_{\mathrm{r}}(t), \quad x_{\mathrm{r}}(0) = x_{\mathrm{r},0}, \tag{7}$$

where $x_{\mathrm{r}}(t) \in \mathbb{R}^n$ is the reference model state and $u_{\mathrm{r}}(t) \in \mathbb{R}^m$ is the reference model control signal that satisfies

$$u_{\mathrm{r}}(t) = -K_1 x_{\mathrm{r}}(t) + K_2 c(t). \tag{8}$$

The final form of the reference model is determined by using (8) in (7), which yields

$$\dot{x}_{\mathrm{r}}(t) = A_{\mathrm{r}} x_{\mathrm{r}}(t) + B_{\mathrm{r}} c(t), \tag{9}$$

where $A_{\mathrm{r}} \triangleq A - BK_1 \in \mathbb{R}^{n\times n}$ and $B_{\mathrm{r}} \triangleq BK_2 \in \mathbb{R}^{n\times m}$. In addition, the Lyapunov equation given by

$$0 = A_{\mathrm{r}}^{\mathrm{T}} P + PA_{\mathrm{r}} + R \tag{10}$$

holds for some $R \in \mathbb{R}_+^{n\times n}$ and $P \in \mathbb{R}_+^{n\times n}$.

From (5) and (9), it follows that

$$\lim_{t\to\infty} \big(Ex_{\mathrm{r}}(t) - c(t)\big) = 0 \tag{11}$$

only when the command $c(t)$ is constant (once again, details in [4]). From a practical standpoint, (11) approximately holds as well for the case when the command $c(t)$ is slowly time-varying, yet otherwise (11) does not hold. Output regulation theory can be adopted for the nominal control design in order to accurately following time-varying commands. However, it can demand a redesign of the control architecture to accommodate different time-varying command profiles or it can result in nominal control architectures with gain matrices of significantly increased dimensions. In contrast, we can achieve time-varying command following through a gradient descent approach in the next section.

C. Adaptive Control

The closed-loop system dynamics can be written in a new form by considering (1), (2), and (3) as

$$\dot{x}(t) = A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) + B\big(u_{\mathrm{a}}(t) + W_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}(x(t))\big). \tag{12}$$

In (12), the goal of the adaptive control signal $u_{\mathrm{a}}(t)$ is to suppress the negative effect of the system uncertainty

$W_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}(x(t))$ such that the state of (12) approaches the state of (9). To this standpoint, the adaptive control signal is chosen to take the form

$$u_{\mathrm{a}}(t) = -\hat{W}_{\mathrm{p}}^{\mathrm{T}}(t)\sigma_{\mathrm{p}}(x(t)), \tag{13}$$

where $\hat{W}_{\mathrm{p}}(t) \in \mathbb{R}^{s\times m}$ is the estimate of the unknown weight W_{p} that satisfies the parameter adjustment mechanism of the form given by

$$\dot{\hat{W}}_{\mathrm{p}}(t) = \gamma\sigma_{\mathrm{p}}(x(t))e^{\mathrm{T}}(t)PB, \quad \hat{W}_{\mathrm{p}}(0) = \hat{W}_{\mathrm{p}0}. \tag{14}$$

In (14), the learning rate is denoted as $\gamma \in \mathbb{R}_+$ and dictates the rate of adaption for the adaptive control signal. The error is now defined as

$$e(t) \triangleq x(t) - x_{\mathrm{r}}(t) \in \mathbb{R}^n, \tag{15}$$

which has corresponding dynamics of

$$\dot{e}(t) = A_{\mathrm{r}}e(t) - B\tilde{W}_{\mathrm{p}}^{\mathrm{T}}(t)\sigma_{\mathrm{p}}(x(t)), \quad e(0) = e_0. \tag{16}$$

In (16), the weight error is defined as

$$\tilde{W}_{\mathrm{p}}(t) \triangleq \hat{W}_{\mathrm{p}}(t) - W_{\mathrm{p}} \in \mathbb{R}^{s\times m}, \tag{17}$$

which also has corresponding dynamics of

$$\dot{\tilde{W}}_{\mathrm{p}}(t) = \gamma\sigma_{\mathrm{p}}(\cdot)e^{T}(t)PB, \quad \tilde{W}_{\mathrm{p}}(0) = \tilde{W}_{\mathrm{p}0}. \tag{18}$$

Note that the trajectories of the error dynamics given by (16) and the weight error dynamics given by (18) are then bounded, and $\lim_{t\to\infty} e(t) = 0$. Note also that the structure of the adaptive control signal (13) and parameter adjustment mechanism (14) are determined through a Lyapunov stability analysis that considers the error (15) and weight error (17).

A gradient descent approach is implemented in the next section to aid time-varying command following. This objective is achieved by altering the dynamics of the reference model, which also requires an adjustment to be made in the nominal control signal. This allows for the stability of the closed-loop system maintained and the condition $\lim_{t\to\infty} e(t) = 0$ is preserved.

III. Gradient Descent-Based Model Reference Adaptive Control

This section introduces a gradient descent-based term to enable time-varying command profile following and the corresponding stability of the closed-loop system.

A. Gradient Descent Minimization

The reference model control signal given by (8) is updated to

$$u_{\mathrm{r}}(t) = -K_1x_{\mathrm{r}}(t) + K_2c(t) + u_{\mathrm{g}}(t), \tag{19}$$

where $u_{\mathrm{g}}(t) \in \mathbb{R}^m$ is defined as

$$u_{\mathrm{g}}(t) \triangleq B^{\mathrm{T}}\psi_{\mathrm{r}}(t). \tag{20}$$

In (20), the low-pass filter state $\psi_{\mathrm{r}} \in \mathbb{R}^n$ satisfies the dynamics given by

$$\dot{\psi}_{\mathrm{r}}(t) = -\mu(\psi_{\mathrm{r}}(t) - \phi_{\mathrm{r}}(t)), \quad \psi_{\mathrm{r}}(0) = \psi_{\mathrm{r},0}, \tag{21}$$

where $\mu \in \mathbb{R}_+$ is the low-pass filter gain and $\phi_{\mathrm{r}}(t) \in \mathbb{R}^n$ is the proposed gradient descent-based term (details below). The updated reference model is constructed by using (20) in (19) and then using the resulting expression in (7). This yields

$$\dot{x}_{\mathrm{r}}(t) = A_{\mathrm{r}}x_{\mathrm{r}}(t) + B_{\mathrm{r}}c(t) + BB^{\mathrm{T}}\psi_{\mathrm{r}}(t), \tag{22}$$

where (22) represents the reference model referred to in this section. Rather than directly implementing the gradient descent-based term $\phi_{\mathrm{r}}(t)$ in (20), its low-pass filtered state $\psi_{\mathrm{r}}(t)$ is utilized instead. While gradient descent is a powerful method for online function minimization problems, oscillations may be induced [9]–[11]. During the development process of this research paper, we have observed such oscillations as well when we directly employ the gradient descent-based term in (20). Motivated by this standpoint, we have introduced the low-pass filter given by (21) and use the low-pass filtered version of this term in (20). This discussion also implies that the low-pass filter gain μ should not be high.

The proposed gradient descent-based term for (21) takes the form

$$\begin{aligned}\phi_{\mathrm{r}}(t) = -\xi A_{\mathrm{r}}^{\mathrm{T}}E^{\mathrm{T}}\big(&EA_{\mathrm{r}}x_{\mathrm{r}}(t) + EB_{\mathrm{r}}c(t)\\ &+EBB^{\mathrm{T}}\psi_{\mathrm{r}}(t) - \dot{c}(t)\big)\end{aligned} \tag{23}$$

where $\xi \in \mathbb{R}_+$ is the gradient descent gain. The structure of the term in (23) is a result of the gradient descent process for minimizing an error criterion of the difference between the reference model state time rate of change and the given command profile time rate of change. With this being said, we now present the first theorem of this paper, where its proof will be reported elsewhere.

Theorem 1. *The proposed term $\phi_r(t)$ is the negative gradient of the cost function*[1] *as*

$$\phi_{\mathrm{r}}(t) = -\xi\frac{\partial J(\cdot)}{\partial x_{\mathrm{r}}(\cdot)}, \tag{24}$$

whose cost function is given by

$$J(\cdot) = \frac{1}{2}(E\dot{x}_{\mathrm{r}}(t) - \dot{c}(t))^{\mathrm{T}}(E\dot{x}_{\mathrm{r}}(t) - \dot{c}(t)). \tag{25}$$

The reference model in (22) can follow various time-varying command profiles without specificity as the gain ξ increases. This is owing to the fact that this gain multiplies the negative gradient of the cost function $J(\cdot)$ according to (24), which allows for faster online minimization of this cost function. Concurrently, potential oscillations induced by the

[1] One needs to choose $Ex_{\mathrm{r}}(0) = c(0)$ to achieve $Ex_{\mathrm{r}}(t) = c(t)$ when $J(\cdot) = 0$. Because, $J(\cdot) = 0$ implies $E\dot{x}_{\mathrm{r}}(t) = \dot{c}(t)$, where taking the integral of both sides we have $Ex_{\mathrm{r}}(t) - Ex_{\mathrm{r}}(0) = c(t) - c(0)$. Under $Ex_{\mathrm{r}}(0) = c(0)$, this expression now yields $Ex_{\mathrm{r}}(t) = c(t)$.

gradient descent process are filtered through (21). We refer to Section IV for the numerical examples of the effectiveness of the proposed gradient descent-based term.

B. Closed-Loop System Stability

To ensure the closed-loop system stability and the condition $\lim_{t\to\infty} e(t) = 0$, we change the nominal control signal given by (4) to

$$u_{\mathrm{n}}(t) = -K_1 x(t) + K_2 c(t) + u_{\mathrm{g}}(t), \tag{26}$$

which also changes (12) to

$$\dot{x}(t) = A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) + BB^{\mathrm{T}} \psi_{\mathrm{r}}(t) + B\big(u_{\mathrm{a}}(t) + W_{\mathrm{p}}^{\mathrm{T}} \sigma_{\mathrm{p}}(x(t))\big). \tag{27}$$

Now with the error signal given in (15), one can write the identical error dynamics given in (16). Note that because the error dynamics do not change, the adaptive control signal given in (13) and the parameter adjustment mechanism given in (14) remain the same. Finally, we make the following assumption that is required for the following theorem of this paper.

Assumption 1. *The matrix given by*

$$A_{\mathrm{z}} \triangleq \begin{bmatrix} A_{\mathrm{r}} & BB^{\mathrm{T}} \\ -\mu\xi A_{\mathrm{r}}^{\mathrm{T}} E^{\mathrm{T}} E A_{\mathrm{r}} & -\mu(I + \xi A_{\mathrm{r}}^{\mathrm{T}} E^{\mathrm{T}} E B B^{\mathrm{T}}) \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \tag{28}$$

is Hurwitz.

We are now ready to present the second main theorem of this paper, where its proof will be reported elsewhere.

Theorem 2. *Consider the uncertain dynamical system given by (1) with (2) and the control signal given by (3) with (26), (20), (13), (14), (22), and (21). The trajectories of the error dynamics given by (16) and the weight error dynamics given by (18) are then bounded. If, in addition, Assumption 1 holds, the trajectories of the reference model dynamics given by (22) and the low-pass filter dynamics (21) are also bounded, and* $\lim_{t\to\infty} e(t) = 0$.

IV. Illustrative Numerical Examples

Two numerical examples are now presented to demonstrate the efficacy of the proposed gradient descent-based model reference adaptive control architecture for following time-varying command profiles.

A pendulum system with unity parameters is considered for command profile testing, where $x_1(t)$ is the angular position (i.e., $x_1(t) \equiv 0$ denotes the pendulum arm pointing in the downwards direction) and $x_2(t)$ is the angular velocity. Specifically, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are considered for (1). In addition, the presented example focuses on regulating the angular position of the pendulum, and therefore, we set $E = [1 \quad 0]$. For the nominal control signal gain matrices, we choose $K_1 = [0.1547 \quad 0.8017]$, $K_2 = [1.1547]$. Furthermore, $R = I$ is used to solve the Lyapunov equation given in (10). For the injected uncertainty, we use $W_{\mathrm{p}} = [1 \quad 2]^{\mathrm{T}}$ for the unknown weight and $\sigma_{\mathrm{p}}(x(t)) = x(t)$ for the known basis function, where the learning rate is set to $\gamma = 15$. Finally, all initial conditions are set to zero.

We are now ready to present the two aforementioned numerical examples, where a sinusoidal command profile is considered for the first one and a parabola command profile is considered for the second one.

A. Following Sinusoidal Command

The first case presented here is of a sinusoidal signal for command following. Figures 1 and 2 respectively display the position command following performance and the control signal for the standard model reference adaptive control architecture as described in Section II. Though the standard architecture suppresses the negative effect of system uncertainties in the sense that the angular position asymptotically approaches the reference model position, the angular position experiences a large error when compared to the time-varying, sinusoidal command profile.

We now present the numerical results for the proposed gradient descent-based model reference adaptive control architecture as described in Section III. Figures 3 and 4 respectively display the position command following performance and the control signal, where the gains $\xi = 165$ and $\mu = 0.35$ are chosen to allow Assumption 1 to hold. The proposed architecture maintains the system uncertainty suppression seen for the standard adaptive case, while also following the sinusoidal command profile sufficiently close. Lastly, Figure 5 displays the change in the reference model angular position performance when the gradient gain is altered from $\xi = 0$ to $\xi = 165$. Here, it is clear that improved command following performance is seen as we increase ξ.

B. Following Parabola Command

The second case presented here is of a parabola signal for command following. Figures 6 and 7 respectively display the position command following performance and the control signal for the standard model reference adaptive control architecture as described in Section II. Once again, the standard architecture suppresses the negative effect of system uncertainties in the sense that the angular position asymptotically approaches the reference model position, but the angular position again experiences a large error when compared to the time-varying parabola command profile.

Now we give the numerical results for the proposed gradient descent-based model reference adaptive control architecture as described in Section III. Figures 8 and 9 respectively display the position command following performance and the control signal, where the gains $\xi = 120$ and $\mu = 0.2$ are chosen to allow Assumption 1 to hold. The proposed architecture can again maintain the system uncertainty suppression seen for the standard adaptive case, while also following the parabola command profile sufficiently close. Lastly, Figure 10 displays the change in the reference model angular position

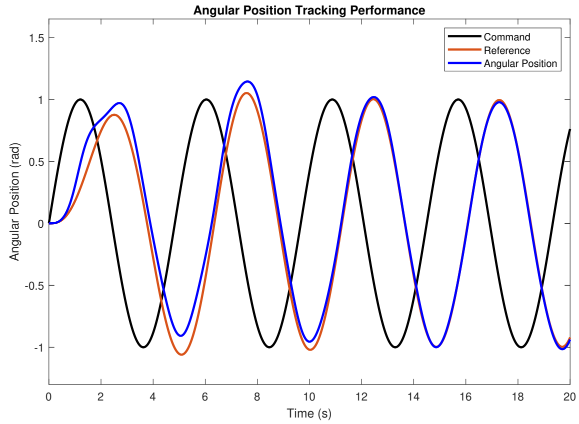

Fig. 1. Sinusoidal command following performance for the standard model reference adaptive control architecture.

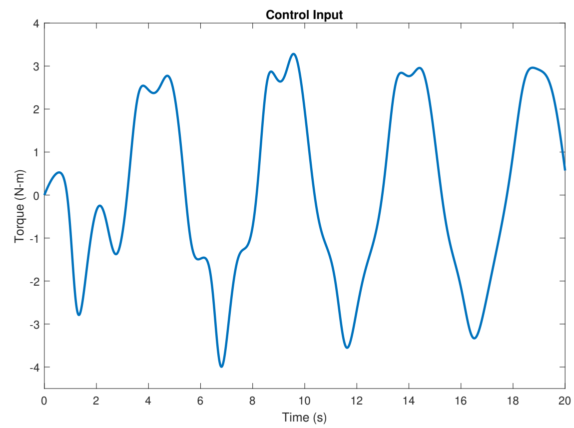

Fig. 2. Control signal for the standard adaptive control architecture for the sinusoidal command following case.

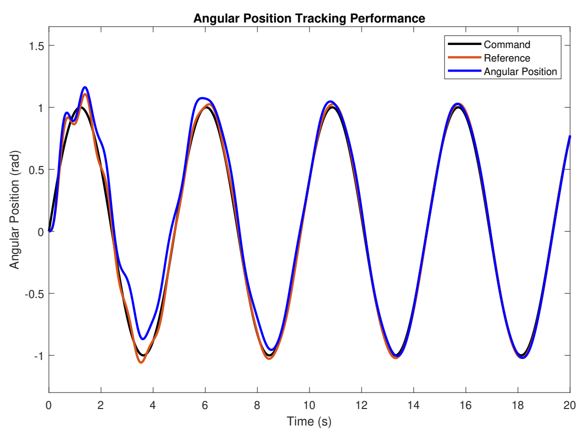

Fig. 3. Sinusoidal command following performance for the gradient descent model reference adaptive control architecture.

performance when the gradient gain is altered from $\xi = 0$ to $\xi = 120$. It is again clear that improved command following performance is seen as we increase ξ.

V. Conclusion

This paper proposed a new term for model reference adaptive control architectures to aid in time-varying command following. This was accomplished by updating both the ref-

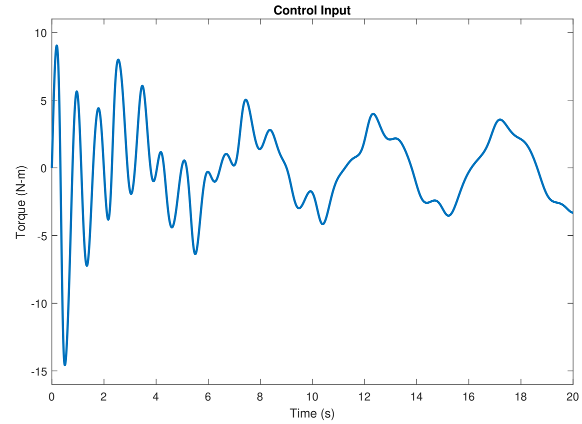

Fig. 4. Control signal for the gradient descent adaptive control architecture for the sinusoidal command following case.

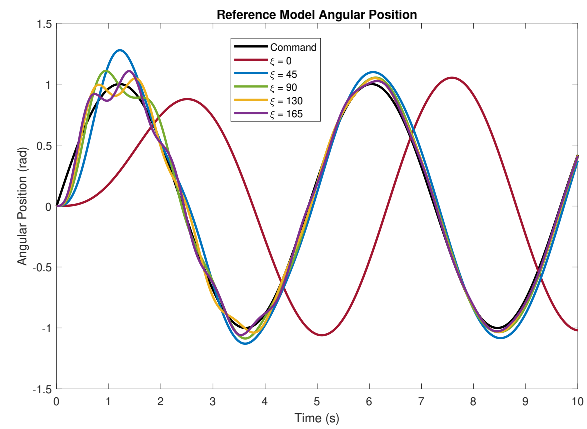

Fig. 5. Changes in the reference model angular position state as a function of ξ for the sinusoidal command following case.

erence model trajectories and the control signal. In particular, the proposed term was determined by using the gradient descent approach for minimizing an error criterion. The error criterion that was implemented captured the difference between the reference model state time rate of change and the given command profile time rate of change. The key focus of the proposed gradient descent term was to aid in time-varying command profile following, where improved performance is seen as the gain of this term increases. As compared to output regulation theory, the proposed term does not require a redesign of the control architecture for accommodating different time-varying command profiles, which provides a unique benefit by allowing any time-varying command profile to be followed without specificity. Illustrative numerical examples were further presented to demonstrate the benefits of the gradient descent-based approach.

Future research will consider applying the proposed gradient descent approach to the recently proposed symbiotic control framework [12] to allow for efficient time-varying command following. It will also be directed towards the implementation of state-dependent design gains to allow for automatic adjustments when following time-varying command profiles.

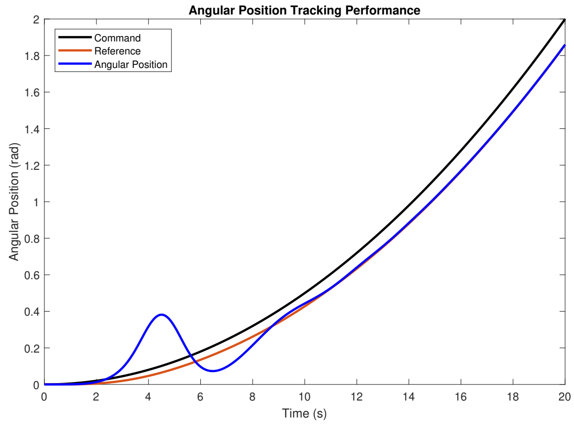

Fig. 6. Parabola command following performance for the standard model reference adaptive control architecture.

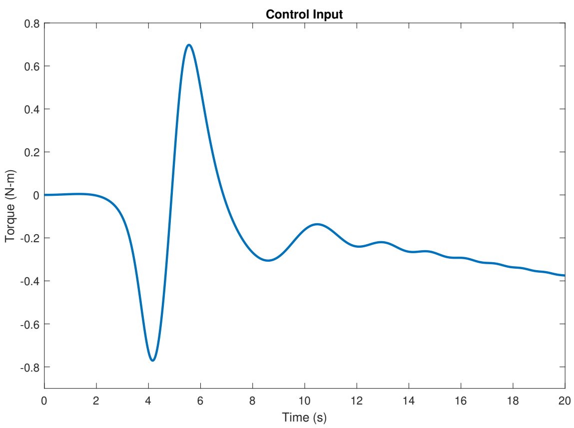

Fig. 7. Control signal for the standard adaptive control architecture for the parabola command following case.

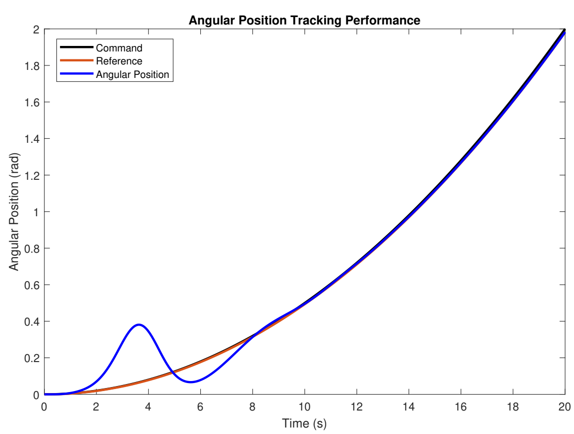

Fig. 8. Parabola command following performance for the gradient descent model reference adaptive control architecture.

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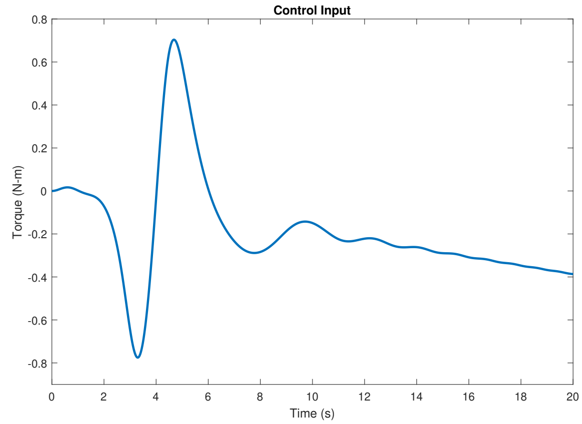

Fig. 9. Control signal for the gradient descent adaptive control architecture for the parabola command following case.

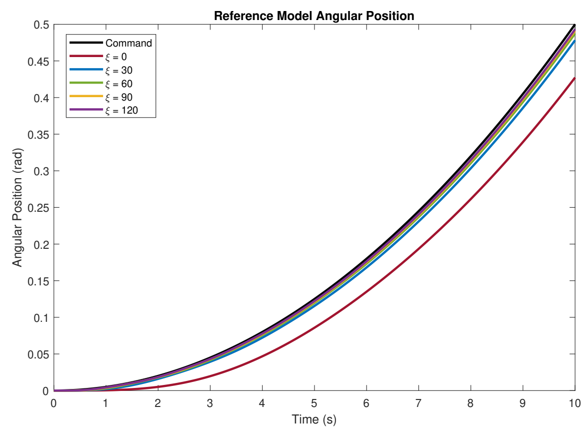

Fig. 10. Changes in the reference model angular position state as a function of ξ for the parabola command following case.

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